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单纯形表与初始解
Simple Tableau and Initial Solution

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Outline

- 单纯形表及算法

- 初始可行解

- 单纯形法总结

标准LP问题的等价问题

$$\zeta^T = c_B^T B^{-1} A - c^T$$

$$\min \quad z = z_0 - \zeta^T x$$

$$s.t. \quad x_B + B^{-1} N x_N = \bar{b}$$

$$x \geq 0$$

典式

最优性准则?

检验数

更新基本可行解?

基变换

基变换

$$\begin{cases} \min & z = c^T x \\ s.t. & Ax = b \\ & x \geq 0 \end{cases} \Leftrightarrow \begin{cases} \min & z = c_B^T \bar{b} - \zeta_N^T x_N \\ s.t. & x_B = \bar{b} - B^{-1} N x_N \\ & x_B \geq 0 \, , \, x_N \geq 0 \end{cases}$$

$$\boxed{\text{几何意义} \frac{\partial z}{\partial x_{N_j}} = -\zeta_{N_j}}$$

$$\begin{pmatrix} x_{B_1} \\ \vdots \\ x_{B_i} \\ \vdots \\ x_{B_m} \end{pmatrix} = \begin{pmatrix} \bar{b}_1 - x_{N_k} \bar{a}_{1,N_k} \\ \vdots \\ \bar{b}_i - x_{N_k} \bar{a}_{i,N_k} \\ \vdots \\ \bar{b}_m - x_{N_k} \bar{a}_{m,N_k} \end{pmatrix}$$

若有多个$\zeta_{N_k} > 0$时，选哪一个？

若有多个$\bar{a}_{i,N_k} > 0$时，选哪一个？

$$\boxed{x_{N_k} \to \theta = \min \left\{ \frac{\bar{b}_i}{\bar{a}_{i,N_k}} \middle| \bar{a}_{i,N_k} > 0 \right\}}$$

设最小比在$i = r$处取得，

则$x_{B_r} \to 0$，$x_{N_k} \to \theta$

单纯形法步骤

- 获取初始可行基
- 计算检验数向量 $\boxed{\zeta^T = c_B^T B^{-1} A - c^T}$
- 求 $\zeta_k = \max\{\zeta_j \mid 1 \le j \le n\}$
- 若$\zeta_k \le 0$，停止，已找到最优解
- 若 $\bar{A}_k \le 0$，停止，原问题无界
- 求 $\min\left\{\dfrac{\bar{b}_i}{\bar{a}_{ik}} \mid \bar{a}_{ik} > 0, i = 1, \ldots, m\right\} = \dfrac{\bar{b}_r}{\bar{a}_{rk}}$
- 以A_k代替A_{Br}得到新基，转第2步

线性规划标准形式的变换

$$\begin{aligned} \min \quad & c^T x \\ s.t. \quad & Ax = b \\ & x \geq 0 \end{aligned} \quad \Longleftrightarrow \quad \begin{aligned} \min \quad & z = C_B^T x_B + C_N^T x_N \\ s.t. \quad & Bx_B + Nx_N = b \\ & x \geq 0 \end{aligned}$$

$$\Longleftrightarrow \quad \begin{aligned} \min \quad & z = C_B^T x_B + C_N^T x_N \\ s.t. \quad & \boxed{\begin{aligned} z - C_B^T x_B - C_N^T x_N &= 0 \\ 0 \cdot z + Bx_B + Nx_N &= b \end{aligned}} \\ & x \geq 0 \end{aligned}$$

单纯形表

$$z - C_B^T x_B - C_N^T x_N = 0$$

$$0 \cdot z + B x_B + N x_N = b$$

基变量	系数			右端项
	z	x		
z	1	$-C_B^T$	$-C_N^T$	0
x_B	0	B	N	b

单纯形表

$$\zeta_N^T = c_B^T B^{-1} N - c_N^T$$

$$z_0 = c_B^T B^{-1} b$$

基变量	系数			右端项
	z	x		
z	1	$-C_B^T$	$-C_N^T$	0
x_B	0	B	N	b

单纯形表

$$\zeta_N^T = c_B^T B^{-1} N - c_N^T$$

$$z_0 = c_B^T B^{-1} b$$

基变量	系数			右端项
	z	x		
z	1	$-C_B^T$	$-C_N^T$	0
x_B	0	I	$B^{-1}N$	$B^{-1}b$

单纯形表

$$\zeta_N^T = c_B^T B^{-1} N - c_N^T$$

$$z_0 = c_B^T B^{-1} b$$

基变量	系数			右端项
	z	x		
z	1	0	$C_B^T B^{-1} N - C_N^T$	$C_B^T B^{-1} b$
x_B	0	I	$B^{-1} N$	$B^{-1} b$

