Digital System Design and Lab: HW1

Lo Chun, Chou
R13922136

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Since the following operation is correct:

$$024 + 043 + 013 + 033 = 201$$

Let base $= k \in \mathbb{Z}^+, \quad k > 4$
We can formulate the following equations:

$$\begin{align}
4+3+3+3 \pmod{k} &= 13 \pmod{k} = 1 \tag{1}\\
2+4+1+3+c_1 \pmod{k} &= 10+c_1 \pmod{k} = 0 \tag{2}\\
0+0+0+0+c_2 \pmod{k} &= c_2 \pmod{k} = 2 \tag{3}
\end{align}$$

Where c_1, c_2 are the carry-out generated by the addition of the previous digits.

By (1), and $k > 4$, we knew that the possible values of k are $6, 12$.

Case 1: $k = 6$

Suppose $k = 6$, we have:

$$\begin{align}
4+3+3+3 \pmod{6} &= 13 \pmod{6} = 1 \tag{1}\\
2+4+1+3+c_1 \pmod{6} &= 10+c_1 \pmod{6} = 0 \tag{2}\\
0+0+0+0+c_2 \pmod{6} &= c_2 \pmod{6} = 2 \tag{3}
\end{align}$$

By (1), we knew that $c_1 = 2$, thus equation (2) holds since $10 + 2 = 12 \equiv 0 \pmod{6}$. Similarly, we knew that $c_2 = 2$, thus equation (3) holds since $0 + 2 = 2 \equiv 2 \pmod{6}$.

Therefore, $k = 6$ is a valid solution.

Case 2: $k = 12$

Suppose $k = 12$, we have:

$$
\begin{align}
4 + 3 + 3 + 3 \pmod{12} &= 13 \pmod{12} = 1 \tag{1}\\
2 + 4 + 1 + 3 + c_1 \pmod{12} &= 10 + c_1 \pmod{12} = 0 \tag{2}\\
0 + 0 + 0 + 0 + c_2 \pmod{12} &= c_2 \pmod{12} = 2 \tag{3}
\end{align}
$$

By (1), we knew that $c_1 = 1$ since $\lfloor 13 \div 12 \rfloor = 1$, however, this contradicts with equation (2) since $10 + 1 = 11 \not\equiv 0 \pmod{12}$.

Therefore, $k = 12$ is not a valid solution.

The only possible base is $k = 6$. $\square$

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(1)

Decimal	weighted code			
	8	4	-2	-1
0	0	0	0	0
1	0	1	1	1
2	0	1	1	0
3	0	1	0	1
4	0	1	0	0
5	1	0	1	1
6	1	0	1	0
7	1	0	0	1
8	1	0	0	0
9	1	1	1	1

(2)

Decimal (d)	$9-d$	weighted code for $9-d$			
		8	4	-2	-1
0	9	1	1	1	1
1	8	1	0	0	0
2	7	1	0	0	1
3	6	1	0	1	0
4	5	1	0	1	1
5	4	0	1	0	0
6	3	0	1	0	1
7	2	0	1	1	0
8	1	0	1	1	1
9	0	0	0	0	0

By obersving the table, and by the fact that $9-d$ is the complement of d, we can see that the weighted code for $9-d$ is the complement of the weighted code for d.

Which means that to obtain the code of $9-d$, we could simply flip the bits of the code of d.

3

Find the complemet of the function by only DeMorgan's laws and involution law:

$$F(A,B,C,D) = AB'C + (A' + B + D)(ABD' + B')$$

As required, we can only use the below properties:

$$\begin{aligned} (AB)' &= A' + B' \\ (A+B)' &= A'B' \\ (A')' &= A \end{aligned}$$

$$\begin{aligned} F'(A,B,C,D) &= (AB'C)' \cdot [(A' + B + D) \cdot (ABD' + B')]' \\ &= \left[(AB')' + C'\right] \cdot \left[(A' + B + D)' + (ABD' + B')'\right] \\ &= (A' + B + C') \cdot \left[(A' + B)' D' + (ABD')' B\right] \\ &= (A' + B + C') \cdot \left[AB'D' + \big((AB)' + D\big) B\right] \\ &= (A' + B + C') \cdot [AB'D' + (A' + B' + D) B] \\ &= (A' + B + C') \cdot (AB'D' + A'B + BD) \end{aligned}$$

Ask is further derivation is needed.

4

5

Obtain sum of product form:

$$
\begin{aligned}
&(A+B+C+D)(A'+B'+C+D')(A'+C)(A+D)(B+C+D)\\
=&\{[A+(B+C+D)]\,(B+C+D)\}\,(A'+B'+C+D')(A'+C)(A+D)\\
=&[A(B+C+D)+(B+C+D)(B+C+D)]\,(A'+B'+C+D')(A'+C)(A+D)\\
=&[AB+AC+AD+(B+C+D)]\,(A'+B'+C+D')(A'+C)(A+D)\\
=&(B+C+D)(A'+B'+C+D')(A'+C)(A+D)\\
=&(B+C+D)\,[A'+(B'+C+D')C]\,(A+D)\\
=&(B+C+D)\,[A'D+A(B'+C+D')C]\\
=&(B+C+D)\,[A'D+AB'C+AC+ACD']\\
=&(B+C+D)\,[A'D+AB'C+AC]\\
=&(B+C+D)\,[A'D+AC]\\
=&A'BD+A'CD+A'D+ABC+AC+ACD\\
=&A'D+AC \quad \square
\end{aligned}
$$

6

Obtain product of sums form:

$$
\begin{aligned}
& BCD + C'D' + B'C'D + CD \\
=& C'D' + B'C'D + CD \\
=& (C' + D)(C + D') + B'(C + D')' \\
=& \left[(C' + D) + B'\right]\left[(C + D')' + (C' + D)\right] \\
=& \left(B' + C' + D\right)\left(C'D + C' + D\right) \\
=& (B' + C' + D)(C' + D) \\
=& B'(C' + D) + (C' + D)(C' + D) \\
=& B'(C' + D) + (C' + D) \\
=& C' + D \quad \square
\end{aligned}
$$

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Inputs			Output
A	B	C	
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	1
0	1	1	1
1	0	1	1
1	1	1	1

From the table we can form the sum of products expression:

$$
\begin{aligned}
&ABC' + A'BC + AB'C + ABC \\
=&ABC' + AB'C + BC \quad \square
\end{aligned}
$$