习题2.

1. (1) $P(X=-1)=\frac{2}{6}=\frac{1}{3}$ $\qquad P(X\le -1)=\frac{1}{3}$

$P(X=1)=\frac{3}{6}=\frac{1}{2}$ $\qquad P(X\le 1)=\frac{5}{6}$

$P(X=3)=\frac{1}{6}$ $\qquad P(X\le 3)=\frac{5}{6}+\frac{1}{6}=1$

则 $F(x)=\begin{cases}0 & (x<-1)\\ \frac{1}{3} & (-1\le x<1)\\ \frac{5}{6} & (1\le x<3)\end{cases}$ $\qquad F(x)=1\ (3\le x)$

(2) $P\{X\le 0\}=\frac{1}{3}$

$P\{-1<X\le 2\}=\frac{1}{2}$

$P\{-1\le x\le 2\}=\frac{5}{6}$

2. $P(x\le R)=k\cdot\pi R^2=1$

$\Rightarrow k=\frac{1}{\pi R^2}$

$\Rightarrow P(x\le r)=\frac{1}{\pi R^2}\times\pi r^2=\frac{r^2}{R^2}$

$F(x)=\begin{cases}\frac{x^2}{R^2}, & (0\le x\le R)\\ 0 & (x<0)\\ 0 & (x>R)\end{cases}$

5. (1) $P\{x<+\infty\}=\lim_{n\to\infty}a\left(1+\frac{1}{1}+\frac{1}{2!}+\cdots+\frac{1}{n!}\right)$

$=ae=1$

$\Rightarrow a=\frac{1}{e}$

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(2) $P\{x<\infty\} = \lim_{n\to\infty} a\left(\frac{1}{1\times2}+\frac{1}{2\times3}+\cdots+\frac{1}{n\times(n+1)}\right)$

$= \lim_{n\to\infty} a\left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots+\frac{1}{n}-\frac{1}{n+1}\right)$

$= \lim_{n\to\infty} a\left(1-\frac{1}{n+1}\right) = a = 1$

$a=1$

6. $P(x=-1)=0.125$

$P(x=0)=0.5$

$P(x=0.5)=0.25$

$P(x=1)=0.125$

X	-1	0	0.5	1
P	0.125	0.5	0.25	0.125

8. (1) 不放回: $P(X=0)=\frac{C_{12}^5}{C_{15}^5}=\frac{24}{91}$

$P(X=1)=\frac{C_3^1\times C_{12}^4}{C_{15}^5}=\frac{45}{91}$

$P(X=2)=\frac{C_3^2\times C_{12}^3}{C_{15}^5}=\frac{20}{91}$

$P(X=3)=\frac{C_3^3\times C_{12}^2}{C_{15}^5}=\frac{2}{91}$

X	0	1	2	3
P	$\frac{24}{91}$	$\frac{45}{91}$	$\frac{20}{91}$	$\frac{2}{91}$

(2) 放回. $P(X=0)=C_5^0\times\left(\frac{1}{5}\right)^0\times\left(\frac{4}{5}\right)^5=\frac{1024}{3125}$

$P(X=1)=C_5^1\times\left(\frac{1}{5}\right)^1\times\left(\frac{4}{5}\right)^4=\frac{256}{625}$

$P(X=2)=C_5^2\times\left(\frac{1}{5}\right)^2\times\left(\frac{4}{5}\right)^3=\frac{128}{625}$

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$$P(X=3) = C_5^3 \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right)^2 = \frac{32}{625}$$

$$P(X=4) = C_5^4 \left(\frac{1}{5}\right)^4 \times \left(\frac{4}{5}\right)^1 = \frac{4}{625}$$

$$P(X=5) = C_5^5 \left(\frac{1}{5}\right)^5 \times \left(\frac{4}{5}\right)^0 = \frac{1}{625}$$

9. (1)

$$P = C_5^0 (0.3)^0 \times (0.7)^5 + C_5^1 (0.3)^1 \times (0.7)^4 + C_5^2 \times (0.3)^2 \times (0.7)^3 = 0.1631$$

