Goodness-of-fit in economic models
¿How much are we losing?

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> (...) all models are approximations. Essentially, all models are wrong, but some are useful. However, the approximate nature of the model must always be borne in mind.
>
> *Empirical Model-Building and Response Surfaces (1987)*
> George Box and Norman Draper

Abstract

Aggregation is a tool used to reduce the complexity of economic models to draw more clear and succinct conclusions or simplify analyses. As any approximation, its use may be accompanied with errors researches may not be willing to tolerate if they were aware of them. In this work I present how these errors appear in simple models using aggregation across goods and across consumers. I also show some of their determinants in order to find ways to bound them. Finally, I briefly discuss a methodology to study the goodness-of-fit of aggregate models in more general settings.

1. Introduction

Every model in science is by definition a simplified reality. On the bright side, abstracting from the complexity of the real world has allowed society to understand the sometimes subtle mechanisms that rule nature and human behavior. This does not mean that a model is useful for every purpose. Evidently, whilst some of them may be very useful to expose certain dynamics of the real world, the approximation may carry errors that harm future predictions. The previous comment points directly to the question of which model is the most useful for some given problem. In particular, when the answer is *many* the modeler needs to make a choice based on the results she expects to highlight and the channels to study. The dilemma is by no means alien to the field of economics. When describing an economy, the researcher is faced with several possible assumptions that shape the complexity of the model. Although some of them may be made by feasibility reasons (for example, because a highly detailed model cannot be solved or simulated or because data will not be available to calibrate it) there may be others that serve a transparency purpose, that is, they intend to make clear the results without dwelling on the unnecessary details. Consequently, in the process of constructing a model, the investigator may choose to follow the Occam's Razor principle: among the models that are consistent with the evidence, choose the one that makes the fewest possible assumptions. This criterion implies that the measure of a (correct) model is its complexity. However, as Milton Friedman said, "The ultime goal of a positive science is the development of a 'theory' or, 'hypothesis' that yields valid and meaningful (...) predictions about phenomena not yet observed" and thus "Its performance is to be judged by the precision, scope, and conformity with experience of the predictions it yields".[1]

Different strands in the economic literature have studied when predictions of some models are robust to different specifications. In Sutton (2007), the author discusses which mechanisms in the context of industrial organization still hold in conditions outside the classical models of, for example, Cournot and Bertrand. A similar motivation can be found in Kajii and Morris (1997), where they study how sensitive game theory conclusions are to the assumption of common knowledge of payoffs in a game. The interest in robustness in the context of mechanism design can be also found in ter Vehn and Morris (2011). An interesting approach is the one in Basu and Fernald (1997) where the authors try to estimate discrepancies due to "aggregation effects" when considering a model with a representative firm and one where heterogeneous effects are considered. Similarly, in Hanushek et al. (1996) the authors try to

[1] Friedman (1953).

reconcile contradictory results in the school literature arguing an important role of aggregation in the magnitude of omitted variable bias, which can in principle invalidate previous estimations. Related to these aggregation literature there is a concern with models that make use of aggregated data and different authors have studied what is called "aggregation bias" arguing that these models hide important mechanisms that could explain these differences.[2]

Back to the economic theory, consider the example of the representative agent model. The assumption that there is only one consumer in the economy is useful and has been key to understand important qualitative results, especially in macroeconomics. Nevertheless, employing this model to predict future realizations of certain key variables such as aggregate demand or marginal propensity to consume (MPC) may be inaccurate if heterogeneity effects are in place. In other words, there is a shadow price in the approximation (which the investigator could be willing to pay or not) if she wishes to use the model for another, more quantitative-driven purpose. This point is made clearly in Carroll (2000) in the context of the buffer-stock model: "Representative-agent models are typically calibrated to match an aggregate wealth-to-income ratio" but "the typical household's wealth is much smaller than the wealth of such a representative agent (...), this would lead one to expect that the behavior of the median household may not resemble the behavior of a representative agent with a wealth-to-income ratio similar to the aggregate ratio". The evidence quickly backs up this view: while the annual MPC predicted by the representative agent model is about 0.04, many empirical analysis estimate this parameter to lie between 0.2 and 0.5.[3]

