5. Let $\vec{w} = [w_0 \cdots w_d]^T$

$$\boxed{\sum_{k=1}^{K} (\vec{w}^T \tilde{x}_k - \tilde{y}_k)^2}$$

$k-1$

$$= \sum_{k=1}^{d+1} \left([w_0\ w_1\ \cdots\ w_d] \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \sqrt{\lambda \alpha_{k-1}} \\ 0 \\ \vdots \\ 0 \end{bmatrix} - 0 \right)^2$$

$$= \sum_{k=1}^{d+1} \left(\sqrt{\lambda \alpha_{k-1}}\, w_{k-1} \right)^2$$

$$= \sum_{k=1}^{d+1} \lambda \alpha_{k-1} w_{k-1}^2 = \lambda \alpha_0 w_0^2 + \lambda \alpha_1 w_1^2 + \cdots + \lambda \alpha_d w_d^2 = \underline{\lambda \sum_{i=0}^{d} \alpha_i w_i^2}$$

PV is to solve:

$$\min_{\vec{w} \in \mathbb{R}^{d+1}} \frac{1}{N+K} \left(\sum_{n=1}^{N} (\vec{w}^T \vec{x}_n - y_n)^2 + \boxed{\sum_{k=1}^{K} (\vec{w}^T \tilde{x}_k - \tilde{y}_k)^2} \right)$$

$$= \min_{\vec{w} \in \mathbb{R}^{d+1}} \frac{1}{N+K} \left(\sum_{n=1}^{N} (\vec{w}^T \vec{x}_n - y_n)^2 + \lambda \sum_{i=0}^{d} \alpha_i w_i^2 \right)$$ $\because K = d+1$

$$= \min_{\vec{w} \in \mathbb{R}^{d+1}} \frac{1}{N+d+1} \left(\sum_{n=1}^{N} (\vec{w}^T \vec{x}_n - y_n)^2 + \lambda \sum_{i=0}^{d} \alpha_i w_i^2 \right)$$

PR is to solve:

$$\min_{\vec{w} \in \mathbb{R}^{d+1}} \frac{1}{N} \sum_{n=1}^{N} (\vec{w}^T \vec{x}_n - y_n)^2 + \frac{\lambda}{N} \sum_{i=0}^{d} \alpha_i w_i^2$$

equivalent

$\because \frac{1}{N}, \frac{1}{N+d+1} > 0$

$$= \min_{\vec{w} \in \mathbb{R}^{d+1}} \frac{1}{N} \left(\sum_{n=1}^{N} (\vec{w}^T \vec{x}_n - y_n)^2 + \lambda \sum_{i=0}^{d} \alpha_i w_i^2 \right)$$

$\square$