Aggregation Without Separability: A Generalized Composite Commodity Theorem

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This paper provides general conditions for aggregating commodities without separable utility. These conditions impose weaker and more empirically plausible restrictions on price movements than the currently existing alternative to separability, the Hicks-Leontief composite commodity theorem. The idea is to allow departures from Hicks-Leontief that take the form of well behaved error terms. Utility functions that permit generalized composite commodity aggregation include the AIDS model, the translog, all homothetic utility functions, and any utility function when demands are aggregated to two groups of goods. Implications of empirical nonstationarity of relative prices for aggregation and demand estimation are considered. (*JEL* D11, D12, C43, C32, E31, I30, H31)

This paper investigates conditions for aggregating commodities without separable utility, based on weaker and more empirically plausible restrictions on price movements than the Hicks-Leontief composite commodity theorem (John R. Hicks, 1936; and Wassily Leontief, 1936).

This issue might seem inconsequential, since prices of different goods tend to move similarly over time, making the differences between correct and incorrect aggregate price indices relatively small. However, model estimates depend on these differences between similar prices, so small departures from valid aggregation (or small differences between separable and inseparable utility functions) can produce large mistakes in elasticity and welfare estimates. This multicollinearity of prices also makes it difficult to test if separability assumptions used for constructing groups are valid. To the extent that separability can be tested, it is generally rejected. These problems with separability are documented both empirically and in Monte Carlo studies. See, for example, Frederick W. Derrick and John D. Wolken (1985), William A. Barnett and Seungmook Choi (1989), Christopher J. Nicol (1991), and W. Erwin Diewert and Terence J. Wales (1995). A generalized composite commodity theory to relax the constraints of separability and Hicks-Leontief is therefore of practical as well as theoretical importance.

Let $\mathbf{p}$ and $\mathbf{w}$ be vectors of prices p_i and budget shares w_i of individual commodities (or goods or services), for $i = 1, \ldots, n$. Let I index groups of commodities, so, for example, groups I might correspond to food, clothing, shelter, and so on, and the set of all $i \in I$ for I being food could be the commodities bread, milk, and so on. Let $\mathbf{P}$ and $\mathbf{W}$ be vectors of group price indices P_I and group budget shares $W_I = \sum_{i \in I} w_i$.

Because the number n of different commodities that consumers buy is vast, empirical demand analysis is commonly carried out at some level of aggregation across commodities, that is, it is usually necessary to work with groups of goods I rather than individual commodities i. Assuming consumers are rational, so they choose budget shares $\mathbf{w}$ to maximize some utility function given prices $\mathbf{p}$, under what conditions will group budget shares $\mathbf{W}$ maximize some utility function given group price indices $\mathbf{P}$, as is usually assumed (or tested) in empirical practice? Also, how do group demand functions, group elasticities, and the group utility function relate to the corresponding functions of the individual goods?

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Two types of results exist to answer these questions. One is the separability and two-stage budgeting literature, which imposes restrictions on utility functions to obtain group demand function rationality. In general the separability assumptions required for rationality of group demand functions are rather strong, difficult to test powerfully (because of multicollinearity of disaggregate prices $\mathbf{p}$), and require group price indices $\mathbf{P}$ that depend on the parameters of the individual's utility function.

Although progress has been made in relaxing its restrictions (see, for example, Richard Blundell and Jean-Marc Robin, 1995 and Charles Blackorby et al., 1995), even weak forms of separability impose very strong elasticity equality restrictions among every good in every group. Nevertheless, separability is usually assumed for lack of any reasonable alternative. That is the problem this paper addresses.

The only currently existing alternative to separability is the Hicks-Leontief composite commodity theorem. Define $\rho_i = \log(p_i/P_I)$, so ρ_i is the log of the ratio of the price of commodity i to the price of the group I (for each good i it is implicit here that I is the group which contains i). Let $\boldsymbol{\rho}$ be the n vector of elements ρ_i. The Hicks-Leontief theorem states that $\mathbf{W}$ maximizes a utility function given group prices $\mathbf{P}$ if $\boldsymbol{\rho}$ is constant.

While prices of related goods do tend to be strongly correlated over time, the Hicks-Leontief theorem requires them to move absolutely synchronously and so have a correlation of one, which clearly does not hold empirically. The generalized composite commodity theorem proposed here relaxes Hicks-Leontief by allowing $\boldsymbol{\rho}$ to vary over time, and instead assumes only that the distribution of $\boldsymbol{\rho}$ is independent of $\mathbf{P}$ (and of income). This assumption generally accords with the widely observed multicollinearity of prices over time, and more specifically requires that changes in the relative prices of goods within a group be unrelated to the general rate of inflation of the group. Movements in U.S. group prices $\mathbf{P}$ and relative within group prices $\boldsymbol{\rho}$ are empirically examined in Section IV below.

Separability or Hicks-Leontief yields rational group demand functions without error terms. To relax the strict assumptions of separability or Hicks-Leontief, the generalized composite commodity theorem proposed here permits the aggregation to induce a well behaved error term in the aggregate group demand equations. This aggregation error then becomes part of the ordinary error term in models of demand.

An implication of generalized composite commodity aggregation is that the errors in estimated group demand functions will depend in part on $\boldsymbol{\rho}$. This implication is investigated using a demand model estimated with U.S. data in Section V. The results are found to be consistent with generalized composite commodity aggregation and inconsistent with separability.

Given the required behavior of group prices, many commonly used utility function specifications satisfy the conditions needed for generalized composite commodity aggregation, including all homothetic utility functions (for example, the Cobb-Douglas and constant elasticity of substitution (CES)), Angus S. Deaton and John Muellbauer's (1980b) almost ideal demand system (AIDS), Dale W. Jorgenson et al. (1982) translog demand system, and any utility function in which goods are aggregated to only two groups. Arbitrary utility functions with an arbitrary number of groups can come close to satisfying the required conditions if certain functions of $\boldsymbol{\rho}$ are small.

A complicating feature of the empirical work here is stochastic nonstationarity of prices. Prices $\mathbf{P}$ are nonstationary as one would expect, given that inflation rates are nonzero. More surprisingly, relative prices $\boldsymbol{\rho}$ are also found to be nonstationary. The integration and cointegration properties of prices are analyzed in Section IV. Section V discusses some implications of these price movements for estimating demand systems, testing separability, and composite commodity aggregation.

The next section presents the generalized composite commodity theorem, Section II discusses interpretation of models based on it. Section III gives example applications, while Sections IV and V provide some empirical analyses and tests of the composite commodity theorem with U.S. data. Section VI concludes and includes discussion of potential applications for production models and for replacing intertemporal separability assumptions in macroeconomic models.

I. The Generalized Composite Commodity Theorem

Define p_i and P_I to be the price of good i and the price index of group I, respectively. Define $r_i = \ln(p_i)$, $R_I = \ln(P_I)$, $\rho_i = \ln(p_i/P_I) = r_i - R_I$, and let $\mathbf{r}$, $\boldsymbol{\rho}$, and $\mathbf{R}$ be the vectors of elements r_i, ρ_i, and R_I. Let z be the log of a consumer's total consumption expenditures.

Following standard empirical practice, let $w_i = g_i(\mathbf{r}, z) + e_i$, so observed budget shares w_i equal some systematic Marshallian demand function $g_i(\mathbf{r}, z)$ plus an error term e_i having conditional mean zero, so $E(e_i \mid \mathbf{r}, z) = 0$.

Define s_{ij}, the budget share analog to Slutsky matrix elements, by

$$(1) \quad s_{ij}(\mathbf{r}, z) = \frac{\partial g_i(\mathbf{r}, z)}{\partial r_j} + \frac{\partial g_i(\mathbf{r}, z)}{\partial z} g_j(\mathbf{r}, z).$$

Definitions: The following definitions refer to any set of Marshallian budget share demand functions $g_i(\mathbf{r}, z)$ for $i = 1, \ldots, n$. The *Adding up* condition is $\Sigma_{i=1}^{n} g_i(\mathbf{r}, z) = 1$. The *homogeneity* (or lack of money illusion) condition is $g_i(\mathbf{r}, z) = g_i(\mathbf{r} - z, 0)$. The *Slutsky symmetry* condition is $s_{ij}(\mathbf{r}, z) = s_{ji}(\mathbf{r}, z)$ for all i and j from 1 to n. The *negative semidefiniteness* condition is that the matrix $\tilde{\mathbf{s}}$ is negative semidefinite, where for all i, j from 1 to n the elements $\tilde{s}_{ij}$ of $\tilde{\mathbf{s}}$ are defined by $\tilde{s}_{ij} = s_{ij} + g_i(\mathbf{r}, z)g_j(\mathbf{r}, z)$ for $i \neq j$ and $\tilde{s}_{ii} = s_{ii} + g_i(\mathbf{r}, z)^2 - g_i(\mathbf{r}, z)$. A set of demand functions are defined to be *integrable* if they satisfy the adding up, homogeneity, and Slutsky symmetry conditions. A set of demand functions are defined to be *rational* if they are integrable and they also satisfy the negative semidefiniteness condition.

The above terms were defined for individual goods, but also apply to aggregate goods by replacing small letters with capitals, for example, aggregate demands $\mathbf{G}(\mathbf{R}, z)$ satisfy homogeneity if $G_I(\mathbf{R}, z) = G_I(\mathbf{R} - z, 0)$ where G_I is the budget share demand function for group I.

The above definitions are standard. In particular, $\tilde{s}_{ij}$ equals the compensated derivative of the budget share of i with respect to r_j, and $\tilde{s}_{ij} = \partial c(\mathbf{r}, u)/\partial r_i \partial r_j$ where $z = c(\mathbf{r}, u)$ is the log expenditure (or cost) function.

Rationality as defined above corresponds to the usual conditions for demands to be derived from utility maximization, while integrability implies that the demands satisfy first-order conditions for utility maximization. Demands are typically estimated imposing only integrability. The resulting estimates are then checked to see if they also satisfy negative semidefiniteness, and hence rationality, in the neighborhood of the data.

