Digital System Design and Lab: HW2

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A	B	C	X	Y
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

From the values where $X = 1$, we can form the following equation:

$$
\begin{aligned}
X &= A'BC + AB'C + ABC' + ABC \\
&= m_3 + m_5 + m_6 + m_7 \\
&= \sum m(3,5,6,7) \\
&= \prod M(0,1,2,4)
\end{aligned}
$$

Similarly, from the values where $Y = 1$, we can form the following equation:

$$
\begin{aligned}
Y &= A'B'C + A'BC' + AB'C' + ABC \\
&= m_1 + m_2 + m_4 + m_7 \\
&= \sum m(1,2,4,7) \\
&= \prod M(0,3,5,6)
\end{aligned}
$$

2

(1)

We form the table by first listing all possible combinations of A, B, C, D and their corresponding decimal values.

Then we calculate the decimal values multiplied by 5, and let S, T, U, V present the decimal values, and W, X, Y, Z present the values of $0 \sim 9$ left.

A	B	C	D	decimal	decimal $\times 5$	S	T	U	V	W	X	Y	Z
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	5	0	0	0	0	0	1	0	1
0	0	1	0	2	10	0	0	0	1	0	0	0	0
0	0	1	1	3	15	0	0	0	1	0	1	0	1
0	1	0	0	4	20	0	0	1	0	0	0	0	0
0	1	0	1	5	25	0	0	1	0	0	1	0	1
0	1	1	0	6	30	0	0	1	1	0	0	0	0
0	1	1	1	7	35	0	0	1	1	0	1	0	1
1	0	0	0	8	40	0	1	0	0	0	0	0	0
1	0	0	1	9	45	0	1	0	0	0	1	0	1

(2)

First, we can find that $D = X = Z$, since when $D = 1$, this means that the decimal value is an odd number, therefore the decimal value multiplied by 5 would have a unit digit of 5, which would result in a one in X and Z.

Then, we can find that $C = V$, since when $C = 1$, this means that the decimal value is added by 2, therefore the decimal value multiplied by 5 would result in adding a 10, which is $V = 1$. Similarly, we would have $B = U$.

Next, $A = T$, because when $A = 1$, the decimal value is greater than 8, which would result in 40 when multiplied by 5, thus the tens digit would be 4, which means $T = 1$.

We can observe that $S = 0$, since in order to have a 1 in S, the resulting value after multiplied by 5 should be greater than 80, which is impossible, because the maximum value of a BCD digit $ABCD$ is 9.

Finally, $W = Y = 0$, since first, any value multiplied by 5 would not result in 2 or 8 in the unit digit, also, the maximum value of $ABCD$ is 9, so when we have 10, it won't be $W = Y = 1$ but moving the ten to be presented in the tens digit.

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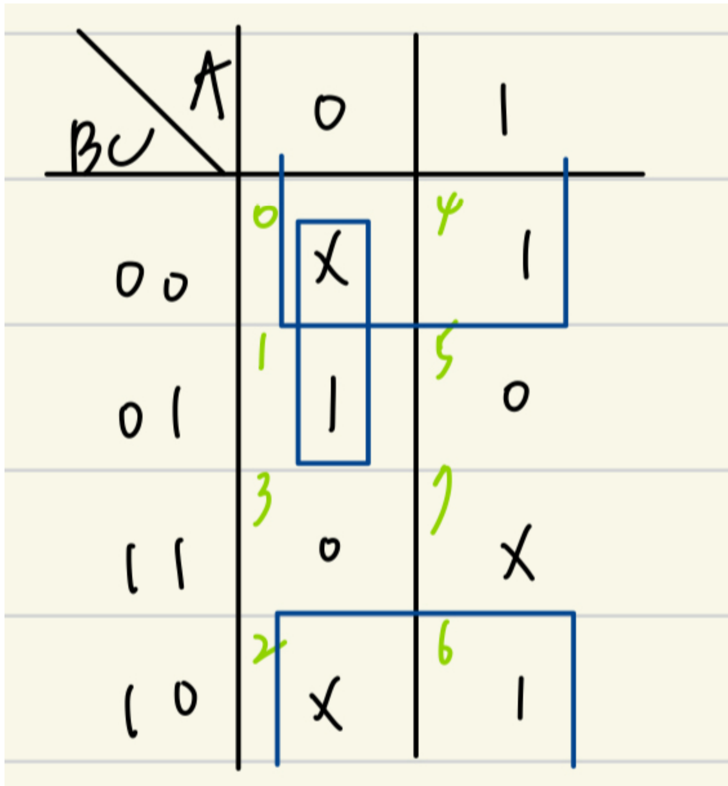


