3. (1) $x \to +\infty,\ y \to +\infty$

$$F(x,y) = A\left(B+\frac{\pi}{2}\right)\left(C+\frac{\pi}{2}\right) = 1$$

$x \to -\infty,\ y \to -\infty$

$$F(x,y) = A\left(B-\frac{\pi}{2}\right)\left(C-\frac{\pi}{2}\right) = 0$$

$x \to -\infty$.

$$F(x,y) = A\left(B-\frac{\pi}{2}\right)\left(C+\arctan\frac{y}{3}\right) = 0$$

$y \to -\infty$

$$F(x,y) = A\left(B+\cancel{\arctan}\,\frac{\pi}{2}\right)\left(C-\frac{\pi}{2}\right) = 0$$

$\arctan\frac{x}{2}$

$\Rightarrow B = C = \frac{\pi}{2}$.

$A = \frac{1}{\pi^2}$.

$$\Rightarrow F(x,y) = \frac{1}{\pi^2}\left(\frac{\pi}{2}+\arctan\frac{x}{2}\right)\left(\frac{\pi}{2}+\arctan\frac{y}{3}\right)$$

(2) $P\{0<X\le 2,\ 0<Y\le 3\} = F(2,3)+F(0,0)-F(2,0)-F(0,3)$

$= \frac{1}{16}$

(3) $P\{X>2, Y>3\} = 1+F(2,3)-F(+\infty,3)-F(2,+\infty)$

$= \frac{1}{16}$

(4) $F_X(x) = \lim\limits_{y\to+\infty} F(x,y) = \lim\limits_{y\to+\infty} \frac{1}{\pi^2}\left(\frac{\pi}{2}+\arctan\frac{x}{2}\right)\cdot\pi = \frac{1}{\pi}\left(\frac{\pi}{2}+\arctan\frac{x}{2}\right)$

$F_Y(y) = \lim\limits_{x\to+\infty} F(x,y) = \frac{1}{\pi^2}\left(\frac{\pi}{2}+\frac{\pi}{2}\right)\left(\frac{\pi}{2}+\arctan\frac{y}{3}\right) = \frac{1}{\pi}\left(\frac{\pi}{2}+\arctan\frac{y}{3}\right)$

4. (1) 不放回.

X \ Y (P)	1	2
1	0	$\frac{1}{3}$
2	$\frac{1}{3}$	$\frac{1}{3}$

$$F(x,y) = \begin{cases} 1 & (x\ge 2,\ y\ge 2) \\ \frac{1}{3} & (1\le x<2,\ y\ge 2 \text{ 或 } x\ge 2,\ 1\le y<2) \\ 0 & \text{其它} \end{cases}$$

(2) 放回

X \ Y (P)	1	2
1	$\frac{1}{9}$	$\frac{2}{9}$
2	$\frac{2}{9}$	$\frac{4}{9}$

$$F(x,y) = \begin{cases} 1 & (x\ge 2,\ y\ge 2) \\ 0 & (x<1 \text{ 或 } y<1) \\ \frac{1}{9} & (1\le x<2,\ 1\le y<2) \\ \frac{1}{3} & (1\le x<2,\ y\ge 2 \text{ 或 } x\ge 2,\ 1\le y<2) \end{cases}$$

8. (1) 令 $f(x,y)=C$

$\frac{2\times2}{2}\times C=1$

$\Rightarrow C=\frac{1}{2}$

$$f(x,y)=\begin{cases}0 & (x<1)\\ 0 & (y<1)\\ \frac{1}{2} & (1\le x\le y\le 3)\\ 0 & (x>3\text{且}y>3)\end{cases}$$

(2) $P\{Y-X\le 1\}=\frac{1}{2}\times\left(\frac{2\times2}{2}-\frac{1\times1}{2}\right)=\frac{3}{4}$

(3) $f_X(x)=\int_{-\infty}^{+\infty}f(x,y)dy$

$=\int_x^3\frac{1}{2}dy=\frac{3-x}{2}\quad(1\le x\le 3)$

$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)dx=\int_1^y\frac{1}{2}dx$

$=\frac{y-1}{2}\quad(1\le y\le 3)$

$$\therefore f_X(x)=\begin{cases}\frac{3-x}{2} & (1\le x\le 3)\\ 0 & \text{其它}\end{cases}$$

$$f_Y(y)=\begin{cases}\frac{y-1}{2} & (1\le y\le 3)\\ 0 & \text{其它}\end{cases}$$

9. (1) 由对称性.