ASSUMPTION 1: *The demand functions $g_i(\mathbf{r}, z)$ for $i = 1, \ldots, n$ are rational.*

ASSUMPTION 2: *The distribution of the random vector $\boldsymbol{\rho}$ is independent of $\mathbf{R}$ and z.*

Assumption 1 is equivalent to assuming that the demand functions $g_i(\mathbf{r}, z)$ arose from utility maximization. Assumption 2 relaxes the Hicks-Leontief assumption that $\boldsymbol{\rho}$ be constant.

Recall that $w_i = g_i(\mathbf{r}, z) + e_i$ where $E(e_i \mid \mathbf{r}, z) = 0$. This corresponds to defining g_i by $g_i(\mathbf{r}, z) = E(w_i \mid \mathbf{r}, z)$. Similarly, define G_I and ε_I by $W_I = G_I(\mathbf{R}, z) + \varepsilon_I$ where $E(\varepsilon_I \mid \mathbf{R}, z) = 0$, so the demand function G_I is just the conditional expectation of $\Sigma_{i \in I} w_i$ given z and the vector of aggregate log price indices $\mathbf{R}$. Given Assumption 1, the aggregation problem addressed here is, under what conditions will the group demands $G_I(\mathbf{R}, z)$ be rational?

Define $G_I^*(\mathbf{r}, z) = \Sigma_{i \in I} g_i(\mathbf{r}, z)$, so G_I^* are the group I demands expressed in terms of z and the log of prices of individual commodities $\mathbf{r}$. Let $\mathbf{R}^* = \mathbf{r} - \boldsymbol{\rho}$. Since $\rho_i = r_i - R_I$ by definition, $\mathbf{R}^*$ is the n vector that has R_I in position i for every group I and every $i \in I$. It follows that

$$(2) \quad G_I(\mathbf{R}, z) = E[G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z) \mid \mathbf{R}, z] = \int G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z)\, dF(\boldsymbol{\rho})$$

where $F(\boldsymbol{\rho})$ is the distribution function of $\boldsymbol{\rho}$. For any two groups I and J, let

$$(3)\quad H_{IJ}(\mathbf{R}, z) = \operatorname{cov}\left[\frac{\partial G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial z},\; G_J^*(\mathbf{R}^* + \boldsymbol{\rho}, z) \,|\, \mathbf{R}, z\right].$$

Also define $\mathbf{H}(\mathbf{R}, z)$ to be the square matrix having elements $H_{IJ}(\mathbf{R}, z)$. Given equation (2), equation (3) can also be written as

$$(4)\quad H_{IJ}(\mathbf{R}, z) = \int \frac{\partial G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial z} \times G_J^*(\mathbf{R}^* + \boldsymbol{\rho}, z)dF(\boldsymbol{\rho}) - \frac{\partial G_I(\mathbf{R}, z)}{\partial z} G_J(\mathbf{R}, z).$$

Analogous to $\mathbf{H}(\mathbf{R}, z)$, define $\tilde{\mathbf{H}}(\mathbf{R}, z)$ to be the matrix of elements $\tilde{\mathbf{H}}_{IJ}(\mathbf{R}, z) = \operatorname{cov}[G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z), G_J^*(\mathbf{R}^* + \boldsymbol{\rho}, z)|\mathbf{R}, z] = \int G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z)G_J^*(\mathbf{R}^* + \boldsymbol{\rho}, z)dF(\boldsymbol{\rho}) - G_I(\mathbf{R}, z)G_J(\mathbf{R}, z)$. Note that $\tilde{\mathbf{H}}(\mathbf{R}, z)$ is the conditional variance matrix of a vector, and therefore is symmetric and positive semidefinite.

THEOREM 1 (Generalized Composite Commodities): *Let Assumptions* 1 *and* 2 *hold. Then group demand functions $G_I(\mathbf{R}, z)$ satisfy the homogeneity and adding up conditions. Group demand functions $G_I(\mathbf{R}, z)$ also satisfy the Slutsky symmetry condition, and so are integrable, if and only if $\mathbf{H}(\mathbf{R}, z)$ is symmetric. Group demand functions $G_I(\mathbf{R}, z)$ are rational if $\mathbf{H}(\mathbf{R}, z)$ is symmetric and $\mathbf{H}(\mathbf{R}, z) + \tilde{\mathbf{H}}(\mathbf{R}, z)$ is negative semidefinite.*

Given Assumptions 1 and 2 in Theorem 1, the symmetry of $\mathbf{H}$ is both necessary and sufficient for group demands to be integrable, while the negative semidefiniteness of $\mathbf{H}(\mathbf{R}, z) + \tilde{\mathbf{H}}(\mathbf{R}, z)$ is sufficient but not necessary for the negative semidefiniteness of group demands.

COROLLARY 1: *Let Assumptions* 1 *and* 2 *hold. Then the group demand functions $G_I(\mathbf{R}, z)$ are integrable if any one of the following conditions hold:*

(i) $\mathbf{H}(\mathbf{R}, z)$ is symmetric.
(ii) $H_{IJ}(\mathbf{R}, z) = 0$ for all groups I and J.
(iii) $var[\partial G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z)/\partial z|\mathbf{R}, z] = 0$ for all groups I.
(iv) $var[\partial g_i(\mathbf{R}^* + \boldsymbol{\rho}, z)/\partial z|\mathbf{R}, z] = 0$ for all commodities i.
(v) $\partial^2 G_I^*(\mathbf{r}, z)/\partial z \partial r_j = 0$ for all commodities j and groups I.
(vi) $\partial^2 g_i(\mathbf{r}, z)/\partial z \partial r_j = 0$ for all commodities i and j.

In Corollary 1, (i) restates part of Theorem 1, (ii) implies (i), and each of the others implies (ii). The conditions are listed roughly in order of increasing restrictiveness. Examples of Theorem 1 and Corollary 1 are provided in Section IV below.

Let S_{IJ} be the IJth element of the log form Slutsky substitution matrix $\mathbf{S}$ of the demand functions $G_I(\mathbf{R}, z)$, that is,

$$(5)\quad S_{IJ}(\mathbf{R}, z) = \frac{\partial G_I(\mathbf{R}, z)}{\partial R_J} + \frac{\partial G_I(\mathbf{R}, z)}{\partial z} G_J(\mathbf{R}, z).$$

To interpret the matrix $\mathbf{H}$, we have from the proof of Theorem 1 that

$$(6)\quad H_{IJ}(\mathbf{R}, z) = S_{IJ}(\mathbf{R}, z) - \sum_{i \in I} \sum_{j \in J} \int s_{ij}(\mathbf{R}^* + \boldsymbol{\rho}, z)dF(\boldsymbol{\rho})$$

so H_{IJ} is the difference between the Slutsky element IJ of the group demands and the sum (over commodities in the groups) of the average Slutsky matrix of individual goods. The latter retains the integrability properties of the individual s_{ij} functions, so (given Assumptions 1 and 2) departures from the integrability of the group demands can only be due to H_{IJ} being nonsymmetric. Even when these conditions on $\mathbf{H}$ do not hold exactly, departures from the integrability of group demands will be small if $\mathbf{H}$ is small relative to $\mathbf{S}$. Examples are given later in Section IV.

II. Interpreting the Aggregate Model

This section discusses relationships between disaggregate and aggregate elasticities, utility functions, and expenditure (or cost) functions using generalized composite commodities. Consider elasticities first.

THEOREM 2: *Assume Theorem* 1 *holds. Then*

$$(7)\qquad \frac{\partial G_I(\mathbf{R}, z)}{\partial R_J} = E\left(\sum_{i \in I} \sum_{j \in J} \frac{\partial g_i(\mathbf{r}, z)}{\partial r_j} \middle| \mathbf{R}, z \right)$$

$$(8)\qquad \frac{\partial G_I(\mathbf{R}, z)}{\partial z} = E\left(\sum_{i \in I} \frac{\partial g_i(\mathbf{r}, z)}{\partial z} \middle| \mathbf{R}, z \right).$$

Theorem 2 shows that the derivatives of the group demand model budget shares are essentially the best unbiased estimates (that is, they are the conditional expectations given aggregate commodity data) of the corresponding derivatives that would be obtained given disaggregate data. These aggregate model budget share derivatives also equal the derivatives that would be obtained if the Hicks-Leontief composite commodity theorem held (since in that case the expectations in Theorem 2 would just equal the functions themselves) and so also equal the derivatives that would be obtained if the utility function were weakly separable (see, for example, Deaton and Muellbauer [1980a p. 129] for the equivalence of aggregate derivatives based on Hicks Leontief versus weak separability).

Theorem 2 works because, by Theorem 1, aggregate budget shares $G_I(\mathbf{R}, z)$ differ from the corresponding sum of disaggregate budget shares $\Sigma_{i \in I}\, g_i(\mathbf{r}, z)$ only by a term that is mean zero conditional on $\mathbf{R}$ and z, so the same is true of the derivatives of these shares with respect to $\mathbf{R}$ or z.

Next consider welfare or other calculations that involve utility levels. Let $u = v(\mathbf{r}, z)$ be the indirect utility function that yields demands $g(\mathbf{r}, z)$, so u is the utility level attained by the consumer. If by Theorem 1 group demand functions $G_I(\mathbf{R}, z)$ are integrable, we can define a corresponding aggregate indirect utility function $U = V(\mathbf{R}, z)$ as the solution to Roy's identity applied to the group demands.

The aggregate model (and by revealed preference theory, aggregate data) yields the utility level U, rather than the true consumer utility level u. Weak separability is, by construction, the situation in which changes in within group relative prices $\boldsymbol{\rho}$ do not change u, and hence make $u = U$. Hicks-Leontief rules out changes in $\boldsymbol{\rho}$ by assumption. Therefore, any aggregation mechanism other than separability or Hicks-Leontief permits movements in $\boldsymbol{\rho}$ that can change u. Changes in $\boldsymbol{\rho}$ do not affect U by construction, so any aggregation mechanism other than separability or Hicks-Leontief can have $u \neq U$. This means that the aggregate model utility level U will not perfectly represent the consumer's actual attained utility level u. The best one could hope for might be

$$(9)\qquad V(\mathbf{R}, z) = E[v(\mathbf{r}, z)\,|\,\mathbf{R}, z].$$

Equation (9) says that U is the best unbiased estimate (that is, it is the conditional expectation) of u given only the aggregate data $\mathbf{R}$ and z.

