9. Let article : $\vec{x}_t = [1 \quad x_{t1} \quad x_{t2} \cdots x_{td}]^T \quad t \in \{1, \ldots, N\}$

\# articles

dictionary : D , where $|D| = d \geq m$

$$D = [\underbrace{w_1 \cdots w_{d_+}}_{\text{more hatred like words}} \underbrace{w_{d_+ +1} \cdots \overset{w_d}{\overset{\|}{w_{d_+ + d_-}}}}_{\text{less hatred like words}}]^T$$

$\forall x_{ti} \quad i \in \{1, \ldots, d\} \quad x_{ti} = 1$ if $w_i \in$ article t

$$\begin{cases} z_+(\vec{x}_t) = \sum_{i=1}^{d_+} x_{ti} \\ z_-(\vec{x}_t) = \sum_{i=d_+ +1}^{d} x_{ti} \end{cases}$$

The mistake bound of PLA $= \dfrac{R^2}{\rho^2}$, where $\begin{cases} R^2 = \max\limits_n \|\vec{x}_n\|^2 \\ \rho = \min\limits_n y_n \dfrac{\vec{w}_f^T}{\|\vec{w}_f\|} \vec{x}_n \end{cases}$

$\because \|\vec{w}_f\| = \sqrt{\vec{w}_f^T \vec{w}_f}$

$\therefore \|\vec{w}_f\|^2 = \vec{w}_f^T \vec{w}_f$

$$\Rightarrow \rho^2 = \min_n \frac{(\vec{w}_f^T \vec{x}_n)^2}{\vec{w}_f^T \vec{w}_f}$$

Since we count number of hatred words to determine if an article is hatred or not, and the bias is -3.5, $\vec{w_f}$ is:

$$\vec{w_f} = [-3.5\ \overbrace{\underbrace{+1\ +1\ \cdots\ +1}_{d_+}\ \underbrace{-1\ -1\ \cdots\ -1}_{d_-}}^{d}]^T \in \mathbb{R}^{d+1}$$

$$(\vec{x} = [x_0\ x_1 \cdots x_d]^T = [1\ x_1 \cdots \boxed{x_d}]^T)$$

$\hookrightarrow$ 如果word i在article $\vec{x}$中 $\Rightarrow x_i = 1$ for $i = 1, \ldots, d$

$$\therefore \|\vec{w_f}\| = \sqrt{\vec{w_f}^T \vec{w_f}} = \sqrt{12.25 + 1^2 \times d_+ + (-1)^2 \times d_-} = \sqrt{12.25 + d}$$

$$\Rightarrow \rho^2 = \min_n \frac{(\vec{w_f}^T \vec{x_n})^2}{\vec{w_f}^T \vec{w_f}} = \min_n \frac{(\vec{w_f}^T \vec{x_n})^2}{12.25 + d} = \frac{0.25}{12.25 + d}$$

suppose $d_+ > d_-$

则min发生在 $\vec{x} =$

$$[1\ \underbrace{1 1 \cdots 1}_{\min(d_+, d_-) = d_-}\ 1 1 1 0 \cdots 0\ \underbrace{1 1 \cdots 1}_{\min(d_+, d_-) = d_-}]^T$$

$$\therefore \min_n (\vec{w_f}^T \vec{x_n})^2 = \min_n (-3.5 + d_- + 3 + 0 - d_-)^2 = \min_n (-0.5)^2 = 0.25$$

$$R^2 = \max_n \|\vec{x_n}\|^2 = \max_n \vec{x_n}^T \vec{x_n}$$

取 $\vec{x_n} = [\underset{x_0}{1}\ \underbrace{1 1 \cdots 1}_{m个}\ 0 \cdots 0]^T$

$$\therefore R^2 = m + 1$$

Therefore, the upperbound of mistake number $= \dfrac{m+1}{\frac{0.25}{12.25 + d}}$ □