Digital Systems Design and Laboratory
[4. Applications of Boolean Algebra]

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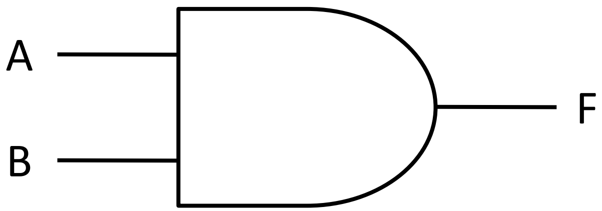
Outline

- **Conversion of English Sentences to Boolean Equations**
- Combinational Logic Design Using a Truth Table
- Minterm and Maxterm Expansions
- General Minterm and Maxterm Expansions
- Incompletely Specified Functions
- Examples of Truth Table Construction
- Design of Binary Adders and Subtracters

Objectives

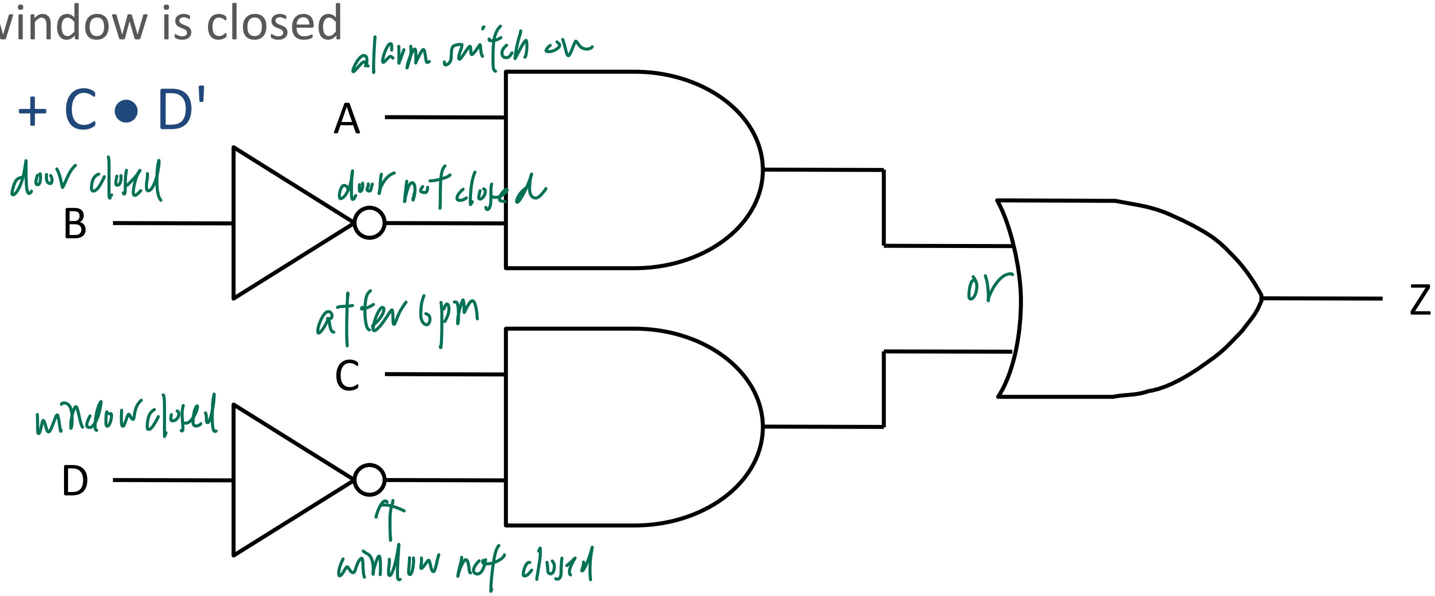
- Design a combinational logic circuit starting with a **word description (specification)** of the desired circuit behavior
- Steps
  - Translate the word description into a switching function
    - Boolean expression or truth table
  - Simplify the function
  - Realize it using available logic gates
- Example

  word description ex:
  - Mary watches TV **if and only if** it is Monday night **and** she has finished her homework
    - F: Mary watches TV
    - A: It is Monday night
    - B: Mary has finished her homework
  - $F = A \bullet B$



Another Example

- The alarm will ring **if and only if** the alarm switch is turned on **and** the door is **not** closed, **or** it is after 6pm **and** the window is **not** closed
  - Z: The alarm will ring
  - A: the alarm switch is on
  - B: The door is closed
  - C: It is after 6pm
  - D: The window is closed
- $Z = A \bullet B' + C \bullet D'$



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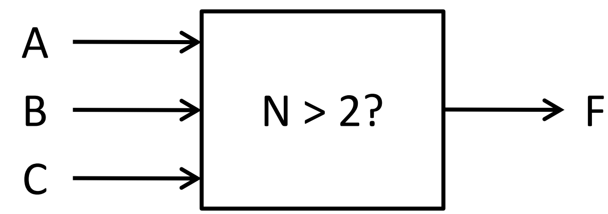
Threshold Detector (1/2)

- Design a detector that outputs 1 when input is greater than 2
  - Inputs $(A, B, C)_2$ represent a binary number N
  - If $N = (A, B, C)_2 \geq 3$, output F = 1; otherwise F = 0

A	B	C	F	F'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Show the condition to make output = 1



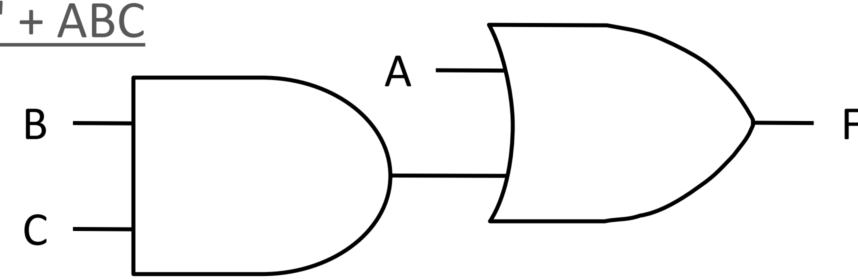
  - $F = A'BC + AB'C' + AB'C + ABC' + ABC$ (SOP)

