磁耦合电路

第13章

13.3 变压器

13.4 理想变压器

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时　间：　　年　月　日

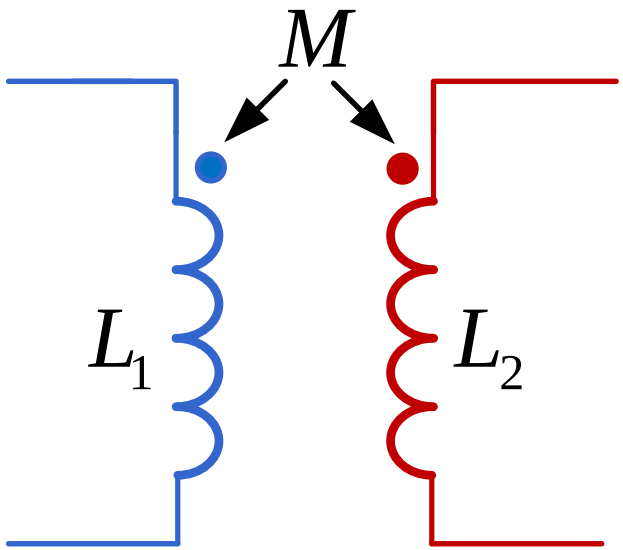
13.3-13.4 变压器、理想变压器——主要内容

- 引言
- 13.3 变压器
- 13.4 理想变压器
- 小结

13.3-13.4 变压器、理想变压器——引言

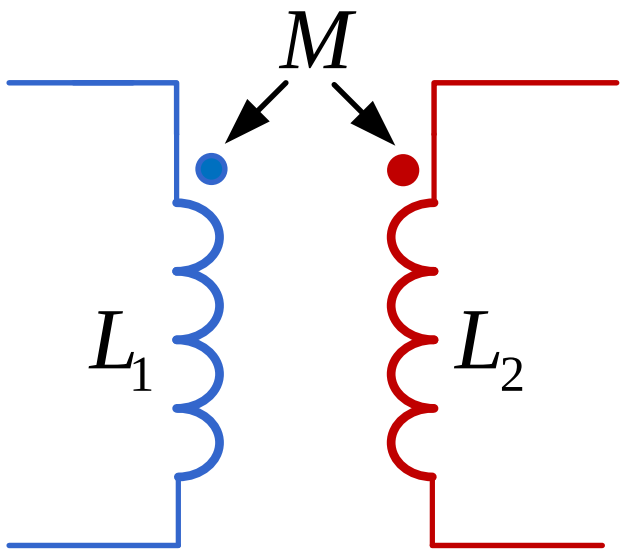
变压器的定义：

变压器即耦合电感，图形符号也相同。



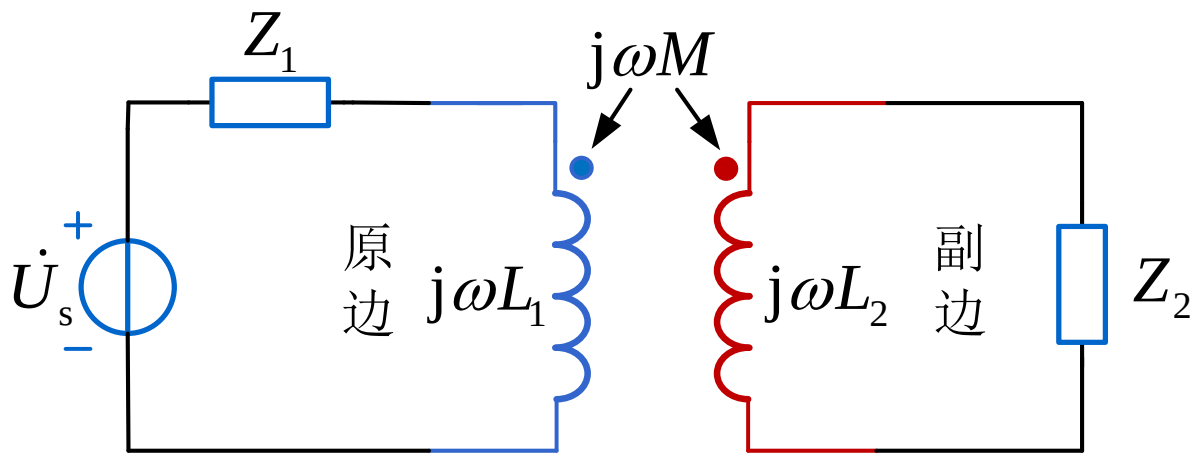
耦合电感（变压器）的耦合系数定义为：

$$k = \frac{M}{\sqrt{L_1 L_2}}$$



- 耦合系数最大值为1，称为全耦合，大于0.5，称为紧耦合，小于0.5称为松耦合
- 耦合系数与线圈尺寸、距离、相对位置、线圈缠绕的磁芯材料等因素有关
- 如果磁芯为非铁磁材料（含空气在内），磁导率近似为真空磁导率，称为空心变压器，如果磁芯为铁磁材料，称为铁心变压器

变压器的原边和副边：



- 变压器电源侧称为变压器的原边
- 变压器负载侧称为变压器的副边

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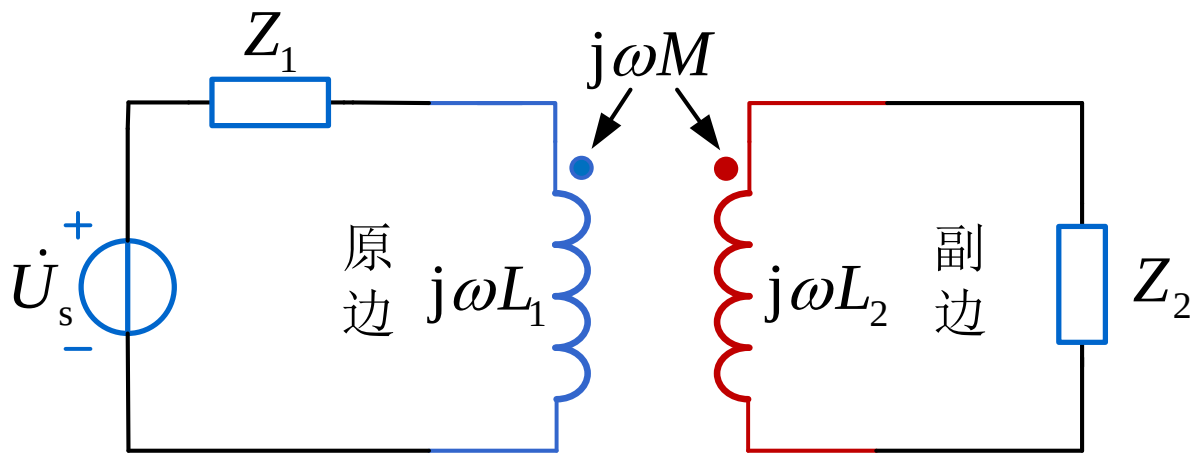

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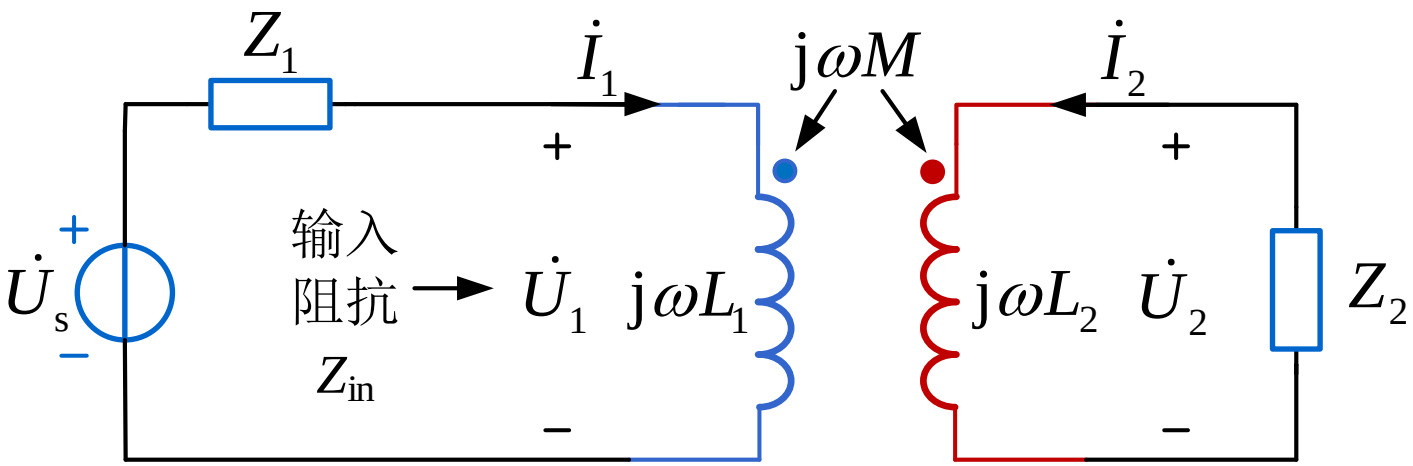
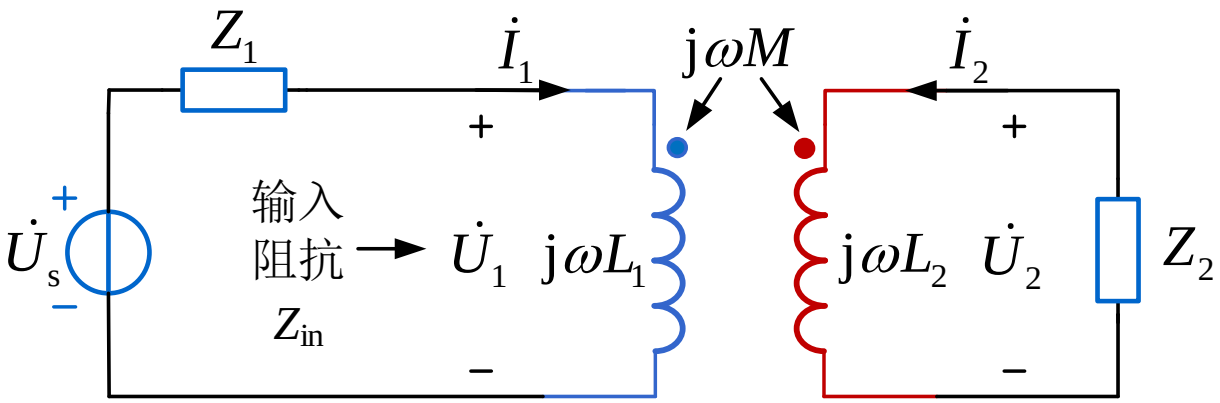
变压器电路的输入阻抗：

