

$$1. E(X) = \int x \cdot \frac{3}{12+3} dx = 1$$

$$\begin{aligned} 3. E(X) &= \int_{-\infty}^{+\infty} x f(x) dx \\ &= \int_0^{+\infty} x \lambda e^{-\lambda x} dx \\ &= \frac{1}{\lambda} \end{aligned}$$

$$\begin{aligned} 5. E(X) &= \int_{-\infty}^{+\infty} x f(x) dx \\ &= \int_0^{+\infty} x \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} dx \\ &= \sqrt{\frac{\pi}{2}} \sigma \end{aligned}$$

$$6. f(x) = \begin{cases} \frac{3a^3}{x^4}, & x \geq a \\ 0, & x < a \end{cases}$$

$$E(X) = \int_{-\infty}^{+\infty} x f(x) dx = \int_a^{+\infty} x \frac{3a^3}{x^4} dx = \frac{3}{2}a$$

$$\begin{aligned} 10. E\left(\frac{1}{2}mX^2\right) &= \int_{-\infty}^{+\infty} \frac{1}{2}mX^2 \cdot f(x) dx = \int_{-\infty}^{+\infty} \frac{1}{2}mX^2 \frac{4x^2}{a^3\sqrt{x}} e^{-\frac{x^2}{a^2}} dx \\ &= \frac{3}{4}ma^2 \end{aligned}$$



$$(1) a + 0.1 + 0 + 0.1 + 0 + b + 0.1 + 0.1 + c = 1$$

$$P(X=-1) = a + 0.1 \quad P(Y=1) = a + 0.2$$

$$P(X=0) = 0.1 + b \quad P(Y=2) = 0.2$$

$$P(X=1) = 0.2 + c \quad P(Y=3) = b + c$$

$$(-1) \times (a + 0.1) + 0 \times (0.1 + b) + 1 \times (0.2 + c) = 0$$

$$(a + 0.2) \times 1 + 0.2 \times 2 + 3 \times (b + c) = 0$$

$$\Rightarrow a = 0.2, b = 0.3, c = 0.1$$

(2)

$(X-Y)^2$	0	1	4	9	16
P	0.1	0.2	0.3	0.4	0

$$E(Z) = 0.2 + 1.2 + 3.6 = 5$$

(3)

$X^2Y$	0	1	2	3
P	0.4	0.3	0.2	0.1

$$E(Z) = 0.3 + 0.3 + 0.4 = 1$$



$$17. (1) f_x(x) = \begin{cases} \frac{1}{2} & (0 < x < 2) \\ 0 & (\text{其他}) \end{cases}$$

$$f_y(y) = \begin{cases} 2e^{-2y} & (y \geq 0) \\ 0 & (\text{其他}) \end{cases}$$

$$f(x, y) = e^{-2y} \quad (0 < x < 2, y \geq 0)$$

$$E(x+y) = \iint f(x, y) d\sigma \quad (x+y)$$

$$= \int_0^2 dx \int_0^{+\infty} (x+y) e^{-2y} dy$$

$$= \frac{3}{2}$$

$$E(x^2 - 2Y + 1) = \iint (x^2 - 2Y + 1) f(x, y) d\sigma$$

$$= \int_0^2 dx \int_0^{+\infty} (x^2 - 2y + 1) e^{-2y} dy d\sigma$$

$$= \frac{4}{3}$$

$$(2) E(XY) = E(X)E(Y) = 1 \times \frac{1}{2} = \frac{1}{2}.$$

$$25. \text{cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$P(X=0) = 0.6$$

$$P(X=1) = 0.4$$

$$E(X) = 0.4$$

$$P(Y=-2) = 0.2$$

$$P(Y=-1) = 0.2$$

$$P(Y=0) = 0.2$$

$$P(Y=1) = 0.2$$

$$P(Y=2) = 0.2$$

$$E(Y) = 0$$

$$P(XY=1) = 0.2$$

$$P(XY=0) = 0.6$$

$$P(XY=-1) = 0.2$$

$$E(XY) = 0$$

$$\therefore \text{cov}(X, Y) = 0, \quad \rho(X, Y) = 0$$



$$28. \text{COV}(X, Y) = E(XY) - E(X)E(Y)$$

X	-1	0	1
P	0.25	0.5	0.25

$$E(X) = 0$$

Y	-1	0	1
P	0.25	0.5	0.25

$$E(Y) = 0$$

XY	-1	0	1
P	0	1	0

$$E(XY) = 0$$

$$\text{COV}(X, Y) = 0. \Rightarrow P_{XY} = 0 \text{ 即 } X, Y \text{ 不相关}$$

$$P(X=-1, Y=-1) = 0 \neq P(X=-1) \times P(Y=-1) = 0.25 \times 0.25 = \frac{1}{8},$$

即 X 与 Y 不独立.

$$32. D(X-Y) = D(X) + D(Y) - 2 \text{COV}(X, Y)$$

$$\rho(X, Y) = \frac{\text{COV}(X, Y)}{\sqrt{D(X)}\sqrt{D(Y)}} \Rightarrow D(X-Y) = D(X) + D(Y) - 2\sqrt{D(X)}\sqrt{D(Y)}$$

$$= 1 + D(Y) - 2\sqrt{D(Y)}$$

$$D(X) = E(X^2) - (E(X))^2 = 1 - 0 = 1$$

$$\text{COV}(X, Y) = E(XY) - E(X)E(Y) = -1 - 0 = -1$$

$$\Rightarrow \frac{-1}{1 \times \sqrt{D(Y)}} = -\frac{1}{2} \Rightarrow D(Y) = 4$$

$$\Rightarrow D(X-Y) = 1 + 4 - 2\sqrt{4} = 1$$

