

9.

From the definition of scaled decision stumps $g_{i,\theta}(x) = \llbracket x_i \geq \theta \rrbracket$, we obtain:

$$\begin{aligned}\Phi_{ds}(\mathbf{x}) &= [g_{1,\theta_1}(\mathbf{x}) \cdots g_{d,\theta_k}(\mathbf{x})]^T \\ &= [\llbracket x_1 \geq \theta_1 \rrbracket \cdots \llbracket x_d \geq \theta_k \rrbracket]^T\end{aligned}$$

where d is the dimension of \mathbf{x} .

Also, we have $\theta \in \{\theta_1 = L + 0.5, \theta_2 = L + 1.5, \dots, \theta_k = R - 0.5\}$, so:

$$\Phi_{ds}(\mathbf{x}) = \begin{bmatrix} \llbracket x_1 \geq L + 0.5 \rrbracket \\ \llbracket x_2 \geq L + 1.5 \rrbracket \\ \vdots \\ \llbracket x_{d-1} \geq R - 1.5 \rrbracket \\ \llbracket x_d \geq R - 0.5 \rrbracket \end{bmatrix}$$

Substituting this result into the definition of $K_{ds}(\mathbf{x}, \mathbf{x}')$, we obtain:

$$\begin{aligned}K_{ds}(\mathbf{x}, \mathbf{x}') &= \Phi_{ds}(\mathbf{x})^T \Phi_{ds}(\mathbf{x}') \\ &= [\llbracket x_1 \geq L + 0.5 \rrbracket \quad \llbracket x_2 \geq L + 1.5 \rrbracket \quad \dots \quad \llbracket x_{d-1} \geq R - 1.5 \rrbracket \quad \llbracket x_d \geq R - 0.5 \rrbracket] \\ &\quad \times \begin{bmatrix} \llbracket x'_1 \geq L + 0.5 \rrbracket \\ \llbracket x'_2 \geq L + 1.5 \rrbracket \\ \vdots \\ \llbracket x'_{d-1} \geq R - 1.5 \rrbracket \\ \llbracket x'_d \geq R - 0.5 \rrbracket \end{bmatrix} \\ &= \sum_{i=1}^d \llbracket x_i \geq \theta_i \rrbracket \llbracket x'_i \geq \theta_i \rrbracket\end{aligned}$$

Therefore, we got the definition of $K_{ds}(\mathbf{x}, \mathbf{x}')$, and the meaning is to count how many dimensions i of the 2 vectors satisfy the condition that both:

$$x_i \geq \theta_i \quad \text{and} \quad x'_i \geq \theta_i$$