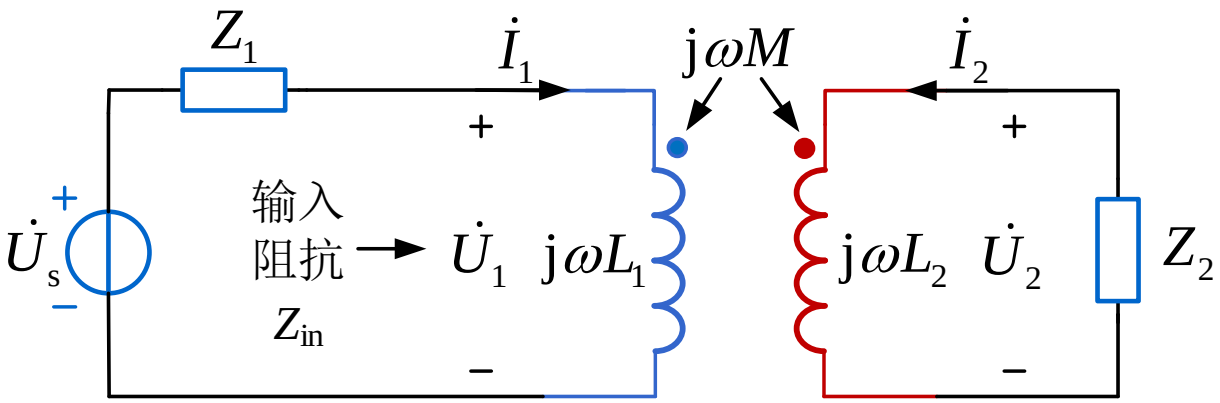


从变压器原边看进去的等效阻抗称为输入阻抗

变压器电路输入阻抗表达式的推导：



变压器电路输入阻抗表达式的推导：



$$-\dot{U}_s + Z_1\dot{I}_1 + j\omega L_1\dot{I}_1 + j\omega M\dot{I}_2 = 0$$

$$Z_2\dot{I}_2 + j\omega L_2\dot{I}_2 + j\omega M\dot{I}_1 = 0$$

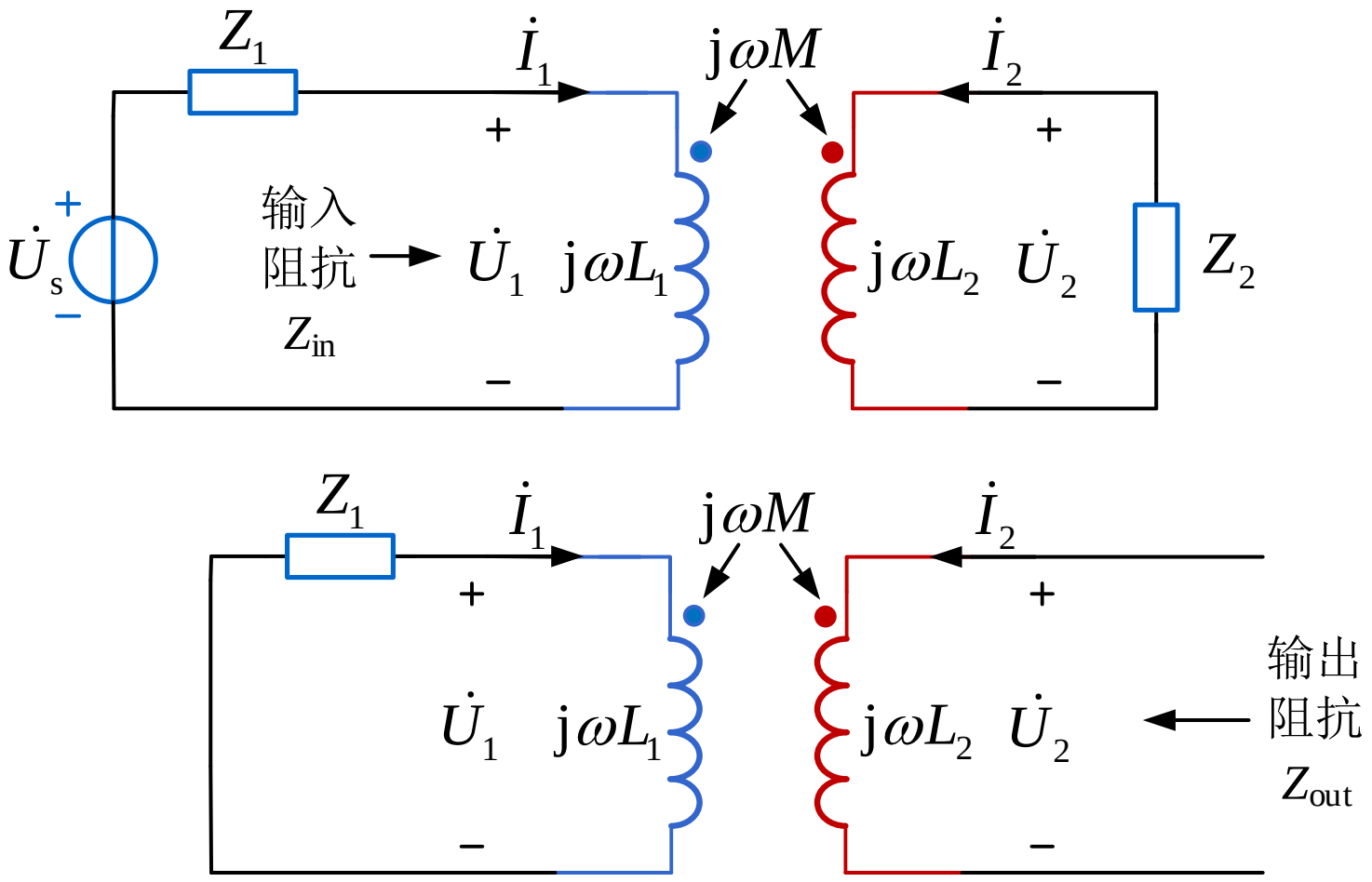
$$\Rightarrow \frac{\dot{U}_s}{\dot{I}_1} = Z_1 + j\omega L_1 + \frac{(\omega M)^2}{Z_2 + j\omega L_2}$$

$$Z_{in} = j\omega L_1 + \frac{(\omega M)^2}{Z_2 + j\omega L_2}$$

$Z_r = \dfrac{(\omega M)^2}{Z_2 + j\omega L_2}$ 称为反映阻抗，

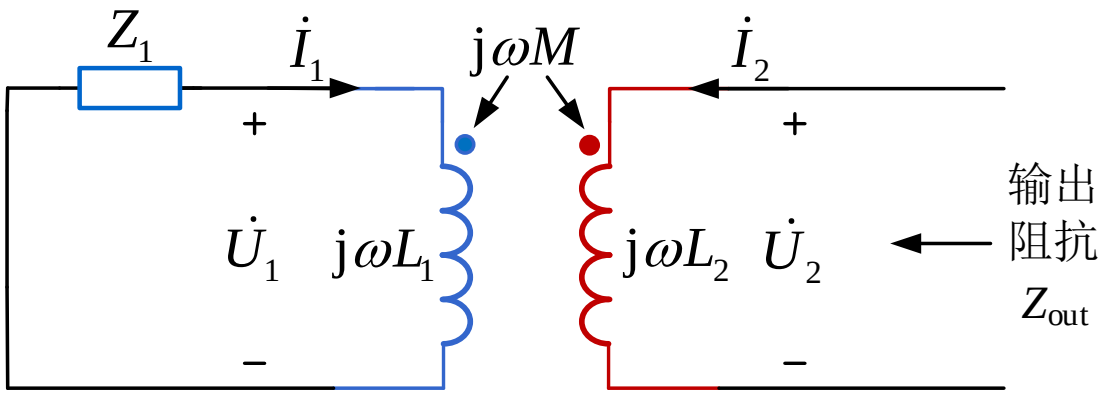
即将负载阻抗反映到原边的阻抗

变压器电路的输出阻抗：



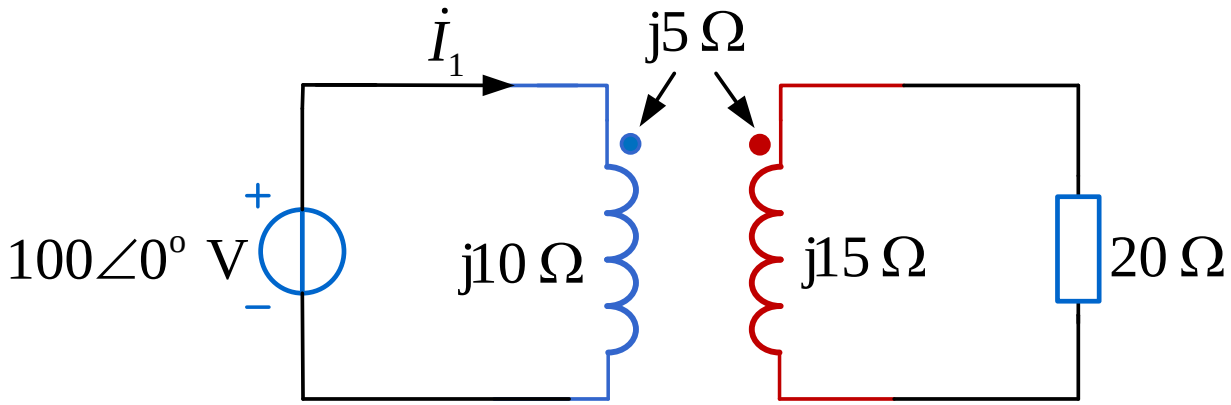
从变压器副边看进去（独立源置零）的等效阻抗称为输入阻抗

变压器电路的输出阻抗：



$$Z_{\text{out}} = \mathrm{j}\omega L_2 + \frac{(\omega M)^2}{Z_1 + \mathrm{j}\omega L_1}$$

