4.4 (1) $\sum_{n=0}^{N-1} \tilde{x}(n+m) W_N^{kn} = \sum_{n=m}^{N-1+m} \tilde{x}(n) W_N^{kn} \cdot W_N^{-km}$

$= \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{kn} \cdot W_N^{-km} = \tilde{X}(k) W_N^{-km}$

(2) $\sum_{n=0}^{N-1} \tilde{x}^*(n) W_N^{kn} = \sum_{n=0}^{N-1} \left(\tilde{x}(n) W_N^{-kn}\right)^*$

$= \tilde{X}^*(-k)$

(3) $\sum_{n=0}^{N-1} \tilde{x}^*(-n) W_N^{kn} = \sum_{n=0}^{N-1} \left(\tilde{x}(-n) W_N^{-kn}\right)^*$

$= \sum_{n=0}^{1-N} \left(\tilde{x}(n) W_N^{kn}\right)^* = \tilde{X}^*(k)$

(4) $\sum_{n=0}^{N-1} Re[\tilde{x}(n)] \cdot W_N^{kn} = \sum_{n=0}^{N-1} \frac{1}{2}(\tilde{x}(n) + \tilde{x}^*(n)) \cdot W_N^{kn}$

$= \frac{1}{2}\tilde{X}(k) + \frac{1}{2}\tilde{X}^*(-k) = \tilde{X}_e(k)$

(5) $\sum_{n=0}^{N-1} j\,Im[\tilde{x}(n)] \cdot W_N^{kn} = \sum_{n=0}^{N-1} \frac{1}{2}\left(\tilde{x}(n) - \tilde{x}^*(n)\right) W_N^{kn}$

$= \frac{1}{2}\tilde{X}(k) - \frac{1}{2}\tilde{X}^*(-k) = \tilde{X}_o(k)$

(2) $\tilde{x}(n)$ 为实周期, $\tilde{x}(n)=\tilde{x}^*(n)$

(a) 则 $\tilde{X}(k)=\tilde{X}^*(-k)$, $\tilde{X}^*(-k)=\tilde{X}^*(k)$

(b) $Re[\tilde{X}(k)]=Re[\tilde{X}^*(-k)]=Re[\tilde{X}(-k)]$

(c) $Im[\tilde{X}(k)]=Im[\tilde{X}^*(-k)]=-Im[\tilde{X}(-k)]$

(d) $|\tilde{X}(k)|=|\tilde{X}(-k)|=|\tilde{X}^*(-k)|$

(e) $\arg[\tilde{X}(k)]=\arg[\tilde{X}(-k)]=-\arg[\tilde{X}^*(-k)]$

4.6 傅里叶级数: 连续、周期信号
连续时间傅里叶变换: 连续 非周期信号
离散时间傅里叶变换: 离散 非周期信号
离散傅里叶变换: 离散 周期信号

CTFT ~~CTFS~~ 可以认为是周期∞的 CTFS.
DTFT 可以认为是周期∞的 DFT

DTFT 可以认为是 CTFT 在时域采样导致的周期延拓(频率), DFT 可以认为是 CTFS 在时域采样导致的周期延拓(频率)

扫描全能王 创建

4.8. (1) $X(e^{j\omega})=\sum_{n=-\infty}^{+\infty} x(n)e^{-j\omega n}$

$=\sum_{n=0}^{+\infty} a^n e^{-j\omega n}=\sum_{n=0}^{+\infty}\left(\frac{a}{e^{j\omega}}\right)^n$,

$=\frac{1}{1-ae^{-j\omega}}$,

(2) $\tilde{X}(k)=\sum_{n=0}^{N-1}\tilde{x}(n)W_N^{kn}=\sum_{n=0}^{N-1}\sum_{r=-\infty}^{\infty}x(n+rN)W_N^{kn}$

$=\sum_{n=0}^{N-1}\sum_{r=-\infty}^{\infty}a^{n+rN}u(n+rN)W_N^{kn}$

$=\sum_{n=0}^{N-1}\sum_{r=0}^{\infty}a^{n+rN}W_N^{kn}=\sum_{n=0}^{N-1}\cdot a^n\cdot\sum_{r=0}^{\infty}a^{rN}W_N^{kn}$

