Claim:

2 $$\underset{\substack{\vec{x}\sim p(\vec{x})\\ y\sim p(y|\vec{x})}}{E}[\![h(\vec{x})\neq y]\!] \le \underset{\vec{x}\sim p(\vec{x})}{E}[\![h(\vec{x})\neq f(\vec{x})]\!] + \underset{\substack{\vec{x}\sim p(\vec{x})\\ y\sim p(y|\vec{x})}}{E}[\![\overset{\text{noise}}{f(\vec{x})\neq y}]\!]$$

Suppose $$\underbrace{\underset{\substack{\vec{x}\sim p(\vec{x})\\ y\sim p(y|\vec{x})}}{E}[\![h(\vec{x})\neq y]\!]}_{S} > \underbrace{\underset{\vec{x}\sim p(\vec{x})}{E}[\![h(\vec{x})\neq f(\vec{x})]\!]}_{S_1} + \underbrace{\underset{\substack{\vec{x}\sim p(\vec{x})\\ y\sim p(y|\vec{x})}}{E}[\![f(\vec{x})\neq y]\!]}_{S_2}$$

Let S be the set that contains all $\vec{x}_i$ s.t. $h(\vec{x}_i)\neq y_i$

S_1 be the set that contains all $\vec{x}_j$ s.t. $h(\vec{x}_j)\neq f(\vec{x}_j)$

S_2 be the set that contains all $\vec{x}_k$ s.t. $f(\vec{x}_k)\neq y_k$

Suppose $\exists\, \vec{x}_t \in S$, then $h(\vec{x}_t)\neq y_t$ $\begin{cases} \text{if } \vec{x}_t\in S_1, \text{ then } f(\vec{x}_t)=y_t \Rightarrow \vec{x}_t\notin S_2 \\ \text{if } \vec{x}_t\in S_2, \text{ then } f(\vec{x}_t)\neq y_t \Rightarrow \vec{x}_t\notin S_1 \end{cases}$

Thus, $S_1\cap S_2=\phi$

If $\vec{x}_t\in S$ and $\vec{x}_t\notin S_1 \cap \vec{x}_t\notin S_2$

$\Rightarrow h(\vec{x}_t)\neq y_t \cap h(\vec{x}_t)\neq f(\vec{x}_t) \cap f(\vec{x}_t)\neq y_t \;\not\Rightarrow$ $\because$ binary classification

Therefore, $\forall\, \vec{x}_t\in S$, either $\vec{x}_t\in S_1$ or $\vec{x}_t\in S_2$

$\therefore S\subseteq S_1\cup S_2 \Rightarrow |S|\le |S_1\cup S_2| \underset{\uparrow}{=} |S_1|+|S_2|$

$\because S_1\cap S_2=\phi$

Which leads to a contradiction.

Thus, $$\underset{\substack{\vec{x}\sim p(\vec{x})\\ y\sim p(y|\vec{x})}}{E}[\![h(\vec{x})\neq y]\!] \le \underset{\vec{x}\sim p(\vec{x})}{E}[\![h(\vec{x})\neq f(\vec{x})]\!] + \underset{\substack{\vec{x}\sim p(\vec{x})\\ y\sim p(y|\vec{x})}}{E}[\![f(\vec{x})\neq y]\!]$$ $\square$