例题1（基础） 求$\dot{I}_1$



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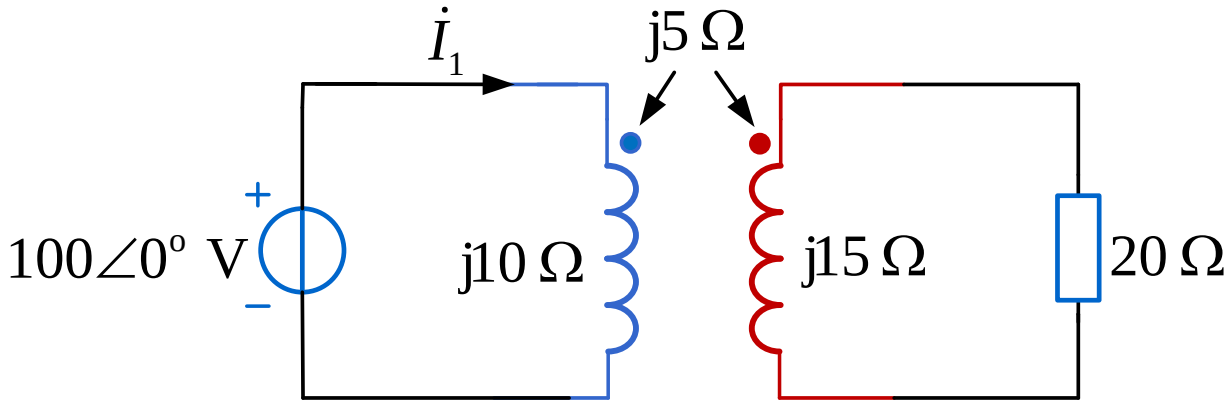
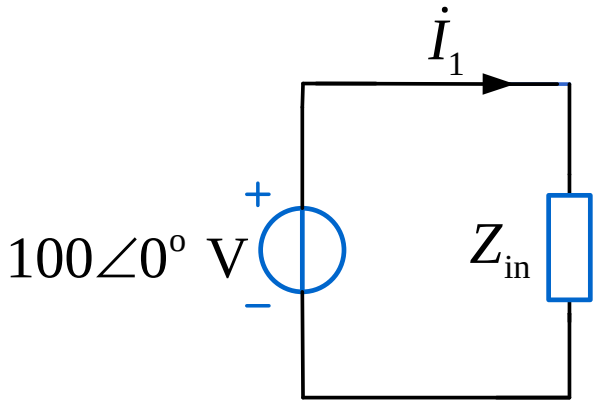
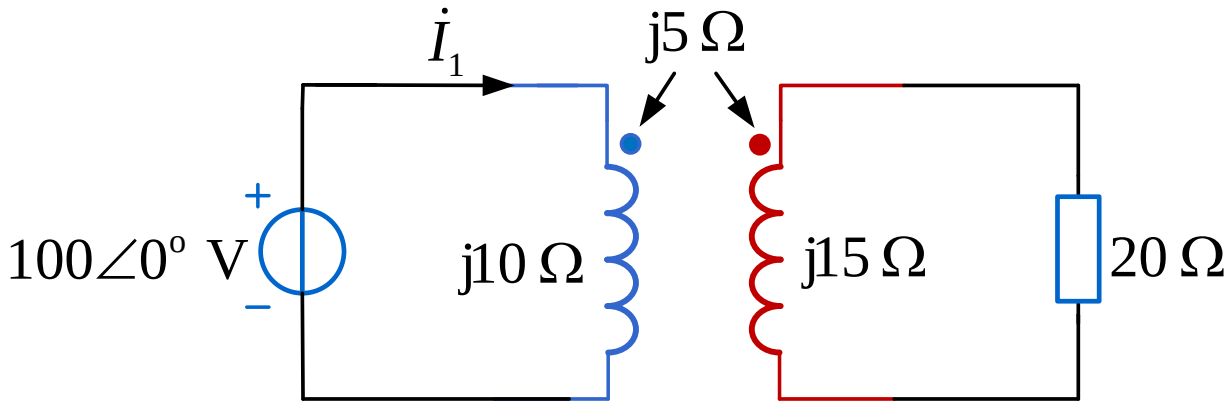




$$\dot{I}_1 = \frac{100}{Z_{in}} = \frac{100}{\mathrm{j}\omega L_1 + \dfrac{(\omega M)^2}{Z_2 + \mathrm{j}\omega L_2}}$$

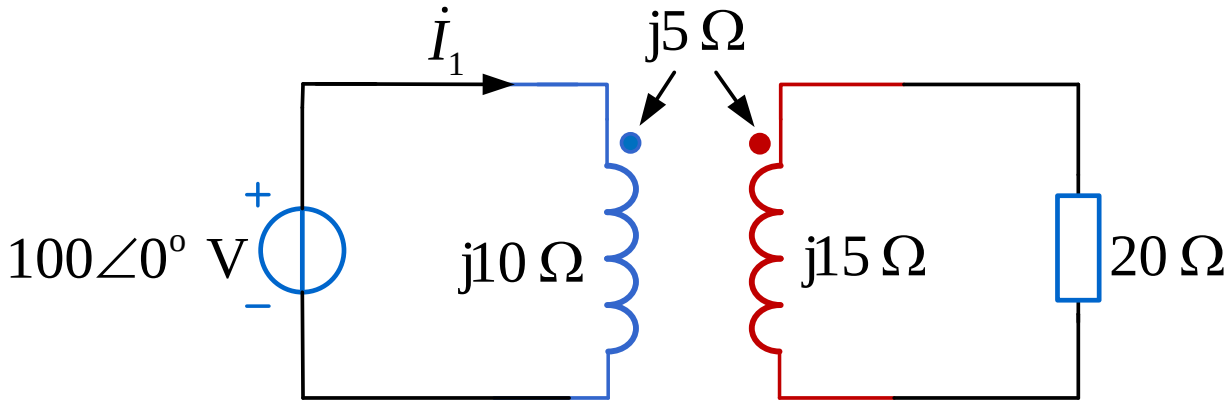
$$= \frac{100}{\mathrm{j}10 + \dfrac{5^2}{20 + \mathrm{j}15}} \approx 14.42\angle 86.8^\circ \ \mathrm{A}$$

