5.

(1) $\mu_1 = E(x) = \frac{a+b}{2}$

$\mu_2 = E(x^2) = D(x) + (E(x))^2$

$= \frac{(b-a)^2}{12} + \left(\frac{a+b}{2}\right)^2$

$\Rightarrow \begin{cases} a+b = 2E(x) \\ \frac{(b-a)^2}{12} + \frac{(a+b)^2}{4} = D(x) + (E(x))^2 \end{cases}$

$E(x) = \frac{1}{6}(11.3+10.6+11.7+12.2+10.3+11.1)$

$= 11.2$

$S^2 = \frac{1}{6} \times [(11.3-11.2)^2 + (10.6-11.2)^2 + (11.7-11.2)^2 + (12.2-11.2)^2 + (10.3-11.2)^2$

$+ (11.1-11.2)^2]$

$= 0.4067$

$\Rightarrow \hat{a} = 10.095, \hat{b} = 12.3045.$

(2) $L(a,b) = \begin{cases} \left(\frac{1}{b-a}\right)^n & (a \le \xi_i \le b) \quad (1 \le i \le 6) \\ 0 & 其它 \end{cases}$

$\hat{a} = 10.3 \qquad \hat{a} = \min \xi_i$

$\hat{b} = 12.2 \qquad \hat{b} = \max \xi_i$

7. (1) $E(X) = -1\times\theta + 0\times\frac{\theta}{2} + 1\times\frac{\theta}{2} + 2\times(1-2\theta)$

$= -\theta + \frac{\theta}{2} + 2 - 4\theta = 2 - \frac{9}{2}\theta$

~~$D(X) =$ $-1-(2-\frac{9}{2}\theta$~~

$E(X) = (-1\times 3 + 0\times 2 + 1\times 5 + 2\times 6) \div (5+5+6)$

$= \frac{-3+5+12}{16} = \frac{14}{16} = \frac{7}{8}$

$\Rightarrow 2 - \frac{9}{2}\theta = \frac{14}{16} = \frac{7}{8}$

$\Rightarrow \theta = \frac{1}{4}$

(2) $L(\theta) = \prod_{i=1}^{4} P\{X_i = x_i\}$

$= P\{X=-1\}P\{X=0\}P\{X=1\}P\{X=2\}$

$= \theta^3 \left(\frac{\theta}{2}\right)^2 \left(\frac{\theta}{2}\right)^5 (1-2\theta)^6 = \theta^3 \times \theta^2 \times \theta^5 \times \frac{1}{2^7} \times (1-2\theta)^6$

$\ln L(\theta) = 10\ln\theta + 6\ln(1-2\theta) + \frac{1}{2^7}$

$\frac{d\ln L(\theta)}{d\theta} = \frac{10}{\theta} + \frac{6}{1-2\theta}\times(-2) = 0$, $\theta = \frac{5}{16}$.

9.

$\sigma^2 = E\left(c\sum_{i=1}^{n-1}(X_{i+1}-X_i)^2\right)$

$= c\sum_{i=1}^{n-1}E\left((X_{i+1}-X_i)^2\right)$

$= c\sum_{i=1}^{n-1}\left(D(X_{i+1}-X_i) - (E(X_{i+1}-X_i))^2\right)$

$= c\sum_{i=1}^{n-1}D(X_{i+1}-X_i) = c\sum_{i=1}^{n-1}\left(D(X_{i+1}) + D(X_i)\right)$

$= c\sum_{i=1}^{n-1}2\sigma^2 = c\times 2\sigma^2(n-1)$

$\Rightarrow c = \frac{1}{2(n-1)}$

9. $E\left(c\sum_{i=1}^{n-1}(X_{i+1}-X_i)^2\right)$

$=cE\left(\sum_{i=1}^{n-1}(X_{i+1}-X_i)^2\right)$

$=c\sum_{i=1}^{n-1}E\left((X_{i+1}-X_i)^2\right)$

$=c\sum_{i=1}^{n-1}E\left(X_{i+1}^2+X_i^2-2X_{i+1}X_i\right)$

$=c\sum_{i=1}^{n-1}\left(E(X_{i+1}^2)+E(X_i^2)-2E(X_{i+1}X_i)\right)$

$=c\sum_{i=1}^{n-1}\left(2E(X^2)-2E(X)E(X)\right)$

$=2c\sum_{i=1}^{n-1}\left(E(X^2)-E(X)E(X)\right)$

$=2c\sum_{i=1}^{n-1}\sigma^2=2c(n-1)\sigma^2=\sigma^2$

$\Rightarrow c=\frac{1}{2(n-1)}$.

