3.11.3.(1) $k\theta_i = J\ddot{\theta}_o + b\theta_o + k\theta_o$

(2) $$\begin{cases} k_2(x_2-x_1) - kx_1 - b_1\dot{x}_1 = m_1\ddot{x}_1 \\ k_2(x_1-x_2) - b_2\dot{x}_2 = m_2\ddot{x}_2 \end{cases}$$

(3) $$\begin{cases} k_2(x_2-x_1) - k_1x_1 - b\dot{x}_1 = m_1\ddot{x}_1 \\ -(x_2-x_1)k_2 = m_2\ddot{x}_2 \end{cases}$$

3.11.7

$$\begin{cases} -k_1\theta_1 + k_0(\theta_2-\theta_1) = J_1\ddot{\theta}_1 \\ -k_0(\theta_2-\theta_1) - k_2\theta_2 = J_2\ddot{\theta}_2 \end{cases}$$

3.11.8　A 动能：$\frac{1}{2}\frac{W_1}{g}\dot{x}_1^2+\frac{1}{2}J_A\cdot w_A^2$

$J_A=\frac{1}{2}\frac{W_1}{g}\cdot R^2$，$\cancel{x_1}$ $w_A=\frac{\dot{x}_1}{R}$

B 动能：$\frac{1}{2}\frac{W_2}{g}\dot{y}_E^2+\frac{1}{2}J_B\cdot w_B^2$

$J_B=\frac{1}{2}\frac{W_2}{g}r^2$，$w_B=\frac{\dot{y}_E+\dot{x}_1}{r}$

$V=-W_1\cdot x_1\cdot\sin\alpha-W_2\cdot y_E$，

$$L=T-V=\frac{1}{2}\frac{W_1}{g}\cdot\dot{x}_1^2+\frac{1}{2}\times\frac{1}{2}\times\frac{W_1}{g}\cdot R^2\cdot\frac{\dot{x}_1^2}{R^2}$$

$$+\frac{1}{2}\frac{W_2}{g}\dot{y}_E^2+\frac{1}{2}\times\frac{1}{2}\times\frac{W_2}{g}\cdot r^2\times\frac{(\dot{y}_E+\dot{x}_1)^2}{r^2}$$

$$+W_1\cdot x_1\cdot\sin\alpha+W_2\cdot y_E$$

$$\frac{\partial L}{\partial \dot{x}_1}=\frac{W_1}{g}\dot{x}_1+\frac{W_1}{4g}\times2\cdot\dot{x}_1+\frac{W_2}{4g}\times2(\dot{y}_E+\dot{x}_1)$$

$$=\frac{3W_1}{2g}\dot{x}_1+\frac{W_2}{2g^2}(\dot{y}_E+\dot{x}_1)$$

$$\frac{\partial L}{\partial \dot{y}_E}=\frac{3}{2}\frac{W_2}{g}\dot{y}_E+\frac{W_2}{2g_2}\dot{x}_1$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x_1}=\left(\frac{3}{2}\frac{W_1}{g}+\frac{1}{2}\frac{W_2}{g}\right)\ddot{x}_1+\frac{1}{2}\frac{W_2}{g}\ddot{y}_E-W_1\sin\alpha=0$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{y}_E}\right)-\frac{\partial L}{\partial y_E}=\frac{3}{2}\frac{W_2}{g}\ddot{y}_E+\frac{1}{2}\frac{W_2}{g}\ddot{x}_1-W_2=0$$

3.11.9 A: 动能 $\frac{1}{2}V_A^2 \cdot m_1 = \frac{1}{2}\dot{x}_1^2 \cdot m_1$

B: 动能 $\frac{1}{2}m_2 V_B^2 = \frac{1}{2}m_2(\dot{x}_1 + \dot{x}_2\cos\alpha)^2 + \frac{1}{2}m_2(\dot{x}_2\sin\alpha)^2$

$$\therefore T = \frac{1}{2}\dot{x}_1^2 \cdot m_1 + \frac{1}{2}m_2[(\dot{x}_1 + \dot{x}_2\cos\alpha)^2] + \frac{1}{2}[\dot{x}_2^2\sin^2\alpha] \cdot m_2$$

$$= \frac{1}{2}\dot{x}_1^2 \cdot m_1 + \frac{1}{2}m_2(\dot{x}_1^2 + \dot{x}_2^2 + 2\dot{x}_1\dot{x}_2\cos\alpha)$$