$B \text{->} I$; $-C^T{}_B \text{->} 0$

基变换

正检验数中最大者对应的列

基变量	系数			右端项
	z	x		
z	1	0	$C_B^T B^{-1} N - C_N^T$	$C_B^T B^{-1} b$
x_B	0	I	$B^{-1} N$	$B^{-1} b$

比值最小的值对应的行(a’>0)

例题求解

$$
\begin{aligned}
\min \quad & z = -6x_1 + 3x_2 - 3x_3 \\
s.t.: \quad & 2x_1 + x_2 \leqslant 8 \\
& -4x_1 - 2x_2 + 3x_3 \leqslant 14 \\
& x_1 - 2x_2 + x_3 \leqslant 18 \\
& x_i \geqslant 0,\ i = 1, 2, 3
\end{aligned}
$$

z	6	-3	3	0	0	0	0
x_4	2^*	1	0	1	0	0	8
x_5	-4	-2	3	0	1	0	14
x_6	1	-2	1	0	0	1	18

$$
\begin{aligned}
\min \quad & z = -6x_1 + 3x_2 - 3x_3 \\
s.t.: \quad & 2x_1 + x_2 + x_4 = 8 \\
& -4x_1 - 2x_2 + 3x_3 + x_5 = 14 \\
& x_1 - 2x_2 + x_3 + x_6 = 18 \\
& x_i \geqslant 0,\ i = 1, 2, \ldots, 6
\end{aligned}
$$

例题求解(续)

z	0	-6	3	-3	0	0	-24
x_1	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	4
x_3	0	0	3	2	1	0	30
x_6	0	$-\frac{5}{2}$	1	$-\frac{1}{2}$	0	1	14

z	6	-3	3	0	0	0	0
x_4	2^*	1	0	1	0	0	8
x_5	-4	-2	3	0	1	0	14
x_6	1	-2	1	0	0	1	18

z	0	-6	3	-3	0	0	-24
x_1	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	4
x_5	0	0	3^*	2	1	0	30
x_6	0	$-\frac{5}{2}$	1	$-\frac{1}{2}$	0	1	14

化为标准
单纯形表

z	6	-3	3	0	0	0	0
x_1	2	1	0	1	0	0	8
x_5	-4	-2	3	0	1	0	14
x_6	1	-2	1	0	0	1	18

例题求解(续)

z	0	-6	3	-3	0	0	-24
x_1	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	4
x_3	0	0	3	2	1	0	30
x_6	0	$-\frac{5}{2}$	1	$-\frac{1}{2}$	0	1	14

化为标准单纯形表

z	0	-6	0	-5	-1	0	-54
x_1	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	4
x_3	0	0	1	$\frac{2}{3}$	$\frac{1}{3}$	0	10
x_6	0	$-\frac{5}{2}$	0	$-\frac{7}{6}$	$-\frac{1}{3}$	1	4

$$\boldsymbol{x}^* = (4, 0, 10, 0, 0, 4)^{\mathrm{T}}$$
$$z^* = -54$$

$$\begin{aligned}
\min\ \ & z = -6x_1 + 3x_2 - 3x_3 \\
s.t.: \ \ & 2x_1 + x_2 + x_4 = 8 \\
& -4x_1 - 2x_2 + 3x_3 + x_5 = 14 \\
& x_1 - 2x_2 + x_3 + x_6 = 18 \\
& x_i \geqslant 0,\ i = 1, 2, \ldots, 6
\end{aligned}$$

例2:

$$\begin{aligned} \min \quad & z = -2x_1 - 4x_2 - 3x_3 \\ s.t. \quad & 3x_1 + 4x_2 + 2x_3 \le 60 \\ & 2x_1 + x_2 + 2x_3 \le 40 \\ & x_1 + 3x_2 + 2x_3 \le 80 \\ & x_j \ge 0, j = 1, 2, 3 \end{aligned}$$

