

$$2. \sigma = 0.3$$

$$P\{E(x) - \varepsilon < X < E(x) + \varepsilon\}$$

$$= P\{-\varepsilon < X - E(x) < \varepsilon\}$$

$$= P\{|X - E(x)| < \varepsilon\} \geq 1 - \frac{D(x)}{\varepsilon^2} = 1 - \frac{0.3^2}{\varepsilon^2} \geq 0.9$$

$$\Rightarrow 1 - \frac{0.09}{\varepsilon^2} \geq 0.9 \Rightarrow \varepsilon \geq \sqrt{0.9}$$

$$\varepsilon_{\min} = \frac{\sqrt{10}}{10}$$

5. 证明: 令 $X_k = \begin{cases} 1, & \text{第 } k \text{ 次试验中事件 } A \text{ 出现} \\ 0, & \text{第 } k \text{ 次试验中事件 } A \text{ 不出现} \end{cases}$

$$E(X_k) = P_k, D(X_k) = P_k(1 - P_k)$$

$$m = \sum_{k=1}^n X_k, E(m) = \sum_{k=1}^n P_k$$

由切比雪夫不等式及大数定律.

$$\forall \varepsilon > 0, \lim_{n \rightarrow \infty} P\left\{\left|\sum_{k=1}^n X_k - \sum_{k=1}^n P_k\right| < \varepsilon\right\} \geq 1 - \frac{D(x)}{\varepsilon^2}$$

$$\text{则 } \lim_{n \rightarrow \infty} P\left\{\left|\frac{1}{n} \sum_{k=1}^n X_k - \frac{1}{n} \sum_{k=1}^n P_k\right| < \varepsilon\right\} = 1$$



$$7. \lim_{n \rightarrow \infty} E(X_n - X)^2 = 0$$

$$\Rightarrow \lim_{n \rightarrow \infty} (E(X_n^2) + E(X^2) - 2E(X_n X)) = 0$$

$$\Rightarrow \lim_{n \rightarrow \infty} E(X_n^2) - 2E(X_n)E(X) + E(X^2) = 0$$

$$\Rightarrow \lim_{n \rightarrow \infty} X_n = X$$

$$\begin{aligned} 9. E(X) &= 0.5 \times 10 + 0.3 \times 9 + 0.8 + 0.05 \times 7 + 6 \times 0.05 \\ &= 5 + 2.7 + 0.8 + 0.35 + 0.3 \\ &= 8.85 + 0.3 = 9.15 \end{aligned}$$

~~$$E(X) = 8.85 \times 100 = 885$$~~

~~$$D(X) = 8.85$$~~

$$D(X_i) = 1.2275$$

$$X \sim N(9.15 \times 100, 1.2275 \times 100)$$

$$X \sim N(915, 122.75)$$

$$\begin{aligned} \therefore P(900 < X < 930) &= P\left(\frac{900 - 915}{\sqrt{122.75}} < \frac{X - 915}{\sqrt{122.75}} < \frac{930 - 915}{\sqrt{122.75}}\right) \\ &= 0.823 \end{aligned}$$

11. 由中心极限定理, n 充分大时,
 $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$



$$13. \quad n = 200$$

$$p = 5\%$$

$$np = 200 \times 5\% = 10$$

$$np(1-p) = 10 \times 95\% = 9.5$$

$$P(X < n) = P\left(\frac{X-10}{\sqrt{9.5}} < \frac{n-10}{\sqrt{9.5}}\right) \geq 90\%$$

$$\Rightarrow n \geq 13.94$$

$$\text{取 } n = 14$$

总机要备 14 条外线。

$$z = 0.3$$

$$P\{E(n) - z < X < E(n) + z\}$$

$$P\{z > (X - E(n)) > z - 1\}$$

$$P\{z > |X - E(n)| > z - 1\}$$

$$P\{z > |X - E(n)| > z - 1\}$$

$$\frac{z}{\sigma} = 0.3$$

证明：
设 A 为事件“中位数不小于 14”，
则 A 的补事件为“中位数小于 14”。

$$P(X < 14) = P(X < 14)$$

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由正态分布的性质可知：

$$P(X < 14) = P\left(\frac{X-10}{\sqrt{9.5}} < \frac{14-10}{\sqrt{9.5}}\right) = P\left(\frac{X-10}{\sqrt{9.5}} < 1.28\right)$$