    $= \underline{A'BC + ABC} + \underline{AB'C' + AB'C + ABC' + ABC}$

    $= A + BC$

如果 A=1 ⇒ 到有中

如果 A=0 ⇒ 唯一可能值 > 2 即 3 ⇒ 代表 B=1 且 C=1



$$F = A'BC + AB'C' + AB'C + ABC' + ABC$$

$$= \underline{A'BC + ABC} + \underline{AB'C' + AB'C + ABC' + ABC}$$

$$= (A'+A)BC + AB'(C'+C) + AB(C'+C)$$

$$= BC + (AB'+AB)(C'+C)$$

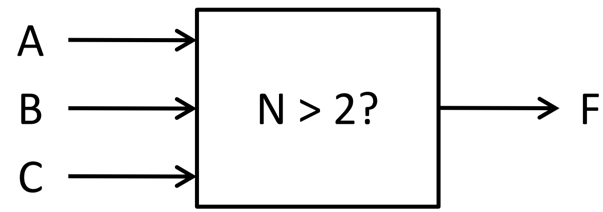
$$= BC + (AB'+AB)$$

$$= BC + A(B'+B)$$

$$= BC + A$$

Threshold Detector (2/2)

- By counting 1's, we have SOP
- What if counting 0's



A	B	C	F	F'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Show the condition to make output = 0

- $F = (ABC \neq 000) \bullet (ABC \neq 001) \bullet (ABC \neq 010)$
  $= (\overset{0}{A}'\overset{0}{B}'\overset{0}{C}')' \bullet (\overset{0}{A}'\overset{0}{B}'\overset{1}{C})' \bullet (\overset{0}{A}'\overset{1}{B}\overset{0}{C}')' = (A + B + C) \bullet (A + B + C') \bullet (A + B' + C)$
- $F = F'' = (A'B'C' + A'B'C + A'BC')'$ ← 0,1,2的情況 or 起來，取'
  $= (A'B'C')' \bullet (A'B'C)' \bullet (A'BC')' = (A + B + C) \bullet (A + B + C') \bullet (A + B' + C)$

$$\overset{000}{}\qquad\quad \overset{001}{}\qquad\quad \overset{010}{}$$

$$F = \underline{(A'B'C')'} \cdot (A'B'C)' \cdot (A'BC')$$

$$(X_1X_2\cdots X_n)' = (X_1' + X_2' + \cdots + X_n')$$

$$= \underline{(A+B+C)} \cdot (A+B+C') \cdot (A+B'+C)$$

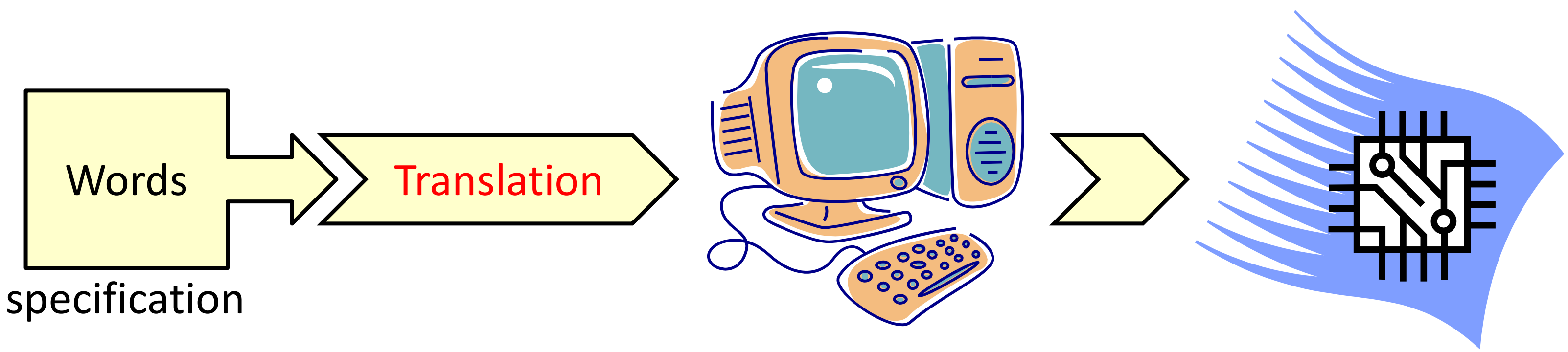
← POS

(product of sums)

Logic Design Using a Truth Table

❑ Steps

- ➢ Make a truth table according to the word description
- ➢ Generate a Boolean expression
  - Sum-of-products (SOP): check 1's 確認 output =1 的情況
  - Product-of-sums (POS): check 0's 〃 〃 =0 〃
    - – Have F' in SOP and then derive F in POS
- ➢ Simplify the Boolean expression



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n variables 要有 n literals，且每個 variable 剛好出現一次

Minterm and Maxterm

一個 variable 或是它的 complement

product　　sum

- Definition: A **minterm/maxterm** of n variables is a product/sum of n $\boxed{\text{literals}}$ in which each variable appears exactly once in either true or complement form (but not both)
  - A literal is a variable or its complement (A or A')
  - Examples of 3 variables
    - Minterm: A'BC, AB'C' ← A 或 A' 一定要剛好出現一個 B B' C C'
    - Maxterm: A + B + C, A + B + C'

3 variables ⇒ 所有可能：$2^3 = 8$ minterms maxterms

m_i 和 M_i 是 NOT 的關係

$(m_i)' = M_i$

Row No.	ABC	Minterm m_i ⟷	Maxterm M_i
0	000	m_0 = A'B'C'	M_0 = A + B + C
1	001	m_1 = A'B'C	M_1 = A + B + C'
2	010	m_2 = A'BC'	M_2 = A + B' + C
3	011	m_3 = A'BC	M_3 = A + B' + C'
4	100	m_4 = AB'C'	M_4 = A' + B + C
5	101	m_5 = AB'C	M_5 = A' + B + C'
6	110	m_6 = ABC'	M_6 = A' + B' + C
7	111	m_7 = ABC	M_7 = A' + B' + C'

