~~3.4~~: 3.3 (1) $x(n)=a^{|n|}$, $0<a<1$

$$X(z)=\sum_{n=-\infty}^{+\infty} a^{|n|}\cdot z^{-n}$$

$$=\sum_{n=-\infty}^{-1} a^{-n}z^{-n}+\sum_{n=0}^{+\infty} a^{n}\cdot z^{-n}$$

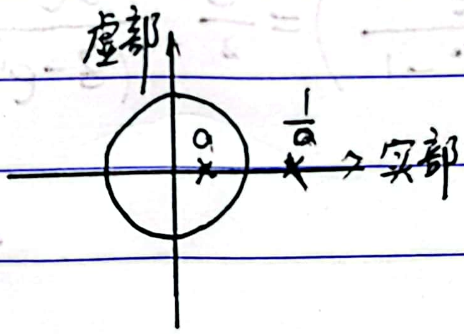
$$=\sum_{n=1}^{+\infty} a^{n}z^{n}+\sum_{n=0}^{+\infty}\left(\frac{a}{z}\right)^{n}$$

$$=\frac{az\times(1-(az)^{n})}{1-az}+\frac{1\times\left(1-\left(\frac{a}{z}\right)^{n}\right)}{1-\frac{a}{z}}$$

$$=\frac{az}{1-az}+\frac{z}{z-a}=\frac{z(1-a^{2})}{(1-az)(z-a)}$$

$$|a|<|z|<\left|\frac{1}{a}\right|$$

~~(2)~~



(2) $x(n) = A r^n \cos(w_0 n+\phi)\, u(n)$, $0<r<1$

$$X(z) = \sum_{n=-\infty}^{+\infty} A r^n \cos(w_0 n+\phi)\, u(n)\, z^{-n}$$

$$= \sum_{n=0}^{+\infty} A r^n \cos(w_0 n+\phi)\, z^{-n}$$

$$= \frac{A}{2}\sum_{n=0}^{+\infty} \left(\frac{r}{z}\right)^n \left(e^{j(w_0 n+\phi)} - e^{-j(w_0 n+\phi)}\right)$$

$$= \frac{A}{2}\left(\sum_{n=0}^{+\infty}\left(\frac{r}{z}e^{jw_0}\right)^n \cdot e^{j\phi} - \sum_{n=0}^{+\infty}\left(\frac{r}{ze^{jw_0}}\right)^n \cdot e^{-j\phi}\right)$$

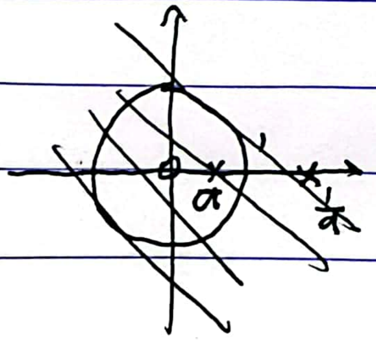
$$= \frac{A}{2}\left(\frac{1\times\left(1-\left(\frac{r}{z}e^{jw_0}\right)^n\right)}{1-\frac{r}{z}e^{jw_0}} \cdot e^{j\phi} - \frac{1\times\left(1-\left(\frac{r}{ze^{jw_0}}\right)^n\right)}{1-\frac{r}{ze^{jw_0}}} \cdot e^{-j\phi}\right)$$