$$V = \frac{1}{2}kx_2^2$$

$$L = T - V = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2(\dot{x}_1^2 + \dot{x}_2^2 + 2\dot{x}_1\dot{x}_2\cos\alpha) - \frac{1}{2}kx_2^2$$

$$\frac{\partial L}{\partial \dot{x}_1} = m_1\dot{x}_1 + m_2\dot{x}_1 + m_2\dot{x}_2\cos\alpha$$

$$\frac{\partial L}{\partial \dot{x}_2} = m_2\dot{x}_2 + m_2\dot{x}_1\cos\alpha$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = m_1\ddot{x}_1 + m_2\ddot{x}_1 + m_2\ddot{x}_2\cos\alpha - 0 = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} = m_2\ddot{x}_1 + kx_2 + m_2\ddot{x}_1\cos\alpha = 0$$

3.11.10. $T = \frac{1}{2}M(R\cdot\dot{\theta})^2 + \frac{1}{2}M(W_0\cdot R\sin\theta)^2$

$V = MgR(1-\cos\theta)$

$L = T - V = \frac{1}{2}MR^2\dot{\theta}^2 + \frac{1}{2}MW_0^2\cdot R^2\cdot\sin^2\theta$
$- MgR(1-\cos\theta)$

$\frac{dL}{d\dot{\theta}} = MR^2\cdot\dot{\theta}$, $\frac{\partial L}{\partial\theta} = \frac{1}{2}MW_0^2R^2\cdot 2\sin\theta\cos\theta$
$- MgR\sin\theta$

$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = MR^2\ddot{\theta} - (MW_0^2R^2\sin\theta\cos\theta$
$-MgR\sin\theta)$

$MR^2\ddot{\theta} + MgR\sin\theta - MW_0^2R^2\sin\theta\cos\theta = 0$

$\Rightarrow R\ddot{\theta} + g\sin\theta - W_0^2\sin\theta\cos\theta = 0$

3.11.11. $A(-a\cos\alpha,\ a\sin\alpha)$

$B(b\cos\alpha,\ -b\sin\alpha)$

$C(-a\cos\alpha + l\cos\beta,\ a\sin\alpha + l\sin\beta)$

$D(b\cos\alpha + l\cos\beta,\ -b\sin\alpha + l\sin\beta)$

① 当 β 改变，α 不变

$\beta \to \beta + \delta\beta$

$\delta W_p = P\cdot[l\sin(\beta+\delta\beta) - l\cdot\sin\beta]$
$= P\cdot l\cdot\cos\beta\ \delta\beta$

$$\delta W_F = -Fl\sin\beta\,\delta\beta$$

$$\delta W = (Pl\cos\beta - Fl\sin\beta)\,\delta\beta$$

② 当 α 改变，β 不变时.

$$\delta W_P = P\,a\cos\alpha \cdot \delta\alpha$$

$$\delta W_F = Fb(-\sin\alpha)\cdot\delta\alpha$$

$$\delta W = (P\,a\cos\alpha - Fb\sin\alpha)\cdot\delta\alpha$$

$$\begin{cases} Pa\cos\alpha - Fb\sin\alpha = 0 \\ Pl\cos\beta - Fl\sin\beta = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \beta = \arctan\left(\frac{P}{F}\right) \\ \alpha = \arctan\left(\frac{Pa}{Fb}\right) \end{cases}$$

3.11.12 $l_{DB} = 600 - l_{AD}$

$$= \left(600 - \frac{300}{\cos\theta}\right)\times 10^{-3}$$

$$d\,l_{DB} = -300 \times (-1)\,\frac{\sin\theta}{\cos^2\theta}\,d\theta \times 10^{-3}$$

$$= \frac{-300\sin\theta}{\cos^2\theta}\,d\theta \times 10^{-3}$$

$$dw = M \cdot d\theta + k\left[300 - \left(600 - \frac{300}{\cos\theta}\right)\right]_{\times 10^{-3}} \cdot (+dr_r)$$

$$= M \cdot d\theta + k\left(\frac{300}{\cos\theta} - 300\right)(+dr_r) \times 10^{-3}$$

$$= M \cdot d\theta + k300\left(\frac{1}{\cos\theta} - 1\right)\frac{-300\sin\theta}{\cos^2\theta} d\theta \times 10^{-3}$$

$$= d\theta\left(M + 300k\frac{1-\cos\theta}{\cos\theta} \times \frac{-300\sin\theta}{\cos^2\theta}\times 10^{-3}\right) = 0$$

$$\Rightarrow M = \frac{450(1-\cos\theta)\tan\theta)}{\cos^2\theta}\ N \cdot m$$