例题求解

$$
\begin{aligned}
\min \quad & z = -2x_1 - 4x_2 - 3x_3 \\
s.t. \quad & 3x_1 + 4x_2 + 2x_3 \le 60 \\
& 2x_1 + x_2 + 2x_3 \le 40 \\
& x_1 + 3x_2 + 2x_3 \le 80 \\
& x_j \ge 0,\ j = 1,2,3
\end{aligned}
$$

z	2	4	3	0	0	0	0
x_4	3	4	2	1	0	0	60
x_5	2	1	2	0	1	0	40
x_6	1	3	2	0	0	1	80

$$
\begin{aligned}
\min \quad & z = -2x_1 - 4x_2 - 3x_3 \\
s.t. \quad & 3x_1 + 4x_2 + 2x_3 + x_4 = 60 \\
& 2x_1 + x_2 + 2x_3 + x_5 = 40 \\
& x_1 + 3x_2 + 2x_3 + x_6 = 80 \\
& x_j \ge 0,\ j = 1,2,3,4,5,6
\end{aligned}
$$

例题求解(续)

z	2	4	3	0	0	0	0
x_4	3	4*	2	1	0	0	60
x_5	2	1	2	0	1	0	40
x_6	1	3	2	0	0	1	80

化为标准单纯形表

z	2	4	3	0	0	0	0
x_2	3	4*	2	1	0	0	60
x_5	2	1	2	0	1	0	40
x_6	1	3	2	0	0	1	80

z	-1	0	1	-1	0	0	-60
x_2	3/4	1	1/2	1/4	0	0	15
x_5	5/4	0	3/2*	-1/4	1	0	25
x_6	-5/4	0	1/2	-3/4	0	1	35

z	-11/6	0	0	-5/6	-2/3	0	-230/3
x_2	7/6	1	0	1/3	-1//3	0	20/3
x_3	5/6	0	1	-1/6	2/3	0	50/3
x_6	-5/3	0	0	-2/3	-1/3	1	80/3

例题求解(续)

z	-11/6	0	0	-5/6	-2/3	0	-230/3
x_2	7/6	1	0	1/3	-1//3	0	20/3
x_3	5/6	0	1	-1/6	2/3	0	50/3
x_6	-5/3	0	0	-2/3	-1/3	1	80/3