Indirect utility is a nonlinear transformation of Marshallian budget shares, and so a conditionally mean zero difference in budget share expressions does not automatically yield a conditionally mean zero difference in indirect utility. Theorem 3 below provides conditions that are sufficient to make Equation (9) hold.

THEOREM 3: *Let Assumptions* 1 *and* 2 *hold, and assume $H(\mathbf{R}, z)$ is symmetric, so by Theorem* 1 *group demand functions $G_I(\mathbf{R}, z)$ are integrable. If $\partial v(\mathbf{r}, z)/\partial z = \theta(\mathbf{R}, z)$ for some function θ, then equation* (9) *holds.*

Theorem 1 places limits on the extent to which the income elasticities $\partial g_i(\mathbf{r}, z)/\partial z$ can depend on $\boldsymbol{\rho}$. Theorem 3 imposes additional restrictions, requiring that the income derivative of the indirect utility function itself, $\partial v(\mathbf{r}, z)/\partial z$, not depend on $\boldsymbol{\rho}$. This can be construed as a step closer to separability, which requires that $v(\mathbf{r}, z)$ itself not depend on $\boldsymbol{\rho}$. Theorem 3 is not required for composite commodity aggregation, but when it holds it permits standard welfare calculations based on the aggregate model.

III. Examples

The examples provided here are applications of Theorems 1 and 3 and Corollary 1.

Theorem 2 also applies whenever Theorem 1 holds. In many cases the conditions required for Theorem 3 and hence equation (9) are stronger than those needed for Theorem 1. In these cases the corollaries below separate the stronger conditions needed for equation (9) welfare calculations from the weaker ones needed only for integrability of generalized composite commodities.

COROLLARY 2 (Hicks-Leontief): *Let Assumption* 1 *hold. If* $\boldsymbol{\rho}$ *is constant, then the group demand functions* $G_I(\mathbf{R}, z)$ *are rational and equation* (9) *holds*.

The Hicks-Leontief composite commodity theorem, Corollary 2, follows from Theorems 1 and 3 because if $\boldsymbol{\rho}$ is constant then Assumption 2 holds, $\mathbf{H}(\mathbf{R}, z) = \tilde{\mathbf{H}}(\mathbf{R}, z) = 0$, and equation (9) simplifies to the stronger condition $V(\mathbf{R}, z) = v(\mathbf{r}, z)$.

COROLLARY 3: *Let Assumptions* 1 *and* 2 *hold. If, for each commodity i and group I,* $g_i(\mathbf{r}, z) = c_i(\mathbf{r}) + d_i(\mathbf{r}, z)$ *and* $\Sigma_{i\in I}\, \partial d_i(\mathbf{r}, z)/\partial z = D_I(\mathbf{R}, z)$ *for some functions* c_i, d_i, *and* D_I, *then group demand functions* $G_I(\mathbf{R}, z)$ *are integrable*.

By the assumptions of Corollary 3, $\partial G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z)/\partial z = D_I(\mathbf{R}, z)$ which is independent of $\boldsymbol{\rho}$, and so Corollary 1(iii) holds. Corollary 3 provides an intermediate case between the extremes of separability and Hicks-Leontief, in that aggregation of the c_i term is handled by the relative prices related Assumption 2, and the assumption that the sum of the d_i functions in each group only depend on prices through $\mathbf{R}$ is essentially a separability type restriction on that part of the functional form.

COROLLARY 4: *Let Assumptions* 1 *and* 2 *hold. If the utility function that* $g_i(\mathbf{r}, z)$ *is derived from is homothetic, so* $v(\mathbf{r}, z) = z - \alpha(\mathbf{r})$ *for some function* α, *then group demand functions* $G_I(\mathbf{R}, z)$ *are integrable and equation* (9) *holds*.

Homothetic utility functions include popular specifications such as the Cobb-Douglas and the CES models. To prove Corollary 4, observe that homotheticity makes the budget share demands $g_i(\mathbf{r}, z)$ independent of z, which implies $\partial g_i(\mathbf{r}, z)/\partial z = 0$, and therefore Corollary 1(vi) holds, which proves integrability. Theorem 3 holds because $v(r, z) = z - \alpha(r)$ makes $\theta = 1$.

Homotheticity over broad aggregates of goods is a common assumption in macroeconomic models. While many problems may exist in making this assumption, Corollary 4 shows that the issue of aggregation across goods in such models can be dealt with via Assumptions 1 and 2, and Assumption 2 is readily amenable to empirical testing or can be checked against the mechanism that generates prices in a given theoretical model.

By ordinality, $\zeta[V(\mathbf{R}, z)]$ and $\zeta[v(\mathbf{r}, z)]$ represent the same sets of preferences as $V(\mathbf{R}, z)$ and $v(\mathbf{r}, z)$ for any monotonically increasing function ζ, but equation (9) generally remains valid only for linear transformations ζ, so applications of Theorem 3 require that the form of the utility function be specified up to a particular linear transformation. For example, all rational homothetic demands have utility functions of the form $v(\mathbf{r}, z) = \zeta[z - \alpha(\mathbf{r})]$ for some monotonically increasing function ζ. Let $\tilde{u} = z - \alpha(\mathbf{r})$. In Corollary 4, integrability of G_I holds for $v = \zeta(\tilde{u})$ for any monotonically increasing ζ, but equation (9) requires $v = \zeta(\tilde{u}) = a + b\tilde{u}$ for some constants a and b. The same applies to Corollary 5 below, with $\tilde{u} = [z - \alpha(\mathbf{r})]/\beta(\mathbf{r})$.

COROLLARY 5: *Let Assumptions* 1 *and* 2 *hold. Let the indirect utility function from which demands* $g_i(\mathbf{r}, z)$ *are derived be of the PIGLOG form* (*see Muellbauer*, 1976) $v(\mathbf{r}, z) = [z - \alpha(\mathbf{r})]/\beta(\mathbf{r})$ *for some functions* α *and* β, *and define* $\beta_i(\mathbf{r}) = \partial \ln \beta(\mathbf{r})/\partial r_i$ *for each commodity i. The following hold*:

(*i*) *If* $\mathrm{var}[\beta_i(\mathbf{R}^* + \boldsymbol{\rho}) | \mathbf{R}] = 0$ *for all commodities i, then the group demand functions* $G_I(\mathbf{R}, z)$ *are integrable*.
(*ii*) *If* $\beta(\mathbf{R}^* + \boldsymbol{\rho}) = \tilde{\beta}(\mathbf{R})$ *for some function* $\tilde{\beta}(\mathbf{R})$, *then the group demand functions* $G_I(\mathbf{R}, z)$ *are integrable and equation* (9) *holds*.

In Corollary 5, (i) is proved by applying Roy's identity to $v(\mathbf{r}, z)$, which yields the PIGLOG demand functions $g_i(\mathbf{r}, z) = [\partial\alpha(\mathbf{r})/$

$\partial r_i] + \beta_i(\mathbf{r})[z - \alpha(\mathbf{r})]$. Therefore $\partial g_i(\mathbf{r}, z)/\partial z = \beta_i(\mathbf{r})$, which shows that Corollary 1(iv) holds and so the G_I are integrable. Alternatively, Corollary 5(i) is the special case of Corollary 3 in which $d_i(\mathbf{r}, z) = \beta_i(\mathbf{r})z$.

The proof of Corollary 5(ii) is that $\beta(\mathbf{r}) = \tilde{\beta}(\mathbf{R})$ makes $\partial v(\mathbf{r}, z)/\partial z = 1/\beta(\mathbf{r}) = 1/\tilde{\beta}(\mathbf{R}) = E[\partial v(\mathbf{r}, z)/\partial z | \mathbf{R}, z]$ so equation (9) holds, and 5(ii) implies 5(i) because $\beta(\mathbf{r}) = \tilde{\beta}(\mathbf{R})$ means that $\beta_i(\mathbf{r}) = \partial \ln \beta(\mathbf{r})/\partial r_i = \beta_i^*(\mathbf{R})$ for each commodity i for some functions β_i^*, and hence $\operatorname{var}[\beta_i(\mathbf{R}^* + \boldsymbol{\rho})|\mathbf{R}] = \operatorname{var}[\beta_i^*(\mathbf{R})|\mathbf{R}] = 0$ for all commodities i.

The usefulness of Corollary 5 is that many popular demand systems have PIGLOG indirect utility functions, as the following Corollaries show.

COROLLARY 6: *Let Assumptions* 1 *and* 2 *hold. Let the utility function from which* $g_i(\mathbf{r}, z)$ *is derived be Deaton and Muellbauer's* (1980b) *almost ideal demand system* (*AIDS*), *so* $v(\mathbf{r}, z) = [z - \alpha(\mathbf{r})]/\beta(\mathbf{r})$ *with* $\ln \beta(\mathbf{r}) = \sum_{i=1}^{n} b_i r_i$. *The following hold*:

(*i*) *The group demand functions* $G_I(\mathbf{R}, z)$ *are integrable*.

(*ii*) *If* $\sum_{i \in I} b_i \rho_i$ *is constant for each group* I, *then equation* (9) *holds also*.

In Corollary 6 (i) follows from Corollary 5 (i) because the AIDS model has $\beta_i(\mathbf{R}^* + \boldsymbol{\rho}) = b_i$, which is a constant and therefore has zero variance. Similarly, 6 (ii) can be immediately derived from 5 (ii). A closely related result to Corollary 6 appears in Lewbel (1993).

COROLLARY 7: *Let Assumptions* 1 *and* 2 *hold. Let the utility function from which* $g_i(\mathbf{r}, z)$ *is derived be Jorgenson et al.'s* (1982) *exactly aggregable translog model, so* $v(\mathbf{r}, z) = [z - \alpha(\mathbf{r})]/\beta(\mathbf{r})$ *with* $1/\beta(\mathbf{r}) = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} r_j - 1$. *The following hold*:

(*i*) *If* $\gamma(\boldsymbol{\rho}) = \sum_{j=1}^{n} \sum_{k=1}^{n} B_{jk} \rho_k$ *is constant, then group demand functions* $G_I(\mathbf{R}, z)$ *are integrable*.

(*ii*) *If* $\sum_{i \in I} \sum_{j \in J} B_{ij} \rho_j$ *is constant for each group* I *and* J, *then group demand functions* $G_I(\mathbf{R}, z)$ *are integrable and equation* (9) *holds*.