Minterm Expansion

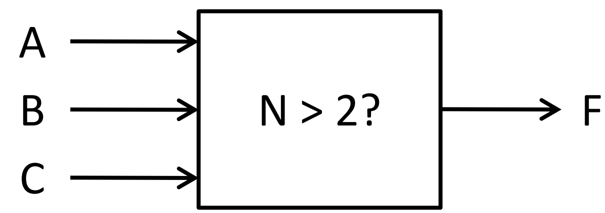
- A **minterm expansion** or a **standard sum of products** is a function written as a sum of minterms
  - Counting 1's

=

minterm expansion (考慮 n 個 variables 的情況下)
每個 product 都有 n 個 variables

- Example
  - F = A'BC + AB'C' + AB'C + ABC' + ABC
    $= m_3 + m_4 + m_5 + m_6 + m_7$ (m-notation)
    $= \sum m(3, 4, 5, 6, 7)$ 表示法.

小m
→ minterm



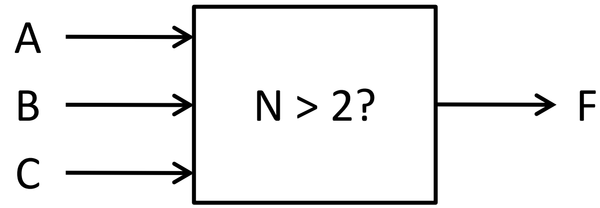
Row No.	ABC	Minterm m_i	Maxterm M_i
0	000	m_0 = A'B'C'	M_0 = A + B + C
1	001	m_1 = A'B'C	M_1 = A + B + C'
2	010	m_2 = A'BC'	M_2 = A + B' + C
3	011	m_3 = A'BC	M_3 = A + B' + C'
4	100	m_4 = AB'C'	M_4 = A' + B + C
5	101	m_5 = AB'C	M_5 = A' + B + C'
6	110	m_6 = ABC'	M_6 = A' + B' + C
7	111	m_7 = ABC	M_7 = A' + B' + C'

Maxterm Expansion

- A **maxterm expansion** or a **standard product of sums** is a function written as a product of maxterms
  - Counting 0's
- Example

每個variable都出現一次

  - $F = \boxed{(A + B + C)} \bullet \boxed{(A + B + C')} \bullet \boxed{(A + B' + C)}$
    $= M_0M_1M_2$ (M-notation)
    $= \prod M(0, 1, 2)$



發0的
情況
↓
前面例子:
≤2時output 0

Row No.	ABC	Minterm m_i	Maxterm M_i
0	000	$m_0 = A'B'C'$	$M_0 = A + B + C$
1	001	$m_1 = A'B'C$	$M_1 = A + B + C'$
2	010	$m_2 = A'BC'$	$M_2 = A + B' + C$
3	011	$m_3 = A'BC$	$M_3 = A + B' + C'$
4	100	$m_4 = AB'C'$	$M_4 = A' + B + C$
5	101	$m_5 = AB'C$	$M_5 = A' + B + C'$
6	110	$m_6 = ABC'$	$M_6 = A' + B' + C$
7	111	$m_7 = ABC$	$M_7 = A' + B' + C'$

Complement by Minterms/Maxterms

- $(m_i)' = M_i$
- Complement of F

$F = \sum m(3, 4, 5, 6, 7)$

$= m_3 + m_4 + m_5 + m_6 + m_7$

  - Counting 0's in F (find F' directly)
    - $F' = m_0 + m_1 + m_2 = \sum m(0, 1, 2)$
    - $F' = M_3M_4M_5M_6M_7 = \prod M(3, 4, 5, 6, 7)$

$F = \prod M(0, 1, 2)$

$= M_0M_1M_2$

  - Counting 1's in F (find F and then complement it)
    - $F' = (m_3 + m_4 + m_5 + m_6 + m_7)'$
      $= m_3'm_4'm_5'm_6'm_7'$ de Morgan
      $= M_3M_4M_5M_6M_7$
      $= \prod M(3, 4, 5, 6, 7)$
    - $F' = (M_0M_1M_2)'$
      $= M_0' + M_1' + M_2'$
      $= m_0 + m_1 + m_2$
      $= \sum m(0, 1, 2)$

A	B	C	F	F'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Another Example

- Example: $F(A, B, C, D) = A'(B' + D) + ACD'$ → 拆 minterm expansion ⇒ sum of products (乘開)

$$F(A, B, C, D)$$
$$= A'(B' + D) + ACD'$$
$$= A'B' + A'D + ACD'$$
$$= A'B'\underbrace{(C + C')}_{}\underbrace{(D + D')}_{} + A'D(B + B')(C + C') + ACD'(B + B')$$

$C + C' = 1$, $D + D' = 1$

$$= A'B'C'D' + A'B'C'D + A'B'CD' + A'B'CD + A'BC'D + A'BCD + ABCD' + AB'CD'$$

0000	0001	0010	0011	0101	0111	1110	1010
0	1	2	3	5	7	14	10

$$= \sum m(0, 1, 2, 3, 5, 7, 10, 14)$$ ... Minterm Expansion

$F(A, B, C, D)$ $\boxed{A'}(B'+D) + \boxed{A}CD'$

$(X+Y)(X+Z) = X+YZ$ (distributive)

$$= (A' + CD')(A + B' + D) = (A' + C)(A' + D')(A + B' + D)$$
$$= (A' + BB' + C + DD')(A' + BB' + CC' + D')(A + B' + CC' + D)$$

$BB' = 0$, $BB' + CC' = 0$

$$= \ldots$$
$$= \prod M(4, 6, 8, 9, 11, 12, 13, 15)$$ ... Maxterm Expansion

$(A' + \underbrace{B}_{Y}\underbrace{B'}_{Z} + C + DD') = (A' + B + C + DD')(A' + B' + C + DD')$

