5.4. (1) $V(k)=DFT[v(n)]$

$$=\sum_{n=0}^{2N-1} v(n)\cdot W_{2N}^{kn}=\sum_{n=0}^{N-1} v(2n)W_{2N}^{k2n}+\sum_{n=0}^{N-1} v(2n+1)\cdot W_{2N}^{k(2n+1)}$$

$$=\sum_{n=0}^{N-1} f(n)\cdot W_N^{kn}+e^{-j\frac{\pi k}{N}}\sum_{n=0}^{N-1} v(2n+1)e^{-j\frac{2\pi kn}{N}}$$

$$=\sum_{n=0}^{N-1} f(n)\cdot W_N^{kn}+e^{-j\frac{\pi k}{N}}\sum_{n=0}^{N-1} \cancel{v(2n+1)}\, g(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$=F(k)+W_{2N}^{k}\cdot G(k) \quad ,k=0,1,2,\cdots,N-1$$

$$V(k+N)=\sum_{n=0}^{N-1} f(n)\cdot W_N^{(k+N)n}+e^{-j\frac{\pi(k+N)}{N}}\sum_{n=0}^{N-1} g(n)e^{-j\frac{2\pi(k+N)n}{N}}$$

$$=F(k)-W_{2N}^{k}G(k) \quad ,k=0,1,2,\cdots,N-1$$

(2) ①首先 $x(n)=f(n)+jg(n)$

$$X(k)=\sum_{n=0}^{N-1}[f(n)+jg(n)]W_N^{kn}$$

②然后. $F(k)=\frac{1}{2}[X_R(k)+X_R(N-k)]+j\cdot\frac{1}{2}[X_I(k)-X_I(N-k)]$

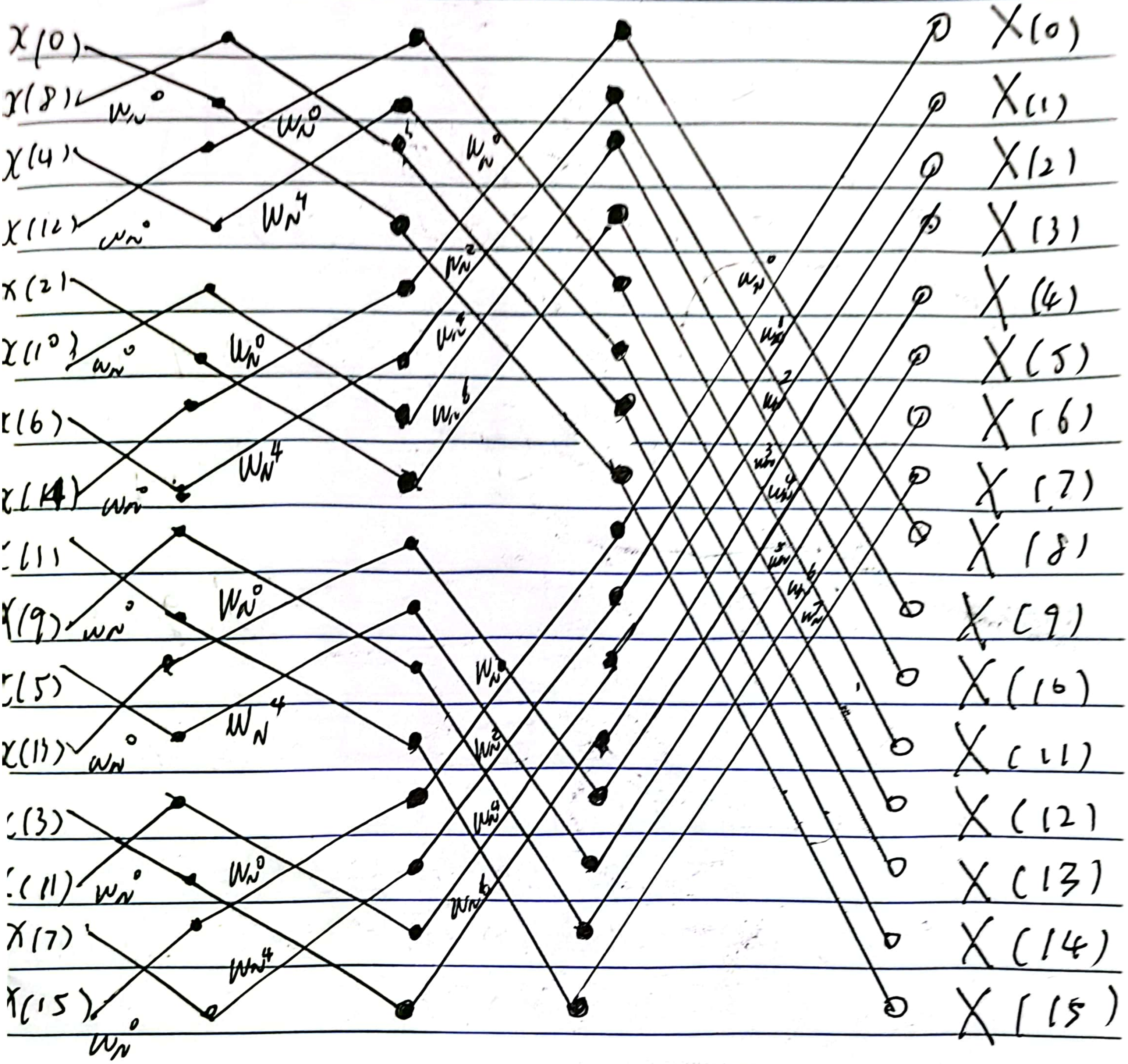
$G(k)=\frac{1}{2}[X_I(k)+X_I(N-k)]+j\frac{1}{2}[X_R(k)-X_R(N-k)]$