Corollary 7 is proved analogously to Corollary 6, since in the translog model $\beta_i(\mathbf{r}) = (\sum_{j=1}^{n} B_{ij})/(1 - \sum_{j=1}^{n} \sum_{k=1}^{n} B_{jk} r_k)$, so $\beta_i(\mathbf{R}^* + \boldsymbol{\rho}) = (\sum_{j=1}^{n} B_{ij})/(1 - \gamma + \sum_{j=1}^{n} \sum_{k=1}^{n} B_{jk} R_k^*)$ which, conditional on $\mathbf{R}$, has zero variance if γ is constant. Unlike the AIDS model, the translog does not immediately aggregate across goods without separability, but it can be made to do so if the log of the relative prices $\boldsymbol{\rho}$ satisfy the one linear constraint implied by having γ constant. This is analogous to the kinds of restrictions needed on the general translog to obtain exact aggregation (see Jorgenson et al., 1982) or to obtain independent of base (IB) equivalence scales (see Lewbel 1991a).

Corollaries 6 and 7 are somewhat more general than the AIDS and translog models, because they both can be applied regardless of what functional form is chosen for the function $\alpha(\mathbf{r})$.

Corollaries 4, 6(i) and 7(i) show that the generalized composite commodity Theorem 1 holds for standard models with little or no restriction. Corollaries 6(ii) and 7(ii) show that welfare analyses using equation (9) (Theorem 3) can require substantial additional restrictions, though they may tend to hold approximately if not exactly in practice, for example, $\sum_{i \in I} b_i \rho_i$ in Corollary 6 may be close to constant. To illustrate, if $R_I = \sum_{i \in I} \kappa_i r_i$ for some constants κ_i, which is a common way to construct group price indices, then $\sum_{i \in I} b_i \rho_i$ will be constant if each $b_i = \kappa_0 + \kappa_i$ for some constant κ_0, and so $\sum_{i \in I} b_i \rho_i$ will be close to constant if either $\boldsymbol{\rho}$ is close to constant or each b_i is close to $\kappa_0 + \kappa_i$.

In cases where Theorem 3 does not hold, it is sometimes still possible to construct a useful estimator of $v(\mathbf{r}, z)$, though that estimate need not equal $V(\mathbf{R}, z)$. For example, if the demand model is AIDS but Corollary 6(ii) does not hold, then the coefficient of z in the group AIDS demand function $G_I(\mathbf{R}, z)$ is still $B_I = \sum_{i \in I} b_i$. Therefore, $\sum_I B_I R_I = \sum_{i=1}^{n} b_i R_i^* = \ln[\beta(\mathbf{R}^*)]$, so $\sum_I B_I R_I$ which can be estimated from group demands can be used to construct an estimate of $\beta(\mathbf{R}^*)$, which in turn is an estimate of $\beta(\mathbf{r})$ for the case of $\boldsymbol{\rho} = 0$. More generally, if $E(\boldsymbol{\rho}) = 0$ then it can be seen by the equations in Section V that $v(\mathbf{R}^*, z)$ can be calculated, up to an additive constant, given z, $\mathbf{R}$, and the parameters from the group AIDS demand function $G_I(\mathbf{R}, z)$. This $v(\mathbf{R}^*, z)$ then represents the attained utility level u of the consumer at average within group prices $E(\mathbf{r} | R)$.

COROLLARY 8: *Let Assumptions* 1 *and* 2 *hold. If the number of groups is two, then the group demand functions* $G_I(\mathbf{R}, z)$ *are integrable.*

It is well known and easily verified that for demand systems consisting of only two goods, the adding up and homogeneity conditions are sufficient to make Slutsky symmetry hold as well. By Theorem 1, Assumptions 1 and 2 alone make group demands satisfy adding up and homogeneity condition, so Corollary 8 follows immediately. One can also directly verify from equation (4) that $H_{12} = H_{21}$ when $G_1^*(\mathbf{r}, z) = 1 - G_2^*(\mathbf{r}, z)$ as implied by the adding up condition when there are only two groups of goods, and hence that $\mathbf{H}$ is symmetric in this case.

Jerry A. Hausman (1981) provides a potential use of Corollary 8. He estimates demands in a two equation system, where one equation is the demand function for the good of interest and the (implicit) other equation is the demand function for all other goods. Hausman justifies the aggregation by assuming rationality over demands for individual goods, Assumption 1, and citing the Hicks-Leontief theorem, since separability is not plausible for such a large group. Corollary 8 shows that Hicks-Leontief is stronger than necessary. Instead, only Assumption 2 is required (albeit over the large group), regardless of the true underlying functional form for utility. The same comment applies to any model that aggregates all demands to two groups. Note, however that Hausman uses the aggregate model for welfare analyses, so one would want to additionally impose $\partial v(\mathbf{r}, z)/\partial z = \theta(\mathbf{R}, z)$, so that Theorem 3 could be applied.

Corollaries 1 to 8 provide examples in which the conditions required for generalized composite commodity aggregation hold exactly. Even when these conditions do not hold exactly, equation (6) shows that departures from integrability of group demands will be small if the matrix $\mathbf{H}$ is small relative to the aggregate Slutsky matrix $\mathbf{S}$. By Corollary 1, this happens when $\mathrm{var}[\partial g_i(\mathbf{R}^* + \boldsymbol{\rho}, z)/\partial z | \mathbf{R}, z]$ is small, that is, when either the variation in $\boldsymbol{\rho}$ itself is small (yielding a situation close to the Hicks-Leontief composite commodity theorem), or when $\partial^2 g_i(\mathbf{r}, z)/\partial z \partial \mathbf{r}$ is small (as in the homothetic and AIDS models where this derivative is zero), or when variations in $\boldsymbol{\rho}$ have only a minor effect on the income term $\partial g_i(\mathbf{r}, z)/\partial z$, as in the translog model where the income effect depends on $\boldsymbol{\rho}$ only through the function $\gamma(\boldsymbol{\rho})$.

To illustrate these points, consider the IQUAIDS model that James Banks et al. (1994) find best describes actual household demands. The IQUAIDS is $v(\mathbf{r}, z)^{-1} = \{[z - \alpha(\mathbf{r})]/\beta(\mathbf{r})\}^{-1} - \lambda(r)$, where α and β are the same as in AIDS and $\lambda(\mathbf{r}) = \Sigma_{i=1}^n \lambda_i r_i$ for constants λ_i. IQUAIDS has budget shares quadratic in z, and $\partial g_i(\mathbf{r}, z)/\partial z = b_i - 2\lambda_i[z - \alpha(\mathbf{r})]/\beta(\mathbf{r})$. Theorem 1 does not hold exactly for this model, but it approximately holds if λ_i is small or if $[z - \alpha(\mathbf{r})]/\beta(\mathbf{r})$ varies little with $\boldsymbol{\rho}$. $\beta(\mathbf{r})$ is independent of $\boldsymbol{\rho}$ if $\Sigma_{i=1}^n b_i \rho_i$ is constant, and a standard approximation for AIDS related models is to replace $\alpha(\mathbf{r})$ in the budget shares with a Stone price index, which need only depend on $\mathbf{R}$. Define an approximated IQUAIDS to be an IQUAIDS model with $\alpha(\mathbf{r})$ replaced by any price index that only depends on $\mathbf{R}$. If $\Sigma_{i=1}^n b_i \rho_i$ is constant then the approximated IQUAIDS satisfies condition (iv) of Corollary 1, and in that way satisfies Theorem 1.

IV. Empirical Relative Price Movements

In this section, movements in log group prices R_I and log relative prices ρ_i are examined empirically, to see if they satisfy Assumption 2.

The data are annual implicit consumption deflators constructed from the Survey of Current Business Tables 2.4 and 2.5 (nominal and real annual personal consumptions expenditures by type of expenditures) of the U.S. national income and product accounts (NIPA), obtained from Citibase (1978). Although the data exists starting in 1947, all analyses here are from 1954 to 1993 to omit the effects of the Korean war and of data problems in the first few years of collection (for example, John Y. Campbell and Gregory N. Mankiw [1990] and Lewbel [1994] do the same).

The groups examined here and the goods that comprise them are as defined by the NIPA tables, except that durables and the rental equivalent of housing are excluded. Also, in cases where a good has zero expenditures in one or more years, that good is combined with other closely related goods. This was done because price indices are constructed by dividing nominal by real expenditures, and this is not possible when expenditures are zero. The resulting goods and their groups are listed in Table 1.

Each group log price index R_I is constructed as the log of the implicit price deflator of the group of goods comprising it, and each relative price variable ρ_i is constructed as $\rho_i = \log p_i - R_I$. Prices are all rescaled to equal 1 in 1954. Figure 1 shows each R_I (solid lines) and the ρ_i's for each good comprising each group I. Some ρ_i's appear jumpy in the early years of the sample. This apparent volatility is due in part to measurement problems on goods having very small nominal expenditures in those years.

In addition to $\mathbf{R}$ and $\boldsymbol{\rho}$, deflated prices for each group of goods I were constructed, by dividing each group price index by the implicit consumption deflator for the sum of all goods in this analysis. Since Assumption 2 requires that $\boldsymbol{\rho}$ be independent of all group prices, an implication is that $\boldsymbol{\rho}$ should be independent of both nominal and deflated log prices.

By inspection of Figure 1, all log prices R_I and many log relative prices ρ_i appear nonstationary with stochastic trends. Table 2 reports the results of applying tests of stationarity to all nominal and deflated R_I's and all ρ_i's. In almost every case, the augmented David A. Dickey and Wayne A. Fuller (1979) tests fail to reject the hypothesis that R_I's and ρ_i's are integrated, and the Denis Kwiatkowski et al. (1992) tests do reject the null hypothesis of stationarity. The tests in Table 2 incorporate a time trend to remove any deterministic trend, and four years of lagged data to compensate for any residual autocorrelation. Similar results are obtained using different numbers of lags or using levels instead of logged data. Removing the time trend regressor from the test regressions strengthens the finding of nonstationarity, as might be expected.

Nominal prices should be nonstationary, reflecting ordinary inflation. The finding that deflated group prices and within group relative prices are also nonstationary is more surprising. Though stationarity is rejected for most of the ρ_i's, a few of them do appear stationary, and most seem closer to stationary than the R_I's. Any ρ_i that is stationary must be uncorrelated with the R_I's, since the R_I's are nonstationary.