例题1（基础） 求$\dot{I}_1$

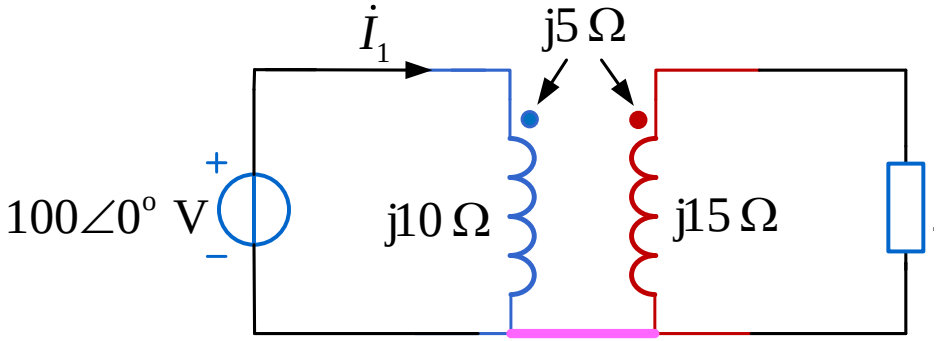


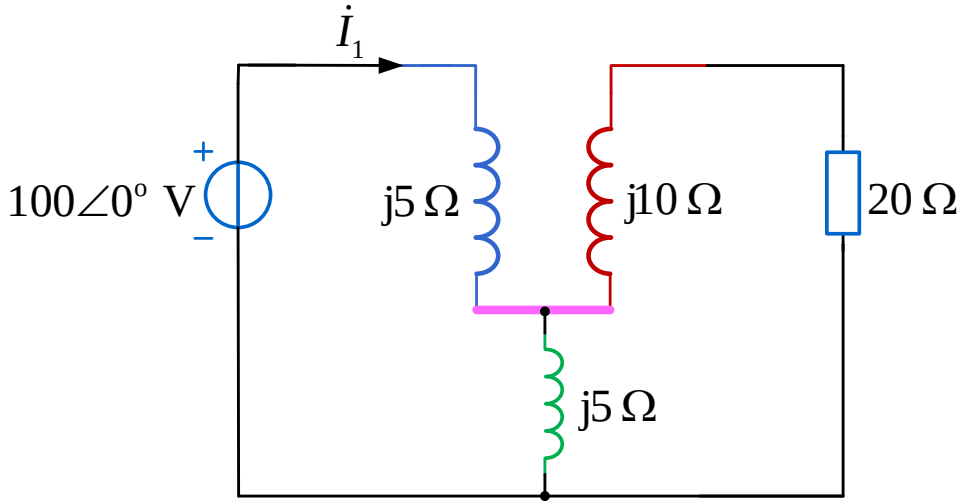
方法2：去耦等效

例题1（基础） 求$\dot{I}_1$

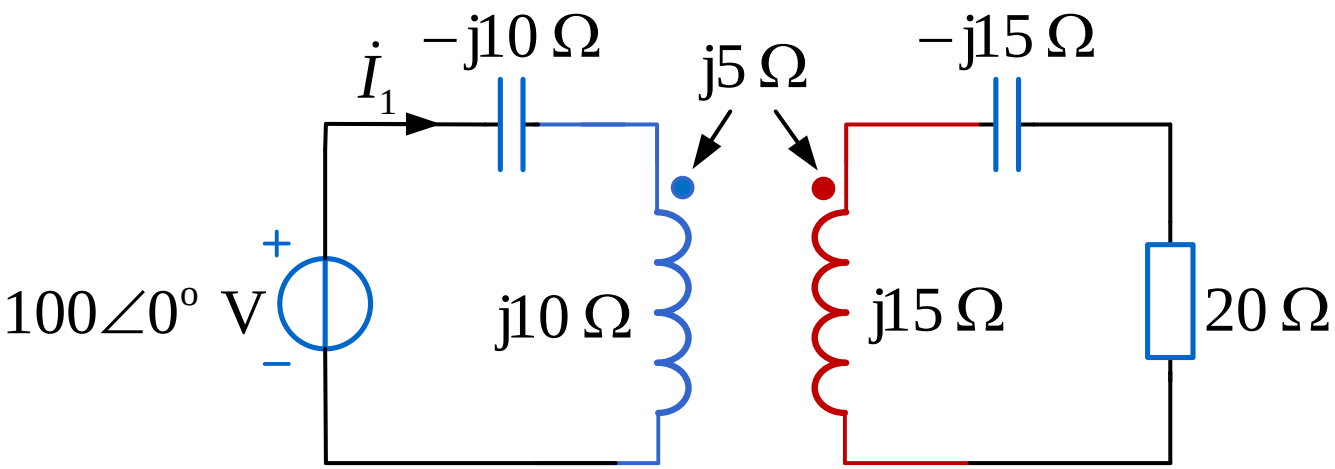


方法2：去耦等效





同步练习题1（基础）　求$\dot{I}_1$



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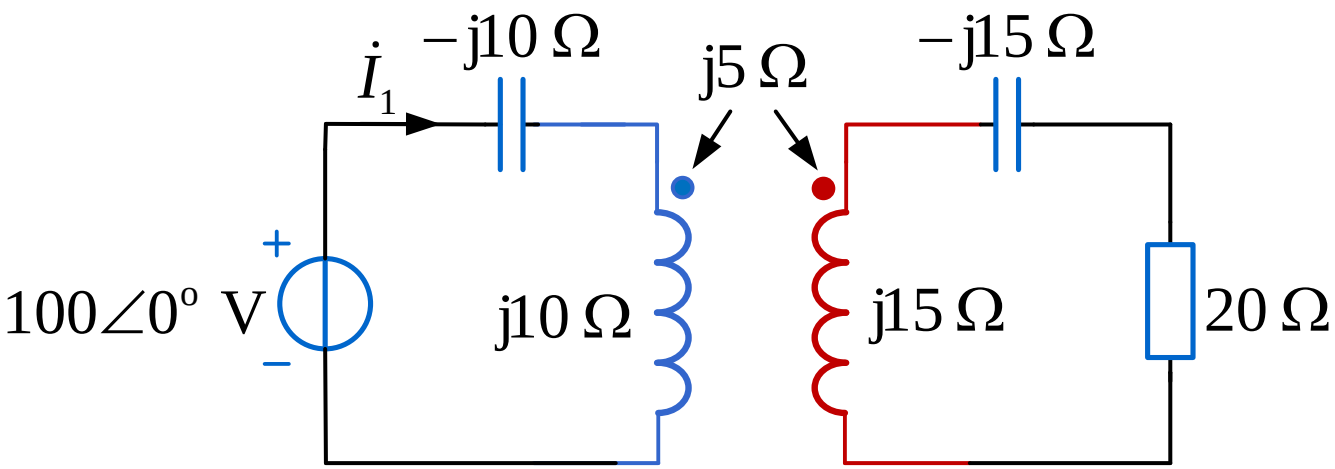
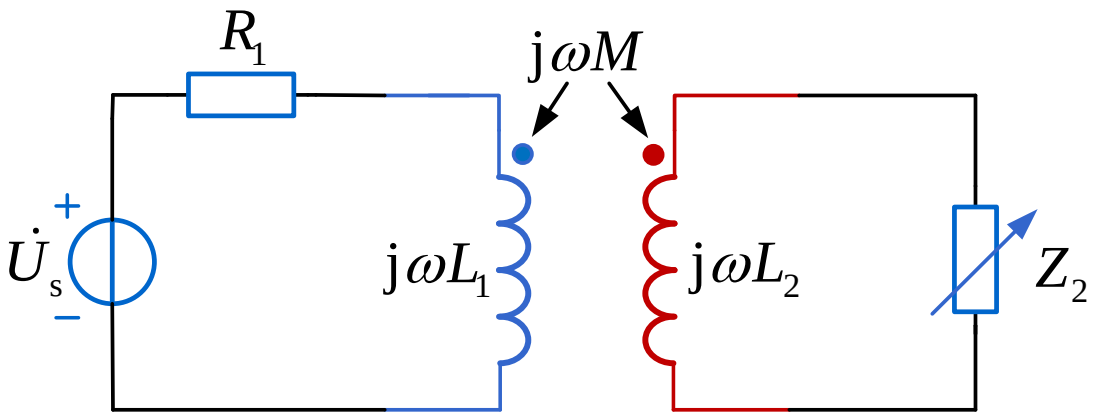


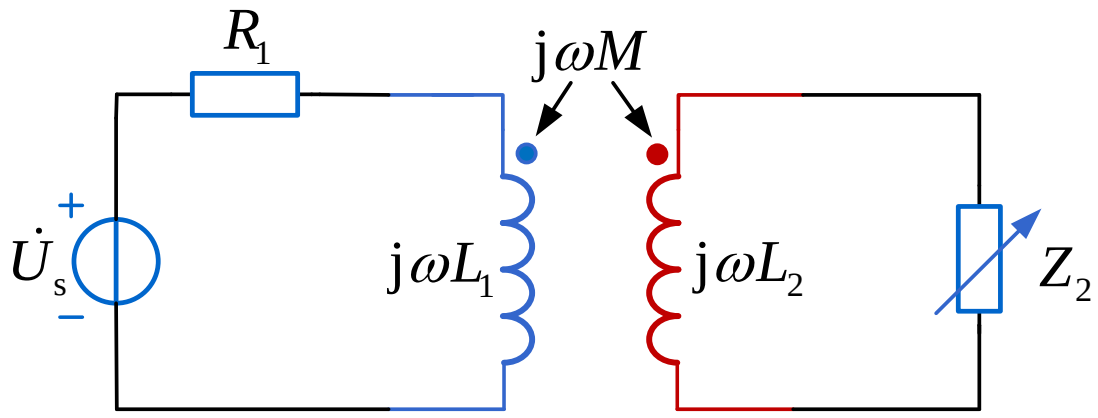
答案：$\dot{I}_1 = 80\angle 0^\circ$ A

例题2（提高）



图示变压器为全耦合变压器。已知$\dot{U}_s=20\angle 0^\circ$ V，$R_1=2\,\Omega$，$\omega L_1=2\,\Omega$，$\omega L_2=8\,\Omega$，求负载阻抗Z_2等于多少时，负载可以获得最大有功功率，并求此最大功率。

例题2（提高）



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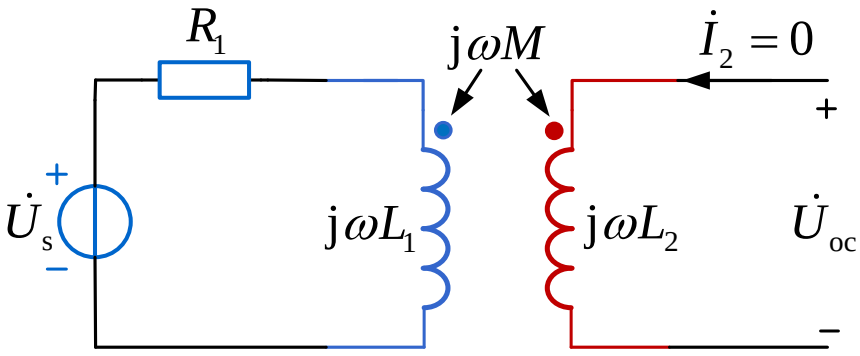
$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{\omega M}{\sqrt{\omega L_1 \omega L_2}} = 1 \quad \Rightarrow \omega M = 4\,\Omega$$