12. ① $\lim_{n\to\infty}P\{|2\bar{X}-\theta|<\varepsilon\}$

$=\lim_{n\to\infty}P\{|\frac{2}{n}\sum_{i=1}^{n}X_i-\theta|<\varepsilon\}$

$=\lim_{n\to\infty}P\{|\frac{1}{n}\sum_{i=1}^{n}X_i-\frac{\theta}{2}|<\frac{\varepsilon}{2}\}=1$

$\cancel{\geq 1-\frac{D(X)}{\frac{\varepsilon^2}{4}}}$

$\lim_{n\to\infty}E[(2\bar{X}-\theta)^2]$

$=\lim_{n\to\infty}E(4(\bar{X})^2+\theta^2-4\theta\bar{X})=0$

② $\lim_{n\to\infty}P\{|X_{(n)}-\theta|<\varepsilon\}$

$=\lim_{n\to\infty}P\{|\max\{X_1,X_2,\cdots X_n\}-\theta|<\varepsilon\}$

$=1$

$\lim_{n\to\infty}E[(X_{(n)}-\theta)^2]$

$=\lim_{n\to\infty}E[(\max\{X_1,X_2,\cdots X_n\}-\theta)^2]$

$=0$.

17. 方差 σ^2 已知.

置信区间: $\left(\overline{X}-\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}, \overline{X}+\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}\right)$

区间长度: $\frac{2\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}$

则 $\frac{2\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}} \le L$

$\Rightarrow n \ge \left(\frac{2\sigma u_{\frac{\alpha}{2}}}{L}\right)^2$

$\Rightarrow n \ge \frac{4\sigma^2 u_{\frac{\alpha}{2}}^2}{L^2}$

18.

$\overline{X} = \frac{1}{5} \times (22.3+21.5+20.0+21.8+21.4)$

$= 20.16$

(1) $\sigma=0.3$ 时 $\alpha=0.05$ 的置信区间:

$\left(\overline{X}-\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}, \overline{X}+\frac{\sigma}{\sqrt{n}}u_{\frac{\alpha}{2}}\right)$

$n=5$, $u_{\frac{\alpha}{2}}$.

则置信区间为 $(21.137, 21.663)$.

(2) σ 未知. $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\overline{X})^2$

置信区间:

$\left(\overline{X}-\frac{S}{\sqrt{n}}t_{\frac{\alpha}{2}}(n-1), \overline{X}+\frac{S}{\sqrt{n}}t_{\frac{\alpha}{2}}(n-1)\right)$

$\Rightarrow (20.335, 22.4645)$

(3) σ 未知, 置信上限.

$\overline{X}+\frac{S}{\sqrt{n}}t_{\alpha}(n-1) = 22.2173$

置信下限 $\overline{X}-\frac{S}{\sqrt{n}}t_{\alpha}(n-1)$

$= 20.5827$

23. $\sigma_1^2=\sigma_2^2=\sigma^2$，则

$\mu_1-\mu_2$ 的一个置信度为 $1-\alpha$ 的置信区间：

$$\left((\overline{X}-\overline{Y})-t_{\frac{\alpha}{2}}(n_1+n_2-2)S_w\sqrt{\frac{1}{n_1}+\frac{1}{n_2}},\ (\overline{X}-\overline{Y})+t_{\frac{\alpha}{2}}(n_1+n_2-2)S_w\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right)$$

$$S_w=\sqrt{\frac{(n_1-1)S_{1n_1}^2+(n_2-1)S_{2n_2}^2}{n_1+n_2-2}}$$

$$\overline{X}=\frac{1}{4}\times(0.143+0.142+0.143+0.137)$$

$$\overline{Y}=\frac{1}{5}\times(0.14+0.142+0.136+0.138+0.14)$$

$$S_{1n_1}=\frac{1}{n_1-1}\sum_{i=1}^{n_1}(X_i-\overline{X})^2$$

$$S_{2n_2}=\frac{1}{n_2-1}\sum_{i=1}^{n_2}(Y_i-\overline{Y})^2$$

则置信区间：$(-0.002,0.006)$

26. $\frac{\sigma_1^2}{\sigma_2^2}$ 的一个置信度为 $1-\alpha$ 的置信区间为

$$\left(\frac{S_{1n_1}^2}{S_{2n_2}^2F_{\frac{\alpha}{2}}(n_1-1,n_2-1)},\ \frac{S_{1n_1}^2}{S_{2n_2}^2F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)}\right)$$

已知 $n_1=n_2=10$.

$S_{1n_1}^2=S_A^2=0.5419$

$S_{2n_2}^2=S_B^2=0.6065$

则置信区间：$(0.222,3.601)$

置信下限：$\frac{S_{1n_1}^2}{S_{2n_2}^2F_{\alpha}(n_1-1,n_2-1)}=0.281$

置信上限：$\frac{S_{1n_1}^2}{S_{2n_2}^2F_{\alpha}(n_1-1,n_2-1)}=2.8413$