

# Digital Systems Design and Laboratory

## [ 14. Derivation of State Graphs and Tables ]

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# Sequential Logic Design

- ❑ Unit 11: Latches and Flip-Flops
- ❑ Unit 12: Registers and Counters
- ❑ Units 13--15: Finite State Machines
- ❑ Unit 16: Summary      如果不能转换成 FSM → 能用 circuit 表示
- ❑ Designing a sequential circuit
  - Construct a state graph or state table (Unit 14)
  - Simplify it (Unit 15)
  - Derive flip-flop input equations and output equations (Unit 12)

# Outline

- ❑ **Design of a Sequence Detector**
- ❑ Guidelines for Construction of State Graphs
- ❑ Serial Data Code Conversion
- ❑ Alphanumeric State Graph Notation

# "101" Detector (1/5)

❑ Output "1" if detecting "101"

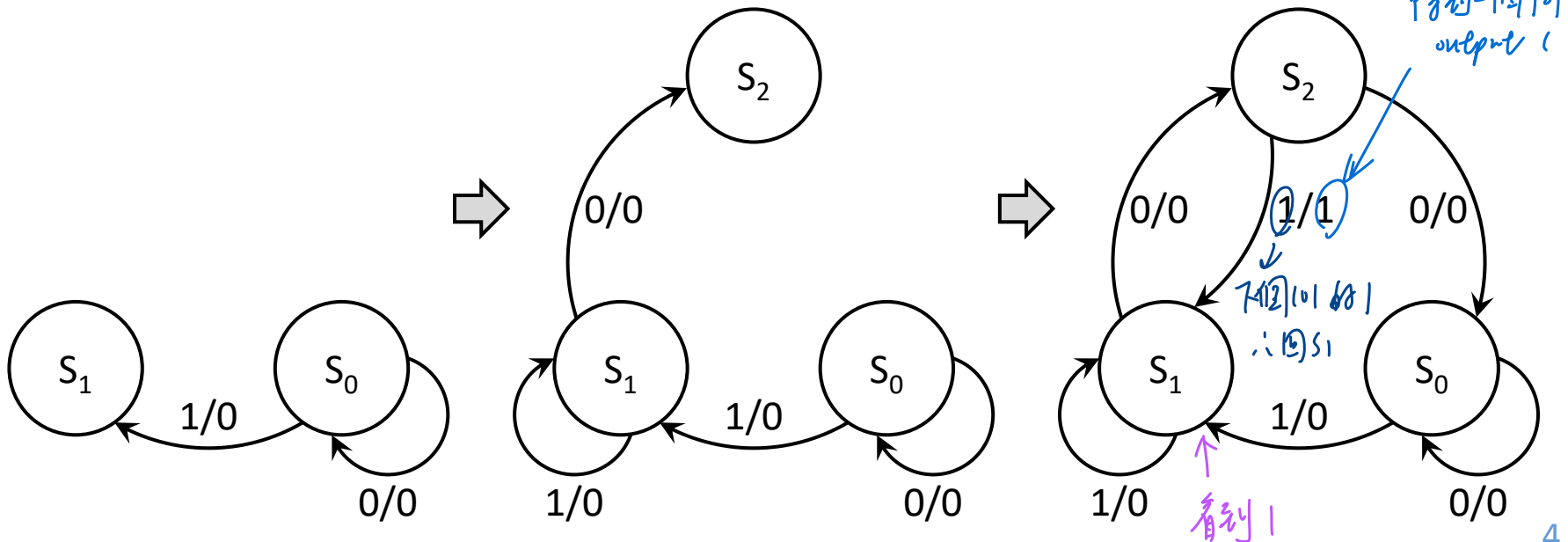
❑ Example

➤ Input X     0 0 1 1 0 1 1 0 0 1 0 1 0 1 0 0

➤ Output Z    0 0 0 0 0 1 0 0 0 0 0 1 0 1 0 0

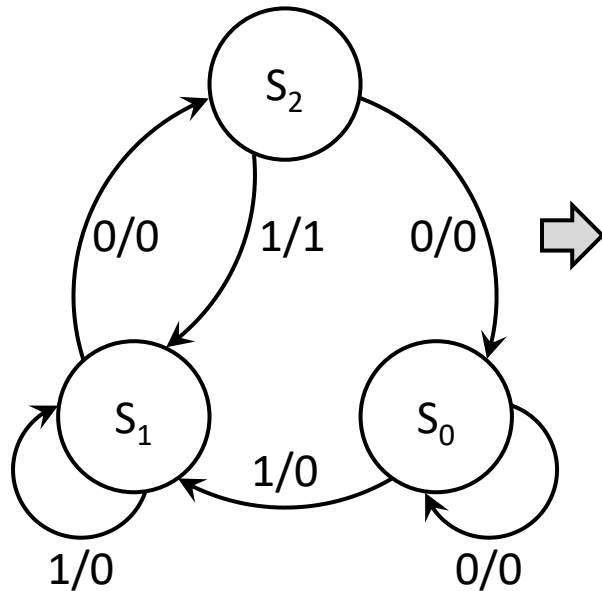
❑ State graph (Mealy) *每個狀態都有輸出*

➤  $S_0$ : initial,  $S_1$ : get "1",  $S_2$ : get "10"



# "101" Detector (2/5)

## State table



Present State	Next State		Present Output	
	X = 0	X = 1	X = 0	X = 1
S <sub>0</sub>	S <sub>0</sub>	S <sub>1</sub>	0	0
S <sub>1</sub>	S <sub>2</sub>	S <sub>1</sub>	0	0
S <sub>2</sub>	S <sub>0</sub>	S <sub>1</sub>	0	1

AB	A+B <sup>+</sup>		Z	
	X = 0	X = 1	X = 0	X = 1
00	00	01	0	0
01	10	01	0	0
10	00	01	0	1
11	XX	XX	X	X

Next state, output don't care

# "101" Detector (3/5)

## State maps

AB	A <sup>+</sup> B <sup>+</sup>		Z	
	X = 0	X = 1	X = 0	X = 1
00	00	01	0	0
01	10	01	0	0
10	00	01	0	1
11	XX	XX	X	X



