1.3 ① $\dddot{y} - 3\ddot{y} + 4\dot{y} - 2y = u$ $\Rightarrow \dddot{y} = u + 3\ddot{y} - 4\dot{y} + 2y$

令 $\begin{cases} x_1 = y \\ x_2 = \dot{y} \\ x_3 = \ddot{y} \end{cases}$ $\cancel{x_4 = y}$ $\Rightarrow \begin{cases} \dot{x}_1 = \dot{y} = x_2 \\ \dot{x}_2 = \ddot{y} = x_3 \\ \dot{x}_3 = \dddot{y} = u + 3\ddot{y} - 4\dot{y} + 2y = u + x_3 - 4x_2 + 2x_1 \end{cases}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \quad D = 0$$

1.3 ②. $2\dddot{y} - 3\dot{y} = \ddot{u} - 2\dot{u}$

令 $\begin{cases} x_1 = 2\ddot{y} - 3\dot{y} - \dot{u} - 2u \\ x_2 = \dot{y} - 3y - u \\ x_3 = y \end{cases}$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

1.6 ① $A=\begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$,

$C=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $D=0$

$$G(s)=\frac{Y(s)}{U(s)}=C(sI-A)^{-1}B$$

$$=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{s+1} & 0 & \frac{1}{(s+1)(s+3)} \\ \frac{1}{(s+1)(2+s)} & \frac{1}{s+2} & \frac{1}{(s+1)(s+2)(s+3)} \\ 0 & 0 & \frac{1}{s+3} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} (s+2)(s+3) & 0 & s+2 \\ s+3 & (s+1)/(s+3) & 1 \\ 0 & 0 & (s+1)(s+2) \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}\times\frac{1}{(s+1)(s+2)(s+3)}$$

$$=\frac{1}{(s+1)(s+2)(s+3)}\begin{bmatrix} 2s^2+9s+10 & 2s^2+9s+10 \\ s^2+7s+12 & 3s^2+15s+18 \end{bmatrix}$$

1.9 ① $A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -25 & -35 & -11 \end{bmatrix}$

$$|\lambda I-A|=\lambda\begin{vmatrix} \lambda & -1 \\ 35 & 11+\lambda \end{vmatrix}+25\begin{vmatrix} -1 & 0 \\ \lambda & -1 \end{vmatrix}$$

$$=\lambda^3+11\lambda^2+35\lambda+25$$

$$=(\lambda+1)(\lambda+5)^2$$

$\lambda_3=-1$, $\lambda_1=\lambda_2=-5$

② $(\lambda_3 I-A)V_3=0 \Rightarrow V_3=\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

$(\lambda_1 I-A)V_1^{(1)}=0$

$\Rightarrow V_1^{(1)}=\begin{bmatrix} 1 \\ -5 \\ -25 \end{bmatrix}$

$(\lambda_2 I-A)V_1^{(2)}=-V_1^{(1)}$

$V_1^{(2)}=\begin{bmatrix} 1 \\ -4 \\ 15 \end{bmatrix}$

$P=\begin{bmatrix} 1 & 1 & 1 \\ -5 & -4 & -1 \\ -25 & 15 & -1 \end{bmatrix}$

$\bar{A}=P^{-1}AP=\begin{bmatrix} -5 & 1 & 0 \\ 0 & -5 & 0 \\ \cdots \end{bmatrix}$, $\bar{B}=P^{-1}B=\begin{bmatrix} \frac{3}{16} \\ -\frac{1}{4} \\ \frac{1}{16} \end{bmatrix}$

1.10. $\lambda_1=-3,\ \lambda_2=-1$

$$\lambda I - A = \begin{bmatrix} \lambda & -1 \\ a & b+\lambda \end{bmatrix}$$

$$|\lambda I - A| = \lambda^2 + b\lambda + a = (\lambda+3)(\lambda+1)$$

$\Rightarrow a=3,\ b=4$

$$\Rightarrow A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$$

$$c = [1\ 0\ \ 0]$$

$$\bar{c} = [-5\ \ 5]$$

令 $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$

$$\bar{C} = c \cdot P$$

$$[-5\ \ 5] = [1\ 0\ \ 0]\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \Rightarrow p_{11} = -\frac{1}{2},\ p_{12} = \frac{1}{2}$$

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix},\ \bar{A} = \begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\bar{A} = P^{-1} A P$$

$$\cancel{\begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}[}$$

$$\begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix} = P^{-1}\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} P$$

$$\Rightarrow P = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ d \end{bmatrix},\ \bar{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\bar{B} = P^{-1} B \Rightarrow d = 1$$

证明：特征值和传递函数的不变性。

① 特征值不变性

$$|\lambda I-\bar{A}|=|\lambda I-P^{-1}AP|$$

$$=|\lambda P^{-1}P-P^{-1}AP|$$

$$=|P^{-1}||\lambda I-A||P|$$

$$=|P^{-1}||P||\lambda I-A|$$

$$=|\lambda I-A|$$

② 传递函数不变性

$$G(s)=C(sI-A)^{-1}B+D$$

$$\bar{G}(s)=\bar{C}(sI-\bar{A})^{-1}\bar{B}+D$$

$$=CP(sI-P^{-1}AP)P^{-1}B+D$$

$$=C(sI-A)B+D$$

$$=G(s)$$