From the K-map, we can derive the following minimum SOP equation:

$$F(A, B, C) = A'B' + C'$$

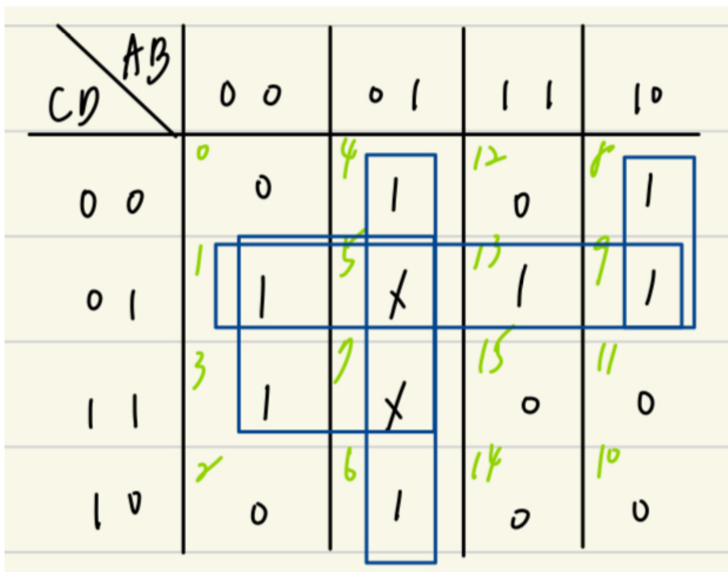
4

(1)

First, we convert the maxterm expression into a minterm expression:

$$
\begin{aligned}
F(A,B,C,D) &= \prod M(0,2,10,11,12,14,15) \cdot \prod D(5,7) \\
&= \sum m(1,3,4,6,8,9,13) \cdot \sum d(5,7)
\end{aligned}
$$

Next, we draw the K-map and find the minimum SOP equation:



From the K-map, we can derive the following minimum SOP equation:

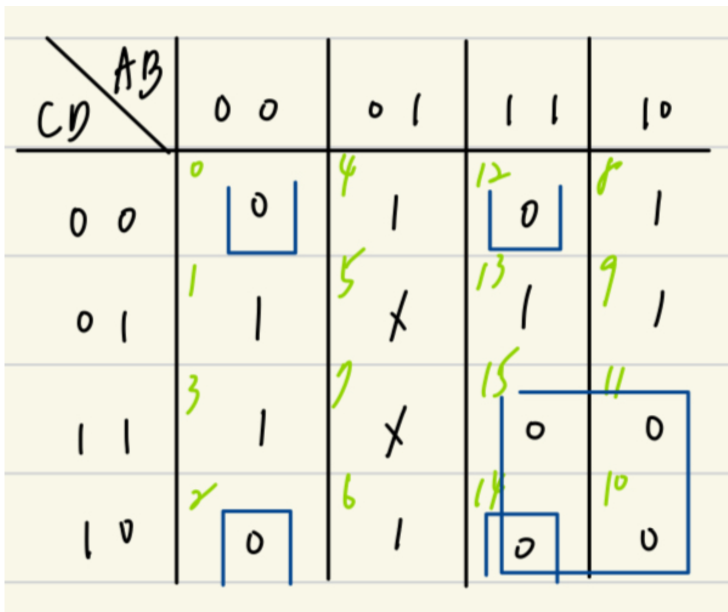
$$F(A,B,C,D) = A'B + C'D + A'D + AB'C'$$

(2)

For this subproblem, we also need to convert the maxterm expression into a minterm expression first, which is the same as the previous subproblem:

$$\begin{aligned} F(A,B,C,D) &= \prod M(0,2,10,11,12,14,15) \cdot \prod D(5,7) \\ &= \sum m(1,3,4,6,8,9,13) \cdot \sum d(5,7) \end{aligned}$$

But in the K-map, we circle 0s instead of 1s:



From the K-map, we can derive the following minimum SOP equation for F':

$$F'(A,B,C,D) = A'B'D' + ABD' + AC$$

Then, we can derive the minimum POS equation for F by using De Morgan's law:

$$\begin{aligned} F(A,B,C,D) &= (F'(A,B,C,D))' \\ &= (A'B'D' + ABD' + AC)' \\ &= (A+B+D)(A'+B'+D)(A'+C') \end{aligned}$$