$(X+YZ) = (X+Y)(X+Z)$

$$= [(A'+B+C+D)(A'+B+C+D')][(A'+B'+C+D)(A'+B'+C+D')]$$

Summary

maxterm (product of sums)

- Convert a Boolean expression to a minterm/maxterm expansion
  - Use truth table
    - Sometimes there are too many terms

minterm ⇒ 有1的狀況 ⇒ OR起來
maxterm ⇒ 有0的狀況 ⇒ AND起來

- Use Boolean algebra
  - SOP: multiply out and use (X + X') = 1 → minterm expansion (SOP)
  - POS: factor and use XX' = 0 → maxterm expansion (POS)

缺項 ⇒ 補 redundant

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General Truth Table

- Given n Boolean variables, how many different Boolean functions can you produce?
  - Each a_i can be assigned with either 0 or 1
  - Boolean function 共 $2^{(2^n)}$ 種 每個 case 可有 0/1

n個 variables

⇒ 2^n 種 cases

A	B	C	F
0	0	0	a_0
0	0	1	a_1
0	1	0	a_2
0	1	1	a_3
1	0	0	a_4
1	0	1	a_5
1	1	0	a_6
1	1	1	a_7

每個 a_i $i=0\sim7$ 都可有 0/1 兩種值

AND of Minterm Expansions

- Given $F_1 = \sum m(0, 2, 3, 5, 9, 11)$ and $F_2 = \sum m(0, 3, 9, 11, 13, 14)$, find $F_1F_2 = ?$ 要AND 即两边都要是1
  - AND: take the numbers that appear in both expansions
  - $F_1F_2 = \sum m(0, 3, 9, 11)$
- AND for two maxterm expansions? 留只要其中一边出现的数字.
- OR for two minterm expansions? 只要出现一次就留下来
- OR for two maxterm expansions? 都出现才留下来.

$$F_1 = \underline{m_0} + m_2 + \underline{m_3} + m_5 + \underline{m_9 + m_{11}}$$

$$F_2 = \underline{m_0 + m_3 + m_9 + m_{11}} + m_{13} + m_{14}$$

$$F_1 \cdot F_2 = (m_0 + m_3 + m_9 + m_{11} + m_2 + m_5)(m_0 + m_3 + m_9 + m_{11} + m_{13} + m_{14})$$

$$= m_0 + m_3 + m_9 + m_{11} + (m_2 + m_5)(m_{13} + m_{14})$$

Conversion of Forms

- ❑ Convert between a minterm and a maxterm expansion

Given Form \ Desired Form	Minterm Expansion of F	Maxterm Expansion of F	Minterm Expansion of F'	Maxterm Expansion of F'
Minterm Expansion of F	---	The nos. not on the minterm list for F	The minterms not present in F	The same as minterm nos. of F
Maxterm Expansion of F	The nos. not on the maxterm list for F	---	The same as maxterm nos. of F	The maxterms not present in F

Given Form \ Desired Form	Minterm Expansion of F	Maxterm Expansion of F	Minterm Expansion of F'	Maxterm Expansion of F'
$F = \sum m(3, 4, 5, 6, 7)$	---	$\prod M(0, 1, 2)$	$\sum m(0, 1, 2)$	$\prod M(3, 4, 5, 6, 7)$
$F = \prod M(0, 1, 2)$	$\sum m(3, 4, 5, 6, 7)$	---	$\sum m(0, 1, 2)$	$\prod M(3, 4, 5, 6, 7)$

沒出現在F的數字

max 換 min → 翻一次
F 換 F' → 翻一次
⇒ 翻2次 = 原數字

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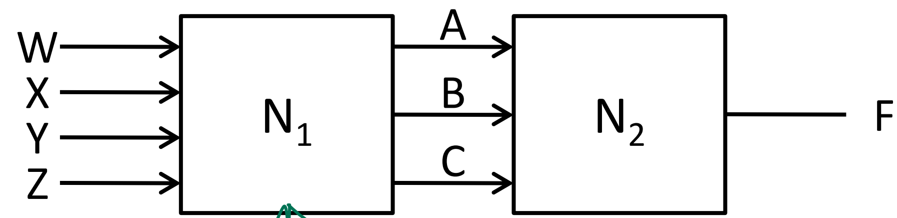
Incompletely Specified Functions (1/2)

- A large digital system is usually divided into subcircuits
- Assume N_1 never generates ABC = 001/110 for any W, X, Y, Z

A	B	C	F
0	0	0	1
0	0	1	X
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	X
1	1	1	1



假設 N1 output
不會產生 001/110

∵ 001/110
不會出現
∴ 001/110 的 input 給 N2 的 output → x
(don't care)

- F: Incompletely specified function
- A'B'C, ABC': don't care terms
  - "don't care" (DC) terms can be assigned with either 0 or 1

↓
可為 0 也可為 1

Incompletely Specified Functions (2/2)

❑ Impact of don't care terms on Boolean simplification

- ➢ Try exhaustive combinations of DCs to find the best
  - (may be stupid but works for now)

i 001/110 的 output 是 0/1 皆可

⇒ 試所有的情況

- ➢ Assign 0 to both "X"
  - F = A'B'C' + A'BC + ABC = A'B'C' + BC
- ➢ Assign 1 to 1st "X" and 0 to 2nd "X" (seems to be the simplest)
  - F = A'B'C' + <u>A'B'C</u> + A'BC + ABC = A'B' + BC
- ➢ Assign 1 to both "X"
  - F = A'B'C' + <u>A'B'C</u> + A'BC + <u>ABC'</u> + ABC = A'B' + BC + AB
- ➢ ...
- ➢ 2. is the simplest solution

A	B	C	F
0	0	0	1
0	0	1	X
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	X
1	1	1	1