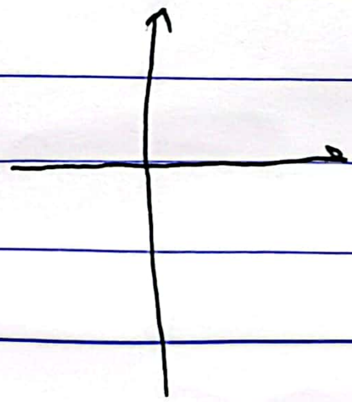
$$= \frac{A}{2}\left(\frac{z}{z-re^{jw_0}} \cdot e^{j\phi} - \frac{ze^{jw_0}}{ze^{jw_0}-r}\, e^{-j\phi}\right)$$

$$= \frac{Az}{2}\left(\frac{e^{j\phi}}{z-re^{jw_0}} - \frac{e^{jw_0}\cdot e^{-j\phi}}{ze^{jw_0}-r}\right)$$

$$= \frac{Az}{2}\, \frac{ze^{jw_0}\cdot e^{j\phi} - r\cdot e^{j\phi} - z\cdot e^{jw_0}\cdot e^{-j\phi} + re^{-j\phi}\cdot e^{2jw_0}}{(z-re^{jw_0})(ze^{jw_0}-r)}$$

$|z|>r$





(3) $x(n)=a^n u(n)+b^n u(-n-1)$

$$X(z)=\sum_{n=-\infty}^{+\infty} x(n) z^{-n}$$

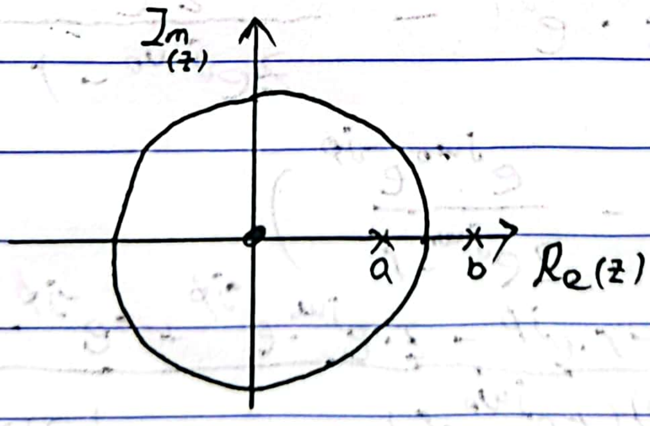
$$=\sum_{n=-\infty}^{+\infty}\left(a^n u(n)+b^n u(-n-1)\right) z^{-n}$$

$$=\sum_{n=0}^{+\infty} a^n z^{-n}+\sum_{n=-\infty}^{-1} b^n z^{-n}$$

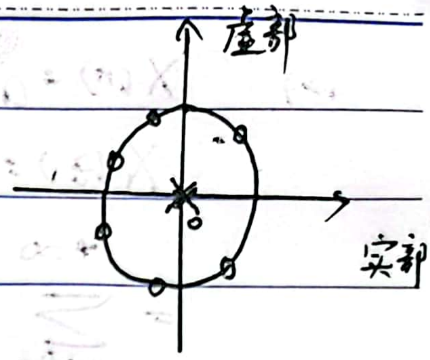
$$=\frac{1}{1-\frac{a}{z}}+\sum_{n=1}^{\infty} b^{-n} z^{n}$$

$$=\frac{1}{1-\frac{a}{z}}+\sum_{n=1}^{\infty}\left(\frac{z}{b}\right)^n=\frac{1}{1-\frac{a}{z}}+\frac{\frac{z}{b}\left(1-\left(\frac{z}{b}\right)^n\right)}{1-\frac{z}{b}}$$

$$=\frac{z}{z-a}+\frac{z}{b-z}=\frac{z(a-b)}{(z-a)(z-b)}\qquad |a|<|z|<|b|$$



(4) $x(n)=u(n)-u(n-N)$



$$X(z)=\sum_{n=-\infty}^{+\infty}\left(u(n)-u(n-N)\right)z^{-n}$$

$$=\sum_{n=1}^{+\infty}z^{-n}-\sum_{n=N}^{+\infty}z^{-n}$$

$$=\frac{\frac{1}{z}\times\left(1-\left(\frac{1}{z}\right)^{n}\right)}{1-\frac{1}{z}}-\frac{\frac{1}{z^{N}}\times\left(1-\left(\frac{1}{z}\right)^{n}\right)}{1-\frac{1}{z}}$$