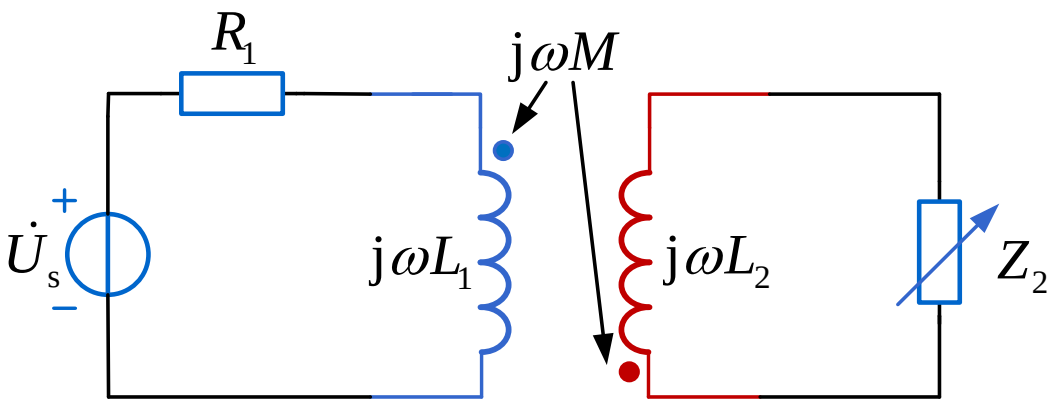
$$\dot{U}_{oc} = \mathrm{j}\omega M \frac{\dot{U}_s}{R + \mathrm{j}\omega L_1} = \mathrm{j}4 \times \frac{20}{2 + \mathrm{j}2} = 20\sqrt{2}\angle 45^\circ \text{ V}$$

$$Z_{eq} = Z_{out} = \mathrm{j}\omega L_2 + \frac{(\omega M)^2}{R_1 + \mathrm{j}\omega L_1} = \mathrm{j}8 + \frac{4^2}{2 + \mathrm{j}2} = 4 + \mathrm{j}4\,\Omega$$

$Z_2 = Z_{eq}^{\ *} = 4 - \mathrm{j}4\,\Omega$时获最大功率

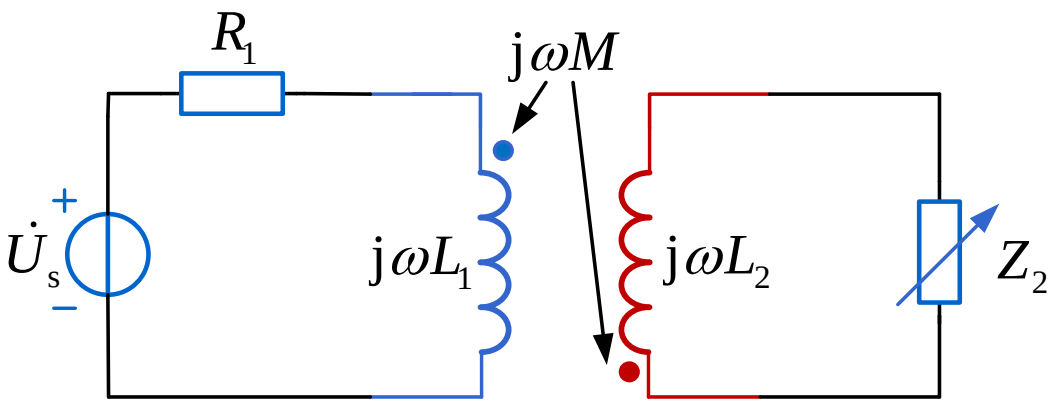
$$P_{max} = \frac{U_{oc}^{\ 2}}{4R_{eq}} = \frac{\left(20\sqrt{2}\right)^2}{4 \times 4} = 50 \text{ W}$$

同步练习题2（提高）



图示变压器为全耦合变压器。已知$\dot{U}_s = 20\angle 0^\circ$ V，$R_1 = 8\,\Omega$，$\omega L_1 = 8\,\Omega$，$\omega L_2 = 2\,\Omega$，求负载阻抗Z_2等于多少时，负载可以获得最大有功功率，并求此最大功率。

同步练习题2（提高）



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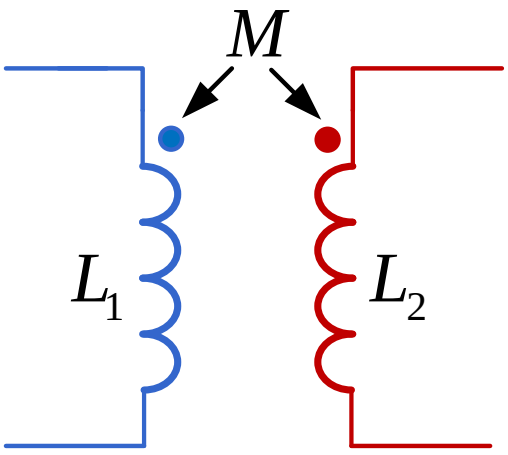
答案：$Z_2 = 1 - \mathrm{j}1\,\Omega$时获最大功率，$P_{\max} = 12.5$ W

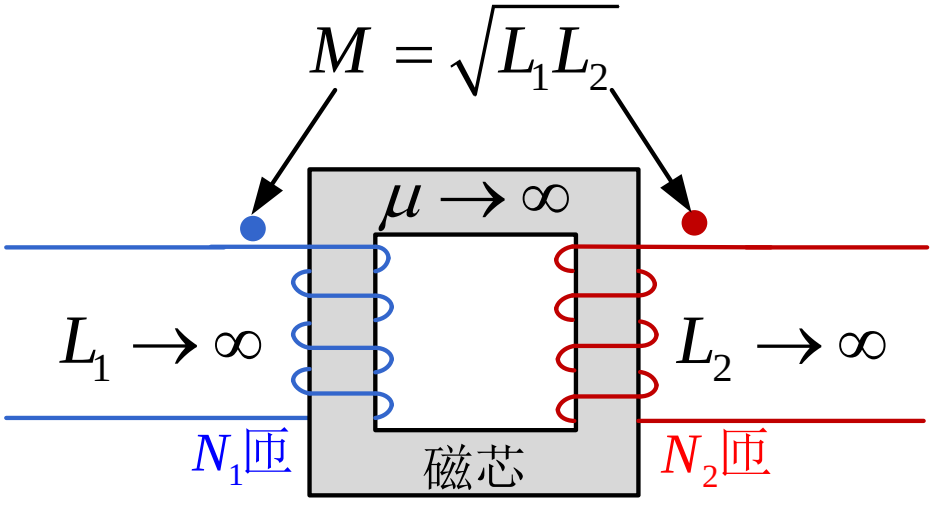
理想变压器的定义

满足以下3个条件的变压器称为理想变压器：

- 无损耗，即忽略线圈电阻损耗和所有其他损耗
- 磁场全耦合，即耦合系数等于1
- 自感值和互感值趋于无穷大



变压器



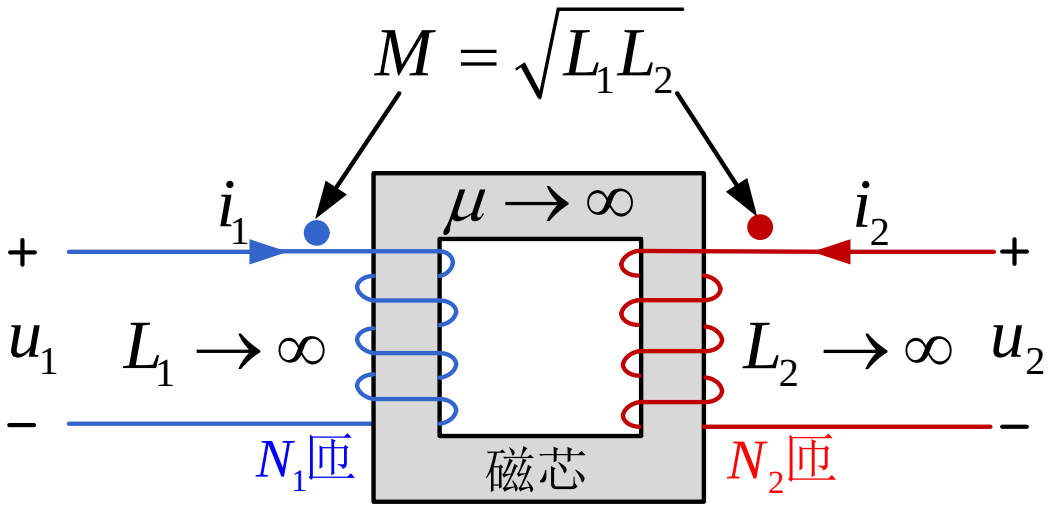
理想变压器

理想变压器特性的推导

$$\psi_1 = N_1\phi \qquad \psi_2 = N_2\phi$$

$$u_1 = \frac{\mathrm{d}\psi_1}{\mathrm{d}t} = \frac{\mathrm{d}(N_1\phi)}{\mathrm{d}t} = N_1\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

$$u_2 = \frac{\mathrm{d}\psi_2}{\mathrm{d}t} = \frac{\mathrm{d}(N_2\phi)}{\mathrm{d}t} = N_2\frac{\mathrm{d}\phi}{\mathrm{d}t}$$



$$\frac{u_1}{u_2} = \frac{N_1}{N_2}$$

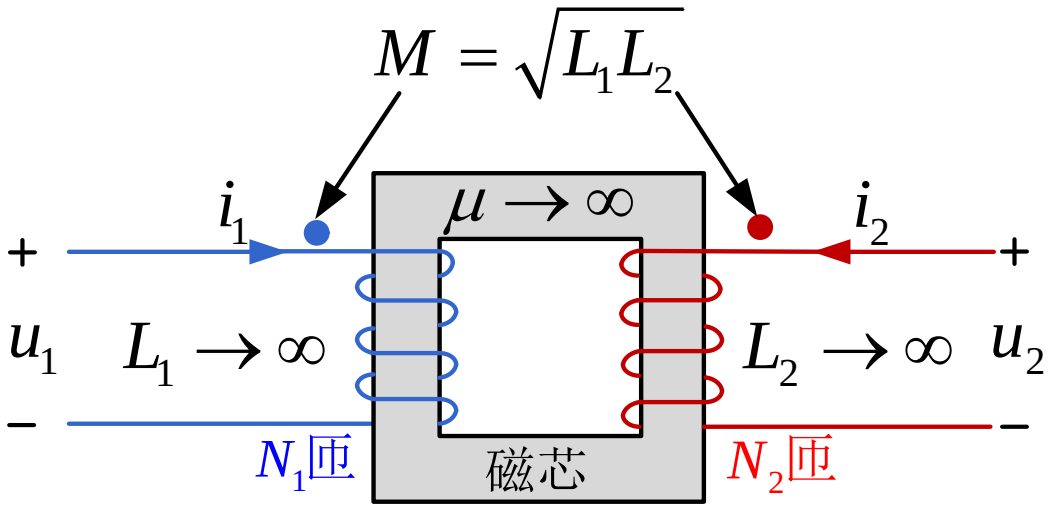
$$u_1 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t} = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + \sqrt{L_1 L_2}\frac{\mathrm{d}i_2}{\mathrm{d}t} = \sqrt{L_1}\left(\sqrt{L_1}\frac{\mathrm{d}i_1}{\mathrm{d}t} + \sqrt{L_2}\frac{\mathrm{d}i_2}{\mathrm{d}t}\right)$$

