1 Introduction

flq is a task in application level. It generates the Floquet Hamiltonian¹ Note that, the example attached in this file uses Haldane_flq.plb instead because NiO is not a good system to have Floquet effects.

2 Dictionary

2.1 Input

flq.Frequency This parameter gives the floquet frequency.

flq.Order This parameter gives the order of photon process.

flq.Amplitude This parameter gives the amplitude of the x, y and z direction, e.g [1,1,1]. The periodic wave vector is assume to have the form: $A = [A_x * sin(\omega t + \phi_x), A_x * sin(\omega t + \phi_y), A_x * sin(\omega t + \phi_z)],$ this parameters is essentially $[A_x, A_y, A_z]$.

flq.Amplitude This parameter gives the phase of each components, $[\phi_x, \phi_y, \phi_z]$ in unit of radian.

2.2 Output

flq.state_info This variable shows the Flqouet states used in the calculation. There are 6 column in this variable [state_label, order, site, identifier, l, SubOrb]. They have similar meaning as shown in the hop.state_info. The only difference is there is an additional column which shows the "order". Since all Floquet states come with an additional quantum number which describes the Fourier component of time domain. Usually we call this quantum number as "order" because it relates to the number of photons involved.

flq.H_onsite(n) This variable shows the onsite energy of Floquet Hamiltonian. Like the Floquet state, the Hamiltonians also requires an additional quantum number to label them, so flq.H_onsite(n) describes the onsite energy of the (n-1)-th order.

flq.hop_size This variable shows the size of the variable flq.hop_mat. This is just for PiLab to preset memory and most users can ignore this variable.

 $flq.hop_mat(p)(n)(:,:,m)$ This variable shows the hopping between the n-th sublattice and its m-th neighbor as shown in the hop.hop_mat. Also, there is an additional variable p to describe the order of the Floquet Hamiltonian.

¹For detail formulation of Floquet theory, see PRB 88, 245422 (2013) and PRL110,200403 (2013)

```
flq.Frequency=[10]
                          // field frequency, 1x1 real
                       // order of photon process, 1x1, int
flq.Order=[0]
flq.Amplitude=[1,1]
                          // AC amplitude, 1x1 / 1x2 / 1x3, real
                            // AC phase 1x1 / 1x2 / 1x3, real
flq.Phase=[0,\%pi/2]
====== PiLib Variable =======
flg.state info, @full, [state label, order, site, identifier, l, SubOrb]
ORDER= 0, SIZE=[ 2, 6], TYPE=INTEGER
   1
        2
             3
                  4
                       5
                            6
                            5
   1
             1
                       1
                  1
   2
             2
                  1
                       1
                            5
        ===== PiLib Variable ======
flq.H onsite(1), @t-sp, Floquet H onsite of order 0
ORDER= 0, SIZE=[ 1, 3], TYPE=SPARSE
   1
        2
                     3
   2
        2 0.000000 0.000000
   ======= PiLib Variable ========
flq.hop size, @full, size of flq.hop mat [order+1,sublatt,hop mat size]
ORDER= 0, SIZE=[ 2, 5], TYPE=INTEGER
   1
             3
                  4
                       5
        1
                  2
                       9
             2
    1
        2
            ==== PiLib Variable ======
flq.hop mat(1)(1)(:,:,1), @a-sp, Floquet hop_mat(1)(:,:,1) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
        2
                     3
   2
        2 0.000000 0.000000
   1
        2 0.719622 0.000000
      ====== PiLib Variable =======
flq.hop mat(1)(1)(:,:,2), @a-sp, Floquet hop mat(1)(:,:,2) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
        2
                     3
```

Figure 1: page 1 of Haldane_flq.plb

```
2 0.000000 0.000000
   2
        2 0.719622 0.000000
   1
     ====== PiLib Variable ======
flq.hop mat(1)(1)(:,:,3), @a-sp, Floquet hop mat(1)(:,:,3) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
        2
                    3
   2
        2 0.000000 0.000000
        2 0.719622 0.000000
    ======= PiLib Variable ======
flq.hop_mat(1)(1)(:,:,4), @a-sp, Floquet hop_mat(1)(:,:,4) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
        2
                    3
        2 0.000000 0.000000
        1 0.000000 0.027876
            === PiLib Variable ==
flq.hop mat(1)(1)(:,:,5), @a-sp, Floquet hop mat(1)(:,:,5) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
                    3
        2
        2 0.000000 0.000000
   1
        1 0.000000 -0.027876
       ====== PiLib Variable =======
flq.hop_mat(1)(1)(:,:,6), @a-sp, Floquet hop_mat(1)(:,:,6) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
    1
                    3
        2
        2 0.000000 0.000000
   1
        1 0.000000 -0.027876
        ===== PiLib Variable =====
flq.hop mat(1)(1)(:,:,7), @a-sp, Floquet hop mat(1)(:,:,7) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
        2
                    3
   2
        2 0.000000 0.000000
```

Figure 2: page 2 of Haldane_flq.plb

1	1	0.000000	0.027876
====		===== PiLil	o Variable ======
flq.hop_mat(1)(1)(:,:,8), @a-sp, Floquet hop_mat(1)(:,:,8) of order 0 ORDER= 0, SIZE=[2, 3], TYPE=SPARSE			
1	2		3
2	1	0.000000	0.027876
======================================			
1	2		3
2	2	0.000000 0.000000	0.000000 -0.027876
====== PiLib Variable ======			
flq.hop_mat(1)(2)(:,:,1), @a-sp, Floquet hop_mat(2)(:,:,1) of order 0 ORDER= 0, SIZE=[2, 3], TYPE=SPARSE			
1	2		3
2 2	2	0.000000 0.719622	
====== PiLib Variable =====			
flq.hop_mat(1)(2)(:,:,2), @a-sp, Floquet hop_mat(2)(:,:,2) of order 0 ORDER= 0, SIZE=[2, 3], TYPE=SPARSE			
1	2		3
2 2	2		
======================================			
1	2		3
2 2	2		

Figure 3: page 3 of Haldane_flq.plb

```
======= PiLib Variable ======
flq.hop mat(1)(2)(:,:,4), @a-sp, Floquet hop mat(2)(:,:,4) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
        2
   2
        2 0.000000 0.000000
   2
        2 0.000000 -0.027876
      ====== PiLib Variable ======
flq.hop_mat(1)(2)(:,:,5), @a-sp, Floquet hop_mat(2)(:,:,5) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
        2
                    3
   2
        2 0.000000 0.000000
   2
        2 0.000000 0.027876
           ==== PiLib Variable ======
flq.hop mat(1)(2)(:,:,6), @a-sp, Floquet hop mat(2)(:,:,6) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
        2
                    3
        2 0.000000 0.000000
   2
   2
        2 0.000000 0.027876
   ====== PiLib Variable ======
flq.hop mat(1)(2)(:,:,7), @a-sp, Floquet hop mat(2)(:,:,7) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
        2
                    3
        2 0.000000 0.000000
   2
        2 0.000000 -0.027876
   ======= PiLib Variable =======
flq.hop mat(1)(2)(:,:,8), @a-sp, Floquet hop mat(2)(:,:,8) of order 0
ORDER= 0, SIZE=[ 2, 3], TYPE=SPARSE
   1
        2
                    3
   2
        2 0.000000 0.000000
   2
        2 0.000000 -0.027876
```

Figure 4: page 4 of Haldane_flq.plb

Figure 5: page 5 of Haldane_flq.plb