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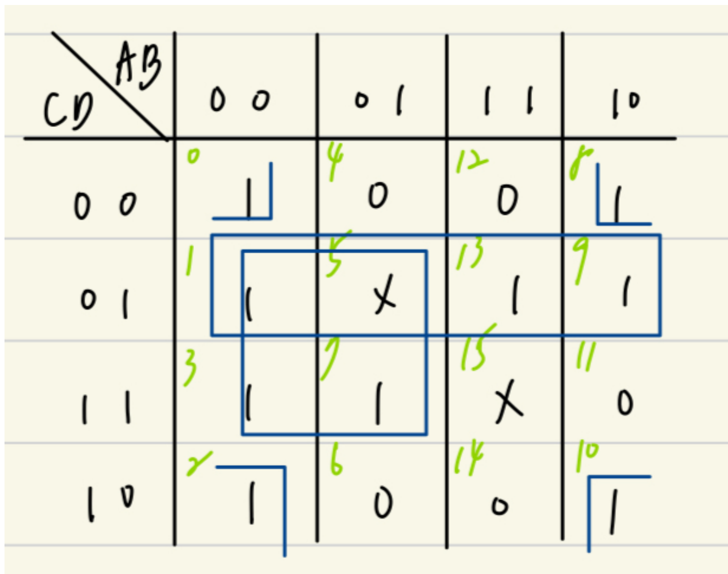
By the given restriction, we knew that $ABCD = 1111$ and $ABCD = 0101$ would never occur, so they are the don't care terms.

We then construct the K-map:



From the K-map, we can derive the following simplified equation:

$$F(A, B, C, D) = B'D' + A'D + C'D$$

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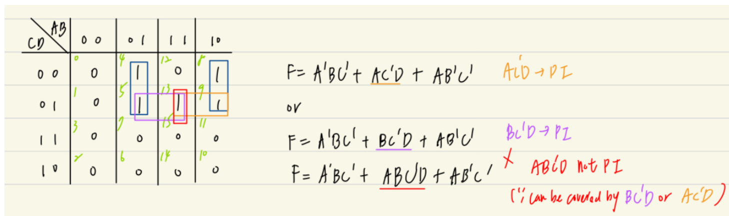
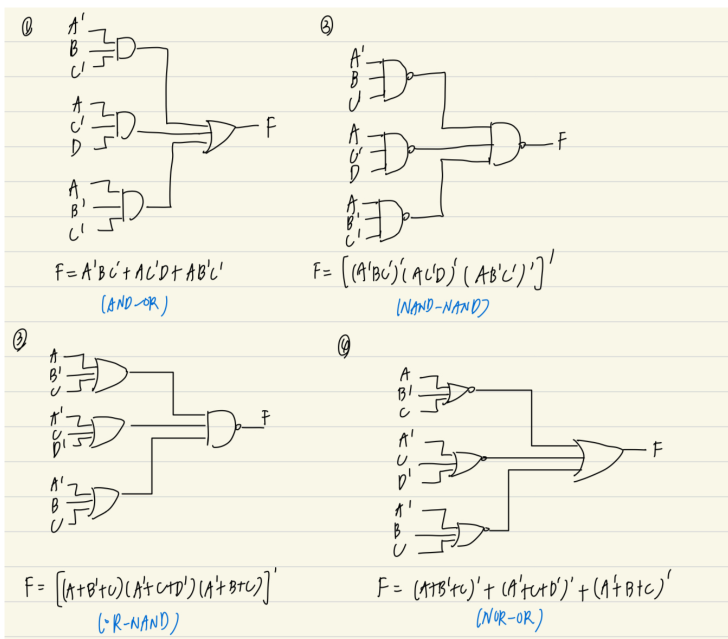
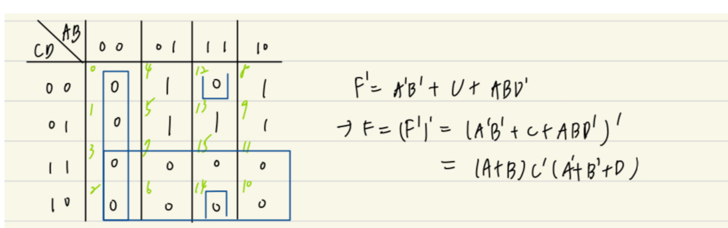
K-map using SOP (circle 1s):