❑ Notation

minterm ⇒ OR

- ➢ F = ∑ m(0, 3, 7) + ∑ d(1, 6)
- ➢ F = ∏ M(2, 4, 5) • ∏ D(1, 6)

maxterm ⇒ AND

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Error Detector for 6-3-1-1 Codes (1/2)

invalid時output F=1

- Design an error detector for 6-3-1-1 codes:
  - The output F = 1 iff inputs (A, B, C, D) represent an **invalid** code combination
  - Step 1: construct the truth table

Decimal Digit	6-3-1-1 Code
0	0000
1	0001
2	0011
3	0100
4	0101
5	0111
6	1000
7	1001
8	1011
9	1100

	ABCD (6311)		F	
0	0000	m_0	0	
1	0001	m_1	0	
1	0010	m_2	1	$A'B'CD'$
2	0011	m_3	0	
3	0100	m_4	0	
4	0101	m_5	0	
4	0110	m_6	1	$A'BCD'$
5	0111	m_7	0	
6	1000	m_8	0	
7	1001	m_9	0	
7	1010	m_{10}	1	$AB'CD'$
8	1011	m_{11}	0	
9	1100	m_{12}	0	
10	1101	m_{13}	1	$ABC'D$
10	1110	m_{14}	1	$ABCD'$
11	1111	m_{15}	1	$ABCD$

m_2 or m_6 or m_{10} or m_{13} or m_{14} or m_{15} 时为 valid

$$\therefore\ F = m_2 + m_6 + m_{10} + m_{13} + m_{14} + m_{15}$$

$$= \sum m(2, 6, 10, 13, 14, 15)$$

$$= \underline{A'B'CD' + A'BCD'} + AB'CD' + ABC'D + ABCD' + ABCD \Rightarrow \text{simplify}$$

$$= A'CD'(B'+B) + ACD'(B'+B) + ABD(C+C')$$

$XY + XY' = X$ (uniting)

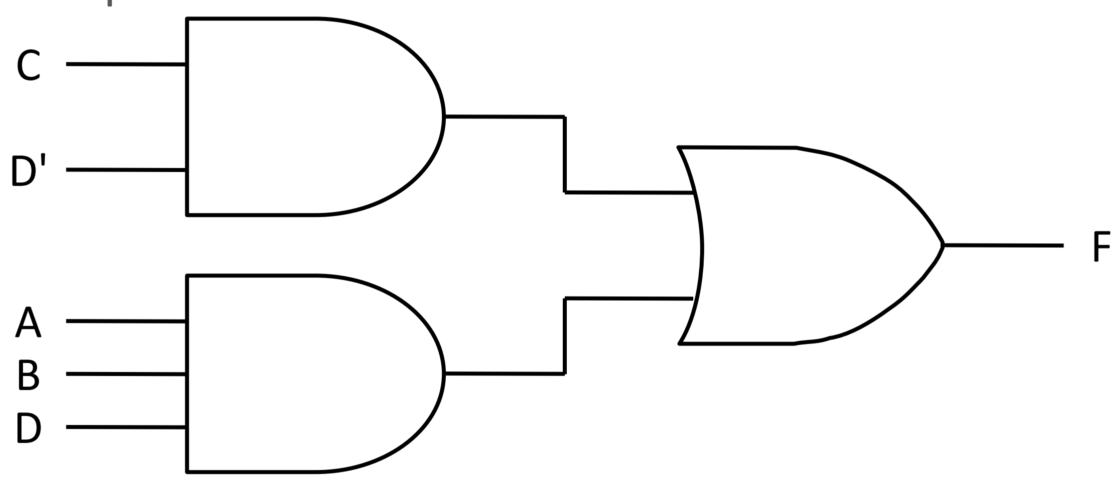
$$= \underline{A'CD' + ACD'} + ABD$$

$$= \underline{CD'} + ABD$$

Error Detector for 6-3-1-1 Codes (2/2)

- Design an error detector for 6-3-1-1 codes:
  - The output F = 1 iff inputs (A, B, C, D) represent an **invalid** code combination
  - Step 2: simplify the function

    F(A, B, C, D)
    = ∑ m(2, 6, 10, 13, 14, 15)
    = A'B'CD' + A'BCD' + AB'CD' + ABCD' + ABC'D + ABCD
    = A'CD' + ACD' + ABD
    = CD' + ABD
  - Step 3: realize it



ABCD	F
0000	0
0001	0
0010	1
0011	0
0100	0
0101	0
0110	1
0111	0
1000	0
1001	0
1010	1
1011	0
1100	0
1101	1
1110	1
1111	1

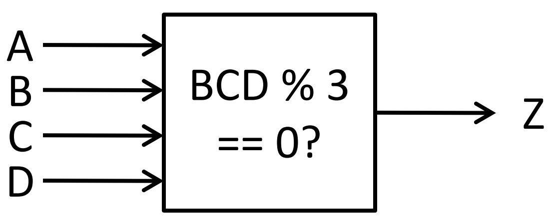
Another Example

- The output Z = 1 iff the 8-4-2-1 BCD number (A, B, C, D) is divisible by 3
  - Step 1: construct the truth table
  - Step 2: simplify the function

    Z(A, B, C, D)

    $= \sum m(0, 3, 6, 9) +$

    $\sum d(10, 11, 12, 13, 14, 15)$
  - Step 3: realize it
    - ?
    - Unit 5!