Given nonstationarity, ordinary covariances or correlations cannot be used to test if the ρ_i's are related to the R_I's. Instead, cointegration tests are required. Table 2 reports results of applying Robert F. Engle and Clive W. J. Granger (1987) cointegration tests. Each one of the 46 ρ_i's fails to reject (most by a large margin) the null hypothesis of being not cointegrated with its corresponding R_I, both nominal and deflated. In short, there is no evidence of a linear relationship between each ρ_i and either its nominal or deflated group price R_I. Similar results are obtained when the variables are not logged.

While not ruling out the possibility of some nonlinear relationship between the stochastic trends driving ρ_i's and R_I's, these results provide empirical evidence supporting Assumption 2.

Figure 1 shows that the change in each R_I over the sample is larger than the corresponding movement in almost every ρ_i, which suggests (though does not guarantee) that the matrix $\mathbf{H} + \tilde{\mathbf{H}}$ in Theorem 1 will tend to be small. As discussed near the end of the previous section, a relatively small $\mathbf{H} + \tilde{\mathbf{H}}$ implies that any departures from group demand rationality, if present due to violations of Theorem 1, will be small.

V. Generalized Composite Commodities Versus Separability

Hicks-Leontief is easily tested and easily rejected, since it would require that every dotted line in Figure 1 be perfectly flat and horizontal. The evidence of nonstationarity of these ρ_i's strengthens the rejection.

In theory, separability is also easily tested by estimating models for individual goods without imposing separability, then testing whether the required elasticity restrictions are satisfied. The problem is that without separability, each one of these demand equations must include the prices of all the dozens or hundreds of goods people buy. Even when

TABLE 1—NATIONAL INCOME AND PRODUCT ACCOUNTS (NIPA) DEFINED GOODS AND GROUPS

I	Group	i	Good or service
1	Food	1	Food for off premise consumption
		2	Purchased meals and beverages excluding alcohol
		3	Alcohol for off premise consumption
		4	Purchased alcohol for on premise consumption
		5	Tobacco
		6	Other food (includes food for employees, food produced and consumed on farms)
2	Clothing	7	Shoes
		8	Womens and childrens clothes
		9	Mens and boys clothes
		10	Clothes and shoes cleaning, storage, and repair
		11	Other clothing related (includes military issued clothes)
3	Household operation	12	Semidurable furnishings
		13	Household cleansers, paper products, supplies
		14	Stationary and writing supplies
		15	Electricity
		16	Gas
		17	Water and sanitary services
		18	Fuel oil and coal
		19	Telephone
		20	Domestic service
		21	Other household operation
4	Medical care	22	Drugs
		23	Opthalmic products
		24	Physicians
		25	Dentists
		26	Other professional services
		27	Hospitals, nursing homes, other medical care
		28	Health insurance
5	Transportation	29	Repair, washing, parking, rental
		30	Gasoline and oil
		31	Tolls
		32	Insurance
		33	Mass transit
		34	Taxis
		35	Intercity railway
		36	Intercity bus
		37	Airlines
6	Recreation	38	Magazines and newspapers
		39	Nondurable toys and sports equipment
		40	Radio, television repair
		41	Flowers and plants
		42	Movie theaters
		43	Live theater
		44	Spectator sports
		45	Commercial participant amusements
		46	Other recreation
7	Other consumption		Personal care, business, education, religion, foreign

enough degrees of freedom are available to estimate these models, multicollinearity among the prices causes the resulting tests to have little or no power. Multiple survey or panel data sets with thousands of observations of different consumers does not alleviate this problem, because all of the consumers face the same or similar prices in each region and time period.

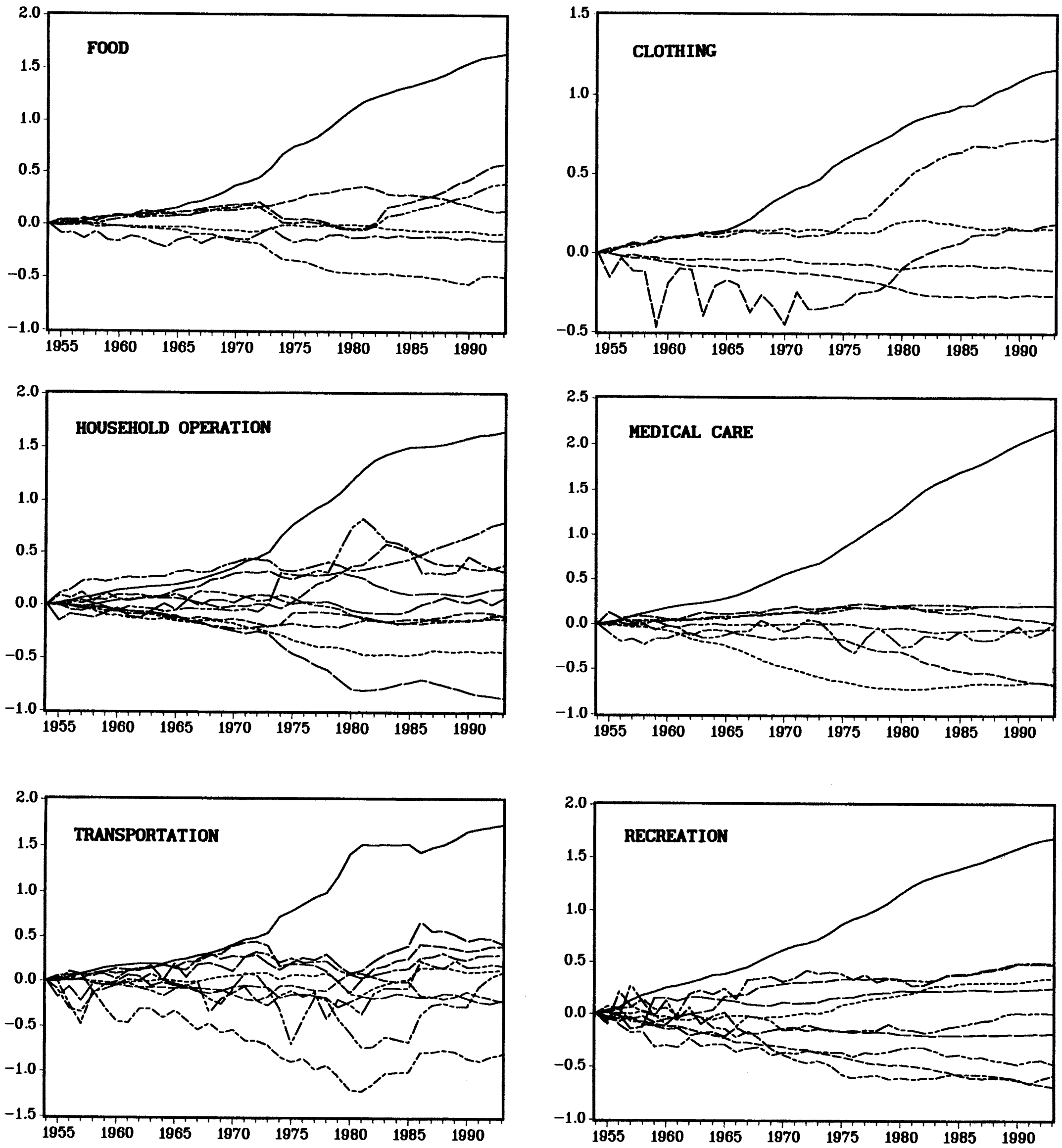

FIGURE 1. LOG GROUP PRICES AND WITHIN GROUP RELATIVE PRICES

Note: In each graph the solid line is the logged group price index R_I, and the other lines are the logged within group relative prices ρ_i for each good comprising each group, as listed in Table 1.

Separability is also difficult to test powerfully because it involves relatively complicated cross equation parameter restrictions.

In a Monte Carlo study Barnett and Choi (1989) find that all of the standard tests fail to reject separability much of the time even with data constructed from utility functions that are far from separable. This "difficulty to reject" may be one reason why separability is so commonly assumed in practice, despite its implausible restrictions. Even with these power problems, separability is often empirically re-

TABLE 2—UNIT ROOT, STATIONARITY, AND COINTEGRATION TESTS

Null hypotheses:	$I(1)$	$I(0)$		$I(1)$	$I(0)$
Nominal:			Deflated:		
R_1	−2.51	0.397	R_1	−1.95	0.316
R_2	−2.33	0.319	R_2	−2.13	0.462
R_3	−2.74	0.355	R_3	−2.01	0.221
R_4	−2.63	0.504	R_4	−0.45	0.420
R_5	−2.14	0.299	R_5	−2.26	0.197
R_6	−2.35	0.367	R_6	−2.16	0.444

