4. 证明: $E(X(t)) = E(\sin(At+\theta)) = 0$

$R_x(\tau) = E[X(t)X(t+\tau)]$

$= E(\sin(At+\theta)\sin(At+\theta+A\tau))$

$= E(-\frac{1}{2}(\cos(2At+2\theta+A\tau) - \cos(A\tau)))$

$= -\frac{1}{2}(E(\cos(2At+2\theta+A\tau)) - E(\cos A\tau))$

只与 τ 有关

$\therefore X(t) = \sin(At+\theta)$ $-\infty < t < -\infty$ 是平稳过程

5. (1) $E(Y_T) = E(aX(t)+b) = b + aE(X(t)) = b + am_x$

$R_Y(\tau) = E((aX(t)+b)(aX(t+\tau)+b)) = E(a^2X(t)X(t-\tau) + abX(t) + abX(t+\tau) + b^2)$

$= a^2E(X(t)X(t-\tau)) + 2abm_x + b^2 = a^2R_X(\tau) + 2abm_x + b^2$

$\therefore$ 是平稳过程

(2) $E(Y_T) = E(AX(t)+B) = E(A)E(X(t)) + E(B)$

$= a m_x + b$

$R_Y(\tau) = E\left((AX(t)+B)(AX(t+\tau)+B)\right) = E\left(A^2 X(t)X(t+\tau) + AB X(t) + AB X(t+\tau) + B^2\right)$

$= E(A^2) R_X(\tau) + 2ab\, m_x + E(B^2)$

$= (\sigma_1^2 + a^2) R_X(\tau) + 2ab\, m_x + (\sigma_2^2 + b^2)$

(3) $E(Y_T) = E(X(t+a) - X(t)) = 0$

$R_Y(\tau) = E\left((X(t+a)-X(t))(X(t+\tau+a) - X(t+\tau))\right)$

$= E\left(X(t+a)X(t+\tau+a) - X(t+a)X(t+\tau) - X(t)X(t+\tau+a) + X(t)X(t+\tau)\right)$

$= 2R_X(\tau) - E\left(X(t+a)X(t+\tau)\right) - E\left(X(t)X(t+\tau+a)\right)$

$= 2R_X(\tau) - R_X(\tau - a) - R_X(\tau + a)$

12. (1) $R_{XY}(-\tau) = E(X(t+\tau)Y(t)) = E(Y(t)X(t+\tau)) = R_{YX}(\tau)$

(2) $|R_{XY}(\tau)|^2 = \{E[X(t)Y(t+\tau)]\}^2 \le E[X^2(t)]\,E[Y^2(t+\tau)] = R_X(0)R_Y(0$

$R_{XY}(\tau) \le \sqrt{R_X(0)}\cdot\sqrt{R_Y(0)}$.