$=\sum_{r=0}^{\infty}a^{rN}\sum_{n=0}^{N-1}a^nW_N^{kn}=\sum_{r=0}^{\infty}a^{rN}\frac{1-a^Ne^{-j2\pi k}}{1-ae^{-j\frac{2\pi k}{N}}}$

$=\frac{1-a^Ne^{-j2\pi k}}{1-ae^{-j\frac{2\pi k}{N}}}\cdot\frac{1}{1-a^N}=\frac{1-a^N}{1-ae^{-j\frac{2\pi k}{N}}}\cdot\frac{1}{1-a^N}$

$=\frac{1}{1-ae^{-j\frac{2\pi k}{N}}}$

(3) $\tilde{X}(k)=X(e^{j\omega})$，当 $\omega=\frac{2\pi k}{N}$ 时.

4.9. (1) $\sum_{n=0}^{N-1} \delta(n) W_N^{kn} = \sum_{n=0}^{N-1} \delta(n) e^{-\frac{2\pi}{N}kn} = 1$

$\therefore X(k) = R_N(k)$

(2) $x(n) = \delta(n-n_0)$

$X(k) = \sum_{n=0}^{N-1} \delta(n-n_0) W_N^{kn} \cdot R_N(k)$

$= \sum_{n=-n_0}^{N-1-n_0} \delta(n) W_N^{kn} \cdot W_N^{kn_0} \cdot R_N(k)$

$= W_N^{kn_0} \cdot R_N(k)$

(3) $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} R_N(k)$

$= \left[ W_N^{k\cdot 0} + W_N^{k\cdot 2} + W_N^{k\cdot 4} + \cdots + W_N^{k(N-2)} \right] R_N(k)$

$= \sum_{n=0}^{\frac{N-2}{2}} W_N^{2kn} \cdot R_N(k)$

$= \frac{1-e^{-j\cdot 2\pi k}}{1-e^{-j\cdot\frac{4\pi k}{N}}} \cdot R_N(k)$

$1-e^{-j\frac{4\pi k}{N}} = 0 \Rightarrow e^{-j\frac{4\pi k}{N}} = 1$

$\frac{4\pi k}{N} = 2\pi m$

$k = \frac{N}{2} \cdot m$, $k < N$, ~~$X(k) = 1$~~

则 $k=0$, $k=\frac{N}{2}$, ~~则~~

$$\lim_{k\to 0}\frac{1-e^{-j2\pi k}}{1-e^{-j\frac{4\pi k}{N}}}=\frac{e^{j\cdot 2\pi k}\cdot j\cdot 2\pi}{e^{j\frac{4\pi k}{N}}\cdot j\cdot\frac{4\pi}{N}}=\frac{N}{2}$$

$$\lim_{k\to\frac{N}{2}}\frac{1-e^{-j2\pi k}}{1-e^{-j\frac{4\pi k}{N}}}=\frac{N}{2}.$$

则 $X(k)=\begin{cases}\frac{N}{2}, & (k=0, k=2)\\ 0 & \text{others}.\end{cases}$

(4) $X(k)=\sum_{n=0}^{N-1}x(n)W_N^{k-1}\cdot R_N(k)=\sum_{n=0}^{\frac{N}{2}-1}W_N^{k-1}\cdot R_N(k)$

$$=\frac{1-e^{-j\pi k}}{1-e^{-j\frac{2\pi k}{N}}}\cdot R_N(k)$$

$1-e^{-j\frac{2\pi k}{N}}=0 \Rightarrow e^{-j\frac{2\pi k}{N}}=1$, $k=0, N$(舍去)

$1-e^{-j\pi k}=0 \Rightarrow e^{-j\pi k}=0 \Rightarrow k=2.4.6\cdots$

$$\lim_{k\to 0}\frac{1-e^{-j\pi k}}{1-e^{-j\frac{2\pi k}{N}}}=\frac{e^{-j\pi k}\cdot\pi}{e^{-j\frac{2\pi k}{N}}\cdot\frac{2\pi}{N}}=\frac{N}{2}$$