The aforementioned model is a particular case of a common practice in economics: aggregation. The other canonical example of its use is aggregation across goods, where instead of describing the myriad of goods available in an economy they are grouped into one or several categories. Regarding these two implementations, previous theoretical literature focused in one side of the problem: When is it possible to carry out this practice and describe precisely the same economy?. In the case of the representative agent, the necessary and sufficient condition is that the indirect utility function of every consumer has the Gorman form.[4] When aggregation is applied to goods, the answer has been more elusive but two results arise. First, the Hicks-Leontief (composite commodity) theorem allows aggregation if relative prices are constant in the group of goods that are to be bundled.[5] Although a somewhat weaker requirement is proposed by Lewbel (1996): bundling is possible if all group relative prices are

[2]See for example Lee et al. (1990); Ravallion (1998); Feenstra and Hanson (2000); Imbs et al. (2005).
[3]Carroll (2000).
[4]Gorman (1953).
[5]Leontief (1936); Hicks (1946).

independent of price indexes and income. The second answer states that grouping some goods is possible if preferences between them are "independent" of the remaining goods present in the economy.[6] For the two kinds of aggregation, the conditions are highly restrictive and not typically met in econometric or theoretical applications. As mentioned previously, the literature has used them both in constructing models and making econometric estimations and this practice comes at a cost. Thus, understanding and quantifying possible approximation errors is crucial in determining and measuring their goodness-of-fit. In contrast with some of the articles mentioned earlier, in this work I intend to give a theoretical look at how these deviations appear using simple models and try to bound them in terms of available variables in the economy. Next, drawing on the previous results, I will discuss a methodology about approaching this problem in more general settings.

The rest of the paper proceeds as follows. To motivate the following discussion, in Section 2 I present a summary of the problems and previous results about conditions under which aggregation is possible. Then, I present 3 settings to illustrate how approximation errors appear when using aggregate models to estimate future economic variables. First, in Section 3, I present a representative agent model in the context of aggregate demand estimation. Second, In Section 4 I study aggregation across goods [7]. The third model is presented in Section 5 and mixes the previous two by presenting an economy of several individuals consuming various goods, where the good aggregation takes a different form. More sections?. Finally, in Section 6 I discuss some final thoughts about model fitness.

2. Previous results in aggregation

The economic literature has recognized two forms of aggregation that are usually taught in microeconomics courses around the world. First, the problem of consumer aggregation or the representative agent problem seeks to describe the aggregate demand of a multi-person economy by focusing only on the aggregate determinants of demand (i.e., the aggregate income), as opposed to the distribution of such variables. The second class of aggregation focuses on describing demands for categories of goods without distinguishing the individual consumption on each element in the category. I will refer to this last problem as the "Hicksian aggregation problem".

Both kinds of aggregation are widely used in economic models. The representative model

[6] See for example Gorman (1959).

[7] when used in the context of demand forecasting

agent was a salient feature of macroeconomic models in the decade of which one? I need literature here.. On the other hand, the two good setting (one particular example of aggregation of goods) has been widely used in buffer-stock models where their main conclusions arise from the interaction between consumption and savings.[examples] Empirical studies also make use of both forms of aggregation. Some of them assume all consumers are equal which is an example of consumer aggregation and others overcome problems in the data (e.g., availability or comprehensiveness) by assuming that agents choose consumption on the category and not on individual goods.[examples].

In what follows I will formally describe the problems about aggregation the early literature tried to answer. These questions aimed at finding conditions for which aggregation is possible. In order to describe both problems I will rely heavily on Varian (1992).