$$x^* = (0, \frac{20}{3}, \frac{50}{3}, 0, 0, \frac{80}{3})^T$$

$z^*=-230/3$

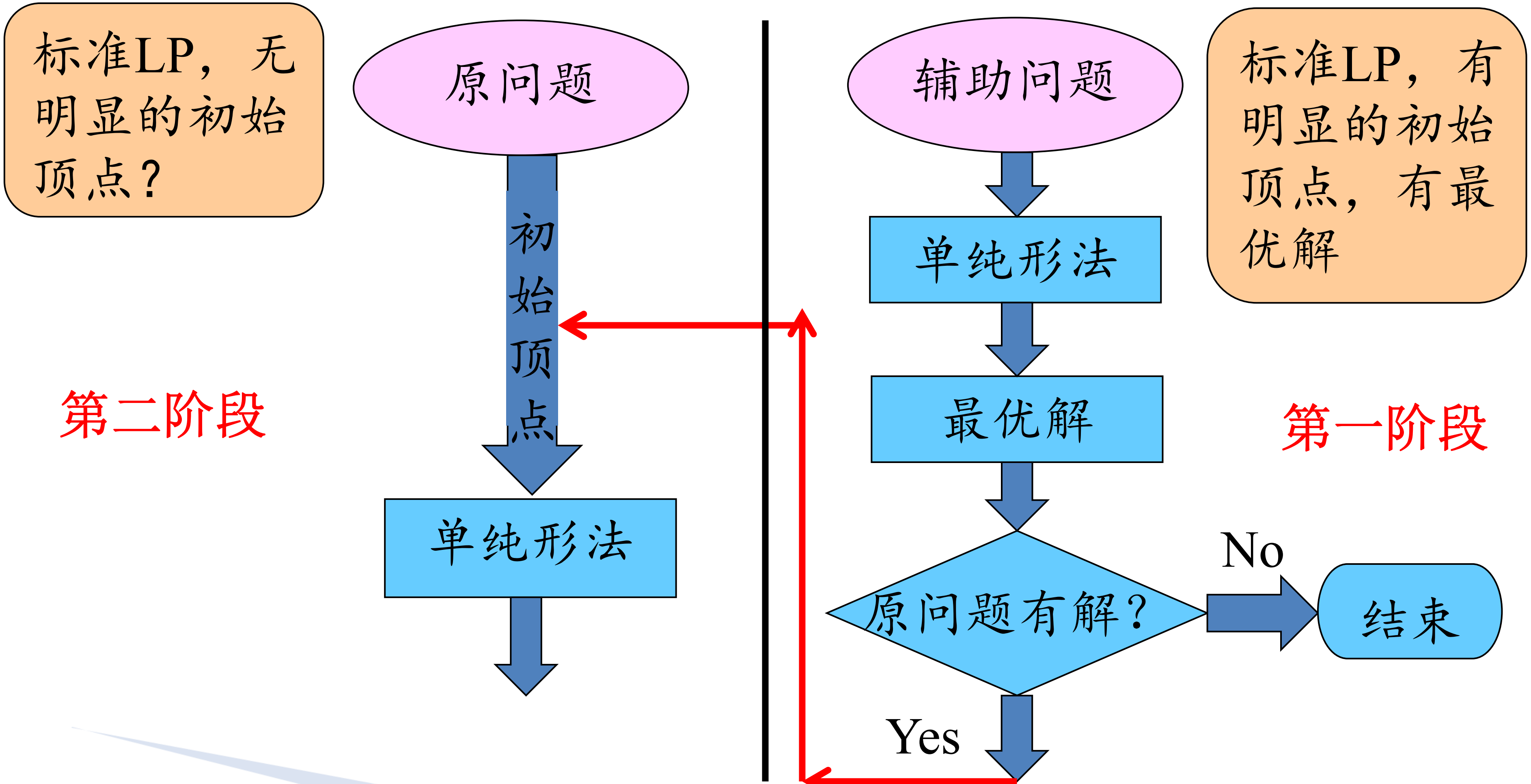
$$\begin{aligned}
\min \quad & z = -2x_1 - 4x_2 - 3x_3 \\
s.t. \quad & 3x_1 + 4x_2 + 2x_3 + x_4 = 60 \\
& 2x_1 + x_2 + 2x_3 + x_5 = 40 \\
& x_1 + 3x_2 + 2x_3 + x_6 = 80 \\
& x_j \geq 0,\ j = 1,2,3,4,5,6
\end{aligned}$$

Outline

- 单纯形表及算法
- 初始可行解
- 单纯形法总结

初始解：两阶段法的核心思想

单纯形法步骤中为什么没有出现问题无解的情况？



两阶段法

$$\min \quad z = c^T x$$

$$s.t. \quad Ax = b (b \geq 0)$$

$$x \geq 0$$

$$x_a = (x_{n+1}, \ldots, x_{n+m})^T$$

$$\min \quad g = x_{n+1} + x_{n+2} + \ldots + x_{n+m}$$

$$s.t. \quad Ax + x_a = b (b \geq 0)$$

$$x \geq 0, x_a \geq 0$$

两阶段法的基本原理

原问题

$$\begin{cases} \min & z = c^T x \\ s.t. & Ax = b \\ & x \geq 0 \end{cases}$$

?

辅助问题

$$\begin{cases} \min & g = x_{n+1} + x_{n+2} + \cdots + x_{n+m} \\ s.t. & Ax + x^a = b \\ & x \geq 0,\ x^a \geq 0 \end{cases}$$

定理1. 原问题有可行解(有顶点)的充分必要条件是辅助问题最优目标函数值为0.

辅助问题可能情况
- 无可行解 ×
- 无下界 ×
- 有最优解(最优顶点) √

由辅助问题最优顶点如何获得原问题一个顶点?