$P\{2X\le Y\}=\frac{1}{2}$

(2) $P=\iint f(x,y)d\sigma$

$=\iint\frac{1}{4\pi^2}e^{-\frac{x^2+y^2}{4\pi}}d\sigma$

$=\int_0^{2\pi}d\theta\int_{\sqrt{4\pi}}^{\sqrt{4\pi}}\frac{1}{4\pi^2}e^{-\frac{\rho^2}{4\pi}}\rho d\rho$

$=e^{-\frac{\pi}{4\pi}}-e^{-\frac{\pi}{\pi}}=e^{-\frac{1}{4}}-e^{-1}$

10. (1) $\iint f(x,y)d\sigma=1$

$\int_{-1}^1dx\int_{x^2}^1c\cdot x^2y\,dy=1$

$\Rightarrow c=\frac{21}{4}$.

(2) $$f(x,y)=\begin{cases}\frac{21}{4}x^2y, & x^2\le y\le 1\\ 0 & \text{其它}.\end{cases}$$

$P\{|X|\le Y\}=\int_{-1}^0dx\int_{-x}^1\frac{21}{4}x^2y\,dy+\int_0^1dx\int_x^1\frac{21}{4}x^2y\,dy$

$=\frac{7}{10}$

(3) $f_X(x)=\int_{x^2}^1f(x,y)dy=\frac{21}{8}(x^2-x^6)\quad(-1\le x\le 1)$

$$f_X(x)=\begin{cases}\frac{21}{8}(x^2-x^6) & (-1\le x\le 1)\\ 0 & \text{其它}\end{cases}$$

$f_Y(y)=\int_{-\sqrt{y}}^{\sqrt{y}}f(x,y)dx=\frac{7}{2}y^{\frac{5}{2}}\quad(0\le y\le 1)$

$$f_Y(y)=\begin{cases}\frac{7}{2}y^{\frac{5}{2}} & (0\le y\le 1)\\ 0 & \text{其它}\end{cases}$$

12. (1)

当 $1<y\le 3$ 时 $f_{X|Y}(x|y)=\dfrac{f(x,y)}{f_Y(y)}=\dfrac{\frac{1}{2}}{\frac{y-1}{2}}=\dfrac{1}{y-1}\ (1\le x\le y\le 3)$

$=0$ (其它)

当 $1\le x<3$ 时 $f_{Y|X}(y|x)=\dfrac{f(x,y)}{f_X(x)}=\dfrac{\frac{1}{2}}{\frac{3-x}{2}}=\dfrac{1}{3-x}\ (1\le x\le y\le 3)$

$=0$ (其它)

(2)

当 $0<y\le 1$ 时 $f_{X|Y}(x|y)=\dfrac{f(x,y)}{f_Y(y)}=\dfrac{\frac{21}{4}x^2y}{\frac{7}{2}y^{\frac{5}{2}}}=\dfrac{3}{2}x^2y^{-\frac{3}{2}}\ (x^2\le y\le 1)$

$=0$ (其它)

当 $-1<x<1$ 时, $f_{Y|X}(y|x)=\dfrac{f(x,y)}{f_X(x)}=\dfrac{\frac{21}{4}x^2y}{\frac{21}{8}(x^2-x^6)}=\dfrac{2y}{1-x^4}\ (x^2\le y\le 1)$

$=0$ 其它.