Decimal Digit	8-4-2-1 Code
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

	ABCD	Z
	0000	1
	0001	0
	0010	0
	0011	1
	0100	0
	0101	0
	0110	1
	0111	0
	1000	0
	1001	1
10	1010	X
11	1011	X
12	1100	X
13	1101	X
14	1110	X
15	1111	X

Outline

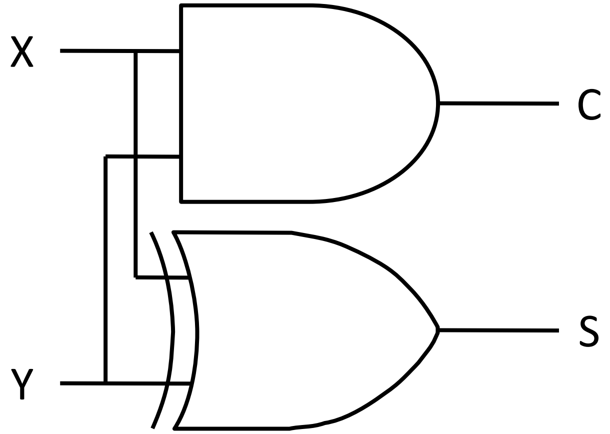
- ❑ Conversion of English Sentences to Boolean Equations
- ❑ Combinational Logic Design Using a Truth Table
- ❑ Minterm and Maxterm Expansions
- ❑ General Minterm and Maxterm Expansions
- ❑ Incompletely Specified Functions
- ❑ Examples of Truth Table Construction
- ❑ **Design of Binary Adders and Subtracters**

1-Bit Half Adder (HA)

- Step 1: construct the truth table

X	Y	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

- Step 2: simplify the function
  - C = XY
  - S = X'Y + XY' = X $\oplus$ Y
- Step 3: realize it



1-Bit Full Adder (FA)

兩個1時進位

三個1時進位,

- ❑ Step 1: construct the truth table
- ❑ Step 2: simplify the function
  - ➢ $C_{out} = X'YC_{in} + XY'C_{in} + XYC_{in}' + XYC_{in}$
    $= XY + XC_{in} + YC_{in}$
  - ➢ $S = \boxed{X'Y'C_{in} + X'YC_{in}' + XY'C_{in}' + XYC_{in}}$

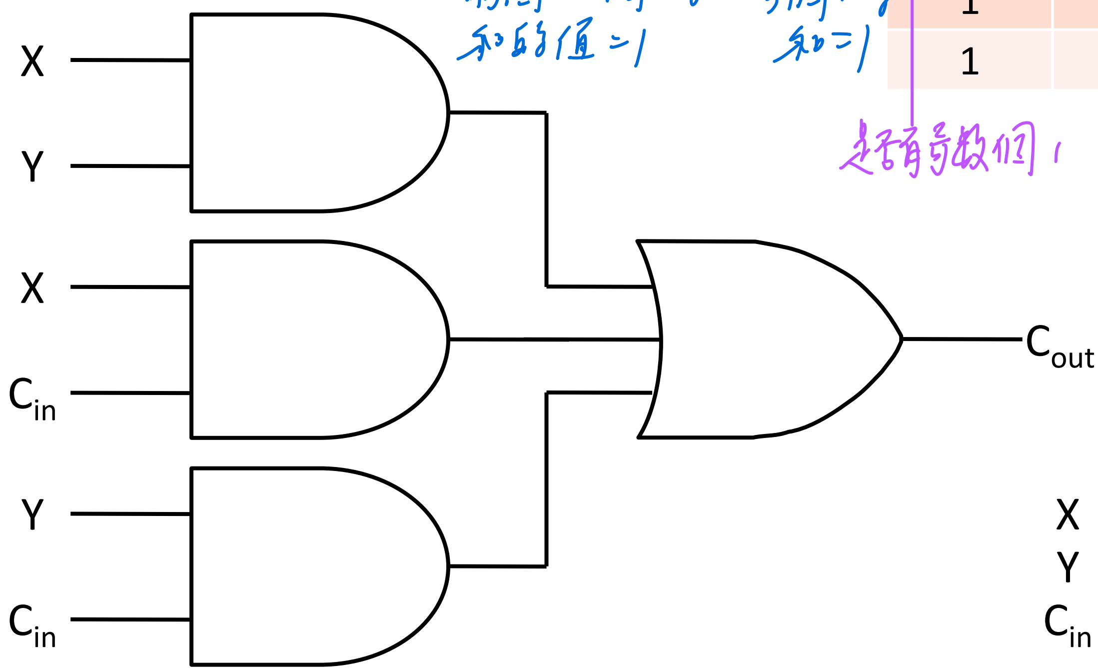
兩個0一個1時 和的值=1

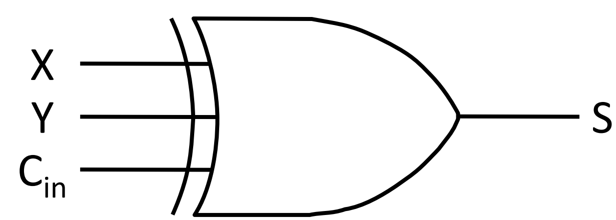
3個1時 和=1

是否有奇數個1

- ❑ Step 3: realize it

X	Y	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1





$$C_{out} = \underline{X'YC_{in}} + \underline{XY'C_{in} + XYC_{in}'} + \underline{XYC_{in}}$$