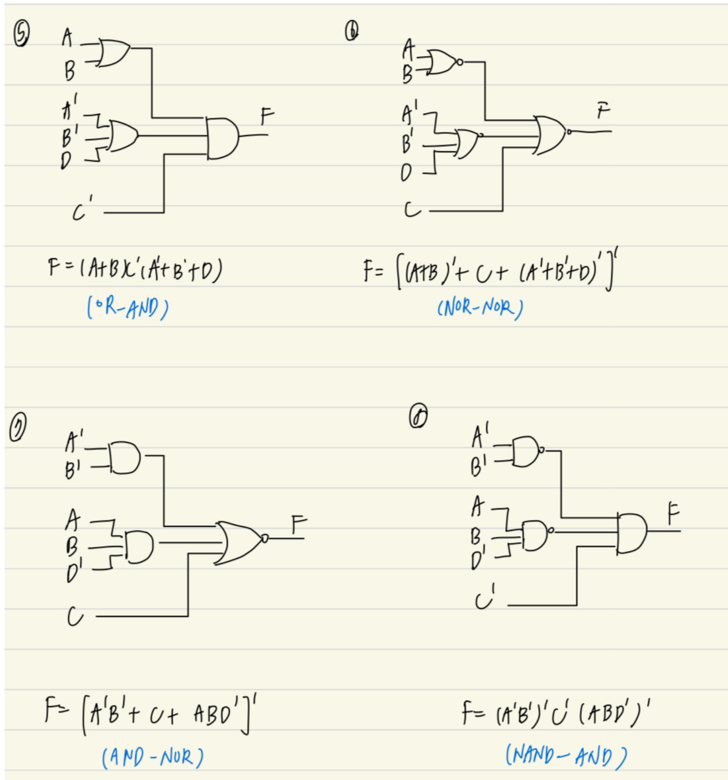
From the order $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$, we can formulate the first four minimum two-level gate circuits:



K-map using POS (circle 0s):



From the order $5 \rightarrow 6 \rightarrow 7 \rightarrow 8$, we can formulate the first four minimum two-level gate circuits:



Since we're using K-map to ensure that both SOP / POS expressions are minimum, we can conclude that the eight two-level gate circuits are minimum.

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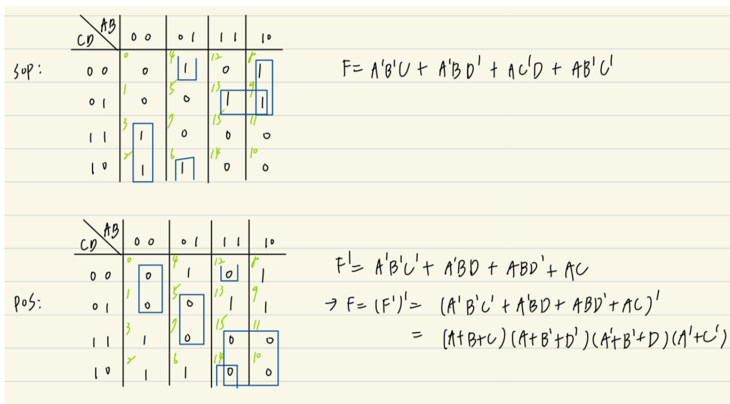
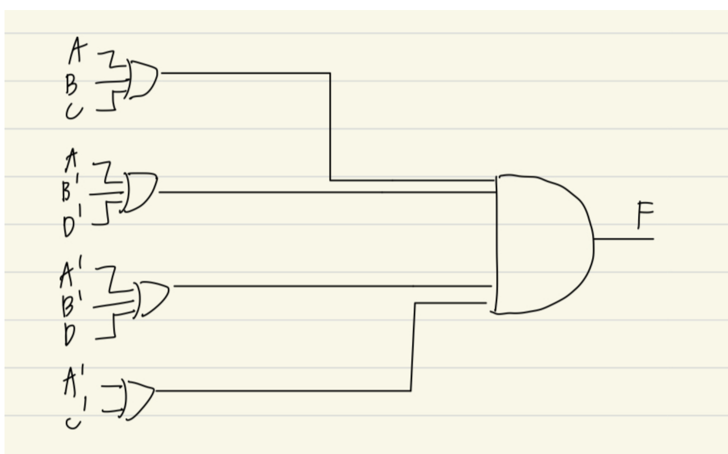
First, we convert the maxterm expression into a minterm expression:

$$\begin{aligned} F(A,B,C,D) &= \prod M(0,1,3,13,14,15) \\ &= \sum m(2,4,5,6,7,8,9,10,11,12) \end{aligned}$$

Then, we draw the K-map:



From the K-map, we can formulate the curcuit with only AND, OR gates:



There are 5 gates and 11 gate inputs in the circuit.