		X	
		0	1
AB	00	0	0
	01	1	0
	11	X	X
	10	0	0
	00	0	0

$$A^+ = X'B$$

		X	
		0	1
AB	00	0	1
	01	0	1
	11	X	X
	10	0	1
	00	0	0

$$B^+ = X$$

		X	
		0	1
AB	00	0	0
	01	0	0
	11	X	X
	10	0	1
	00	0	0

$$Z = XA$$

# "101" Detector (4/5)

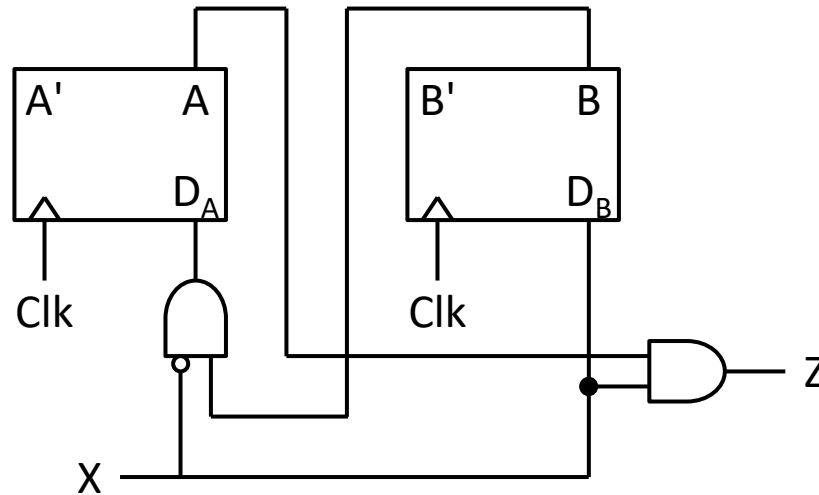
化簡後，用必要的 flip-flop 做出來。

## □ Realize it

➤  $A^+ = X'B$

➤  $B^+ = X$

➤  $Z = XA$

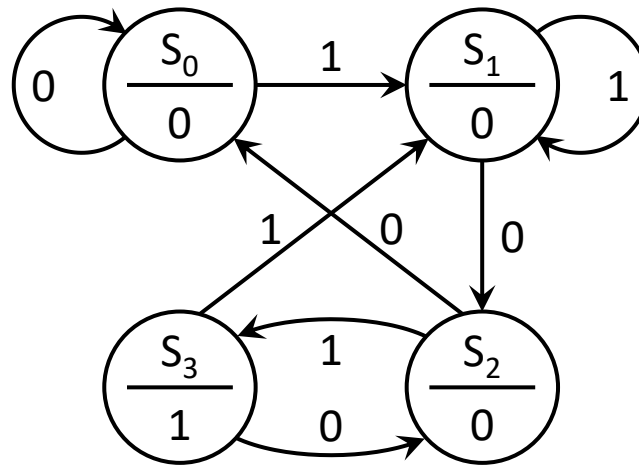


# "101" Detector (5/5)

## □ Some variants

➤ Moore machine?