$$= \underline{YC_{in}} + X(\underline{Y'C_{in} + YC_{in}'})$$

$$= YC_{in} + X$$

$$YC_{in} + XY'C_{in} + XYC_{in}'$$

$$= Y(C_{in} + XC_{in}')$$

$$= Y(C_{in} + X)$$

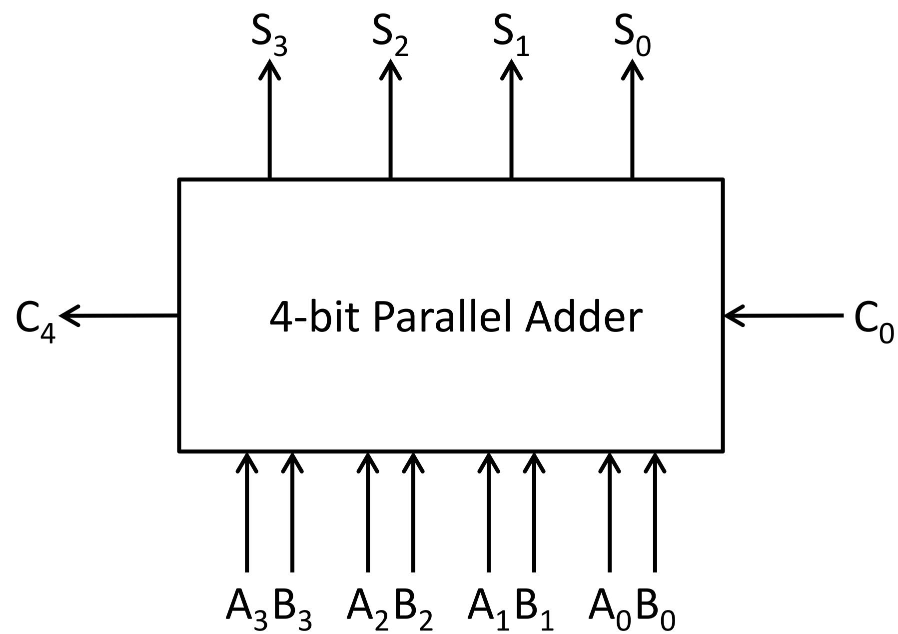
$$= YC_{in} + XY$$

4-Bit Parallel Adder (1/3)

- $A = (A_3A_2A_1A_0)$, $B = (B_3B_2B_1B_0)$

$$
\begin{array}{lcccccccc}
 & & & A_3 & A_2 & A_1 & A_0 & & \\
 & + & & B_3 & B_2 & B_1 & B_0 & & \\
\hline
C_4 & \leftarrow & & S_3 & S_2 & S_1 & S_0 & \leftarrow & C_0
\end{array}
$$

carry-out carry-in



- Example

$$
\begin{array}{lcccccccc}
 & & 1 & 0 & 1 & 1 & 0 & \leftarrow & \text{carries} \\
 & & & 1 & 0 & 1 & 1 & & \\
 & + & & 1 & 0 & 1 & 1 & & \\
\hline
1 & \leftarrow & 1 & 0 & 1 & 1 & 0 & \leftarrow & 0
\end{array}
$$

- How?
  - Step 1: construct the truth table
  - ...

4-Bit Parallel Adder (2/3)

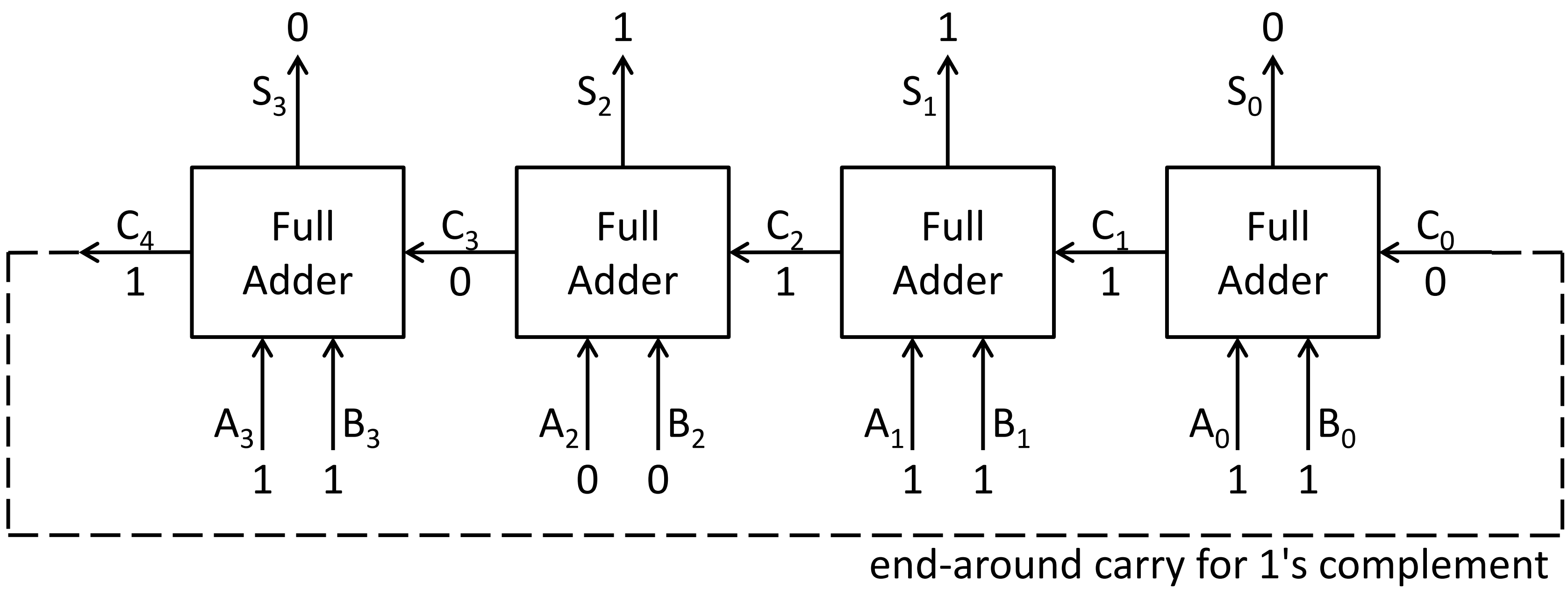
把4 bit adder 分成4個 1 bit 的 subcomponent 即 full adder

- Decompose the 4 bit adder into four modules
  - Each module adds two bits and a carry → use full adder
- Extend to negative numbers
  - Consider 1's complement
    - Add just as if all numbers are positive
    - Add the carry out back to the rightmost bit
  - How to detect overflow?
    - Check the sign
      - (+) + (+) becomes (-)
      - (-) + (-) becomes (+)

Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
+3 0011	+5 0101	+5 0101	−5 1010	−3 1100	−5 1010
+4 0100	+6 0110	−6 1001	+6 0110	−4 1011	−6 1001
+7 0111	1011	−1 1110	+1 (1) 0000	−7 (1) 0111	(1) 0011
			1	1	1
			0001	1000	0100

正加正 變負的 ⇒ overflow

4-Bit Parallel Adder (3/3)



- Overflow detection?
  - $V = \overset{0}{A_3}'\overset{0}{B_3}'\overset{1}{S_3} + \overset{1}{A_3}\overset{1}{B_3}\overset{0}{S_3}'$ 3 → 檢查最前面的bit
  - Why?