			Not cointegrated with					Not cointegrated with	
Null hypotheses:	$I(1)$	$I(0)$	Deflated R_I	Nominal R_I		$I(1)$	$I(0)$	Deflated R_I	Nominal R_I
ρ_1	−2.11	0.224	−2.43	−1.85	ρ_{24}	−2.18	0.344	−3.19	−2.43
ρ_2	0.13	0.388	−2.09	−1.36	ρ_{25}	−2.79	0.179	−1.67	−2.00
ρ_3	−1.93	0.267	−1.67	−1.12	ρ_{26}	0.49	0.494	−1.80	−1.15
ρ_4	−1.32	0.322	−2.57	−1.36	ρ_{27}	0.07	0.468	−2.10	−1.14
ρ_5	−1.04	0.348	−2.41	−1.36	ρ_{28}	−1.67	0.129	−1.90	−1.73
ρ_6	−5.26	0.150	−2.94	−2.50	ρ_{29}	−2.59	0.146	−2.22	−2.50
ρ_7	−2.04	0.350	−2.78	−1.98	ρ_{30}	−1.72	0.195	−2.43	−1.33
ρ_8	−2.22	0.184	−2.34	−3.08	ρ_{31}	−0.59	0.349	−1.70	−1.47
ρ_9	−3.23	0.148	−2.87	−2.81	ρ_{32}	−0.68	0.221	−2.83	−0.98
ρ_{10}	−2.79	0.434	−3.11	−3.28	ρ_{33}	−2.41	0.182	−3.21	−2.55
ρ_{11}	−1.36	0.463	−2.34	−1.55	ρ_{34}	−1.81	0.239	−1.07	−2.65
ρ_{12}	−1.22	0.284	−2.32	−1.85	ρ_{35}	−1.06	0.403	−1.26	−1.46
ρ_{13}	−1.89	0.326	−2.47	−3.11	ρ_{36}	−2.87	0.166	−2.15	−2.45
ρ_{14}	−2.06	0.330	−2.09	−2.88	ρ_{37}	−3.62	0.113	−3.25	−3.08
ρ_{15}	−0.95	0.517	−1.38	−0.90	ρ_{38}	−2.18	0.442	−2.40	−2.70
ρ_{16}	−2.46	0.270	−1.74	−2.44	ρ_{39}	−1.88	0.392	−1.94	−0.79
ρ_{17}	−1.69	0.262	−1.72	−0.83	ρ_{40}	−1.20	0.451	−0.87	−0.51
ρ_{18}	−1.58	0.223	−2.66	−1.42	ρ_{41}	−3.45	0.290	−3.29	−3.45
ρ_{19}	−1.87	0.233	−2.90	−2.57	ρ_{42}	−1.76	0.286	−2.66	−2.35
ρ_{20}	−1.15	0.495	−1.60	−1.52	ρ_{43}	−1.75	0.279	−2.19	−2.67
ρ_{21}	−2.02	0.236	−2.01	−2.52	ρ_{44}	−0.36	0.384	−1.06	−1.01
ρ_{22}	−2.28	0.510	−2.07	−2.00	ρ_{45}	−2.23	0.122	−2.86	−3.26
ρ_{23}	−1.30	0.487	−1.55	−1.79	ρ_{46}	−3.39	0.222	−3.17	−3.21
Critical values:	$I(1)$	$I(0)$							
1 percent	−4.24	0.216	−4.80	−4.80					
5 percent	−3.54	0.146	−4.06	−4.06					
10 percent	−3.20	0.119	−3.71	−3.71					

Notes: $I(1)$ tests the null that the variable is integrated with a stochastic trend (against the alternative of trend stationarity). Reported is the augmented Dickey-Fuller (1979) t statistic of the lagged-level variable in the regression of the differenced variable on a constant, a time trend, and four lags of the differenced variable. $I(0)$ tests the null that the variable is trend stationary against the alternative of a stochastic trend. Reported is the $\hat{\eta}_T$ statistic of Kwiatkowski et al. (1992), which is the sum of squared partial sums of residuals from regressing the variable on a constant and a time trend, divided by a Newey-West (1987) error variance estimator (using four lags and the Bartlett spectral window). Cointegration tests are Engle-Granger (1987) tests of the null that the variables are not cointegrated. This is a Dickey-Fuller $I(1)$ test of the residual of regressing ρ_i on the real or nominal R_I and a time trend. Estimates and critical values for the Dickey-Fuller and Engle-Granger tests obtained using MicroTSP. Critical values for $I(0)$ tests are from Table 1 in Kwiatkowski et al.

jected (see, for example, Diewert and Wales, 1995).

Generalized composite commodity aggregation provides a new way to test for separability, by explicitly considering the role of the error term in the aggregation. Demand system errors e_i are typically interpreted as random deviations from rationality, unobserved preference heterogeneity, or measurement errors in demands. Without separability or Hicks

Leontief, group demands also contain aggregation errors, which under generalized composite commodity aggregation are well behaved but are still present.

If utility functions are separable, then there is no aggregation bias when summing demands for individual goods to obtain group demands. Group demand errors ε_I will just equal the sum of the errors of the individual goods, that is, $\varepsilon_I = \Sigma_{i \in I} e_i$, and therefore with separability ε_I should be independent of the within group relative prices $\boldsymbol{\rho}$.

If demands are not separable, then by definition of g_i and G_I the residuals from estimated demand systems for groups of goods are given by

$$\text{(10)} \quad \varepsilon_I = \left[\sum_{i \in I} e_i + g_i(\mathbf{R}^* + \boldsymbol{\rho}, z) \right] - G_I(\mathbf{R}, z),$$

which is partly comprised of functions of $\boldsymbol{\rho}$, and therefore is not independent of $\boldsymbol{\rho}$. Given that $\boldsymbol{\rho}$ is not constant (Hicks-Leontief does not hold), separability can tested by seeing if the errors in group demands depend on $\boldsymbol{\rho}$.

To give an example, the AIDS demand model is

$$\text{(11)} \quad w_i = a_i + \sum_{j=1}^{n} c_{ij} r_j + b_i[z - d(\mathbf{r})] + e_i,$$

where a_i, b_i and c_{ij} are parameters and $d(\mathbf{r}) = a_0 + \Sigma_{i=1}^{n} a_i r_i + \Sigma_{i=1}^{n} \Sigma_{j=1}^{n} c_{ij} r_i r_j / 2$. Without separability, by equations (2), (10) and (11) aggregation across goods yields the group AIDS model demands

$$\text{(12)} \quad W_I = A_I + \sum_J C_{IJ} R_J + B_I[z - D(\mathbf{R})] + \varepsilon_I$$

where $A_I = \Sigma_{i \in I} [a_i + E(\rho_i)]$, $B_I = \Sigma_{i \in I} b_i$, $C_{IJ} = \Sigma_{j \in J} \Sigma_{i \in I} c_{ij}$, $A_0 = a_0 + [\Sigma_I a_I \Sigma_{i \in I} E(\rho_i)] + [\Sigma_I \Sigma_J C_{IJ} \Sigma_{i \in I} \Sigma_{j \in J} E(\rho_i \rho_j)/2]$, $D(\mathbf{R}) = A_0 + \Sigma_I a_I R_i + \Sigma_J C_{IJ} R_I R_J / 2$, and

$$\text{(13)} \quad \varepsilon_I = \left\{ \sum_{i \in I} e_i + \sum_{j=1}^{n} c_{ij}[\rho_j - E(\rho_j)] \right\} - b_I \left(\sum_{i=1}^{n} a_i[\rho_i - E(\rho_i)] + \sum_{j=1}^{n} c_{ij} \{ R_i^*[\rho_j - E(\rho_j)] + R_j^*[\rho_i - E(\rho_i)] + [\rho_i \rho_j - E(\rho_i \rho_j)] \}/2 \right).$$

Equation (13) shows that when an AIDS model without separability is aggregated across goods, the resulting errors are linear in $\boldsymbol{\rho}$ except for a quadratic term arising from aggregation of the deflator $d(\mathbf{r})$. Therefore, for the AIDS model we can test for generalized composite commodity aggregation versus separability by testing if the errors in the AIDS model for groups of goods are correlated with $\boldsymbol{\rho}$. This is done by simply including $\boldsymbol{\rho}$ linearly (or quadratically) in the AIDS model equations and testing for the statistical significance of its coefficients.

Lewbel (1991b) showed that the AIDS and translog demand models fit aggregate U.S. consumption data well, because individual households have demands that are roughly of these forms, and regularities in the distribution of total expenditures cause the exact aggregation of AIDS or translog models across households to be almost identical to aggregate AIDS or translog models. More precisely, individual households appear to have IQUAIDS demands (see Banks et al., 1994), but differences between the QUAIDS and AIDS forms are concentrated in the tails of the income distribution, and so cancel out to some extent when the data are aggregated across households.

Given these results, the U.S. NIPA data analyzed in the previous section were used to construct log per capita total expenditures z and group budget shares W_I. An unrestricted (neither symmetry nor homogeneity is imposed) aggregate AIDS model, equation (12), is estimated by regressing each W_I on a constant, the seven log group price indices com-

TABLE 3—AIDS MODEL TESTS

	Not cointegrated Levels data		ρ coefficients all equal to zero					
			Levels data		Differenced data		Error correction	
Null	Without ρ	With ρ	F	L-R	F	L-R	F	L-R
W_1	−4.08	−4.39	1.53	52.91	2.18	60.79	7.22	106.72
W_2	−3.57	−4.32	6.30	100.52	3.37	74.98	3.67	82.69
W_3	−3.17	−5.01	5.88	98.00	3.06	71.67	2.89	74.59
W_4	−3.79	−3.57	1.65	55.11	1.37	46.98	3.58	81.90
W_5	−3.70	−3.63	0.98	40.67	2.26	61.90	3.29	78.94
W_6	−4.81	−4.05	2.09	62.50	2.72	67.75	2.03	63.23
W_7	−2.98	−3.78	3.50	79.55	6.13	66.04	5.73	98.39
Critical values:								
1 percent	−6.43	−6.43	4.15	34.81	3.91	34.81	4.15	34.81
5 percent	−5.58	−5.58	2.67	28.87	2.57	28.87	2.67	28.87
10 percent	−5.17	−5.17	2.13	25.99	2.07	25.99	2.13	25.99

Notes: The cointegration tests are Engle-Granger (1987) tests of the null hypothesis that the variables are not cointegrated (see notes to Table 2). In the first column (without $\boldsymbol{\rho}$) the variables tested are the budget share, every R_I, and $z - d(\mathbf{R})$ as in equation (13). A time trend and a squared time trend are also included. In the second column the same set variables are tested, along with 18 ρ_i's (the ρ_i's for the three largest average expenditure components of each group). Critical values for cointegration tests with this many variables have not been tabulated. The critical values provided above for the first two columns of tests are for cointegration tests with only six variables, and so are lower bounds on the true critical values for the statistics in these two columns. The levels data model is the regression of the budget share on every R_I, $z - d(\mathbf{R})$, a constant, a time trend, a squared time trend, and the 18 ρ_i's, with a first-order autocorrelation correction. The null hypothesis tested is that the coefficients of all the ρ_i's are zero. Under the null (and ignoring effects of regressors being integrated) the F statistic has an F distribution with 18 and 11 degrees of freedom, and the likelihood ratio (L-R) statistic has a chi-squared distribution with 18 degrees of freedom. The differenced-data model is the regression of the change in budget share on the change in every R_I, the change in $z - d(\mathbf{R})$, a constant, a time trend and the change in the 18 ρ_i's, with a first-order autocorrelation correction. The error-correction model is the same as the differenced-data model, but with the residuals from the levels data model added as another regressor. The null hypotheses and statistic distributions are the same as in the levels-data model, but in the differenced-data model the F distribution has 12 degrees of freedom in the denominator, and integrated regressors should not be a problem for test statistic distributions in the differenced-data and error-correction models.

prising $\mathbf{R}$, and on $z - D(\mathbf{R})$. The usual approximation is to replace $D(\mathbf{R})$ with the log of a general price index (the implicit price deflator for total consumption z). The same model is then reestimated including the ρ_i's as additional regressors, and the joint significance of the ρ_i's is tested. Statistical significance of the ρ_i's is evidence against separability. The results of this and related tests are reported in Table 3.