- One more state

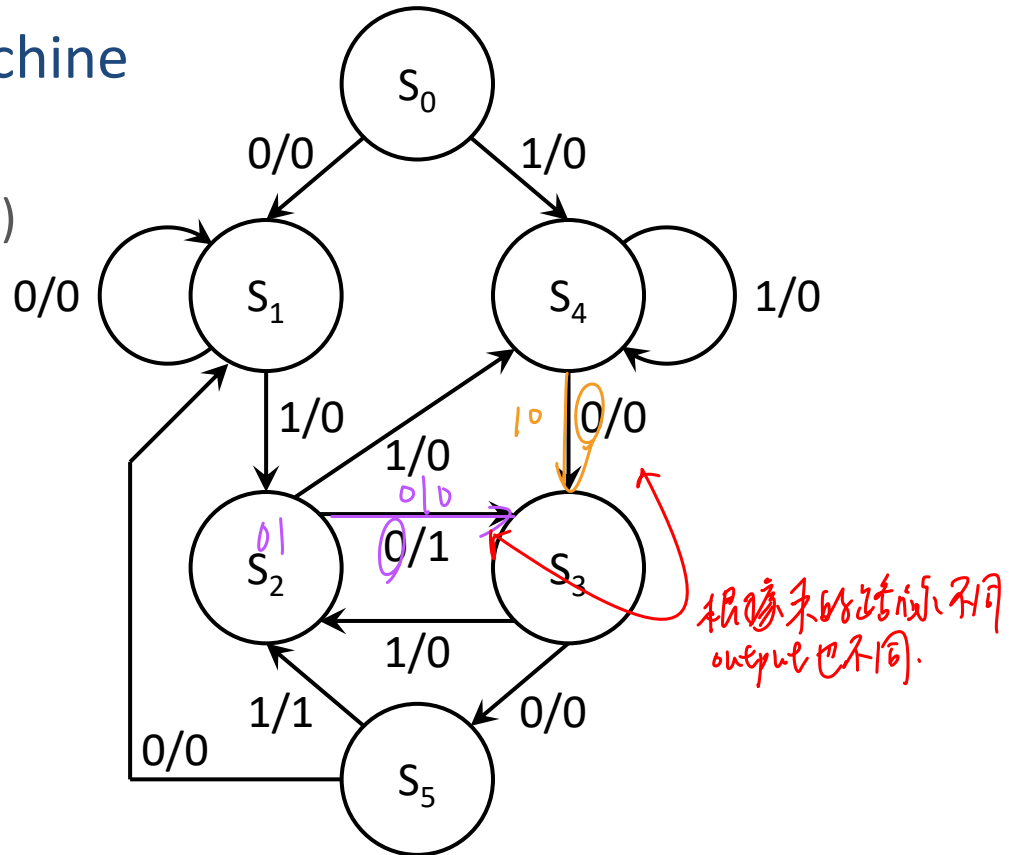




# "010" and "1001" Detector

## □ A more complicated machine

- $S_0$ : reset
- $S_1$ : "0" (but not 10 nor 100)
- $S_2$ : "01"
- $S_3$ : "10"
- $S_4$ : "1" (but not 01)
- $S_5$ : "100"



# Outline

- ❑ Design of a Sequence Detector
- ❑ **Guidelines for Construction of State Graphs**
- ❑ Serial Data Code Conversion
- ❑ Alphanumeric State Graph Notation

# Guidelines for Construction of State Graphs

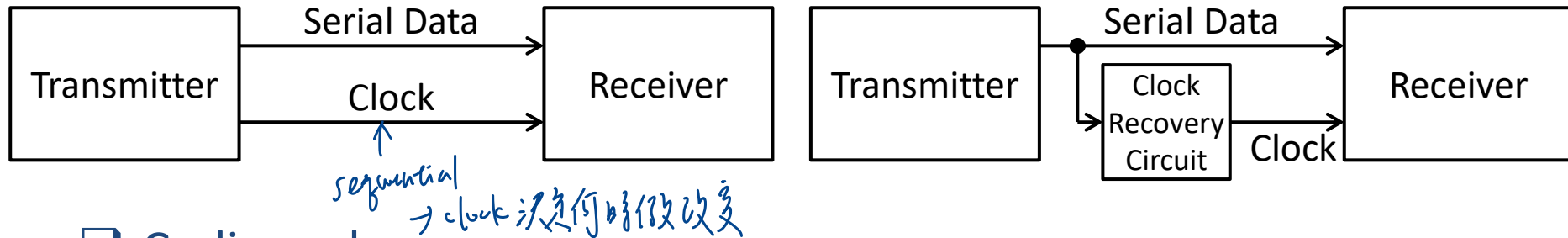
## □ Steps

- Construct sample sequences to help you understand the problem
- Determine under what conditions it should reset *回到 initial state*
- If only one or two sequences lead to a nonzero output, construct a partial state graph
  - Another way, determine what sequences or groups of sequences must be remembered by the circuit and set up states accordingly
- Each time you add an arrow to the state graph, determine whether it can go to one of the previously defined states or whether a new state must added
- Check your graph to make sure there is one and only one path leaving each state for each combination of values of the input variables
- When your graph is complete, verify it by applying the input sequences formulated in step 1