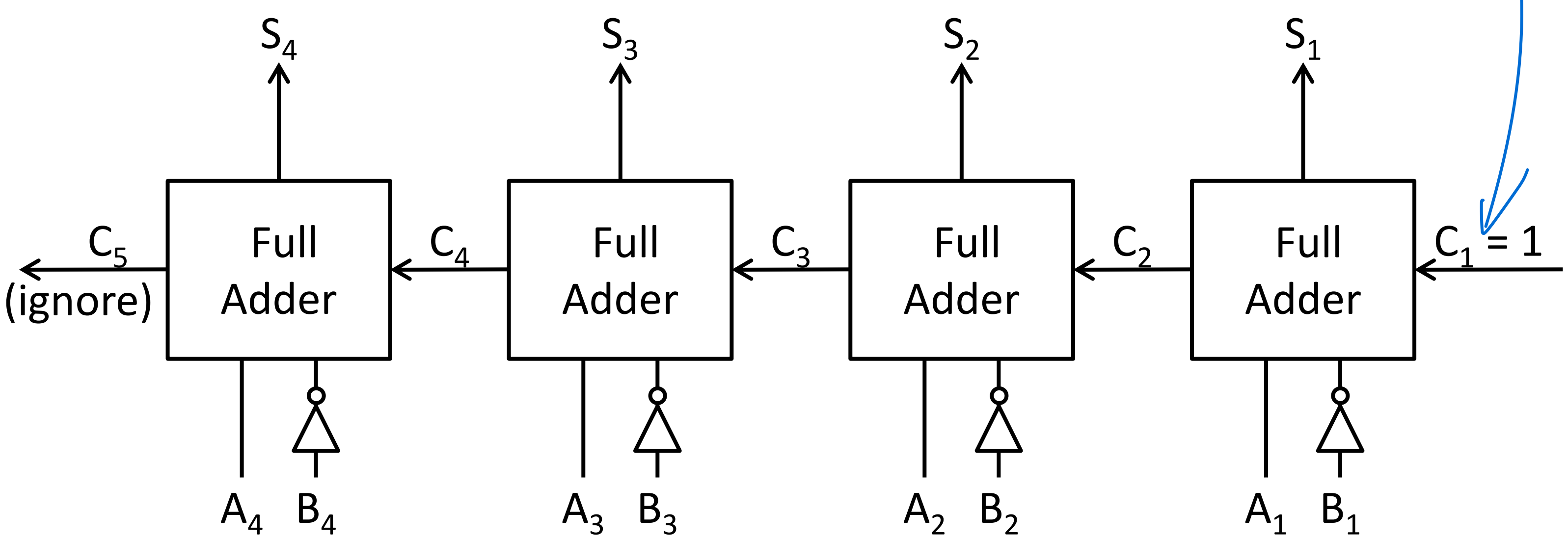
$A_3'B_3' \rightarrow$ 正+正
$S_3 \rightarrow$ 負

$A_3B_3 \rightarrow$ 負+負
$S_3' \rightarrow$ 正

Binary Subtracter (1/2)

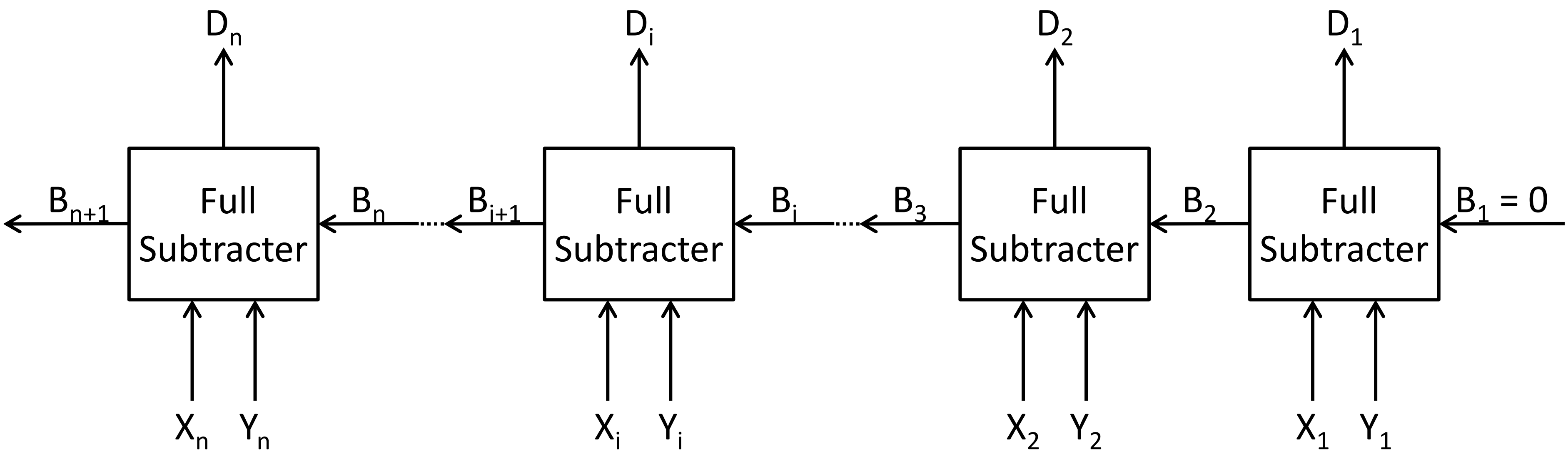
所有bit反過來再加1

- Consider A – B = A + (-B) in 2's complement
  - $A - B = A + (-B) = A + B^* = A + \overline{B} + 1$
  - Convert B to 2's complement: inverse and then add 1
- Discard the carry from the sign bit



Binary Subtracter (2/2)

- Or design a full subtracter
  - D = X – Y: difference
  - B: borrow



Q&A