(2)

$$P = C_7^0 (0.3)^0 \times (0.7)^5 + C_7^1 (0.3)^0 \times (0.7)^4 + C_7^2 \times (0.3)^2 \times (0.7)^3 = 0.3529$$

11.

$$P(X=k) = \frac{4^k}{k!} e^{-4} \quad (k=0,1,2,\cdots)$$

(1)

$$P(X=6) = \frac{4^6}{6!} e^{-4} = 0.1042$$

(2)

$$P = P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= \frac{4^5}{5!} e^{-4} + \frac{4^6}{6!} e^{-4} + \frac{4^7}{7!} e^{-4} + \frac{4^8}{7!} e^{-4} + \frac{4^9}{9!} e^{-4} + \frac{4^{10}}{10!} e^{-4} = 0.3683$$

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14. (1) $P\{X=k\}=\frac{\lambda^k}{k!}e^{-\lambda}\quad (k=0,1,2,\cdots)$

$\frac{P\{X=k+1\}}{P\{X=k\}}=\frac{\lambda}{k+1} \Rightarrow \begin{cases}\frac{\lambda}{k}\geq 1\\ \frac{\lambda}{k+1}\leq 1\end{cases} \Rightarrow \lambda-1\leq k\leq\lambda$

由于k为整数，即 $k=[\lambda]$（λ不为整数），$k=\lambda$或$\lambda-1$，（λ为整数）

(2) $\lambda=4$.

$$P(X=k)=\frac{4^k}{k!}e^{-4}$$

$k=3$. $k=4$. 最可能的呼唤次数为4，3.

16. (1) $f(x)=Ae^{-|x|}=\begin{cases}Ae^{-x}, & x\geq 0\\ Ae^{x}, & x<0\end{cases}$

$$\int_{-\infty}^{+\infty}f(x)dx=A\left(\int_0^{+\infty}e^{-x}dx+\int_{-\infty}^{0}e^{+x}dx\right)$$

$$=2A=1 \Rightarrow A=\frac{1}{2}$$

(2) $P(-1<X<2)=\int_0^2\frac{1}{2}e^{-x}dx+\int_{-1}^{0}\frac{1}{2}e^{x}dx$

$$=\frac{1}{2}\left(2-e^{-1}-e^{-2}\right)$$

(3) $F(x)=\int_{-\infty}^{x}f(t)dt$

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① $t<0$ 时. $F(x)=\int_{-\infty}^{x}f(t)dt=\int_{-\infty}^{x}Ae^{t}dt=\frac{1}{2}\int_{-\infty}^{x}e^{t}dt$

$=\frac{1}{2}(e^{x})=\frac{1}{2}e^{x}$

② $t\geq 0$ 时 $F(x)=\frac{1}{2}\int_{-\infty}^{0}e^{t}dt+\frac{1}{2}\int_{0}^{x}e^{-t}dt$

$=\frac{1}{2}+\frac{1}{2}(1-e^{-x})=1-\frac{1}{2}e^{-x}$

则 $F(x)=\begin{cases}\frac{1}{2}e^{x} & (x<0)\\ 1-\frac{1}{2}e^{-x} & (x\geq 0)\end{cases}$

19. 证明：(1) $F(x)=P(X<x)$

$F(2\mu-x)=P(X<2\mu-x)$

$P(X<x)+P(X<2\mu-x)=1$

$\Rightarrow F(x)+F(2\mu-x)=1\Rightarrow F(x)=1-F(2\mu-x)$

当 $x=\mu$ 时, 有 $2F(\mu)=1\Rightarrow F(\mu)=\frac{1}{2}$.

(2) $\Phi(x)=P(X<x)$

$\Phi(-x)=P(X<-x)$

$P(X<x)+P(X<-x)=1$

$\Rightarrow \Phi(x)=1-\Phi(-x)$ 当 $x=0.5$ 时,

$\Rightarrow \Phi(0)=0.5$.