2.1. The representative agent problem

Consider an economy composed by n consumers indexed by $i = 1, \ldots, n$. Their demand functions of the k goods in the economy are summarized in the vector $\mathbf{x}_i(\mathbf{p}, y_i)$, where $\mathbf{p}$ is the price vector of the goods and y_i is the income of agent i. The aggregate demand vector is defined by

$$\mathbf{X}(\mathbf{p}, y_1, \ldots, y_n) = \sum_{i=1}^{n} \mathbf{x}_i(\mathbf{p}, y_i). \tag{1}$$

The question that automatically arises is: Can this aggregate demand function be regarded as generated by a single (or "representative") consumer?. In terms of equation (1), the previous question is equivalent to looking for conditions under which $\mathbf{X}$ does not depend on the distribution but on the aggregate income

$$Y := \sum_{i=1}^{n} y_i.$$

The definitive answer to this problem came with Gorman (1953). According to his result, $\mathbf{X}$ is a function of M if and only if for every $i \in \{1, \ldots, n\}$ the indirect utility function has the Gorman form

$$v_i(\mathbf{p}, y_i) = a_i(\mathbf{p}) + b(\mathbf{p}) y_i,$$

where a_i, b are functions that must only depend on $\mathbf{p}$ and b has to be the same across consumers. This functional requirement is somewhat restrictive but at least two particular examples are worth mentioning: homothetic and quasilinear utility functions. For the first, the indirect utility functions is

$$v(\mathbf{p}, y) = v(\mathbf{p})y, \tag{2}$$

while for the second

$$v(\mathbf{p}, y) = v(\mathbf{p}) + y.$$

Both examples clearly have the Gorman form. However, some homothetic functions could differ in the function v between consumers and thus aggregation is not possible. For example, when $k = 2$, an economy of consumers with Cobb-Douglas utility functions

$$u(x_1, x_2) = x_1^{\alpha_i} x_2^{1-\alpha_i},$$

but where at least two α_i are different. In that case the function $v(\mathbf{p})$ in (2) is different between individuals and thus aggregation is not possible.

In spite of the previous comment, the widespread use of these two classes of utility functions in the literature has induced an overuse of representative agent models in settings where it is not possible to propose them. Examples maybe?. As a final remark, note that functional separability is a sufficient but not a necessary condition for aggregation.

2.2. Hicksian aggregation problem

For this problem consider the following setting. Assume the consumption vector of some agent is divided in two bundles $(\mathbf{x}, \mathbf{z})$. Accordingly, the price vector is separated into $(\mathbf{p}, \mathbf{q})$. Thus, if the utility function of the consumer is u, then the demand for the $\mathbf{x}$ goods is

$$\begin{aligned} \mathbf{x}(\mathbf{p}, \mathbf{q}, y) = \arg\max_{\mathbf{x},\mathbf{z}} \quad & u(\mathbf{x}, \mathbf{z}) \\ \text{s.a.} \quad & \mathbf{px} + \mathbf{qz} = y. \end{aligned} \tag{3}$$

In numerous models, there is no interest in the consumption of each of the $\mathbf{x}$-goods but only in the demand for the group (i.e., focus on expediture in savings against expenditure in different financial instruments). Hence, the Hicksian aggregation problem is finding conditions under which this approximation can be made without losing information. This implies finding a quantity index $X = g(\mathbf{x})$, a price index $P = f(\mathbf{p})$ and a new utility function $U(X, \mathbf{z})$ such that the solution

$$\begin{aligned} X(P, \mathbf{q}, y) = \arg\max_{X,\mathbf{z}} \quad & U(X, \mathbf{z}) \\ \text{s.a.} \quad & PX + \mathbf{qz} = y. \end{aligned} \tag{4}$$

satisfies

$$X(f(\mathbf{p}), \mathbf{q}, y) = g(\mathbf{x}(\mathbf{p}, \mathbf{q}, y)).$$

At least two situations exist under which the three requirements can be found: functional and Hicksian separability. In the first, assume that the preference relation represented by u has the following "independence" property

$$(\mathbf{x}, \mathbf{z}) \succ (\mathbf{x}', \mathbf{z}) \Longleftrightarrow (\mathbf{x}, \mathbf{z}') \succ (\mathbf{x}', \mathbf{z}') \qquad \forall\ \mathbf{x}, \mathbf{x}', \mathbf{z}, \mathbf{z}'.$$