③最后: $V(k)=F(k)+W_{2N}^{k}G(k)$

$V(k+N)=F(k)-W_{2N}^{k}G(k)$

FFT的计算复杂度为 $O(N\log N)$，由2N降为N.

计算效率提高.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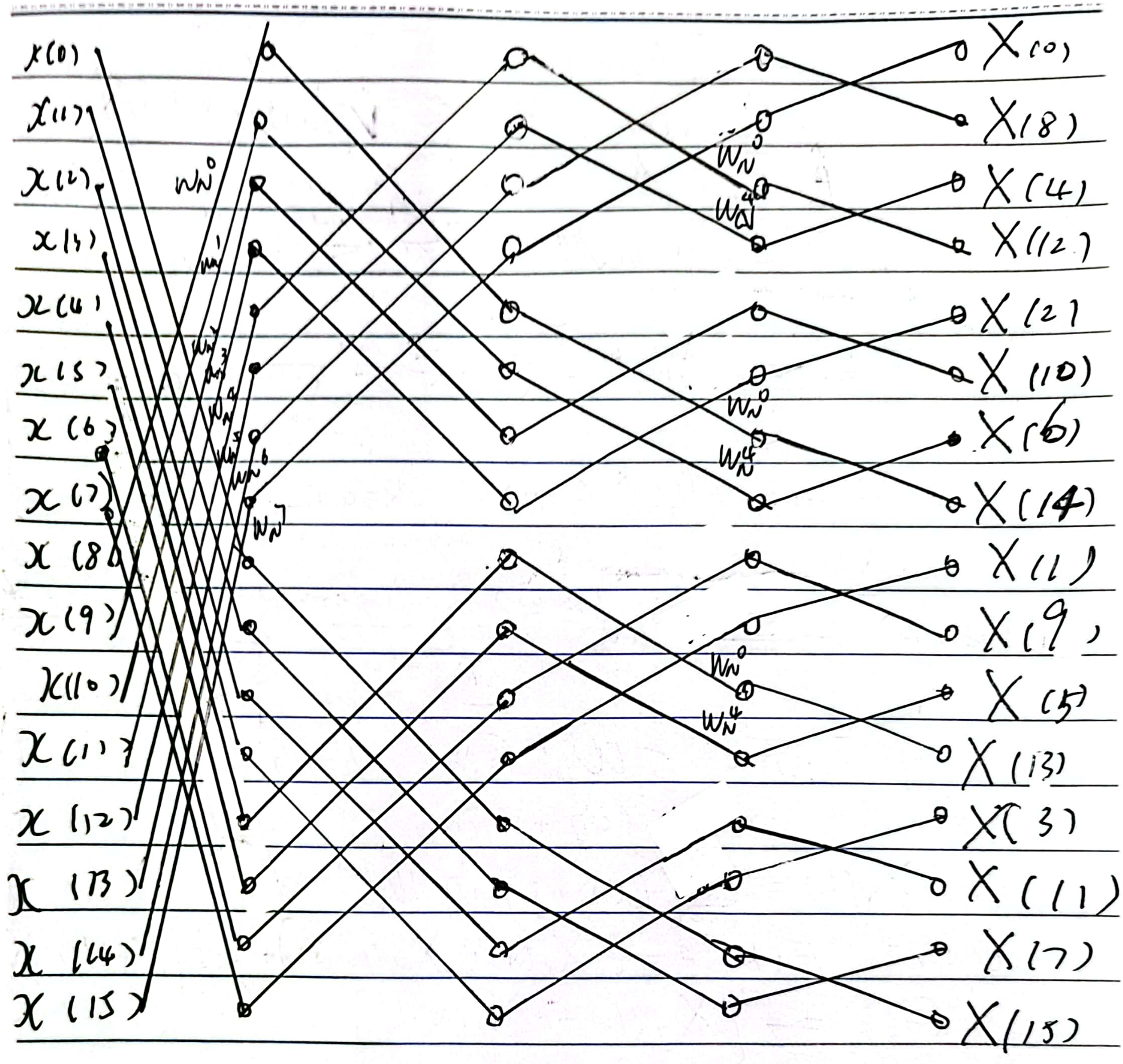