$$=\frac{\frac{1}{z}}{1-\frac{1}{z}}-\frac{\frac{1}{z^{N}}}{1-\frac{1}{z}}=\frac{\frac{1}{z}-\frac{1}{z^{N}}}{1-\frac{1}{z}}=\frac{z^{N-1}-1}{z^{N}-z^{N-1}}$$

$$=\frac{z^{N-1}-1}{z^{N-1}(z-1)}$$

$|z|>0$

3.4. (1) $X(z)=\sum_{n=-\infty}^{+\infty}x(n)z^{-n}$

$$\frac{dX(z)}{dz}=\sum_{n=-\infty}^{+\infty}(-n)\cdot x(n)z^{-n-1}$$

$$X(z)=\sum_{n=-\infty}^{+\infty}n^{2}a^{n}u(n)z^{-n}=\sum_{n=0}^{+\infty}n^{2}a^{n}z^{-n}$$

$$=-z\frac{d}{dz}\left(\frac{az}{(z-a)^{2}}\right)$$

$$=-z\cdot a\,(-2)\,\frac{(z-a)^{2}-2z(z-a)}{(z-a)^{3}}$$

$$=\frac{az(z+a)}{(z-a)^{3}}\quad |z|>|a|$$

(2) $x(n)=a^{-n}u(-n)$

$$X(z)=\sum_{n=-\infty}^{+\infty} a^{-n}u(-n)z^{-n}=\sum_{n=-\infty}^{0} a^{-n}\cancel{u(-n)}\, z^{-n}$$

$$=\sum_{n=0}^{+\infty} a^{n}z^{n}=\frac{1\times(az)^{n}}{1-az}=\frac{1}{1-az},\quad |z|<\left|\frac{1}{a}\right|$$

3.5. $X(z)=e^{z}+e^{\frac{1}{z}}$

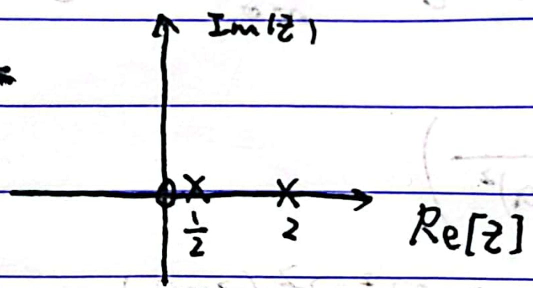
$$=\sum_{n=0}^{\infty}\frac{1}{n!}z^{n}+\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{1}{z}\right)^{n}$$

$$=\sum_{n=-\infty}^{-1}\frac{1}{(-n)!}z^{-n}+1+\sum_{n=0}^{\infty}\frac{1}{n!}z^{-n}$$

$\therefore x(n)=\frac{1}{|n|!}$

3.7 $X(z)=\dfrac{-3z^{-1}}{2-5z^{-1}+2z^{-2}}=\dfrac{-3z}{2z^{2}-5z+2}=\dfrac{-3z}{(z-2)(2z-1)}$

1



$$=\frac{z}{z-\frac{1}{2}}-\frac{z}{z-2}$$

(1) 右边序列, $x(n)=\left[\left(\frac{1}{2}\right)^{n}-2^{n}\right]u(n)$

(2) 左边序列 $x(n)=\left[2^{n}-\left(\frac{1}{2}\right)^{n}\right]u(-n-1)$

(3) 双边序列, $x(n)=\left(\frac{1}{2}\right)^{n}u(n)+2^{n}u(-n-1)$

方法一：部分分式展开法

3.12. (1) $X(z)=\frac{1}{1+0.5z^{-1}}=\frac{z}{z+0.5}=1-\frac{1}{2z+1}$

$=1-\frac{1}{1+2z}$

$x(n)=\delta(n)-2^{n}u(n)$

$X(z)=\frac{1}{1+0.5z^{-1}}=\frac{1}{1+\frac{1}{2}z^{-1}}$, $|z|>\frac{1}{2}$

$=\frac{1}{1-(-\frac{1}{2}z^{-1})}$

$x(n)=(-\frac{1}{2})^{n}u(n)$

(2) $X(z)=\frac{1}{1+0.25z^{-2}}=\frac{4}{4+z^{-2}}=\frac{4}{z^{-2}+4}=\frac{4}{(z^{-1}-2j)(z^{-1}+2j)}$