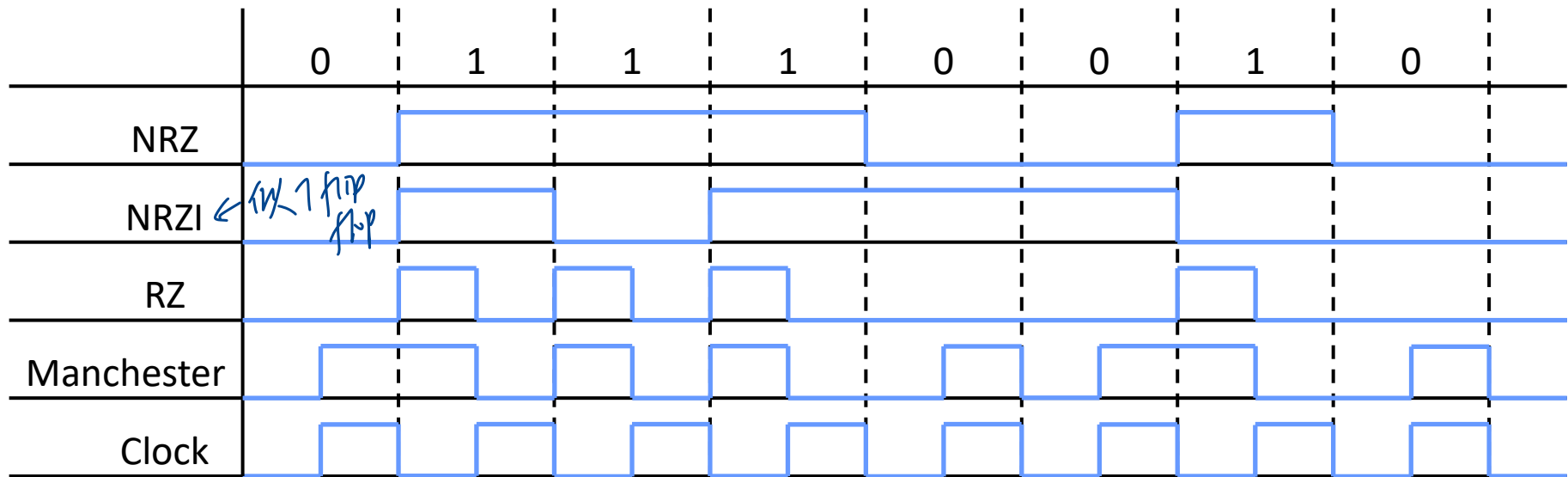
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# Serial Data Transmission



## □ Coding schemes



NRZ: Non-Return-to-Zero

NRZI: Non-Return-to-Zero-Inverted

RZ: Return-to-Zero

Manchester

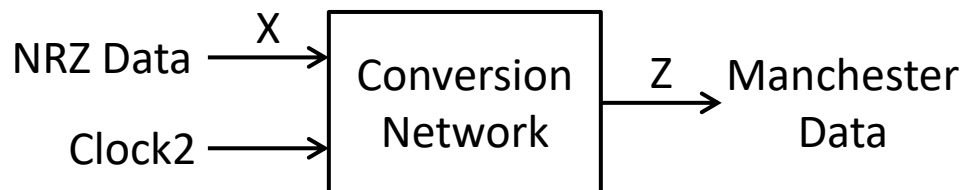
0 → 0 1 → 1

0 → D 1 → D' (D=不變, D'=變)