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5.10. $$Y(k)=\sum_{n=0}^{31} y(n) W_{32}^{kn} = \sum_{n=0}^{N-1} x(n) W_{32}^{kn} + \sum_{n=N}^{31} y(n) W_{32}^{kn}$$

$$= \sum_{n=0}^{N-1} x(n) W_{N}^{\frac{N}{32}kn} + \sum_{n=N}^{31} y(n) W_{32}^{kn}$$

① $2N=32 \Rightarrow N=16$

$$\sum_{n=N}^{31} y(n) W_{32}^{kn} = \sum_{n=N}^{31} x(n-16) W_{32}^{kn} = \sum_{n=0}^{15} x(n) W_{32}^{nk} (-1)^k$$

② $N<16$, $$\sum_{n=N}^{31} y(n) W_{32}^{kn} = \sum_{n=N}^{2N-1} x(n-16) W_{32}^{kn}$$

$$= \sum_{n=0}^{2N-17} x(n) W_{32}^{(n+16)k} = \sum_{n=0}^{2N-17} x(n) W_{32}^{nk} (-1)^k$$

③ $N>16$, $$\sum_{n=N}^{31} y(n) W_{32}^{kn} = \sum_{n=N}^{N+15} x(n-16) W_{32}^{kn}$$

$$= \sum_{n=N-16}^{N-1} x(n) W_{32}^{nk} (-1)^k$$

∴综上, $$Y(k)=\begin{cases} X\left(\frac{N}{32}k\right)+\sum_{n=0}^{2N-17} x(n)\cdot W_{32}^{nk}(-1)^k, & N<16 \\ X\left(\frac{N}{32}k\right) + \sum_{n=N-16}^{N-1} x(n) W_{32}^{nk}(-1)^k, & N>16. \\ [1+(-1)^k]\, X\left(\frac{k}{2}\right) & N=16 \end{cases}$$

5.13. (1) $X(k)=\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

$$=\sum_{n=0}^{\frac{N}{2}-1} x(n) e^{-j\frac{2\pi}{N}\cdot kn}+\sum_{n=\frac{N}{2}}^{N-1} x(n) e^{-j\frac{2\pi}{N}\cdot kn}$$

$$=\sum_{n=0}^{\frac{N}{2}-1} x(n) e^{-j\frac{2\pi}{N}\cdot kn}-\sum_{r=0}^{\frac{N}{2}-1} x[r+\tfrac{N}{2}] e^{-j\cdot\frac{2\pi k}{N}[r+\frac{N}{2}]}$$

$$=\sum_{n=0}^{\frac{N}{2}-1} x(n)[1-(-1)^k] e^{-j\frac{2\pi kn}{N}} =0.$$

k为偶数.

(2) $X(2k+1)=\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}(2k+1)n}$

$$=\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}n}\cdot e^{-j\frac{2\pi}{N}\cdot n\cdot 2k}$$

$$=\sum_{n=0}^{\frac{N}{2}-1} W_N^n[x(n)-x(n+\tfrac{N}{2})] W_{N/2}^{nk}$$

$$=\sum_{n=0}^{\frac{N}{2}-1}[2W_N^n x(n)] W_{\frac{N}{2}}^{nk} \qquad k=0,\cdots,\tfrac{N}{2}-1$$

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