$\therefore X(k)=\begin{cases}\frac{N}{2} & (k=0)\\ \frac{2}{1-e^{-j\cdot\frac{2\pi k}{N}}}, & (k\le N-1, k\text{为奇数})\\ 0 & \text{others}\end{cases}$

(5) $X(k)=\sum_{n=0}^{N-1} a^n W_N^{kn} \cdot R_N(k)$

$=\frac{1-a^N}{1-ae^{-j\frac{2\pi k}{N}}} \cdot R_N(k)$

4.17 (1) $X(k)=\sum_{n=0}^{N-1} x(n) W_N^{kn}$

$k=0$时，

$X(0)=x(0)+x(1)+\cdots+x(N-1)$

① $N$为偶数. $x(n)=-x(N-1-n)$

$\Rightarrow x(0)=-x(N-1)$

$x(1)=-x(N-2)$

$\vdots$

$x(\frac{N}{2}-1)=-x(\frac{N}{2})$

则 $X(0)=0$

② $N$为奇数. 同上，且 $x(\frac{N-1}{2})=-x(\frac{N-1}{2})$

$\Rightarrow x(\frac{N-1}{2})=0$

$\Rightarrow X(0)=0$

(2) $X(\frac{N}{2})=\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}n\cdot\frac{N}{2}}$

$=\sum_{n=0}^{N-1} x(n)\cdot e^{-j\pi n}$

$=x(0)-x(1)+x(2)-x(3)+\cdots+x(N-2)-x(N-1)$

$=0.$

4.23. $\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n) \cdot x^*(n)$

$= \sum_{n=0}^{N-1} x(n) \cdot ( \quad x(n) \quad )^*$

$= \sum_{n=0}^{N-1} x(n) \cdot \left[ \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn} \right]^*$

$= \frac{1}{N} \sum_{n=0}^{N-1} x(n) \cdot \sum_{k=0}^{N-1} X^*(k) W_N^{kn}$

$= \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) \sum_{n=0}^{N-1} x(n) W_N^{kn}$

$= \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) \cdot X(k)$

$= \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$

4.29. (1) $\sum_{n=0}^{N-1} x(N-1-n) e^{-j\frac{2\pi kn}{N}}$ ，令 $N-1-n=m$

$n=N-1-m$

$= \sum_{m=N-1}^{0} x(m) e^{-j\frac{2\pi k(N-1-m)}{N}}$

$= \sum_{m=0}^{N-1} x(m) e^{+j\frac{2\pi k(1+m)}{N}}$

$= e^{j\frac{2\pi k}{N}} \cdot \sum_{m=0}^{N-1} x(m) e^{j\frac{2\pi km}{N}}$

$= e^{j\frac{2\pi k}{N}} X(e^{-j\frac{2\pi k}{N}})$

4.29.(2) $\sum_{n=0}^{\frac{N}{2}-1} x(2n)\, e^{-\frac{2\pi k}{\frac{N}{2}} \cdot n}$

$= \sum_{n=0}^{\frac{N}{2}-1} x(2n)\, e^{-\frac{4\pi \cdot k n}{N}}$ $\quad , 2n = m$

$= \sum_{m=0}^{N-1} x(m)\, e^{-\frac{2\pi k}{N} \cdot m}$

$= \frac{1}{2} \sum_{m=0}^{N-1} x(m)\, e^{-j\frac{2\pi k m}{N}} + \frac{1}{2} \sum_{m=0}^{N-1} (-1)^{m} x(m)\, e^{-j\frac{2\pi k m}{N}}$

$= \frac{1}{2} X\left(e^{j\frac{2\pi k}{N}}\right) + \frac{1}{2} X\left(e^{j\cdot\frac{2\pi(k+\frac{N}{2})}{N}}\right)$

(3) $\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi k n}{2N}} = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{\pi k n}{N}}$

$= X\left(e^{j\frac{\pi k}{N}}\right)$