0 → 0 1 → 10

0 → 01 1 → 10

# Mealy Machine

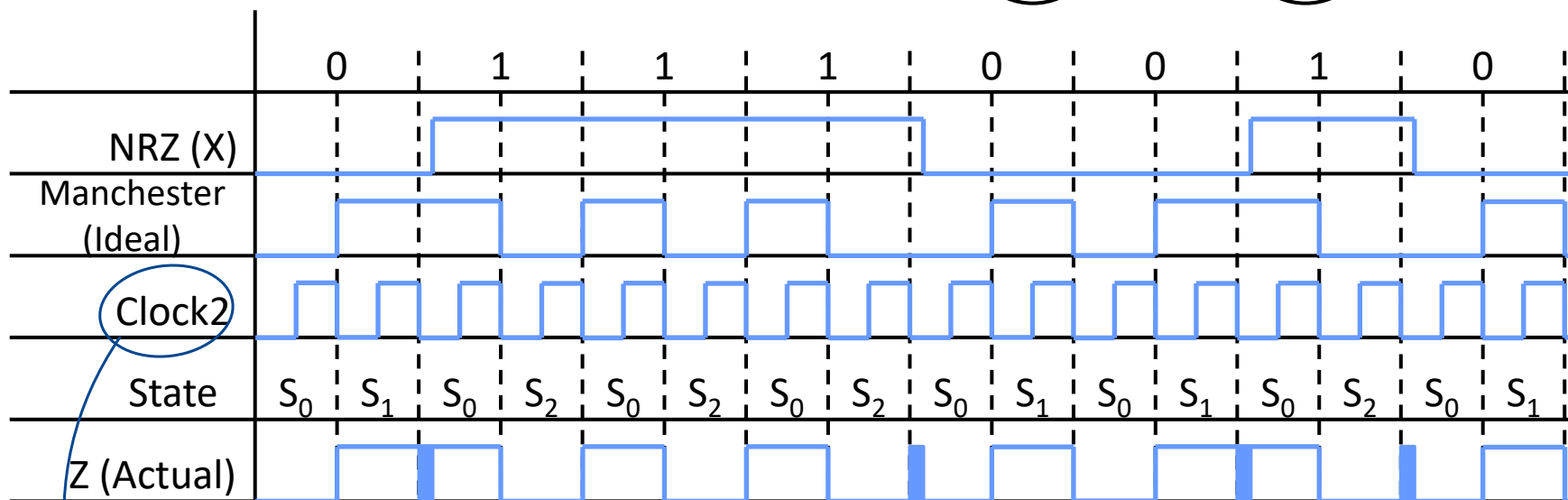
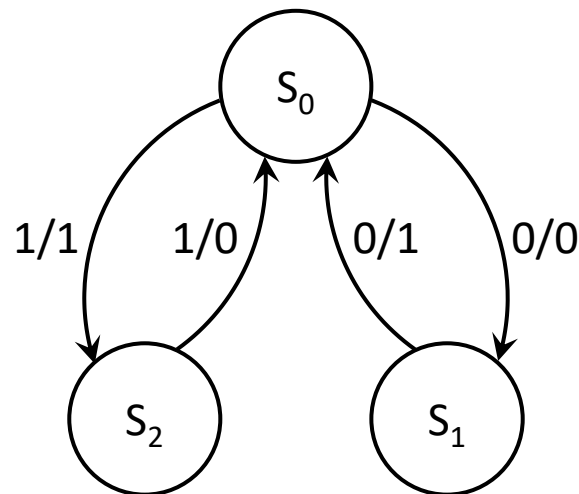


## Output depends on

- Current state (synchronous)
- Input (maybe asynchronous)