$=\frac{4}{4j}\cdot\frac{(z^{-1}+2j)-(z^{-1}-2j)}{(z^{-1}+2j)(z^{-1}-2j)}=-j\left(\frac{1}{z^{-1}-2j}-\frac{1}{z^{-1}+2j}\right)$

$=-j\times\left(\frac{1}{2j}\cdot\frac{1}{\frac{1}{2j}z^{-1}-1}-\frac{1}{2j}\times\frac{1}{1+\frac{1}{2j}z^{-1}}\right)$

$=-j\times\left(-\frac{1}{2j}\times\frac{1}{1-\frac{1}{2j}z^{-1}}-\frac{1}{2j}\times\frac{1}{1+\frac{1}{2j}z^{-1}}\right)$

$=\frac{1}{2}\times\left(\frac{1}{1-\frac{1}{2j}z^{-1}}+\frac{1}{1+\frac{1}{2j}z^{-1}}\right)$

$=-\frac{1}{2}\times(-\frac{1}{2}j)^{n}u(-n-1)-\frac{1}{2}\times(\frac{1}{2}j)^{n}u(-n-1)$

(3) $X(z)=\dfrac{1-az^{-1}}{z^{-1}-a}=\dfrac{1}{z^{-1}-a}-\dfrac{az^{-1}}{z^{-1}-a}$

$=-\dfrac{1}{a-z^{-1}}+\dfrac{az^{-1}}{a-z^{-1}}=-\dfrac{1}{a}\dfrac{1}{1-\frac{1}{a}z^{-1}}+\cancel{a}\cdot\dfrac{z^{-1}}{1-\frac{1}{a}z^{-1}}$

$=-\dfrac{1}{a}\times\left(\dfrac{1}{a}\right)^{n}u(n)+\left(\dfrac{1}{a}\right)^{n-1}u(n-1)$

方法二. 幂级数展开

(1)

$$
\begin{array}{r|l}
 & 1-0.5z^{-1}+0.25z^{-2} \\
\hline
1+0.5z^{-1} & 1 \\
 & 1+0.5z^{-1} \\
\hline
 & -0.5z^{-1} \\
 & -0.5z^{-1}-0.25z^{-2} \\
\hline
 & +0.25z^{-2} \\
 & \cdots\cdots
\end{array}
$$

$x(n)=(-0.5)^{n}u(n)$

(2)

$$
\begin{array}{r|l}
 & 4z^{2}-16z^{4}+64z^{6} \\
\hline
0.25z^{-2}+1 & 1 \\
 & 1+4z^{2} \\
\hline
 & -4z^{2} \\
 & -4z^{2}-16z^{4} \\
\hline
 & 16z^{4} \\
 & \cdots\cdots
\end{array}
$$

$x(n)=\begin{cases} 0 & n\geq 0 \\ 0, & |n|为奇 \\ 4^{-\frac{n}{2}}, & |n|为偶 \quad n<0 \end{cases}$

(3)

$$
\begin{array}{r}
-\frac{1}{a}+\frac{\frac{1}{a}-a}{+a}z^{-1}+\frac{a^2-1}{a^3}z^{-2} \\
-a+z^{-1}\;\overline{\big)\;1-az^{-1}\qquad\qquad\qquad\qquad} \\
\underline{1-\frac{1}{a}z^{-1}\qquad\qquad} \\
\left(\frac{1}{a}-a\right)z^{-1}\qquad\qquad \\
\underline{\left(\frac{1}{a}-a\right)z^{-1}-\frac{\frac{1}{a}-a}{a}z^{-2}} \\
\frac{\frac{1}{a}-a}{a}z^{-2}
\end{array}
$$