This independence property implies that there exists a function v such that

$$u(\mathbf{x}, \mathbf{z}) = U(v(\mathbf{x}), \mathbf{z}),$$

where $U(v, \mathbf{z})$ is increasing in v. By calling $m_{\mathbf{x}} := \mathbf{p} \cdot \mathbf{x}(\mathbf{p}, \mathbf{q}, y)$, then it can be shown that the following equality holds

$$\begin{aligned} \mathbf{x}(\mathbf{p}, \mathbf{q}, y) = \arg\max_{\mathbf{x}} \quad & v(\mathbf{x}) \\ \text{s.a.} \quad & \mathbf{p}\mathbf{x} = m_{\mathbf{x}}. \end{aligned}$$

Thus, if $e(\mathbf{p}, v)$ is the expenditure function of the previous problem, then

$$\begin{aligned} v(\mathbf{x}(\mathbf{p}, \mathbf{q}, y)) = \arg\max_{v, \mathbf{z}} \quad & U(v, \mathbf{z}) \\ \text{s.a.} \quad & e(\mathbf{p}, v) + \mathbf{q}\mathbf{z} = y. \end{aligned}$$

However, note that the latter problem does not have the exact same structure as (4). This only happens if v has a particular structure in order to write

$$e(\mathbf{p}, v) = e(\mathbf{p})v.$$

This property holds if, for example, v is homothetic. The Cobb-Douglas utility function is an example for which this kind of aggregation holds. One more time, since this functions are widely used in models (specially in production functions, where labor and capital are grouped goods), the use of this kind of aggregation has also become a norm in the economic work.

On the other hand, Hicksian separability is present in the following situation. Assume that $\mathbf{p} = t\mathbf{p}_0$, where t is a scalar and $\mathbf{p}_0$ is a fixed price vector. By defining $P := t$ and $X := \mathbf{p}_0\mathbf{x}$, we can define the following indirect utility function

$$V(P, \mathbf{q}, y) = \arg\max_{\mathbf{x},\mathbf{z}} \quad u(\mathbf{x}, \mathbf{z})$$
$$\text{s.a.} \quad P\mathbf{p}_0\mathbf{x} + \mathbf{q}\mathbf{z} = y.$$

By using V we can find another function U by solving

$$U(X, \mathbf{z}) = \arg\min_{P,\mathbf{q}} \quad V(P, \mathbf{q}, y)$$
$$\text{s.a.} \quad PX + \mathbf{q}\mathbf{z} = y,$$

that by definition satisfies

$$V(P, \mathbf{q}, y) = \arg\max_{X,\mathbf{z}} \quad U(X, \mathbf{z})$$
$$\text{s.a.} \quad PX + \mathbf{q}\mathbf{z} = y.$$

and thus aggregation is possible with $g(\mathbf{x}) = \mathbf{p}_0\mathbf{x}$ and $f(\mathbf{p}) = t$. This result was both presented by Leontief (1936) and Hicks (1946) and thus has been called the Hicks-Leontief theorem. As was the case with functional separability, note that the theorem presents sufficient but not necessary conditions to find the functions g, f and U. In his work, Lewbel (1996) relaxes the assumptions of the Hicks-Leontief theorem by not asking for constant relative prices but instead that these are independent of the price index and income. In terms of data analysis, the generalized composite commodity theorem of Lewbel does not ask a correlation of one between the price of the $\mathbf{x}$-goods and their price index.

One of the most common applications of good aggregation under Hicksian separability assumptions are two-good models. Here, the interest is in the demand for one good while bundling the other goods in only one category. These kind of models are frequently used in the macroeconomic literature where the study of aggregate consumption dynamics is greatly simplified by assuming agents just consume and save (and hence there are only two goods available). Observe that for this to happen and in accordance with the conditions in Lewbel (1996) it must be true that relative prices of all goods in the economy have to be independent of income and the price index. I want to explain why this could be a strong assumption but I can't find a convincing argument yet (I was thinking about connecting the assumption to substitutability issues of price indices but I'm not sure if that works.)

