ADSP: HW1

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(2)

(a)

Originally, we have the convolution:

$$y[n] = x[n] * h[n]$$

By uising FT, we can transform this equation into:

$$Y(f) = X(f)H(f)$$

then taking log on both sides, we get:

$$\log Y(f) = \log X(f) + \log H(f)$$

(b)

1. We can use FT to do spectral analysis.

2. We can use FT to convert convolution into multiplication.

(c)

1. It is not a real operation, and we need to deal with irrational numbers, so the computation complexity is high.

2. If the sampling frequency is too low (i.e. $f_s < 2B$), then aliasing will occur. (And if we try to prevent this to happen by increasing f_s, then more computation is needed.)

(3)

We have the given equations:

$$
\begin{aligned}
&y[n] = x[n] * h[n] \\
&h[n] = (0.7^n + (-0.6)^{n+1})u[n] \quad \text{ where } u[n]: \text{ unit step function}
\end{aligned}
$$

We can first find $H(z)$ used in the z-transform:

$$
\begin{aligned}
H(z) &= \sum_{n=-\infty}^{\infty} h[n]z^{-n} \\
&= \sum_{n=-\infty}^{\infty} (0.7^n + (-0.6)^{n+1})u[n]z^{-n} \\
&= \sum_{n=0}^{\infty}(0.7^n + (-0.6)^{n+1})z^{-n} \\
&= \sum_{n=0}^{\infty} 0.7^n z^{-n} + \sum_{n=0}^{\infty}(-0.6)^{n+1}z^{-n} \\
&= \sum_{n=0}^{\infty}(0.7z^{-1})^n + (-0.6)\sum_{n=0}^{\infty}(-0.6z^{-1})^n \\
&= \frac{1}{1-0.7z^{-1}} + \frac{-0.6}{1-(-0.6z^{-1})} \\
&= \frac{1}{1-0.7z^{-1}} - \frac{0.6}{1+0.6z^{-1}} \\
&= \frac{(1+0.6z^{-1}) - (0.6-0.42z^{-1})}{(1-0.7z^{-1})(1+0.6z^{-1})} \\
&= \frac{0.4+1.02z^{-1}}{(1-0.7z^{-1})(1+0.6z^{-1})}
\end{aligned}
$$

Since $Y(z) = H(z)X(z)$, by plugging in this result, and using the inverse z-transform, we get:

$$
\begin{aligned}
&Y(z) = \frac{0.4+1.02z^{-1}}{(1-0.7z^{-1})(1+0.6z^{-1})}X(z) \\
\Rightarrow\ &(1-0.7z^{-1})(1+0.6z^{-1})Y(z) = (0.4+1.02z^{-1})X(z) \\
\Rightarrow\ &(1-0.1z^{-1}-0.42z^{-2})Y(z) = (0.4+1.02z^{-1})X(z) \\
\Rightarrow\ &Y(z) = (0.1z^{-1})Y(z) + (0.42z^{-2})Y(z) + 0.4X(z) + 1.02z^{-1}X(z) \\
\Rightarrow\ &y[n] = 0.1y[n-1] + 0.42y[n-2] + 0.4x[n] + 1.02x[n-1]
\end{aligned}
$$

(4)

(a)

We can use the step invariance method to reduce the aliasing effect because we take the integral of $h_a(t)$. From the equation:

$$H_{a,u}(f) = \frac{H_a(f)}{j2\pi f}$$

we can see that when f is large, the denominator is large, so the high-frequency part is decreased.

(b)

The bilinear transform can fully avoid the aliasing effect because by converting the original frequency to the new frequency through the equation:

$$f_{new} = \frac{f_s}{\pi}\mathbf{atan}(\frac{2\pi}{c}f_{old})$$

We can map the frequency range from $(-\infty, \infty)$ to $(-\frac{f_s}{2}, \frac{f_s}{2})$.

(5)

First, since we're designing a 7-point FIR filter, $N = 7$ and we have

$$k = \frac{N-1}{2} = 3$$

By the formulas in p.51, we have:

$$\begin{aligned} s[0] &= \int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)dF \\ s[n] &= 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi nF)H_d(F)dF \quad \text{for } n = 1, 2, 3 \end{aligned}$$

Plugging in the given facts that:

$$H_d(F) = \begin{cases} 1, & |F| < 0.25 \\ 0, & 0.25 < |F| < 0.5 \end{cases}$$

we can get:

$$s[0] = \int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)dF = 0.5$$

$$s[1] = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi F)H_d(F)dF = 2\int_{-0.25}^{0.25} \cos(2\pi F)dF = 2\frac{\sin(2\pi F)}{2\pi}\Big|_{-0.25}^{0.25} = \frac{(1-(-1))}{\pi} = \frac{2}{\pi}$$

$$s[2] = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(4\pi F)H_d(F)dF = 2\int_{-0.25}^{0.25} \cos(4\pi F)dF = 2\frac{\sin(4\pi F)}{4\pi}\Big|_{-0.25}^{0.25} = \frac{(0-0)}{2\pi} = 0$$

$$s[3] = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(6\pi F)H_d(F)dF = 2\int_{-0.25}^{0.25} \cos(6\pi F)dF = 2\frac{\sin(6\pi F)}{6\pi}\Big|_{-0.25}^{0.25} = \frac{((-1)-1)}{3\pi} = \frac{-2}{3\pi}$$

Therefore, we have:

$$\begin{aligned} h[3] &= s[0] = 0.5 \\ h[2] &= h[4] = \frac{s[1]}{2} = \frac{1}{\pi} \\ h[1] &= h[5] = \frac{s[2]}{2} = 0 \\ h[0] &= h[6] = \frac{s[3]}{2} = \frac{-1}{3\pi} \end{aligned}$$

(6)

(a)

1. We can decrease the error without adding N by enlarging the transition band width ΔF.

2. If we do not ignore the error in transition band, the error will never be smaller than 0.5.

(b)

We're given $N = 21,\ \Delta_t = 0.0002$ and transition band $1950 \sim 2050$ Hz, so the transition band width is:

$$\Delta F = \frac{f_1 - f_2}{f_s} = \frac{f_1 - f_2}{\frac{1}{\Delta_t}} = (2050 - 1950) \times 0.0002 = 0.02$$

By the formula on p.82 and plug in the given values, we have:

$$\delta_1\delta_2 = 10^{-\frac{3}{2}N\Delta F - 1} = 10^{-\frac{3}{2}\times 21\times 0.02 - 1} = 10^{-0.63-1} = 10^{-1.63}$$

Since we assume that pass and stop band ripples are equal, we have:

$$\delta_1 = \delta_2 = \sqrt{\delta_1 \delta_2} = \sqrt{10^{-1.63}} \approx 0.153$$

Thus, the pass and stop band ripples ≤ 0.153.

(c)

Using the same approach in (b), we have:

$N = 31$, $\Delta_t = 0.0001$ and transition band $1750 \sim 2250$ Hz, so the transition band width is:

$$\Delta F = (2250 - 1750) \times 0.0001 = 0.05$$

then:

$$\delta_1 \delta_2 = 10^{-\frac{3}{2} \times 31 \times 0.05 - 1} = 10^{-2.325 - 1} = 10^{-3.325}$$

$$\delta_1 = \delta_2 = \sqrt{\delta_1 \delta_2} = \sqrt{10^{-3.325}} \approx 0.0218$$

Thus, the pass and stop band ripples ≤ 0.0218.

Extra

Given $f_s = 40000, N = 600000$, if $m = 6000$, then $f = ?(Hz)$

$$f = m \frac{f_s}{N} = 6000 \frac{40000}{600000} = 400$$