$\therefore\; x(n)=-\left(\frac{1}{a}\right)^{n+1}u(n)+\left(\frac{1}{a}\right)^{n-1}u(n-1)$

5.10. $H(z)=\dfrac{3-7z^{-1}+5z^{-2}}{1-\frac{5}{2}z^{-1}+z^{-2}}$

$= 5-\dfrac{1}{1-2z^{-1}}-\dfrac{3}{1-\frac{1}{2}z^{-1}}$

$\Rightarrow h(n)=5\delta(n)-2^{n}u(-n-1)-3\times(\frac{1}{2})^{n}u(n)$

(1) $y(n)=h(n)*x(n)=\sum\limits_{m=-\infty}^{+\infty}h(m)\,x(n-m)$

$=\sum\limits_{m=-\infty}^{n}h(m)$

$=\begin{cases}-2^{n+1}, & n<0\\ -2+3\times(\frac{1}{2})^{n}, & n\geq 0\end{cases}$

(2) $x(n)=u(n)$, $Y(z)=X(z)\cdot H(z)$

$X(z)=\dfrac{1}{1-z^{-1}}$, $=\dfrac{-2}{1-z^{-1}}+\dfrac{2}{1-2z^{-1}}+\dfrac{3}{1-\frac{1}{2}z^{-1}}$

$\Rightarrow y(n)=-2u(n)-2\times2^{n}u(-n-1)+3\times(\frac{1}{2})^{n}u(n)$

3.40. (1) $H(z)=\dfrac{1-b^{-1}z^{-1}}{1-bz^{-1}}=\dfrac{Y(z)}{X(z)}$

$\Rightarrow (1-b^{-1}z^{-1})X(z)=(1-bz^{-1})Y(z)$

$\Rightarrow X(z)-b^{-1}z^{-1}X(z)=Y(z)-bz^{-1}Y(z)$

$\Rightarrow x(n)-b^{-1}x(n-1)=y(n)-by(n-1)$

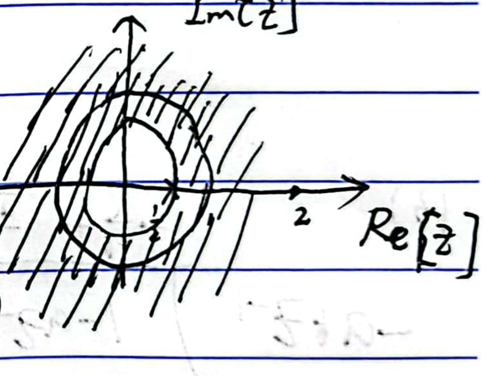
$\Rightarrow y(n)=by(n-1)+x(n)-b^{-1}x(n-1)$

(2) $H(z)=\dfrac{1-b^{-1}z^{-1}}{1-bz^{-1}}=\dfrac{z-b^{-1}}{z-b}$

极点 $z=b$ ，$|b|<1$

(3) $b=\frac{1}{2}$, $H(z)=\dfrac{1-2z^{-1}}{1-\frac{1}{2}z^{-1}}=\dfrac{2-4z^{-1}}{2-z^{-1}}=\dfrac{2z-4}{2z-1}$

零点: $z=2$，极点 $z=\frac{1}{2}$



(4) $H(z)=\dfrac{1-b^{-1}z^{-1}}{1-bz^{-1}}=\dfrac{1}{1-bz^{-1}}-b^{-1}\cdot\dfrac{z^{-1}}{1-bz^{-1}}$

$\Rightarrow h(n)=b^{n}u(n)-b^{-1}\cdot b^{n-1}\cdot u(n-1)$

$=b^{n}u(n)-b^{n-2}u(n-1)$

(5) $H(z)=\dfrac{1-b^{-1}z^{-1}}{1-bz^{-1}}$, $H(e^{jw})=\dfrac{1-b^{-1}e^{-jw}}{1-be^{-jw}}$

$|H(e^{jw})|=\dfrac{|1-b^{-1}e^{-jw}|}{|1-be^{-jw}|}=\dfrac{1}{|b|}$.