$$u_2 = L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} + M\frac{\mathrm{d}i_1}{\mathrm{d}t} = L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} + \sqrt{L_1 L_2}\frac{\mathrm{d}i_1}{\mathrm{d}t} = \sqrt{L_2}\left(\sqrt{L_2}\frac{\mathrm{d}i_2}{\mathrm{d}t} + \sqrt{L_1}\frac{\mathrm{d}i_1}{\mathrm{d}t}\right)$$

$$\frac{u_1}{u_2} = \frac{\sqrt{L_1}}{\sqrt{L_2}} = \frac{N_1}{N_2}$$

理想变压器特性的推导

$$\frac{u_1}{u_2}=\frac{\sqrt{L_1}}{\sqrt{L_2}}=\frac{N_1}{N_2}$$



$$u_1=L_1\frac{\mathrm{d}i_1}{\mathrm{d}t}+M\frac{\mathrm{d}i_2}{\mathrm{d}t}\Rightarrow\frac{\mathrm{d}i_1}{\mathrm{d}t}=\frac{u_1}{L_1}-\frac{M}{L_1}\frac{\mathrm{d}i_2}{\mathrm{d}t}=0-\frac{\sqrt{L_2}}{\sqrt{L_1}}\frac{\mathrm{d}i_2}{\mathrm{d}t}=-\frac{N_2}{N_1}\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$\frac{\mathrm{d}i_1}{\mathrm{d}t}=-\frac{N_2}{N_1}\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$i_1(t)-i_1(0)=-\frac{N_2}{N_1}\left[i_2(t)-i_2(0)\right]$$

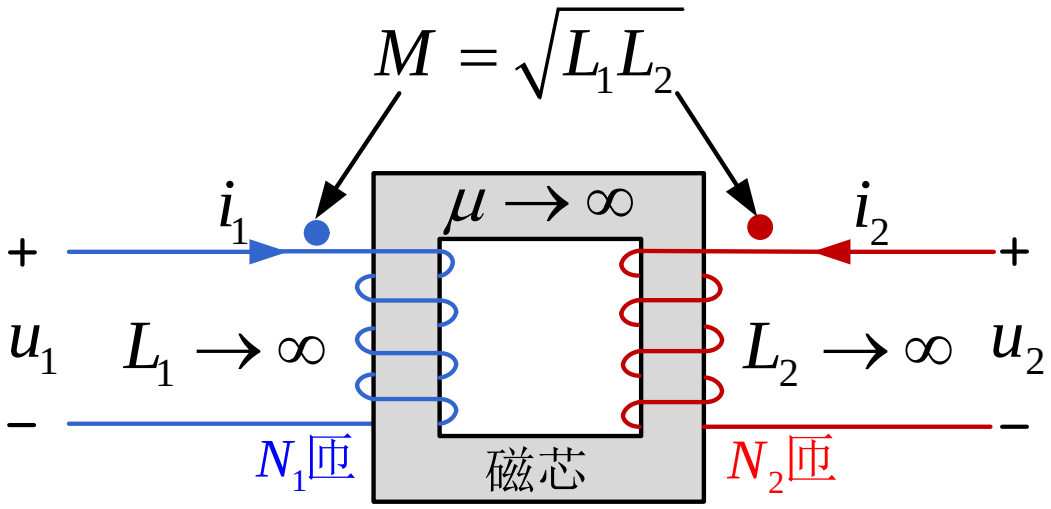
通常初始电流为零 $\Rightarrow\dfrac{i_1(t)}{i_2(t)}=-\dfrac{N_2}{N_1}$

$$\frac{i_1}{i_2}=-\frac{N_2}{N_1}$$

理想变压器特性的推导

$$\frac{u_1}{u_2}=\frac{N_1}{N_2}=n$$

$$\frac{i_1}{i_2}=-\frac{N_2}{N_1}=-\frac{1}{n}$$



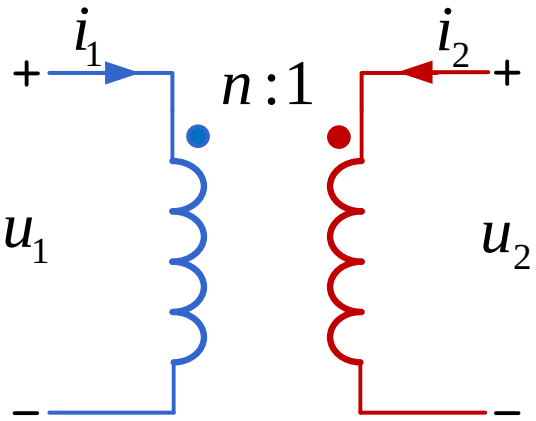
电流之比有负号是因为

副边电流参考方向与中学相反

$$\frac{u_1}{u_2}\times\frac{i_1}{i_2}=-1\Rightarrow u_1i_1+u_2i_2=0$$

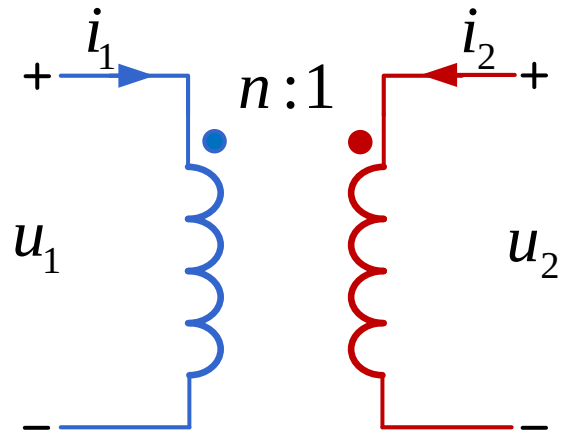
$$\Rightarrow p_1+p_2=0$$

理想变压器输入功率=输出功率

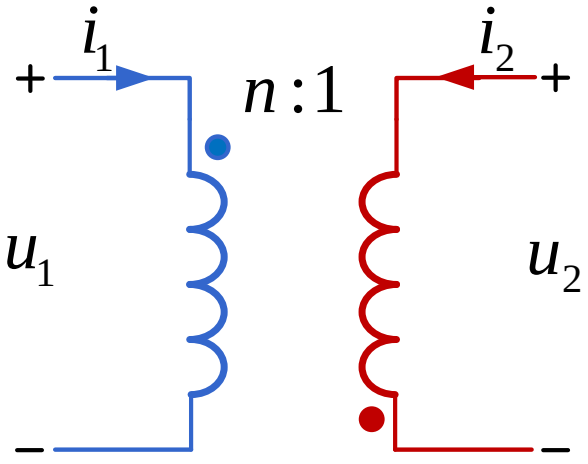


理想变压器的图形符号

理想变压器如果改变同名端位置，电压之比和电流之比的正负号将随之改变

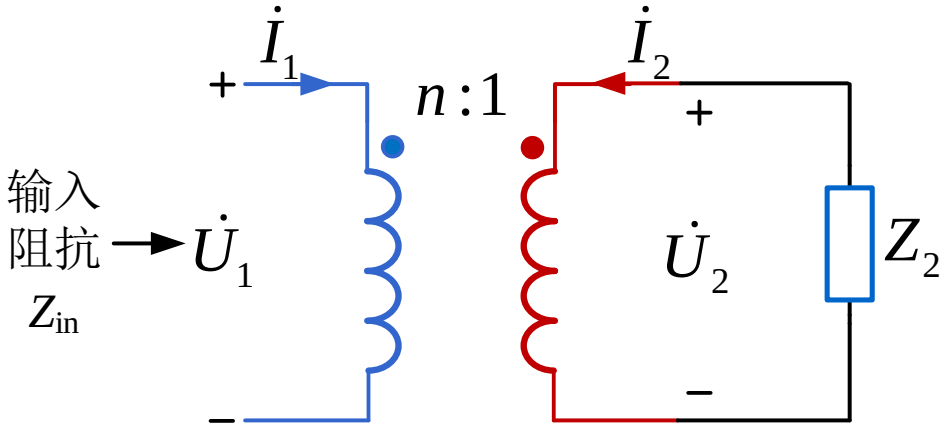


$$\frac{u_1}{u_2}=\frac{N_1}{N_2}=n \qquad \frac{i_1}{i_2}=-\frac{N_2}{N_1}=-\frac{1}{n}$$



