

期中考试模拟题（八）答案 2021. 11

一、 1. 0.8704; 2. 0.7; 3. 9; 4. $\frac{1}{4\sqrt{11\pi}} e^{-\frac{(z-10)^2}{176}}$; 5. $f_{U,V}(u,v) = \begin{cases} \frac{1}{2}, & 3 \geq u \geq v \geq 1 \\ 0, & \text{其它} \end{cases}$

二、 1.D 2.D 3.C 4.B 5.A

三、(1) $A_1 = \{\text{飞到距目标 400}\}, A_2 = \{\text{飞到距目标 200}\}, A_3 = \{\text{飞到距目标 100}\}, B = \{\text{命中目标}\},$

$$P(B) = \sum_{i=1}^3 P(A_i)P(B|A_i) = 0.5 \times 0.01 + 0.3 \times 0.02 + 0.2 \times 0.1 = 0.031$$

$$\begin{aligned} (2) \quad P(A_1|B) &= \frac{P(A_1B)}{P(B)} = \frac{P(A_1)P(B|A_1)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)} \\ &= \frac{0.5 \times 0.01}{0.5 \times 0.01 + 0.3 \times 0.02 + 0.2 \times 0.1} = \frac{5}{31} \approx 0.1613 \end{aligned}$$

$$P(A_2|B) = \frac{P(A_2B)}{P(B)} = \frac{0.3 \times 0.02}{0.5 \times 0.01 + 0.3 \times 0.02 + 0.2 \times 0.1} = \frac{6}{31} \approx 0.1935$$

$$P(A_3|B) = 1 - P(A_1|B) - P(A_2|B) = \frac{20}{31} \approx 0.6452$$

四、 $W \sim U[-2, 2], \therefore f_w(w) = \begin{cases} \frac{1}{4}, & -2 \leq w \leq 2 \\ 0, & \text{其他} \end{cases}$

$$P(X = -1, Y = -1) = P(W \leq -1, W \leq 1) = P(W \leq -1) = \int_{-2}^{-1} f_w(w)dw = \frac{1}{4}$$

$$P(X = -1, Y = 1) = P(W \leq -1, W > 1) = P(\emptyset) = 0$$

$$P(X = 1, Y = -1) = P(W > -1, W \leq 1) = P(-1 < W \leq 1) = \int_{-1}^1 f_w(w)dw = \frac{1}{2}$$

$$P(X = 1, Y = 1) = P(W > -1, W > 1) = P(1 < W) = \int_1^2 f_w(w)dw = \frac{1}{4}$$

	Y		$P_{i\cdot}$
$X \backslash Y$	-1	1	
-1	$\frac{1}{4}$	0	$\frac{1}{4}$
1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$
$P_{\cdot j}$	$\frac{1}{4}$	$\frac{3}{4}$	1

五、令 $W=2X$, 易知 W 的概率密度为 $f_W(w) = \begin{cases} 1 & 0 \leq w \leq 1 \\ 0 & \text{其他} \end{cases}$

$Z=2X+Y=W+Y$, 且 W 与 Y 相互独立, 故 $f_Z(z) = \int_{-\infty}^{+\infty} f_W(w) f_Y(z-w) dw$

当 $z < 0$ 时, $f_Z(z) = 0$; 当 $0 \leq z \leq 1$ 时, $f_Z(z) = \int_0^z 1 \cdot e^{-(z-w)} dw = 1 - e^{-z}$;

当 $z > 1$ 时, $f_Z(z) = \int_0^1 1 \cdot e^{-(z-w)} dw = e^{-z}(e-1)$

综上, 有: $f_Z(z) = \begin{cases} 0, & z < 0 \\ 1 - e^{-z}, & 0 \leq z \leq 1 \\ e^{-z}(e-1), & 1 < z \end{cases}$

六、(1) $f(x, y) = f_{Y|X}(y|x) f_X(x) = \begin{cases} 8xy, & 0 \leq x \leq y \leq 1 \\ 0, & \text{其它} \end{cases}$

$$P(X+Y \geq 1) = \int_{0.5}^1 dy \int_{1-y}^y 8xy dx = \frac{5}{6}$$

$$(2) P(Y < 0.5) = \int_0^{1/2} dy \int_0^y 8xy dx = \frac{1}{16}$$

$$(3) P\left(Y < \frac{2}{3} \middle| X = \frac{1}{2}\right) = \int_{-\infty}^{2/3} f_{Y|X}\left(y \middle| \frac{1}{2}\right) dy = \int_{-\infty}^{2/3} \frac{2y}{1-x^2} \bigg|_{x=\frac{1}{2}} dy = \int_{1/2}^{2/3} \frac{2y}{1-(0.5)^2} dy = \frac{7}{27}.$$

七、(12分) (1) $f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy$, 得 $f_X(x) = \begin{cases} xe^{-x}, & x > 0 \\ 0, & \text{其他} \end{cases}$

同理 $f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx$ 得 $f_Y(y) = \begin{cases} \frac{y^2}{2} e^{-y}, & y > 0 \\ 0, & \text{其他} \end{cases}$

$$(2) F_X(x) = \int_0^x f_X(t) dt = \begin{cases} 1 - xe^{-x} - e^{-x}, & x > 0 \\ 0, & \text{其它} \end{cases}$$

(3) 显然, $f(x, y) = f_X(x) f_Y(y)$ 非几乎处处成立, 因此 X 与 Y 不相互独立.

八 (1) $X \sim B(2, \frac{1}{2})$, X 的分布律为

X	0	1	2
P	0.25	0.5	0.25

$$(2) F_X(x) = \begin{cases} 0 & x < 0 \\ 0.25 & 0 \leq x < 1 \\ 0.75 & 1 \leq x < 2 \\ 1 & 2 \leq x \end{cases} \quad (3) E(X) = 1, D(X) = \frac{1}{2}.$$