The number of ρ_i's exceeds the number of available observations in the sample, so it is impossible to include all or even most of them in the model. In Table 3 only the ρ_i's corresponding to the three largest goods in each group (measured in terms of average within group budget shares) are used. This still results in adding 18 ρ_i's to each AIDS equation.

A second obstacle to estimation is that, by Table 2, most of the ρ_i's and all of the logged group prices (both nominal and deflated) are nonstationary. Using a closely related data set (annual U.S. NIPA consumption data, but grouped differently), Serena Ng (1995) finds that group prices have both deterministic and stochastic trends, with some group prices having linear deterministic trends and others quadratic. Controlling for these deterministic trends, Ng finds that some group AIDS equations appear cointegrated (having a common stochastic trend), while others do not.

The first two columns of Table 3 report Engle-Granger tests of cointegration for the AIDS models here (both excluding and including the 18 largest component ρ_i's), incorporating a linear and a quadratic time trend to remove any deterministic trends, and allowing four years of lags in the residuals equation to compensate for any residual autocorrelation. Based on these tests, none of the AIDS equations appear cointegrated either including or excluding the ρ_i's. However, given the large number of variables and limited degrees of freedom, cointegration cannot be convincingly tested.

The third and fourth columns of Table 3 report ordinary F statistic and likelihood ratio tests for the joint significance of the ρ_i's in each AIDS equation. The models are estimated with a first-order autocorrelation correction. A time trend and a time trend squared are included in the models, which Ng's (1995) results suggest could be needed for cointegration. These time trends can also be economically motivated, for example, Lewbel (1991b) shows that inclusion of a time trend in aggregate demand models can proxy for some of the changes in demographic composition of the population over time. Other authors include time trends as a crude control for dynamic effects (see, for example, Robert A. Pollak and Wales, 1992).

The ρ_i's are jointly statistically significant for some of the level data AIDS equations using F tests, and are significant for all of the AIDS equations using likelihood ratio tests (columns 3 and 4 of Table 3). However, if the AIDS equation variables are in fact not cointegrated then these test results are invalid because the errors are nonstationary.

To deal with this problem the equations were reestimated after differencing all of the variables and the same F and likelihood ratio tests were repeated, with similar results (columns 5 and 6 of Table 3). Differencing removes the stochastic trends and leaves at most a linear deterministic trend, which remains in the model. Finally, the last two columns of Table 3 report the same tests with the same differenced data, but adding the residuals from the levels data equation as an additional regressor, making the result a crude error correction model if the levels data is cointegrated. A true error correction model would also include lags of all the regressors, but this is impractical given the large number of regressors and limited degrees of freedom available. In this case almost all of the F tests (and as before all of the likelihood ratio tests) indicate that the ρ_i's are statistically significant.

The results in Table 3 show significance of the ρ_i's regardless of whether the levels model is cointegrated or not, thereby rejecting separability and supporting generalized composite commodity aggregation. Moreover, the results in Table 3 are likely to underestimate the true impact of the ρ_i's, because most of these within group relative prices had to be left out of the models due to the lack of available degrees of freedom.

The preceding analysis has limitations that should be recognized. The significance of the ρ_i's could be due to model specification errors, aggregation (across households) errors, or spurious correlations of the ρ_i's with other relevant variables, for example, possible omitted dynamic effects. Another potential problem is possible endogeny of prices. The results should also be tempered by consideration of measurement errors, for example, Daniel T. Slesnick (1993) documents numerous shortcomings in the U.S. NIPA consumption data.

Though limited by the above caveats, the empirical finding that ρ_i's are far from constant and are statistically significant in group demand models is consistent with generalized composite commodity aggregation and is inconsistent with either Hicks-Leontief or utility function separability.

VI. Conclusions

This paper provides a generalized composite commodity aggregation theorem, which consists of simple conditions that make the biases from aggregation over goods take the form of well behaved error terms, and thereby provides a new rationalization for aggregation across goods. Aggregate U.S. price data appear to satisfy the required assumptions, and estimated demand models are found to have properties that are consistent with generalized composite commodity aggregation and inconsistent with the alternative assumption of separable utility.

The Hicks-Leontief composite commodity theorem requires that the ratio of the prices of individual goods to group prices be constant. The generalized composite commodity theorem proposed here allows this ratio to vary, requiring only that this ratio be independent of the level of group prices (and of total expenditures). A possible theoretical rationalization of this assumption is that prices are proportional to marginal costs of production, and that marginal costs of individual goods within a group generally move together over time because of similar technologies, while differences in marginal costs result from technology or input price changes. It would be useful to examine more elaborate general equilibrium models to see which ones yield prices that satisfy the conditions needed for generalized composite commodity aggregation.

Empirically, the assumptions on price movements required for generalized composite commodity aggregation seem to hold. For NIPA defined groups of goods, nominal and deflated group prices appear nonstationary and within group relative prices are not cointegrated with either real or nominal group prices.

If the price assumptions were found to be violated in some application, empirical methods like the factor analyses applied to prices by Nicol (1991) might be used to identify alternative groupings of goods that do satisfy the assumptions. Identifying goods that have related production technologies might also be helpful to the extent that production costs determine prices.

Whatever method is used to group goods, groups must be chosen that yield economically meaningful price and income elasticities. One possibility would be to first allocate goods into meaningful groups, such as might be hypothesized for separability. Then empirical methods like factor analysis could be used to subdivide these groups into subgroups that satisfy Assumption 2. Estimation would then be done at the level of these subgroups.

Theorem 1 requires assumptions concerning the matrices $\mathbf{H}$ and $\tilde{\mathbf{H}}$. These matrices are defined as moments of functions of conditional expectations, that is, covariances of demands and their income derivatives. General methods exist for nonparametrically estimating these kinds of moments at root N rates (see, for example, Whitney K. Newey and Daniel L. McFadden, 1994) and consistently testing equality hypotheses based on them (see, for example, Lewbel, 1995). One could therefore in principle directly test the assumptions about $\mathbf{H}$ and $\tilde{\mathbf{H}}$ required for generalized composite commodity aggregation, though this would require more relative price variation than is usually available.

This paper described restrictions that make commodity aggregation errors be mean zero, conditional on group prices and income. It may be possible to relax these restrictions by instead requiring that aggregation errors be mean zero conditional on some set of instruments. The demand system would then be estimated using instrumental variables or the generalized method of moments.

As it stands in Theorem 1, demand models that use generalized composite commodity aggregates can be estimated using instrumental variables or the generalized method of moments (for example, to deal with endogeny of prices), provided that the chosen instruments are uncorrelated with ε_I as defined in equation (10). Given Assumption 2, a sufficient condition would be that the instruments be uncorrelated with the disagregate model errors e_i and be independent of $\boldsymbol{\rho}$.

The framework proposed here could be relaxed to permit aggregation errors that have some functional form, and this aggregation induced form would then be estimated as part of the model. For example, we could require that $E(W_I|\mathbf{R}, z) = G_I(\mathbf{R}, z) + \kappa_I$ where G_I is rational and κ_I is a constant induced by the aggregation. Theorem 1 is the special case of this set up in which κ_I either equals 0 or can be incorporated into G_I while maintaining rationality. Lewbel (1993) estimates a specific demand model having this feature of a constant aggregation error, and finds the resulting constants to be statistically significant.

The generalized composite commodity theorem provided here could be used to estimate factor demand equations in a production context, where they may help justify the aggregation of diverse forms of capital, material, labor, and so on, in standard KLM or KLEM models. Note that separability is typically rejected for such models (see, for example, Diewert and Wales 1995).

Another possible application of generalized composite commodities is to replace questionable intertemporal separability assumptions in macroeconomic models. In these models all goods are combined into a single aggregate called total consumption in each time period, the demand for which is derived from an intertemporal utility function. In this case the disaggregate model is a separate demand equation (and a separate price) for every good in every time period. Each group then corresponds to a time period and consists of all the goods consumed in that time period. Group prices are essentially interest rates, and the macroeconomic intertemporal utility function would be the aggregate utility function over generalized composite commodities.

A problem with this procedure is that the intertemporal analog to Assumption 2 cannot be easily tested. In the usual demand system case a single good is gasoline, the price of which is observed repeatedly, which permits testing of its statistical properties. In the intertemporal problem a single good might be gasoline in 1990 (gasoline in 1991 would be a different good in a different group). One would need to have multiple observations of 1990 gasoline prices to test the macro model analog to Assumption 2. Such data might be obtained by, for example, comparing the prices for the same goods in the same time periods across countries or other regions.

While the intertemporal analog to Assumption 2 may be difficult to test empirically, most macro models have equations that determine prices, and this theoretical mechanism that generates prices can be checked to see if it satisfies Assumption 2. Macro models also typically assume homothetic utility, which by Corollary 4 satisfies the generalized composite commodity theorem.