两阶段法的基本原理

原问题

$$\begin{cases} \min & z = c^T x \\ s.t. & Ax = b \\ & x \geq 0 \end{cases}$$

？

辅助问题

$$\begin{cases} \min & g = x_{n+1} + x_{n+2} + \cdots + x_{n+m} \\ s.t. & Ax + x^a = b \\ & x \geq 0, x^a \geq 0 \end{cases}$$

辅助问题最优解的可能情况：

1. 最优值 g*>0：原问题无可行解
2. 最优值 g*=0，且最优顶点(基本可行解)中人工变量全部为非基变量：
   已得原问题一个顶点，进入第二阶段
3. 最优值 g*=0，但最优顶点(基本可行解)中有些人工变量为基变量：
   构造（旋转变换）出原问题一个顶点，进入第二阶段

构造原问题顶点的旋转变换方法

辅助问题的最优单纯形表：设第r个基变量为人工变量x_{n+k}

g	ζ_1	ζ_2	$\cdots$	ζ_n	ζ_{n+1}	$\cdots$	ζ_{n+k-1}	0	ζ_{n+k+1}	$\cdots$	ζ_{n+m}	g^*
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_{B_r}	$\bar{a}_{r,1}$	$\bar{a}_{r,2}$	$\cdots$	$\bar{a}_{r,n}$	$\bar{a}_{r,n+1}$	$\cdots$	$\bar{a}_{r,n+k-1}$	1	$\bar{a}_{r,n+k+1}$	$\cdots$	$\bar{a}_{n+m}$	$\bar{b}_r$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$

⑴. $\bar{b}_r = ?$ **0**

重复执行直至人工变量全部出基

⑵. 若 $\bar{a}_{r,1} = \bar{a}_{r,2} = \cdots = \bar{a}_{r,n} = 0?$ 冗余约束，秩数问题

⑶. 若存在 $j\,(1 \le j \le n)$ 使得 $\bar{a}_{r,j} \ne 0?$ x_{n+k}出基，x_j入基

两阶段法的几点技巧

1. 视问题需要，引入尽可能少的人工变量，且第一阶段目标函数中只有人工变量
2. 一个人工变量一旦出基，立即删除其所在的列不要保留，更不要在后续迭代中将其再次入基
3. 第一阶段结束后，只需修改单纯形表第一行，即可继续迭代

$$
\begin{array}{c|ccc|c}
g & \zeta_1 & \cdots & \zeta_n & g^* \\
\hline
\vdots & \vdots & \ddots & \vdots & \vdots \\
x_{B_r} & \bar{a}_{r,1} & \cdots & \bar{a}_{r,n} & \bar{b}_r \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\hline
\end{array}
\quad \Longrightarrow \quad
\begin{array}{c|ccc|c}
z & -c_1 & \cdots & -c_n & 0 \\
\hline
\vdots & \vdots & \ddots & \vdots & \vdots \\
x_{B_r} & \bar{a}_{r,1} & \cdots & \bar{a}_{r,n} & \bar{b}_r \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\hline
\end{array}
$$

例：

求解：

$$
\begin{aligned}
\min \quad & z = 5x_1 + 21x_3 \\
s.t. \quad & x_1 - x_2 + 6x_3 - x_4 = 2 \\
& x_1 + x_2 + 2x_3 - x_5 = 1 \\
& x_j \geq 0, j = 1, 2, 3, 4, 5
\end{aligned}
$$

例：

解：增加人工变量x_6，x_7得到辅助LP问题：

$$
\begin{aligned}
\min \quad & g = x_6 + x_7 \\
s.t. \quad & x_1 - x_2 + 6x_3 - x_4 + x_6 = 2 \\
& x_1 + x_2 + 2x_3 - x_5 + x_7 = 1 \\
& x_j \geq 0,\ j = 1,2,3,4,5,6,7
\end{aligned}
$$

$$\min \quad z = 5x_1 + 21x_3$$
$$s.t. \quad x_1 - x_2 + 6x_3 - x_4 = 2$$
$$x_1 + x_2 + 2x_3 - x_5 = 1$$
$$x_j \ge 0, j = 1,2,3,4,5$$

$$\min \quad g = x_6 + x_7$$
$$s.t. \quad x_1 - x_2 + 6x_3 - x_4 + x_6 = 2$$
$$x_1 + x_2 + 2x_3 - x_5 + x_7 = 1$$
$$x_j \ge 0, j = 1,2,3,4,5,6,7$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	–5	0	–21	0	0	0	0	0
g	0	0	0	0	0	–1	–1	0
x_6	1	–1	6	–1	0	1	0	2
x_7	1	1	2	0	–1	0	1	1

$$\begin{aligned} \min \quad & z = 5x_1 + 21x_3 \\ s.t. \quad & x_1 - x_2 + 6x_3 - x_4 = 2 \\ & x_1 + x_2 + 2x_3 - x_5 = 1 \\ & x_j \geq 0, j = 1,2,3,4,5 \end{aligned}$$