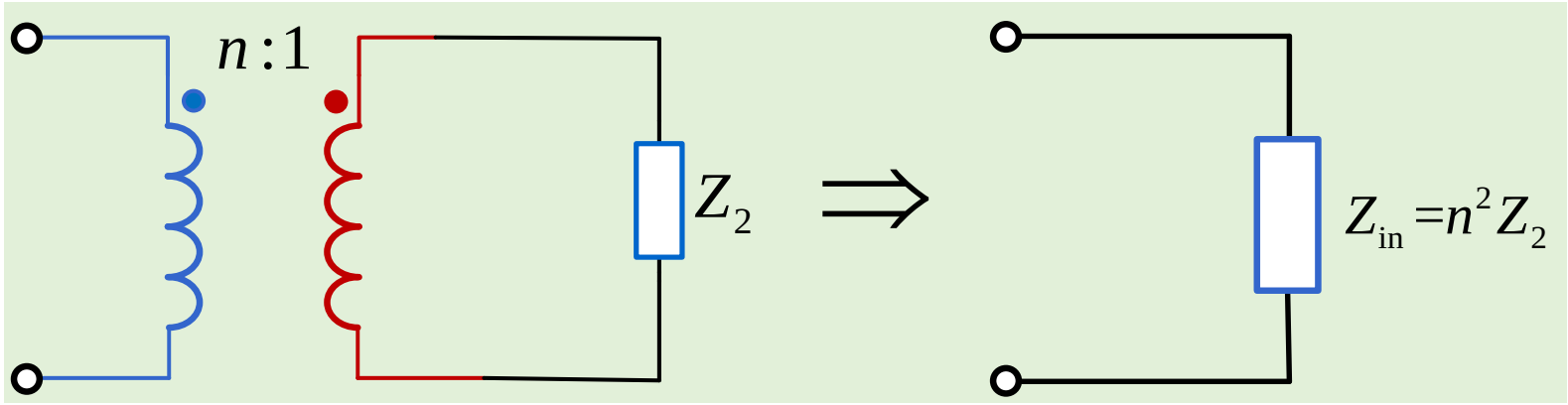
$$\frac{u_1}{u_2}=-\frac{N_1}{N_2}=-n \qquad \frac{i_1}{i_2}=\frac{N_2}{N_1}=\frac{1}{n}$$

理想变压器用于阻抗变换



$$Z_{in}=\frac{\dot{U}_1}{\dot{I}_1}=\frac{n\dot{U}_2}{-\frac{1}{n}\dot{I}_2}=n^2\times\frac{\dot{U}_2}{-\dot{I}_2}$$

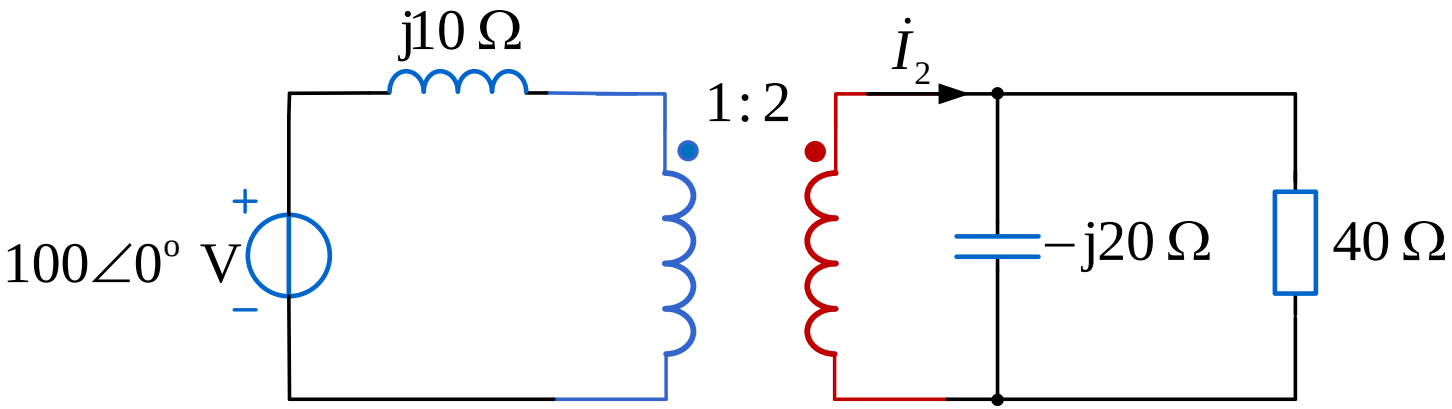
$$=n^2Z_2=\frac{N_1^{\ 2}}{N_2^{\ 2}}Z_2$$



