2. $\sigma = 0.3$

$$P\{E(X)-\varepsilon < X < E(X)+\varepsilon\}$$

$$= P\{-\varepsilon < X-E(X) < \varepsilon\}$$

$$= P\{|X-E(X)| < \varepsilon\} \geq 1-\frac{D(X)}{\varepsilon^2} = 1-\frac{0.3^2}{\varepsilon^2} \geq 0.9$$

$$\Rightarrow 1-\frac{0.09}{\varepsilon^2} \geq 0.9 \Rightarrow \varepsilon \geq \sqrt{0.9}$$

$$\varepsilon_{min} = \frac{3\sqrt{10}}{10}$$

5. 证明: 令 $X_k = \begin{cases} 1, & 第k次试验中事件A出现 \\ 0, & 第k次试验中事件A不出现 \end{cases}$

$$E(X_k) = p_k, \quad D(X_k) = p_k(1-p_k)$$

$$m = \sum_{k=1}^{n} X_k, \quad E(m) = \sum_{k=1}^{n} p_k$$

由切比雪夫~~不等~~大数定律

$$\forall \varepsilon > 0, \lim_{n\to\infty} P\left\{\left|\sum_{k=1}^{n} X_k - \sum_{k=1}^{n} p_k\right| < \varepsilon\right\} \geq 1-\frac{D(x)}{\varepsilon^2}$$

则 $\lim_{n\to\infty} P\left\{\left|\frac{1}{n}\sum_{k=1}^{n} X_k - \frac{1}{n}\sum_{k=1}^{n} p_k\right| < \varepsilon\right\} = 1$

7. $\lim\limits_{n\to\infty} E(X_n-X)^2=0$

$\Rightarrow \lim\limits_{n\to\infty}\left(E(X_n^2)+E(X^2)-2E(X_nX)\right)=0$

$\Rightarrow \lim\limits_{n\to\infty} E(X_n^2)-2E(X_n)E(X)+E(X^2)=0$

$\Rightarrow \lim\limits_{n\to\infty} X_n^{\cancel{E(X)}}=X$

9. $E(x_i)=0.5\times10+0.3\times9+0.8+0.05\times7+6\times0.05$

$=5+2.7+0.8+0.35+0.3$

$=8.85+0.3=9.15$

$\cancel{E(X)=8.85\times100=885}$

$\cancel{D(x_i)=8.85}$

$D(x_i)=1.2275$

$X\sim N(9.15\times100,\ 1.2275\times100)$

$X\sim N(915,\ 122.75)$

$\therefore P\{900<X<930\}=P\left\{\frac{900-915}{\sqrt{122.75}}<\frac{X-915}{\sqrt{122.75}}<\frac{930-915}{\sqrt{122.75}}\right\}$

$=0.823$

11. 由中心极限定理，n充分大时，

$\overline{X}\sim N(\mu,\frac{\sigma^2}{n})$

13. $n = 200$

$p = 5\%$

$np = 200 \times 5\% = 10$

$np(1-p) = 10 \times 95\% = 9.5$

$$P(X < n) = P\left(\frac{X-10}{\sqrt{9.5}} < \frac{n-10}{\sqrt{9.5}}\right) \geq 90\%$$

$\Rightarrow n \geq 13.94$

取 $n = 14$.

总机要备 14条外线.