$$\begin{aligned} \min \quad & g = x_6 + x_7 \\ s.t. \quad & x_1 - x_2 + 6x_3 - x_4 + x_6 = 2 \\ & x_1 + x_2 + 2x_3 - x_5 + x_7 = 1 \\ & x_j \geq 0, j = 1,2,3,4,5,6,7 \end{aligned}$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	−5	0	−21	0	0	0	0	0
g	2	0	8	−1	−1	0	0	3
x_6	1	−1	6*	−1	0	1	0	2
x_7	1	1	2	0	−1	0	1	1

$$\min \quad z = 5x_1 + 21x_3$$
$$\text{s.t.} \quad x_1 - x_2 + 6x_3 - x_4 = 2$$
$$x_1 + x_2 + 2x_3 - x_5 = 1$$
$$x_j \geq 0, j = 1,2,3,4,5$$

$$\min \quad g = x_6 + x_7$$
$$\text{s.t.} \quad x_1 - x_2 + 6x_3 - x_4 + x_6 = 2$$
$$x_1 + x_2 + 2x_3 - x_5 + x_7 = 1$$
$$x_j \geq 0, j = 1,2,3,4,5,6,7$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	−3/2	−7/2	0	−7/2	0	7/2	0	7
g	2/3	4/3	0	1/3	−1	−4/3	0	1/3
$\boxed{x_3}$	1/6	−1/6	1	−1/6	0	1/6	0	1/3
x_7	2/3	4/3*	0	1/3	−1	−1/3	1	1/3

$$\min \quad z = 5x_1 + 21x_3$$
$$s.t. \quad x_1 - x_2 + 6x_3 - x_4 = 2$$
$$x_1 + x_2 + 2x_3 - x_5 = 1$$
$$x_j \geq 0, j = 1,2,3,4,5$$

$$\min \quad g = x_6 + x_7$$
$$s.t. \quad x_1 - x_2 + 6x_3 - x_4 + x_6 = 2$$
$$x_1 + x_2 + 2x_3 - x_5 + x_7 = 1$$
$$x_j \geq 0, j = 1,2,3,4,5,6,7$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	-3/2	-7/2	0	-7/2	0	7/2	0	7
g	2/3	4/3	0	1/3	-1	-4/3	0	1/3
x_3	1/6	-1/6	1	-1/6	0	1/6	0	1/3
$\boxed{x_2}$	2/3	4/3*	0	1/3	-1	-1/3	1	1/3

$$\begin{aligned} \min \quad & z = 5x_1 + 21x_3 \\ s.t. \quad & x_1 - x_2 + 6x_3 - x_4 = 2 \\ & x_1 + x_2 + 2x_3 - x_5 = 1 \\ & x_j \geq 0,\ j = 1,2,3,4,5 \end{aligned}$$

$$\begin{aligned} \min \quad & g = x_6 + x_7 \\ s.t. \quad & x_1 - x_2 + 6x_3 - x_4 + x_6 = 2 \\ & x_1 + x_2 + 2x_3 - x_5 + x_7 = 1 \\ & x_j \geq 0,\ j = 1,2,3,4,5,6,7 \end{aligned}$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	1/4	0	0	-21/8	-21/8	21/8	21/8	63/8
g	0	0	0	0	0	-1	-1	0
x_3	1/4	0	1	-1/8	-1/8	1/8	1/8	3/8
x_2	1/2	1	0	1/4	-3/4	-1/4	3/4	1/4

$$
\begin{aligned}
\min \quad & z = 5x_1 + 21x_3 \\
s.t. \quad & x_1 - x_2 + 6x_3 - x_4 = 2 \\
& x_1 + x_2 + 2x_3 - x_5 = 1 \\
& x_j \geq 0,\ j = 1,2,3,4,5
\end{aligned}
$$

	x_1	x_2	x_3	x_4	x_5	RHS
z	1/4	0	0	−21/8	−21/8	63/8
x_3	1/4	0	1	−1/8	−1/8	3/8
x_2	1/2*	1	0	1/4	−3/4	1/4

$$\min \quad z = 5x_1 + 21x_3$$

$$\begin{aligned} s.t. \quad & x_1 - x_2 + 6x_3 - x_4 = 2 \\ & x_1 + x_2 + 2x_3 - x_5 = 1 \\ & x_j \geq 0,\ j = 1,2,3,4,5 \end{aligned}$$

