

6. sample size  $\Rightarrow \varphi N$  with kind  $\left\{ \begin{array}{l} |A| = N \\ |B| = N \\ |C| = N \\ |D| = N \end{array} \right. \quad N \rightarrow \infty \quad \therefore \text{equal probability}$

Green in A:  $\{1, 3, 5, 7, 9, 11, 13, 15\}$

B:  $\{2, 4, 6, 8, 10, 12, 14, 16\}$

C:  $\{9, 10, 11, 12, 13, 14, 15, 16\}$

D:  $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Let the 5 tickets be  $t_1, t_2, t_3, t_4, t_5$

The cases that 5 tickets with number  $i$  are all green:

$i$

1.  $t_1, \dots, t_5 \in A \cup D$

2.  $t_1, \dots, t_5 \in B \cup D$

3.  $t_1, \dots, t_5 \in A \cup D$

4.  $t_1, \dots, t_5 \in B \cup D$

5.  $t_1, \dots, t_5 \in A \cup D$

6.  $t_1, \dots, t_5 \in B \cup D$

7.  $t_1, \dots, t_5 \in A \cup D$

8.  $t_1, \dots, t_5 \in B \cup D$

$i$

9.  $t_1, \dots, t_5 \in A \cup C$

10.  $t_1, \dots, t_5 \in B \cup C$

11.  $t_1, \dots, t_5 \in A \cup C$

12.  $t_1, \dots, t_5 \in B \cup C$

13.  $t_1, \dots, t_5 \in A \cup C$

14.  $t_1, \dots, t_5 \in B \cup C$

15.  $t_1, \dots, t_5 \in A \cup C$

16.  $t_1, \dots, t_5 \in B \cup C$

Case:

"1 half of the tickets in the bag  $\in A \cup D$

$$\therefore P(t_1, \dots, t_5 \in A \cup D) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

Case 2:

$\therefore$  half of the tickets in the bag  $\in B \cup D$

$$\therefore P(t_1, \dots, t_5 \in B \cup D) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

Similarly, for Case 3, 4,  $P(t_1, \dots, t_5 \in A \cup C) = \frac{1}{32}$

$$P(t_1, \dots, t_5 \in B \cup C) = \frac{1}{32}$$

But all of the tickets  $\in A \cup B \cup C \cup D$  are sampled repeatedly

$$\therefore \text{we need to subtract } \left(\frac{1}{4}\right)^5 \times 4 = \frac{1}{80}$$

Consider the case that if we have 3 kinds in the 5 tickets.

1.  $t_1, \dots, t_5 \in \{B, C, D\} \rightarrow \{1, 3, 5, 7\}$  not all green  $\because \exists t_i \in \{B \cup C\}$

2.  $t_1, \dots, t_5 \in \{A, C, D\} \rightarrow \{2, 4, 6, 8\}$  not all green  $\because \exists t_i \in \{C\}$

3.  $t_1, \dots, t_5 \in \{A, B, D\} \rightarrow \{9, 11, 13, 15\}$  not all green  $\because \exists t_i \in \{B \cup D\}$

4.  $t_1, \dots, t_5 \in \{A, B, C\} \rightarrow \{10, 12, 14, 16\}$  not all green  $\because \exists t_i \in \{D\}$

It is similar for the cases 2~4, and also the case  $t_1, \dots, t_5 \in \{A, B, C, D\}$

Thus, the probability of winning the prize B:

$$\frac{1}{32} \times 4 - \frac{1}{80} = \frac{32}{800} - \frac{1}{800} = \frac{31}{800} \quad \square$$