Technical Appendix

PROOF OF THEOREM 1:

The adding up condition: Summing equation (2) over all groups I gives

$$\sum_I G_I(\mathbf{R}, z) = E\left[\sum_I G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z) | \mathbf{R}, z\right] = E\left[\sum_I \sum_{i \in I} g_i(\mathbf{r}, z) | \mathbf{R}, z\right] = E(1 | \mathbf{R}, z) = 1.$$

The homogeneity condition: By equation (2), for any constant k we have

$$\begin{aligned} G_I(\mathbf{R} - k, z - k) &= \int G_I^*(\mathbf{R}^* - k + \boldsymbol{\rho}, z - k) dF(\boldsymbol{\rho}) \\ &= \int \sum_{i \in I} g_i(\mathbf{R}^* - k + \boldsymbol{\rho}, z - k) dF(\boldsymbol{\rho}) = \int \sum_{i \in I} g_i(\mathbf{R}^* + \boldsymbol{\rho}, z) dF(\boldsymbol{\rho}) \\ &= \int G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z) dF(\boldsymbol{\rho}) = G_I(\mathbf{R}, z). \end{aligned}$$

Now consider the log form Slutsky matrix.

$$\begin{aligned} E\left[\sum_{i \in I} \sum_{j \in J} s_{ij}(\mathbf{r}, z) | \mathbf{R}, z\right] &= E\left\{\sum_{i \in I} \sum_{j \in J} \left[\frac{\partial g_i(\mathbf{r}, z)}{\partial r_j} + \frac{\partial g_i(\mathbf{r}, z)}{\partial z} g_j(\mathbf{r}, z)\right] \middle| \mathbf{R}, z\right\} \\ &= E\left[\sum_{i \in I} \sum_{j \in J} \frac{\partial g_i(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial (R_J^* + \rho_j)} \middle| \mathbf{R}, z\right] + E\left[\sum_{i \in I} \sum_{j \in J} \frac{\partial g_i(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial z} g_j(\mathbf{r}, z) \middle| \mathbf{R}, z\right] \\ &= E\left[\sum_{i \in I} \frac{\partial g_i(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial R_J} \middle| \mathbf{R}, z\right] + E\left[\sum_{i \in I} \frac{\partial g_i(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial z} G_J^*(\mathbf{R}^* + \boldsymbol{\rho}, z) \middle| \mathbf{R}, z\right] \\ &= \int \sum_{i \in I} \frac{\partial g_i(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial R_J} dF(\boldsymbol{\rho}) + \int \frac{\partial G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial z} G_J^*(\mathbf{R}^* + \boldsymbol{\rho}, z) dF(\boldsymbol{\rho}) \\ &= \left[\sum_{i \in I} \frac{\partial \int g_i(\mathbf{R}^* + \boldsymbol{\rho}, z) dF(\boldsymbol{\rho})}{\partial R_J}\right] + \frac{\partial G_I(\mathbf{R}, z)}{\partial z} G_J(\mathbf{R}, z) + H_{IJ}(\mathbf{R}, z) \end{aligned}$$

which shows that equation (5) holds, that is,

$$S_{IJ}(\mathbf{R}, z) = H_{IJ}(\mathbf{R}, z) + \int \left[\sum_{i \in I} \sum_{j \in J} s_{ij}(\mathbf{R}^* + \boldsymbol{\rho}, z) \right] dF(\boldsymbol{\rho}).$$

Slutsky symmetry $S_{IJ} = S_{JI}$ therefore follows from $H_{IJ} = H_{JI}$ and $s_{ij} = s_{ji}$.

Now define $\tilde{\mathbf{S}}(\mathbf{R}, z)$ as the matrix of elements $\tilde{\mathbf{S}}_{IJ}(\mathbf{R}, z) = S_{IJ}(\mathbf{R}, z) + G_I(\mathbf{R}, z)G_J(\mathbf{R}, z) - \delta_{IJ}G_I(\mathbf{R}, z)$, where $\delta_{IJ} = 1$ if $I = J$ and zero otherwise. We now need to show that $\tilde{\mathbf{S}}$ is negative semidefinite. It follows from equation (5) and the definition of $\tilde{\mathbf{s}}$ that

$$\begin{aligned} S_{IJ}(\mathbf{R}, z) = H_{IJ}(\mathbf{R}, z) &+ E\left[\sum_{i \in I} \sum_{j \in J} \tilde{s}_{ij}(\mathbf{R}^* + \boldsymbol{\rho}, z) | \mathbf{R}, z \right] \\ &- E\left[\sum_{i \in I} \sum_{j \in J} g_i(\mathbf{R}^* + \boldsymbol{\rho}, z) g_j(\mathbf{R}^* + \boldsymbol{\rho}, z) - \delta_{ij} g_i(\mathbf{R}^* + \boldsymbol{\rho}, z) | \mathbf{R}, z \right]. \\ &= H_{IJ}(\mathbf{R}, z) + E\left[\sum_{i \in I} \sum_{j \in J} \tilde{s}_{ij}(\mathbf{R}^* + \boldsymbol{\rho}, z) | \mathbf{R}, z \right] \\ &- E[G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z) G_J^*(\mathbf{R}^* + \boldsymbol{\rho}, z) - \delta_{IJ} G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z) | \mathbf{R}, z]. \\ &= H_{IJ}(\mathbf{R}, z) + E\left[\sum_{i \in I} \sum_{j \in J} \tilde{s}_{ij}(\mathbf{R}^* + \boldsymbol{\rho}, z) | \mathbf{R}, z \right] - G_I(\mathbf{R}, z) G_J(\mathbf{R}, z) + \tilde{H}_{IJ}(\mathbf{R}, Z) - \delta_{IJ} G_I(\mathbf{R}, z), \end{aligned}$$

so

$$\tilde{S}_{IJ}(\mathbf{R}, z) = H_{IJ}(\mathbf{R}, z) + E\left[\sum_{i \in I} \sum_{j \in J} \tilde{s}_{ij}(\mathbf{R}^* + \boldsymbol{\rho}, z) | \mathbf{R}, z \right] + \tilde{H}_{IJ}(\mathbf{R}, Z).$$

Negative semidefiniteness of $\tilde{\mathbf{S}}(R, z)$ therefore follows from negative semidefiniteness of $\tilde{\mathbf{s}}(\mathbf{R}^* + \boldsymbol{\rho}, z)$ and of $H_{IJ}(\mathbf{R}, z) + \tilde{H}_{IJ}(\mathbf{R}, Z)$.

PROOF OF THEOREM 2:

For equation (7)

$$E\left(\sum_{i \in I} \sum_{j \in J} \frac{\partial g_i(\mathbf{r}, z)}{\partial r_j} \middle| \mathbf{R}, z \right) = E\left(\sum_{j \in J} \frac{\partial G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial (R_j^* + \rho_j)} \middle| \mathbf{R}, z \right) = E\left(\frac{\partial G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial R_J} \middle| \mathbf{R}, z \right)$$

$$= \frac{\partial E[G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z) | \mathbf{R}, z)}{\partial R_J} = \frac{\partial G_I(\mathbf{R}, z)}{\partial R_J}.$$

For equation (8):

$$E\left(\sum_{i \in I} \frac{\partial g_i(\mathbf{r}, z)}{z} \middle| \mathbf{R}, z \right) = E\left(\frac{\partial G_I^*(\mathbf{R}^* + \rho, z)}{\partial z} \middle| \mathbf{R}, z \right) = \frac{\partial E[G_I^*(\mathbf{R}^* + \boldsymbol{\rho}, z) | \mathbf{R}, z)}{\partial z} = \frac{\partial G_I(\mathbf{R}, z)}{\partial z}.$$

PROOF OF THEOREM 3:

Define the function $\Theta(\mathbf{R}, z) = E[v(\mathbf{r}, z) | \mathbf{R}, z]$. Given that $\partial v(\mathbf{r}, z)/\partial z = \theta(\mathbf{R}, z)$, $\partial \Theta(\mathbf{R}, z)/z = \partial E[v(\mathbf{r}, z) | \mathbf{R}, z]/\partial z = E[\partial v(\mathbf{r}, z)/\partial z | \mathbf{R}, z] = \theta(\mathbf{R}, z)$. Integrating $\partial v(\mathbf{r}, z)/\partial z = \theta(\mathbf{R}, z) = \partial \Theta(\mathbf{R}, z)/z$ with respect to z gives $v(\mathbf{r},$

$z) = \Theta(\mathbf{R}, z) + \phi(\mathbf{r})$ for some function $\phi(\mathbf{r})$, and since $E[v(\mathbf{r}, z)|\mathbf{R}, z] = \Theta(\mathbf{R}, z)$, it follows that $E[\phi(\mathbf{r})|\mathbf{R}, z] = 0$. Now, by Roy's identity

$$g_i(\mathbf{r}, z) = -\frac{\partial v(\mathbf{r}, z)}{\partial r_i} \bigg/ \frac{\partial v(\mathbf{r}, z)}{\partial z} = -\frac{\partial v(\mathbf{r}, z)}{\partial r_i} \bigg/ \frac{\partial \Theta(\mathbf{R}, z)}{\partial z}.$$

Summing over i gives

$$G_I^*(\mathbf{R} + \boldsymbol{\rho}, z) = -\sum_{i \in I} \frac{\partial v(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial (R_I + \rho_i)} \bigg/ \frac{\partial \Theta(\mathbf{R}, z)}{\partial z} = -\frac{\partial v(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial R_I} \bigg/ \frac{\partial \Theta(\mathbf{R}, z)}{\partial z}.$$

Take the conditional expectation of the above with respect to $\mathbf{R}$ and z to get

$$G_I(\mathbf{R}, z) = -E\left[\frac{\partial v(\mathbf{R}^* + \boldsymbol{\rho}, z)}{\partial R_I} \bigg/ \frac{\partial \Theta(\mathbf{R}, z)}{\partial z} \bigg| \mathbf{R}, z\right] = -\frac{\partial E[v(\mathbf{R}^* + \boldsymbol{\rho}, z)|\mathbf{R}, z]}{\partial R_I} \bigg/ \frac{\partial \Theta(\mathbf{R}, z)}{\partial z}$$

so

$$G_I(\mathbf{R}, z) = -\frac{\partial \Theta(\mathbf{R}, z)}{\partial R_I} \bigg/ \frac{\partial \Theta(\mathbf{R}, z)}{\partial z}.$$

This final expression is Roy's identity for the aggregate demands with $\Theta(\mathbf{R}, z)$ equal to the aggregate indirect utility function $V(\mathbf{R}, z)$, so $V(\mathbf{R}, z) = \Theta(\mathbf{R}, z)$ (formally, integrating Roy's identity shows that $V(\mathbf{R}, z) = \zeta[\Theta(\mathbf{R}, z)]$ for some monotonic transformation ζ, but without loss of generality V can be defined such that ζ is the identity function). Since $V(\mathbf{R}, z) = \Theta(\mathbf{R}, z) = E[v(\mathbf{r}, z)|\mathbf{R}, z]$, the theorem is proved.

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