## State changes at a falling edge

## Fewer states



Glitch → 1 X 2 不是 active edge

Glitch

Glitch

Glitch

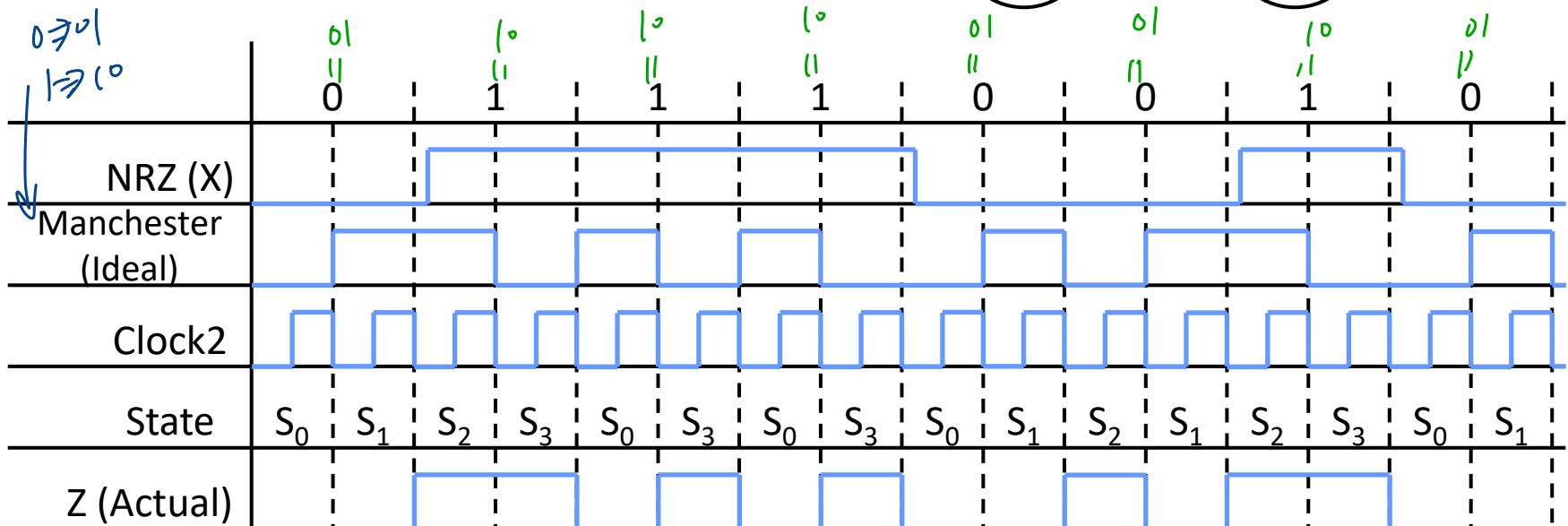
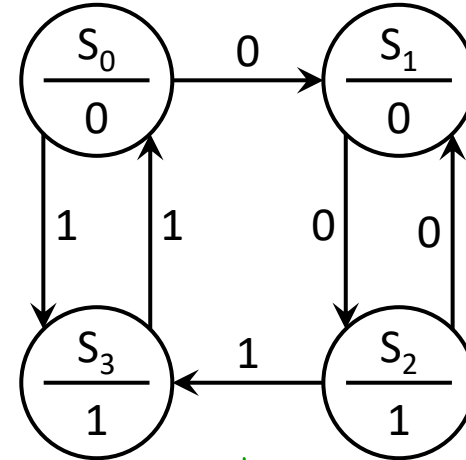
比前角的小更快一些