3. Aggregation across consumers

3.1. Description of the economy

Consider a two period economy ($t = 0, 1$) composed by a continuum of agents that consume two goods $\mathbf{x}_t = (x_{1t}, x_{2t})$ valued at prices $\mathbf{p}_t = (p_{1t}, p_{2t}) \in \mathbb{R}^2_{++}$. For simplicity, all subindices t will be omitted unless needed. Each individual in this setting is identified with a pair (y, α), where $y \in [tm, M]$ is her income and α determines the form of her Cobb-Douglas utility function, that is,

$$u_\alpha(\mathbf{x}) := u_{(y,\alpha)}(\mathbf{x}) = x_1^\alpha x_2^{1-\alpha}.$$

I will further assume that y and α are independent random variables with distributions F_y and F_α, respectively. The joint distribution of (y, α) will be called F, with support $S := [m, M] \times (0, 1)$. For every fixed vector $\mathbf{p}$, all agents choose the consumption bundle $\mathbf{x}(\mathbf{p}, y, \alpha)$ by maximizing u over $\mathbf{x}$ subject to their respective budget restriction. Specifically, the agent identified with the pair (y, α) solves

$$\begin{aligned} \max_{\mathbf{x}} \quad & u_\alpha(\mathbf{x}) \\ \text{s.a.} \quad & \mathbf{p} \cdot \mathbf{x} = y. \end{aligned}$$

Thus,

$$x_j(\mathbf{p}, y, \alpha) = \alpha \frac{y}{p_j}, \qquad j = 1, 2,$$

and thus the aggregate demand is

$$X_j(\mathbf{p}, F_y, F_\alpha) = \frac{1}{p_j} \int_S y\alpha \, dF(\alpha, y) = \frac{1}{p_j} \mathbb{E}[y]\mathbb{E}[\alpha], \qquad j = 1, 2.$$

3.2. The researcher's problem

Suppose now an investigator wishes to describe the aggregate demand of good 1 at time 1. She has access to a full description of the income of the agents in the economy in $t = 0$ (that is, she knows F_y) but preferences at any time are not available to her. She also knows X_{10}, the aggregate demand of good 1 at time 0. In this setting, if X_1 is the aggregate demand for cellphones then our researcher wishes to estimate consumption of mobiles at $t = 1$ by using data available at $t = 0$ on income and aggregate consumption of these devices.

Since individual preferences are unknown to her, she therefore assumes all individuals are equal, that is, there exists $\overline{\alpha}$ such that every agent in this economy has utility function $u_{\overline{\alpha}}$.

This assumption is equivalent as saying there only exists one person in the economy given that aggregation is possible when the parameters of the Cobb-Douglas utility function are the same for all individuals (see Section 2.2).[8] Under this assumption, the representative agent has utility function

$$U_{\overline{\alpha}}(X_1, X_2) = X_1^{\overline{\alpha}} X_2^{1-\overline{\alpha}},$$

and thus, since $E[y_t]$ is the aggregate income of this economy at time t, the best estimation for X_{1t} is

$$\widehat{X}_{1t}(p_{1t}, F_{y_t}, \overline{\alpha}) = \overline{\alpha}\, \frac{E[y_t]}{p_{1t}}.$$

3.3. The estimation error

As mentioned in the introduction, the aggregation assumption imposes a shadow price on the estimation. The error in this case will arise from the differences between the individual demands for x_1 and the ones estimated by the simplified model. Indeed, observe that

$$\begin{aligned}\left|\widehat{X}_1(p_1, F_y, \overline{\alpha}) - X_1(p_1, F_y, F_\alpha)\right| &= \left|\int_S \frac{y\alpha}{p_1} - \frac{y\overline{\alpha}}{p_1}\, dF(y,\alpha)\right| \\ &= \left|\int_S x_1(p_1, y, \alpha) - x_1(p_1, y, \overline{\alpha})\, dF(y,\alpha)\right|,\end{aligned}$$

and thus the accuracy in the estimation, conditional on having the correct distribution of income, will depend only on the choice of $\overline{\alpha}$. Moreover, note that the monetized difference in the estimation is

$$p_1\left|\widehat{X}_1(p_1, F_y, \overline{\alpha}) - X_1(p_1, F_y, F_\alpha)\right| = \mathbb{E}[y]\left|\overline{\alpha} - \mathbb{E}[\alpha]\right|$$

4. Aggregation across goods

5. The mixed setting

6. Concluding remarks

[8] In more subtle interpretation of the mentioned assumption, what the researcher is doing is approximate F_α by $\delta_{\overline{\alpha}}$ where δ_x is the Dirac distribution centered at x.

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