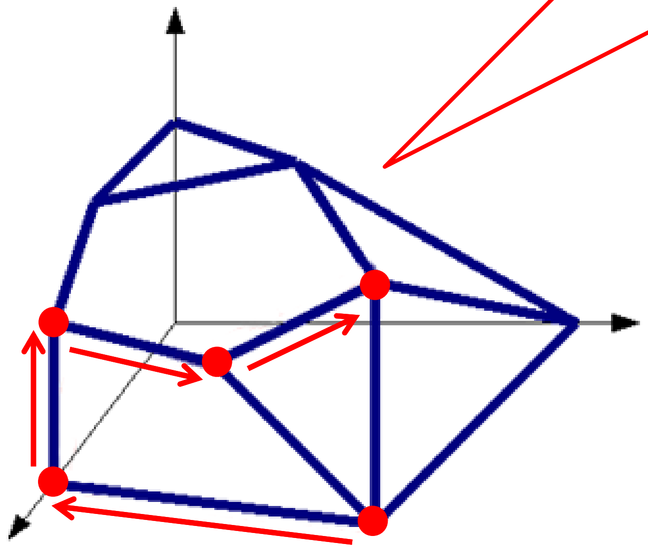
	x1	x2	x3	x4	x5	RHS
z	0	-1/2	0	-11/4	-9/4	31/4
x3	0	-1/2	1	-1/4	1/4	1/4
x1	1	2	0	1/2	-3/2	1/2

Outline

- 单纯形表及算法
- 初始可行解
- 单纯形法总结

几何原理

获取相邻顶点信息比其他顶点快
沿Z的增长率(max)最大的边寻找顶点
所有边Z的增长率为负，则为最优解



基本概念

$$\begin{aligned} \min \quad & c^T x \\ s.t. \quad & Ax = b \\ & x \geq 0 \end{aligned} \Longrightarrow \boldsymbol{A = [B,\ N]} \qquad x = \begin{pmatrix} \overline{x}_B \\ \overline{x}_N \end{pmatrix}$$

$$Ax = b \Longleftrightarrow x_B = \overline{b} - B^{-1} N x_N$$

$$z = c^T x \Longleftrightarrow z = c_B^T \overline{b} - (c_B^T B^{-1} N - c_N^T) x_N$$

$$\begin{aligned} \min \quad & z = z_0 - \zeta^T x \\ s.t. \quad & x_B + B^{-1} N x_N = \overline{b} \\ & x \geq 0 \end{aligned}$$

最优性准则

- 若可行基B对应的检验数向量$\zeta \le 0$，则此可行基对应的基本可行解为**最优解**。且最优值为$c_B^T \overline{b}$
- 若向量ζ的第k个分量$\zeta_k > 0$，而向量$\overline{A}_k = B^{-1} A_k \le 0$ 则原问题**无界**。
- 对于非退化的基本可行解$\overline{x}$，若向量ζ中$\zeta_k > 0$，而其相应的向量$\overline{A}_k$ **至少有一个正分量**，则有一个**新的基本可行解** $\hat{x}$ 使得 $c^T \hat{x} < c^T \overline{x}$

单纯形表

$$\zeta_N^T = c_B^T B^{-1} N - c_N^T$$

$$z_0 = c_B^T B^{-1} b$$

基变量	系数			右端项
	z	x		
z	1	0	$C_B^T B^{-1} N - C_N^T$	$C_B^T B^{-1} b$
x_B	0	I	$B^{-1}N$	$B^{-1}b$

$B \to I$; $-C^T{}_B \to 0$; $\zeta<0$; min(b/a)

两阶段法

目标函数 $\min g = x_{n+1} + \cdots + x_{n+m} + 0x_1 + 0x_2 + \cdots + 0x_n$

约束条件
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots\cdots + a_{1n}x_n + x_{n+1} & = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots\cdots + a_{2n}x_n \qquad + x_{n+2} & = b_2 \\ \qquad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots\cdots + a_{m}x_n \qquad\qquad + x_{n+m} & = b_m \\ x_1, x_2, \cdots\cdots, x_n, x_{n+1}, \cdots, x_{n+m} \geq 0 & \end{cases}$$

作业

- P74 14
- P75 16(1)
- P76 17(1) (3)