暗改变

# Moore Machine

↳ 多一个 state

- ❑ Output only depends on
  - Current state (synchronous)
- ❑ State changes at a falling edge
- ❑ More states (in general)
- ❑ 1 clock period delay



# Outline

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- ❑ **Alphanumeric State Graph Notation**

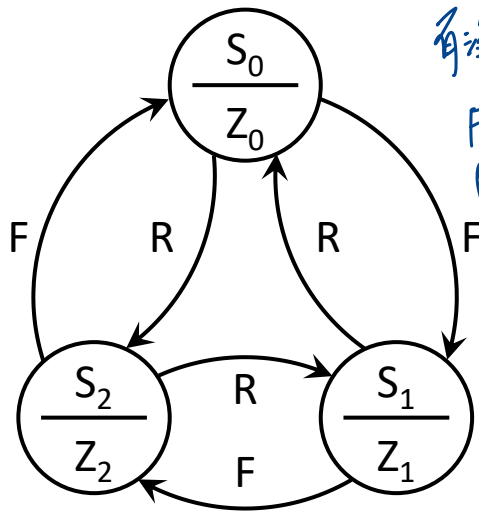


# Alphanumeric State Graph Notation

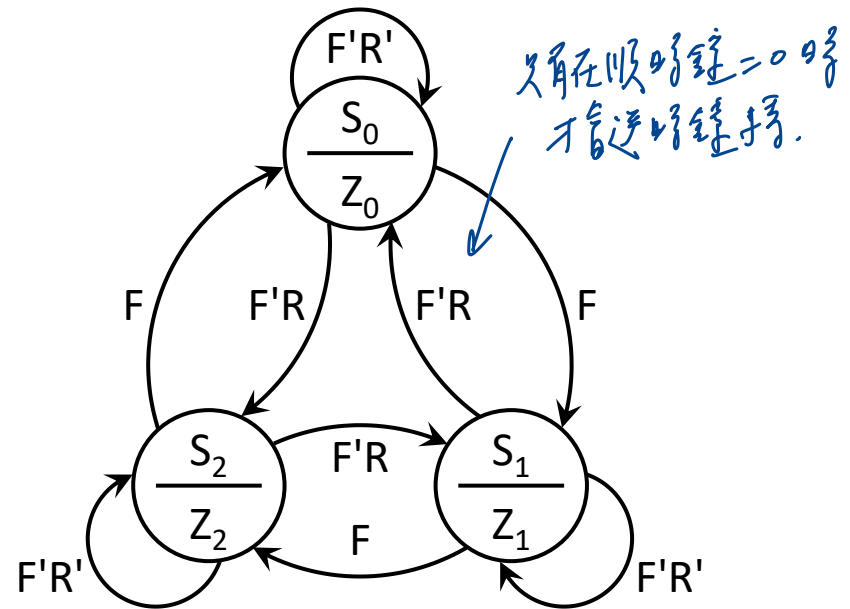
- When a sequential circuit has several inputs, label the state graph arcs with alphanumeric input variable names instead of 0's and 1's