23. (1) $F(x)=\int_{-\infty}^{x} f(t)dt$

① $x<0$时, $F(x)=0$

② $0\le x<1$时, $F(x)=\int_{-\infty}^{x} 2t\,dt=x^2$

③ $x\geq 1$时, $F(x)=1$

$P(X\le\frac{1}{2})=F(\frac{1}{2})=(\frac{1}{2})^2=\frac{1}{4}$

$P\{Y=0\}=C_3^0(\frac{1}{4})\times(\frac{3}{4})^3=\frac{27}{64}$

$P\{Y=1\}=C_3^1(\frac{1}{4})^1\times(\frac{3}{4})^2=\frac{27}{64}$

$P\{Y=2\}=C_3^2(\frac{1}{4})^2\times(\frac{3}{4})^1=\frac{9}{64}$

$P\{Y=3\}=C_3^3\times(\frac{1}{4})^3\times(\frac{3}{4})^0=\frac{1}{64}$

Y	0	1	2	3
P	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

(2) $P\{Y=2\}=\frac{9}{64}$

24. (1)

X	-2	-1	0	1	2	3
Y_1	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
P	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{3}{10}$	$\frac{1}{30}$

(2) 由 Y_2 5 3 1 1 3 5

$\Rightarrow$

Y_2	1	3	5
P	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{10}$

(3) Y_3 -3 0 1 0 -3 -8

$\Rightarrow$

Y_3	-8	-3	0	1
P	$\frac{1}{30}$	$\frac{13}{30}$	$\frac{13}{30}$	$\frac{1}{6}$

(4) Y_4 0 $\frac{\sqrt{2}}{2}$ 1 $\frac{\sqrt{2}}{2}$ 0 $-\frac{\sqrt{2}}{2}$

$\Rightarrow$

Y_4	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1
P	$\frac{1}{30}$	$\frac{11}{30}$	$\frac{13}{30}$	$\frac{1}{6}$

27. (1) $f_1(y)\,dy = f(x)\,dx = \frac{1}{\pi}dx$

$dy = \frac{1}{\cos^2 x}dx$

$\Rightarrow f_1(y) = \frac{\cos^2 x}{\pi} = \frac{1}{\pi}\frac{1}{1+y^2} \quad (-\infty < y < +\infty)$

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30. $y<0$ 时. $F_Y(y)=0$.

$y\geq 1$ 时 $F_Y(y)=1$

$0<y<1$ 时. $F_Y(y)=P(Y\leq y)=\int_0^y \frac{1}{2}dx=\frac{y}{2}$

27. (2) $f_2(y)\cdot dy=\frac{1}{\pi}dx$

$dy=-\sin x\cdot dx$

$f_2(y)=\frac{-1}{\pi}\ \frac{1}{\sin x}=$

$$f(x)=\begin{cases}\frac{1}{\pi}. & x\in(-\frac{\pi}{2},\frac{\pi}{2})\\ 0 & x\leq\frac{\pi}{2},\ x\geq\frac{\pi}{2}\end{cases}$$

$$F(x)=\begin{cases}0 & ,x<-\frac{\pi}{2}\\ \frac{x}{\pi} & ,-\frac{\pi}{2}\leq x<\frac{\pi}{2}\\ 1 & ,x\geq\frac{\pi}{2}\end{cases}$$

设 $y=\cos x$

$x=\arccos y$

$F_2(y)=P\{Y\leq y\}=P\{\cancel{Y\leq\cos x}\}$

$=\cancel{P\{\cos x\leq\arcsin u\}}=P\{X\leq\arccos y\}$

$=\frac{\arccos y}{\pi}$. $f_2(y)=\frac{1}{\pi}\frac{}{\sqrt{1-y^2}}$

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$$F_Y(y) = \int_{-\frac{\pi}{2}}^{-\arccos y} \frac{1}{\pi}\,dx + \int_{\arccos y}^{\frac{\pi}{2}} \frac{1}{\pi}\,dx$$

$$f(y) = F_Y(y)' = \frac{2}{\pi\sqrt{1-y^2}} \quad (0 \le y \le 1)$$

$$\therefore f(y) = \begin{cases} \frac{2}{\pi\sqrt{1-y^2}} & (0 \le y \le 1) \\ 0 & y > 1, y < 0 \end{cases}$$