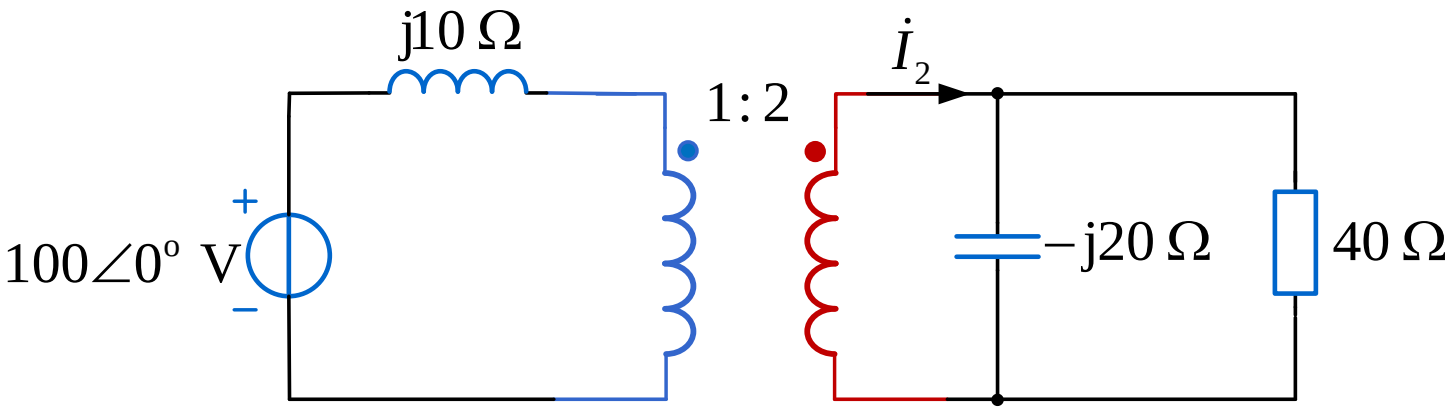
由于理想变压器的输入瞬时功率等于输出瞬时功率，

所以负载阻抗与等效的输入阻抗的功率相等（含无功和有功）

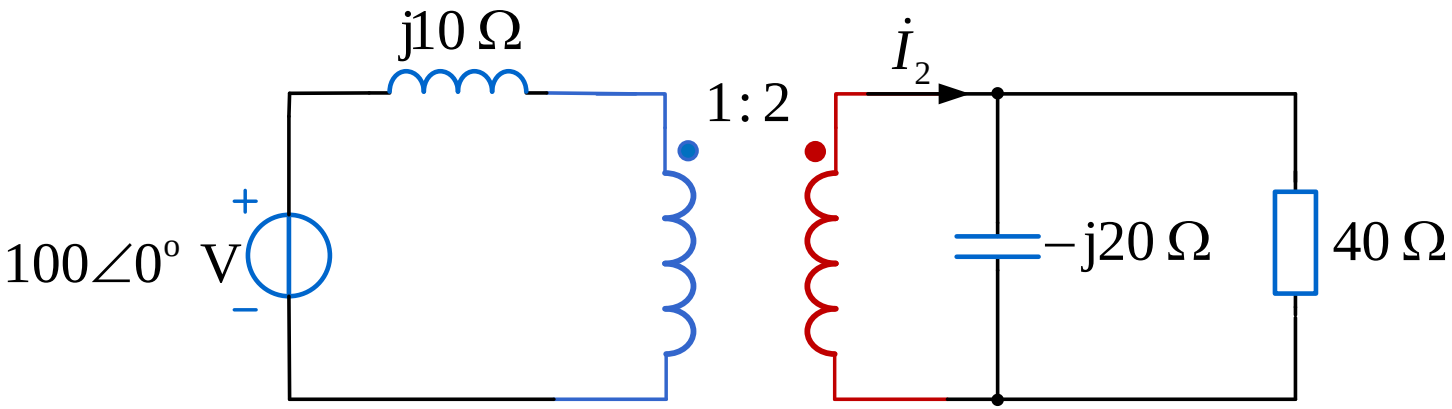
例题3（基础） 求$\dot{I}_2$、电阻的有功功率、电容的无功功率

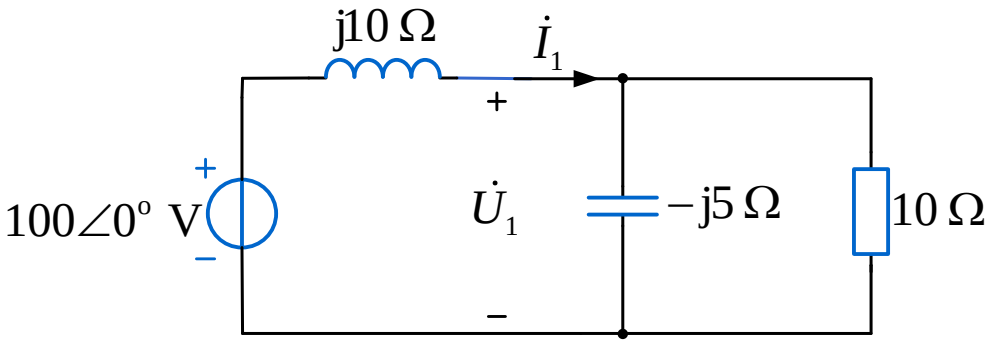


例题3（基础） 求$\dot{I}_2$、电阻的有功功率、电容的无功功率



例题3（基础） 求$\dot{I}_2$、电阻的有功功率、电容的无功功率



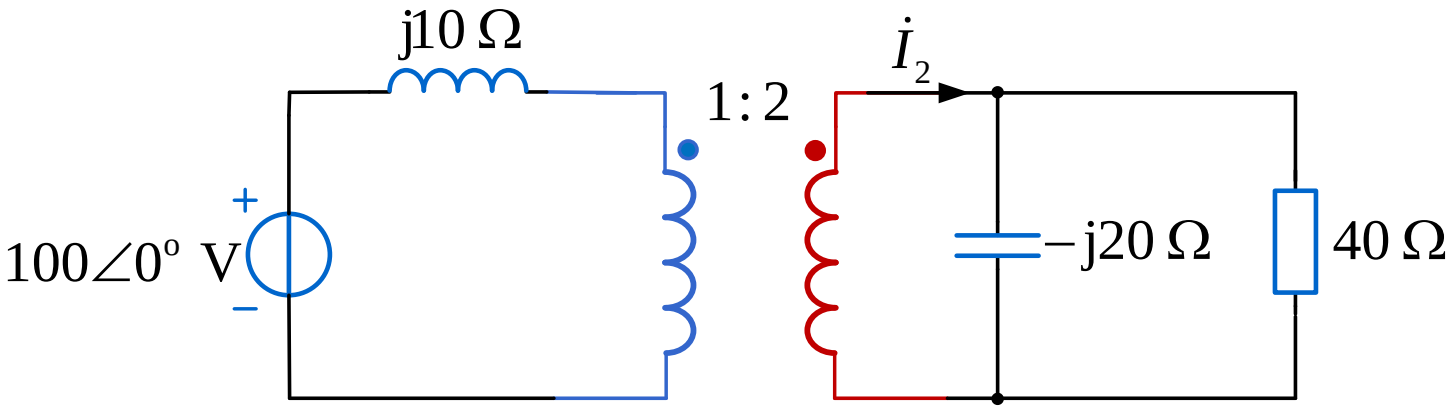


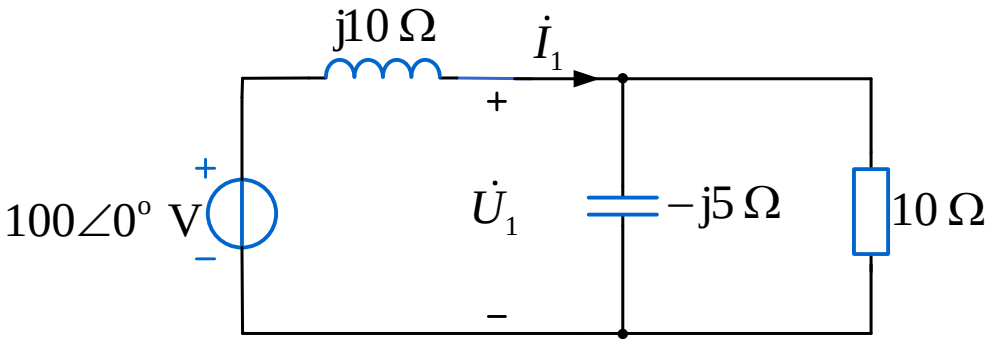
$$\left(\frac{1}{\text{j}10}+\frac{1}{-\text{j}5}+\frac{1}{10}\right)\dot{U}_1=\frac{100}{\text{j}10}$$

$$\dot{U}_1=50\sqrt{2}\angle-135^\circ\ \text{V}$$

$$P_R=\frac{U_1^{\,2}}{R}=\frac{\left(50\sqrt{2}\right)^2}{10}=500\ \text{W}\qquad Q_C=-U_CI_C=-\frac{U_1^{\,2}}{\dfrac{1}{\omega C}}=-\frac{\left(50\sqrt{2}\right)^2}{5}=-1000\ \text{var}$$

例题3（基础） 求$\dot{I}_2$、电阻的有功功率、电容的无功功率





$$\dot{U}_1 = 50\sqrt{2}\angle -135^\circ \text{ V}$$

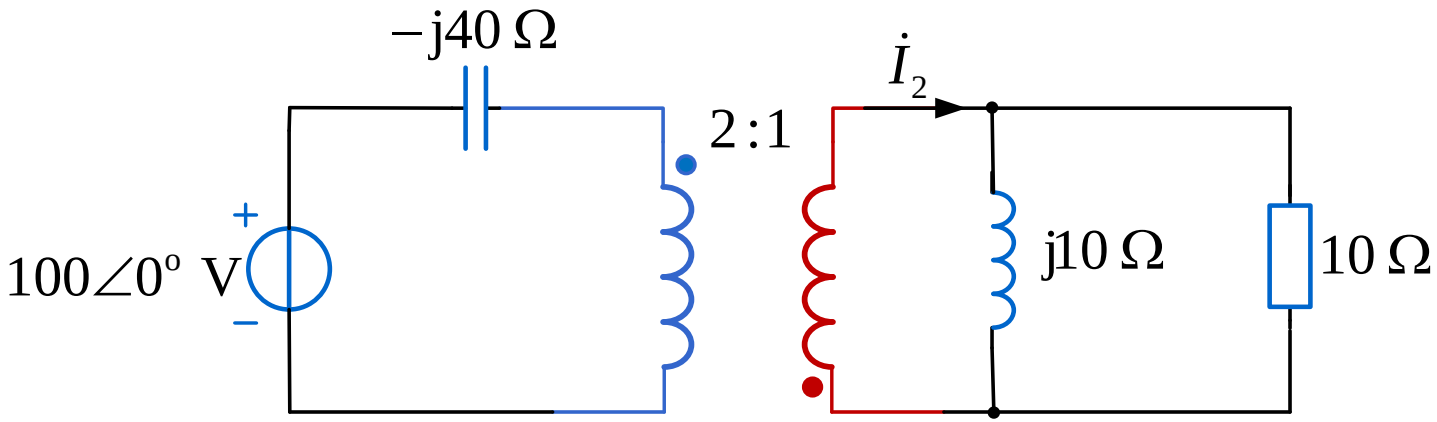
$$\dot{I}_2 = \frac{1}{2}\dot{I}_1$$

$$\dot{I}_1 = \frac{\dot{U}_1}{-\text{j}5} + \frac{\dot{U}_1}{10} = 5\sqrt{10}\angle -71.57^\circ \text{ A}$$

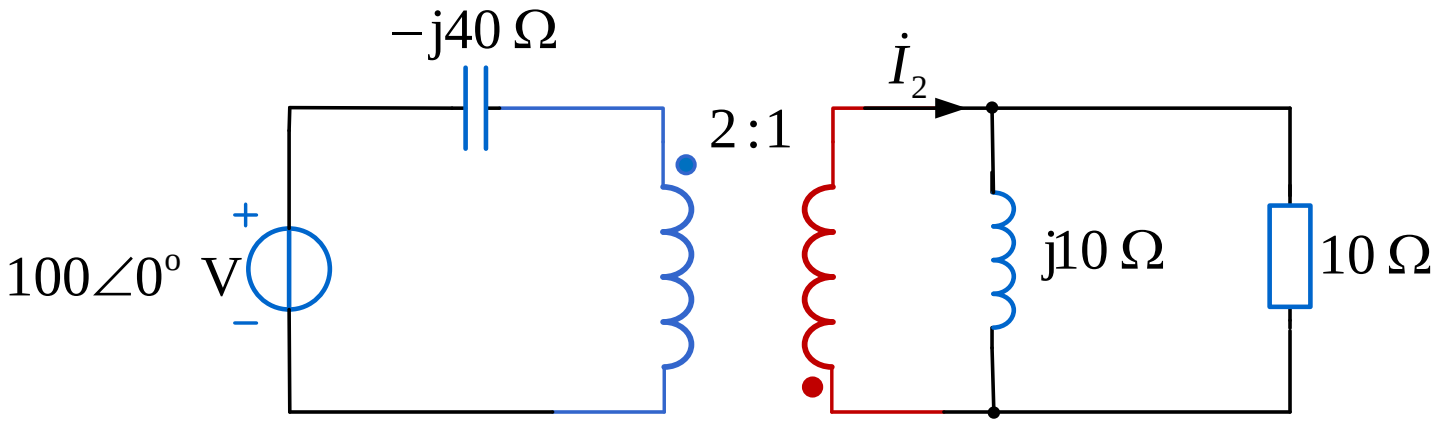
$$= 2.5\sqrt{10}\angle -71.57^\circ$$

$$\approx 7.91\angle -71.57^\circ \text{ A}$$

同步练习题3（基础） 求$\dot{I}_2$、电阻有功功率、电感无功功率



同步练习题3（基础） 求$\dot{I}_2$、电阻有功功率、电感无功功率



答案：$\dot{I}_2 = 5\angle -90^\circ$ A，电阻有功250 W，电感无功250 var

13.3-13.4 变压器、理想变压器——小结

- 变压器即耦合电感，变压器是从耦合电感可以变压的角度定义
- 变压器从磁芯材料角度分为两类：空心变压器、铁心变压器
- 耦合系数可定量衡量磁场耦合的程度，最大为1，称全耦合
- 变压器电路输入阻抗是从原边看进去的等效阻抗
- 变压器电路输出阻抗是从副边看进去（独立源置零）的等效阻抗
- 反映阻抗是副边阻抗反映到原边后的阻抗
- 满足三个条件的变压器称为理想变压器：无损耗、全耦合、自感互感无穷大
- 理想变压器电压之比等于匝数之比，电流之比等于匝数反比，比值的正负与同名端位置有关
- 理想变压器输入功率等于输出功率
- 理想变压器可以将负载阻抗等效变换到原边，等效阻抗等于负载阻抗乘以匝数比的平方，且等效阻抗功率等于负载阻抗功率

感谢大家聆听

主讲人：邹建龙

时　间：　　年　月　日