Hicksian aggregation and a world full of derivatives

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1. Taylor is that you?

To tackle the problem of aggregation across goods, I am considering an agent with utility function $u : \mathbb{R}^{n+m}_{++} \to \mathbb{R}$ that solves the following optimization problem

$$\begin{aligned} \max_{\mathbf{x},\mathbf{z}} \quad & u(\mathbf{x},\mathbf{z}) \\ \text{s.a.} \quad & \mathbf{px} + \mathbf{qz} = y. \end{aligned}$$

With solution $\mathbf{x}(\mathbf{p},\mathbf{q},y)$. Assume we have a rule to aggregate the consumption bundle $\mathbf{x}$ using a function $g : \mathbb{R}^n \to \mathbb{R}$ and call $X := g(x)$. The Hicks aggregation problem seeks to find $P := f(\mathbf{p})$ and $U : \mathbb{R}^{1+m} \to \mathbb{R}$ such that the function

$$\begin{aligned} X(f(\mathbf{p}),\mathbf{q},y) := \arg\max_{X,\mathbf{z}} \quad & U(X,\mathbf{z}) \\ \text{s.a.} \quad & PX + \mathbf{qz} = y \end{aligned}$$

satisfies

$$X(f(\mathbf{p}),\mathbf{q},y) = g\big(\mathbf{x}(\mathbf{p},\mathbf{q},y)\big).$$

Typically this is not possible except in very restrictive cases. Suppose the model imposes some function U to the agent and some price index f. We would like to measure the error in the estimation of the aggregate demand $X = g(x)$. Note that the Lagrangians of both problems are

$$\mathcal{L}_d = u(\mathbf{x},\mathbf{z}) + \lambda_d(y - \mathbf{px} - \mathbf{qz}), \tag{1}$$

$$\mathcal{L}_a = U(X,\mathbf{z}) + \lambda_a(y - f(\mathbf{p})X - \mathbf{qz}), \tag{2}$$

And the corresponding first order conditions are

$$\begin{cases} \nabla_{\mathbf{x}} u - \lambda_d \mathbf{p} = \mathbf{0}, \\ \nabla_{\mathbf{z}} u - \lambda_d \mathbf{q} = \mathbf{0} \end{cases} \tag{3}$$

$$\begin{cases} \dfrac{\partial U}{\partial x} - \lambda_a f(\mathbf{p}) = 0, \\ \nabla_{\mathbf{z}} U - \lambda_a \mathbf{q} = \mathbf{0} \end{cases} \tag{4}$$

We are interested in the difference

$$\Big| X(f(\mathbf{p}), \mathbf{q}, y) - g(\mathbf{x}(\mathbf{p}, \mathbf{q}, y)) \Big|.$$

Assuming the investigator has access to both aggregate and disaggregate data in consumption and a set of prices $\mathbf{p_0}, \mathbf{q_0}$ and income y_0. It is possible to approximate each function using a Taylor expansion around $(\mathbf{p_0}, \mathbf{q_0}, y_0)$. Explicitly

$$X(f(\mathbf{p}), \mathbf{q}, y) = X(f(\mathbf{p_0}), \mathbf{q_0}, y_0) + \frac{\partial X}{\partial P} \nabla f(\mathbf{p} - \mathbf{p_0}) + \nabla_q X(\mathbf{q} - \mathbf{q_0}) + \frac{\partial X}{\partial m}(m - m_0) + \mathcal{O}(2)$$
$$g(x(\mathbf{p}, \mathbf{q}, m)) = g(x(\mathbf{p_0}, \mathbf{q_0}, m_0)) + \nabla g \cdot D_{\mathbf{x}} \Big((\mathbf{p}, \mathbf{q}, m) - (\mathbf{p_0}, \mathbf{q_0}, m_0) \Big) + \mathcal{O}(2)$$

The derivatives of each function could in principle be obtained by implicit differentiation of equations (3) and (4). In particular, the derivatives of X and $\mathbf{x}$ with respect to m can be obtained using the envelope theorem.

2. In other news...

The Schechtman y Escudero paper seems promising. Theorem 1.1 provides bounds on the MPC, but they depend on the solution of another (though deterministic) problem. Besides, the bounds work on a finite horizon problem although I think it may be enough if we take t large enough. Again, I have more questions than answers in this point.