全通系统.

3.52. (1) $x(n)=\left(\frac{1}{2}\right)^n u(n) + 2^n \cdot u(-n-1)$

$$X(z)=\frac{1}{1-\frac{1}{2}z^{-1}}+\sum_{n=-\infty}^{+\infty}2^n u(-n-1)\cdot z^{-n}$$

$$=\frac{z}{z-\frac{1}{2}}+\sum_{n=-\infty}^{-1}\left(\frac{2}{z}\right)^n$$

$$=\frac{z}{z-\frac{1}{2}}+\sum_{n=1}^{+\infty}\left(\frac{z}{2}\right)^n=\frac{z}{z-\frac{1}{2}}+\frac{\frac{z}{2}\times\left(1-\left(\frac{z}{2}\right)^n\right)}{1-\frac{z}{2}}$$

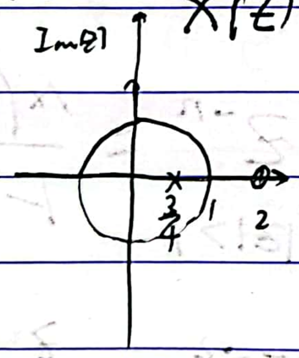
$$=\frac{z}{z-\frac{1}{2}}-\frac{z}{2-z}=\frac{2z}{2z-1}-\frac{z}{2-z}=-\frac{3}{2}\frac{z}{(z-\frac{1}{2})(z-2)}$$

$$Y(n)=6\times\left(\frac{1}{2}\right)^n u(n)-6\times\left(\frac{3}{4}\right)^n u(n)$$

$$Y(z)=6\times\frac{1}{1-\frac{1}{2}z^{-1}}-6\times\frac{1}{1-\frac{3}{4}z^{-1}}$$

$$=6\times\left(\frac{z}{z-\frac{1}{2}}-\frac{z}{z-\frac{3}{4}}\right)=6z\,\frac{-12}{(2z-1)(4z-3)}$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{z-2}{z-\frac{3}{4}},\quad |z|>\frac{3}{4}$$



(2) $H(z)=\frac{z-2}{z-\frac{3}{4}}=\frac{z-\frac{3}{4}+\frac{3}{4}-2}{z-\frac{3}{4}}=1+\left(2-\frac{3}{4}\right)\frac{1}{\frac{3}{4}-z}$

$$=1+\frac{\frac{5}{4}}{\cancel{1}}\frac{z^{-1}}{\frac{3}{4}z^{-1}-1}=1-\frac{5}{4}\frac{z^{-1}}{1-\frac{3}{4}z^{-1}}$$

$$h(n)=\delta(n)-\frac{5}{4}\times\left(\frac{5}{4}\right)^{(n-1)}u(n-1)$$

$$=\delta(n)-\left(\frac{5}{4}\right)^n u(n-1)$$

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(3) $H(z)=\dfrac{z-2}{z-\frac{3}{4}}=\dfrac{Y(z)}{X(z)}$

$\Rightarrow zX(z)-X(z)=zY(z)-\frac{3}{4}Y(z)$

$\Rightarrow X(n+1)-2X(n)=Y(n+1)-\frac{3}{4}Y(n)$

$\Rightarrow X(n)-2X(n-1)=Y(n)-\frac{3}{4}Y(n-1)$

(4) 具有稳定性、因果性