➤ Example

- 2 inputs: F for "forward" and R for "reverse"



有没有可能:  
 $F=R=1$  ?  
 $F=R=0$  ?

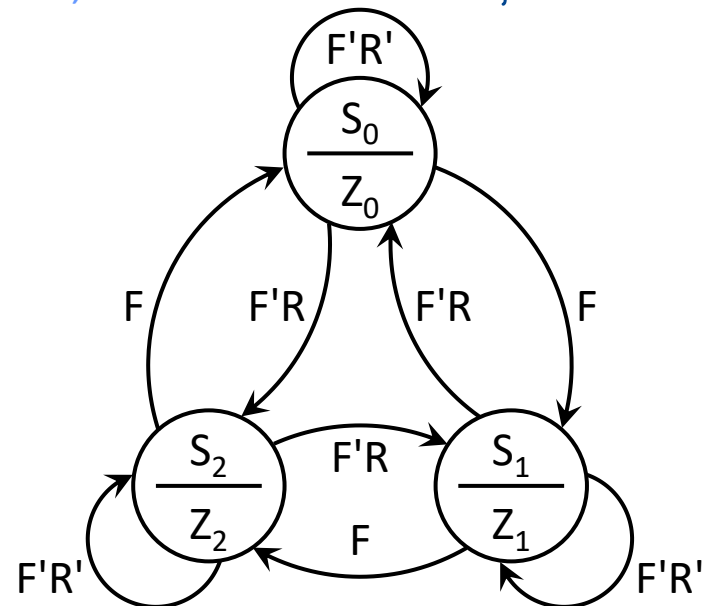
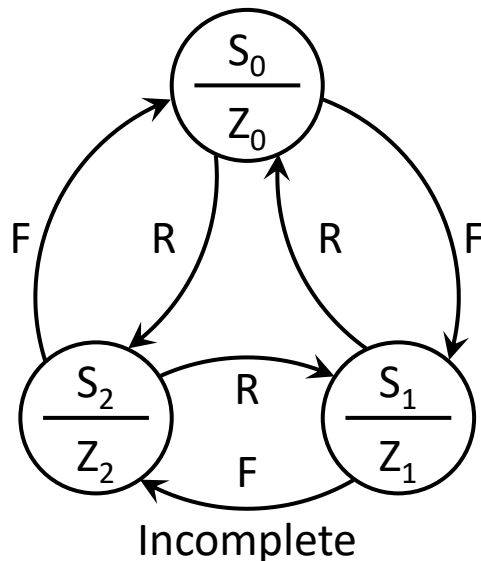


只有在顺时转=0时  
才会逆时针转。

# Completely Specified State Graph

## □ Properties

- **OR** together all input labels on arcs emanating from a state, the result can reduce to 1 *保證至少一條路可走*
  - Cover all conditions:  $F + F'R + F'R' = F + F' = 1$
- **AND** together any pair of input labels on arcs emanating from a state, the result can reduce to 0 *相同條件發生時不可能同時發生。也保證最多一條路可走。*
  - Only one arc is valid:  $F \cdot F'R = 0, F \cdot F'R' = 0, F'R \cdot F'R' = 0$



# Q&A