(3) $P\{X\le\frac{1}{4}\mid Y=\frac{1}{4}\}$

$=\int_{\frac{1}{2}}^{\frac{1}{4}}\frac{3}{2}\times x^2\times(4^{-1})^{-\frac{3}{2}}dx$

$=\frac{9}{16}$

$P\{Y>\frac{1}{2}\mid X=\frac{1}{2}\}=\int_{\frac{1}{2}}^{1}\frac{2y}{1-x^4}\cdot dy=\int_{\frac{1}{2}}^{1}\frac{32}{15}y\,dy=\frac{3}{4}$

(1) $f_{Y|X}(y|x)=\begin{cases}x\cdot e^{-xy}, & 0<x\le 1,\ y>0\\ 0 & 其它\end{cases}$

$f_X(x)=\begin{cases}1 & (0<x<1)\\ 0 & 其它\end{cases}$

$f(x,y)=f_{Y|X}(y|x)\times f_X(x)$

$=\begin{cases}xe^{-xy} & 0<x\le 1,\ y>0\\ 0 & 其它\end{cases}$

(2) $f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)\,dx$

$=\int_0^1 xe^{-xy}\,dx$

$=\frac{1}{y^2}(1-e^{-y}-e^{-y}\cdot y)$

(3) $F_{X|Y}(x|y)=\int_{-\infty}^{x}f_{X|Y}(u|y)\,du$

$=\int_{-\infty}^{x}\dfrac{f(u,y)}{f_Y(y)}\,du=\int_{-\infty}^{x}\dfrac{xe^{-xy}}{\frac{1}{y^2}(1-e^{-y}(1+y))}\,du$

$\Rightarrow$ 当 $y>0$ 时, $f_{X|Y}(x|y)=\dfrac{xe^{-xy}y^2}{1-e^{-y}(1+y)},\ 0<x\le 1.$

$=0$ 其它

16. $P\{X=2, Y=0\} = \frac{1}{9} = \frac{1}{3} \times (\frac{1}{9} + a) \Rightarrow a = \frac{2}{9}$

$P\{X=$[redacted]$, Y=1\} = \frac{1}{18} = (\frac{1}{6} + \frac{1}{9} + \frac{1}{18}) \times (\frac{1}{18} + b)$

$= \frac{1}{3} \times (\frac{1}{18} + b)$

$\Rightarrow b = \frac{1}{9}$

21.

X	0	1
P	$\frac{1}{2}$	$\frac{1}{2}$

Y	0	1
P	$\frac{1}{2}$	$\frac{1}{2}$

X+Y	0	1	2
P	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

2X	0	2
P	$\frac{1}{2}$	$\frac{1}{2}$

XY	0	1
P	$\frac{3}{4}$	$\frac{1}{4}$

X^2	0	1
P	$\frac{1}{2}$	$\frac{1}{2}$

24. (1) $F_Z(z) = P(Z \le z) = P(X - Y \le z)$

① $z > a$, $F_Z(z) = 1$

② $z < -a$, $F_Z(z) = 0$

$f_X(x) = \begin{cases} \frac{1}{a} & (0 \le x \le a) \\ 0 & 其它 \end{cases}$

$f_Y(y) = \begin{cases} \frac{1}{a} & (0 \le y \le a) \\ 0 & 其它 \end{cases}$

$f(x,y) = \begin{cases} \frac{1}{a^2} & (0 \le x \le a,\ 0 \le y \le a) \\ 0 & 其它 \end{cases}$

③ $-a < z < 0$ $F_Z(z) = \iint \frac{1}{a^2} d\sigma = \int_0^{a+z} \frac{1}{a^2} dx \int_{x-z}^{a} dy$

$= \frac{1}{2a^2}(a+z)^2$

$f(z) = \frac{a+z}{a^2}$

④ $0 < z < a$ 时. $F_Z(z) = \frac{1}{a^2}\left(a^2 - \frac{(a-z)^2}{2}\right) = 1 - \frac{(a-z)^2}{2a^2}$

$f(z) = \frac{a-z}{a^2} \Rightarrow f(z) = \begin{cases} \frac{a+z}{a^2} & (-a<z<0) \\ \frac{a-z}{a^2} & (0<z<a) \\ 0 & 其它 \end{cases}$

(2) $z < 0$ 时, $f(z) = 0$

$z > 0$ 时, $f_2(z) = 2f(z)$ $\therefore f(z) = \begin{cases} \frac{2(a-z)}{a^2} & (0<z<a) \\ 0 & 其它 \end{cases}$

27. (1) $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma}}$, $f(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma}}$

$f(x,y) = \frac{1}{(\sqrt{2\pi})^2\sigma^2} e^{-\frac{x^2+y^2}{2\sigma}} = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma}}$

