

7.

Plugging in $\alpha = 1$ and $b = 0$ into $h_{\alpha,b}(\mathbf{x})$, we get:

$$\hat{h}(\mathbf{x}) = \text{sign} \left(\sum_{n=1}^N y_n K(\mathbf{x}_n, \mathbf{x}) \right)$$

Consider arbitrary \mathbf{x}_n and \mathbf{x}_m , the result of the Gaussian kernel is:

$$K(\mathbf{x}_n, \mathbf{x}_m) = \begin{cases} \exp(-\gamma \|\mathbf{x}_n - \mathbf{x}_m\|^2) & \text{if } n \neq m \\ 1 & \text{if } n = m \end{cases}$$

Since the problem assumed that $\|\mathbf{x}_n - \mathbf{x}_m\| \geq \epsilon \quad \forall n \neq m$, for the case $n \neq m$, we have:

$$K(\mathbf{x}_n, \mathbf{x}_m) = \exp(-\gamma \|\mathbf{x}_n - \mathbf{x}_m\|^2) \leq \exp(-\gamma \epsilon^2)$$

Consider the condition that $\gamma > \frac{\ln(N-1)}{\epsilon^2}$, we have:

$$\exp(-\gamma \|\mathbf{x}_n - \mathbf{x}_m\|^2) \leq \exp(-\frac{\ln(N-1)}{\epsilon^2} \epsilon^2) = \exp(-\ln(N-1)) = \frac{1}{N-1}$$

Thus, $\exp(-\gamma \|\mathbf{x}_n - \mathbf{x}_m\|^2) \rightarrow 0$ when γ sufficiently large.

Therefore, we can observe that when γ is large enough, for the prediction of an arbitrary \mathbf{x}_m , we have:

$$\begin{aligned} \hat{h}(\mathbf{x}_m) &= \text{sign} \left(\sum_{n=1}^N y_n K(\mathbf{x}_n, \mathbf{x}_m) \right) \\ &= \text{sign} \left(\sum_{n \in \{1 \dots N\} \setminus m} y_n \exp(-\gamma \|\mathbf{x}_n - \mathbf{x}_m\|^2) + y_m \times 1 \right) \\ &\approx \text{sign}(y_m) \end{aligned}$$

This means that the prediction made by \hat{h} is the same as the label of \mathbf{x}_m , which results in $E_{in}(\hat{h}) = 0$.