Hicksian aggregation across consumers

or (the rebound problem after derivation went wrong)

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1. Do you remember this?

Consider an economy composed by a continuum of agents that consume $n+1$ goods: an essential good z (i.e., water) and n different goods grouped in the vector $\mathbf{x}$. Good z is valued at price $q \in \mathbb{R}_{++}$ and the $\mathbf{x}$ goods are valued at prices $\mathbf{p} \in \mathbb{R}^n_{++}$. Each individual in this setting is identified with a pair (y, α), where $y \in [m, M]$ is her income and $\alpha \in (0,1)$ determines the form of her pseudo-Cobb-Douglas (PCD)[1] utility function, that is,

$$u_\alpha(\mathbf{x}, z) := u_{(y,\alpha)}(\mathbf{x}, z) = \sum_{j=1}^{n} x_j^\alpha z^{1-\alpha} = \left(\sum_{j=1}^{n} x_j^\alpha\right) z^{1-\alpha}.$$

Assume further that (y, α) follows a certain distribution $F(\mathbf{p}, q)$ over its support $S := [m, M] \times (0,1)$ that may depend on the prices $\mathbf{p}$ and q. Suppose that for simplicity the investigator is not interested in a model with disaggregate consumption of the $\mathbf{x}$ goods but instead considers a single good X given by:

$$g_\alpha(\mathbf{x}) = \left(\sum_{j=1}^{n} x_j^\alpha\right)^{1/\alpha}.$$

In this scenario, we can write

$$U(g_\alpha(\mathbf{x}), z) := g_\alpha(\mathbf{x})^\alpha z^{1-\alpha},$$

which is the usual Cobb-Douglas utility function. By defining $\varepsilon := (1-\alpha)^{-1}$ and

$$P_\alpha(\mathbf{p}) := \left(\sum_{j=1}^{n} p_j^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}},$$

[1] The choice of this name will be clear shortly.

we have that the solutions

$$\begin{aligned} \mathbf{x}(\mathbf{p}, q, y) := \arg\max_{\mathbf{x}, z} \quad & u_\alpha(\mathbf{x}, z) \\ \text{s.a.} \quad & \mathbf{p}\mathbf{x} + qz = y. \end{aligned}, \tag{1}$$

and

$$\begin{aligned} X(P(\mathbf{p}), q, y) := \arg\max_{X, z} \quad & U(X, z) \\ \text{s.a.} \quad & P_\alpha(\mathbf{p})X + qz = y \end{aligned} \tag{2}$$

satisfy

$$X(P_\alpha(\mathbf{p}), q, y) = g_\alpha\big(\mathbf{x}(\mathbf{p}, q, y)\big).$$

Note that the functions g_α and P_α depend explicitly on the value of α. Hence, in order to correctly aggregate the goods $\mathbf{x}$ a different aggregator and price index for every person is needed. However, if the investigator is not aware of (or concerned with) the heterogeneity of the population then she will use a single function $g := g_{\overline{\alpha}}$ and $P := P_{\overline{\alpha}}$. If she is interested in estimating the demand for category $X_\alpha := g_\alpha(\mathbf{x})$ for ea