$f(\rho,\theta) = \frac{\rho}{2\pi\sigma^2} e^{-\frac{\rho^2\cos^2\theta + \rho^2\sin^2\theta}{2\sigma}}$ $|J| = \rho$

$= \frac{\rho}{2\pi\sigma^2} e^{-\frac{\rho^2}{2\sigma}}$

$\therefore f(\rho,\theta) = \begin{cases} \frac{\rho}{2\pi\sigma^2} e^{-\frac{\rho^2}{2\sigma}} & (\rho \ge 0,\ 0 < \theta \le 2\pi) \\ 0 & (其它) \end{cases}$

(2) $f_\theta(\theta) = \begin{cases} \frac{1}{2\pi} & (0 \le \theta \le 2\pi) \\ 0 & (其它) \end{cases}$ $f_\rho(\rho) = \begin{cases} \frac{\rho}{\sigma^2} e^{-\frac{\rho^2}{2\sigma}} & (\rho > 0) \\ 0 & (其他) \end{cases}$

$f_\theta(\theta) f_\rho(\rho) = \frac{1}{2\pi\sigma^2} e^{-\frac{\rho^2}{2\sigma}} = f(\rho,\theta)$, 则 ρ, θ 相互独

33.

(1) $P(Z=X+Y=K)=\sum_{n=1}^{k-1}P(X=n)\times P(Y=k-n)$

$\Rightarrow\sum_{n=1}^{k-1}(1-p)^{n-1}\cdot p\times(1-p)^{k-n-1}\cdot p$

$\Rightarrow\sum_{n=1}^{k-1}(1-p)^{k-2}\cdot p^2=(k-1)(1-p)^{k-2}\cdot p^2$

(2) $P\{X|Z\}=\frac{P(ZX)}{P(Z)}=\frac{p(1-p)^{n-1}p(1-p)^{k-n-1}}{(k-1)(1-p)^{k-2}p^2}=\frac{1}{k-1}$

$P\{X|Z\}=\frac{1}{z-1}$.

(3) $P\{W=\max\{X,Y\}=k\}=P\{X=k,Y<k\}+P\{X<k,Y=k\}+P\{X=k,Y=k\}$

$=P\{X=k\}P\{Y<k\}+P\{Y=k\}P\{X<k\}+P\{X=k\}P\{Y=k\}$

$=2P\{X=k\}P\{Y<k\}+P^2\{X=k\}$

$=2\times(1-p)^{k-1}\times p\times\sum_{n=1}^{k-1}(1-p)^{n-1}p+((1-p)^{k-1}\times p)^2$

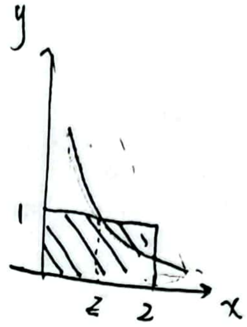
$=2p(1-p)^{k-1}-2p(1-p)^{2k-2}+p^2(1-p)^{2k-2}\quad(k\geq1)$

(4) $P(V=\min\{X,Y\}=k)=P\{X=k,Y>k\}+P\{Y=k,X>k\}+P\{X=k,Y=k\}$

$=2P\{X=k\}P\{Y>k\}+P^2\{X=k\}$

$=2P(1-p)^{2k-1}+p^2(1-p)^{2k-2}\quad(k\geq1)$

35.



$f(x,y)=\frac{1}{2}$

$F_Z(z)=P(xy\leq z)=\iint_{xy\leq z}f(x,y)\,d\sigma$

① $z\geq2$. $F_Z(z)=1$

② $2>z>0$, $F_Z(z)=\frac{1}{2}\times z+\frac{1}{2}\times\int_z^2\frac{z}{x}dx$

$=\frac{1}{2}z+\frac{z}{2}\ln\frac{2}{z}$

$f_Z(z)=\frac{1}{2}+\frac{z}{2}\cdot(-\frac{1}{z})+(\ln2-\ln z)\frac{1}{2}$

$=\frac{\ln2}{2}-\frac{1}{2}\ln z$

③ $z\leq0$ $f_Z(z)=0$

$\therefore f(s)=\begin{cases}\frac{\ln2}{2}-\frac{\ln s}{2} & 0<s\leq2\\ 0 & (其它)\end{cases}$