第3章习题

3.11.3

解:(1) 依题 $k(\theta_i-\theta_o)-b\cdot\dot{\theta}_o=J\ddot{\theta}_o$

整理得 $J\ddot{\theta}_o+b\dot{\theta}_o+k\theta_o=k\theta_i$

(2) 依题 $\begin{cases} k_2(x_2-x_1)-k_1x_1-b_1\dot{x}_1=m_1\ddot{x}_1 \\ -k_2(x_2-x_1)-b_2\dot{x}_2=m_2\ddot{x}_2 \end{cases}$

整理得 $\begin{cases} m_1\ddot{x}_1+b_1\dot{x}_1+(k_1+k_2)x_1=k_2x_2 \\ m_2\ddot{x}_2+b_2\dot{x}_2+k_2x_2=k_2x_1 \end{cases}$

(3) 依题 $\begin{cases} k_2(x_2-x_1)-k_1x_1-b_1\dot{x}_1=m_1\ddot{x}_1 \\ -k_2(x_2-x_1)=m_2\ddot{x}_2 \end{cases}$

整理得 $\begin{cases} m_1\ddot{x}_1+b_1\dot{x}_1+(k_1+k_2)x_1=k_2x_2 \\ m_2\ddot{x}_2+k_2x_2=k_2x_1 \end{cases}$

3.11.5

解. 匀质圆柱体动能 $T=\frac{1}{2}m\dot{x}^2+\frac{1}{2}J\dot{\theta}^2$

$=\frac{1}{2}m\dot{x}^2+\frac{1}{2}\cdot\frac{1}{2}mR^2\cdot\dot{\theta}^2$

$=\frac{3}{4}m\dot{x}^2$

弹簧位能 $U=\frac{1}{2}kx^2$

能量守恒,故 $T+U=$常数

对上式两边同时对时间t求导得 $\frac{3}{2}m\dot{x}\ddot{x}+kx\cdot\dot{x}=0$

$\dot{x}$不恒为0,因此 $m\ddot{x}+\frac{2}{3}kx=0$

振动频率 $w=\sqrt{\frac{2k}{3m}}$

综上,系统的运动方程式为 $m\ddot{x}+\frac{2}{3}kx=0$

振动频率 $w=\sqrt{\frac{2k}{3m}}$

3.11.7

解:系统动能 $T=\frac{1}{2}J_1\dot{\theta}_1^2+\frac{1}{2}J_2\dot{\theta}_2^2$

系统位能 $U=\frac{1}{2}k_1\theta_1^2+\frac{1}{2}k_0(\theta_2-\theta_1)^2+\frac{1}{2}k_2\theta_2^2$

$L=T-U=\frac{1}{2}J_1\dot{\theta}_1^2+\frac{1}{2}J_2\dot{\theta}_2^2-\frac{1}{2}k_1\theta_1^2-\frac{1}{2}k_0(\theta_2-\theta_1)^2-\frac{1}{2}k_2\theta_2^2$

由拉格朗日方程式

$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_1}\right)-\frac{\partial L}{\partial\theta_1}=0 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}_2}\right)-\frac{\partial L}{\partial\theta_2}=0 \end{cases} \Rightarrow \begin{cases} J_1\ddot{\theta}_1+(k_1\theta_1+k_0(\theta_1-\theta_2))=0 \\ J_2\ddot{\theta}_2+(k_2\theta_2+k_0(\theta_2-\theta_1))=0 \end{cases}$

综上,系统的运动方程式为

$\begin{cases} J_1\ddot{\theta}_1+(k_1+k_0)\theta_1-k_0\theta_2=0 \\ J_2\ddot{\theta}_2+(k_2+k_0)\theta_2-k_0\theta_1=0 \end{cases}$

3.11.8

解. 系统动能 $T=\frac{1}{2}\frac{W_1}{g}\dot{x}_1^2+\frac{1}{2}J_Aw_A^2+\frac{1}{2}\frac{W_2}{g}\dot{y}^2+\frac{1}{2}J_Bw^2$

$=\frac{1}{2}\frac{W_1}{g}\dot{x}_1^2+\frac{1}{2}\cdot\frac{1}{2}\frac{W_1}{g}R^2w_A^2+\frac{1}{2}\frac{W_2}{g}\dot{y}^2+\frac{1}{2}\cdot\frac{1}{2}\frac{W_2}{g}r^2w^2$

$=\frac{3}{4}\frac{W_1}{g}\dot{x}_1^2+\frac{1}{2}\frac{W_2}{g}\dot{y}^2+\frac{1}{4}\frac{W_2}{g}(\dot{x}_1+\dot{y})^2$

系统位能 $U=-W_1x_1\sin\alpha-W_2y$

$L=T-U=\frac{3}{4}\frac{W_1}{g}\dot{x}_1^2+\frac{1}{2}\frac{W_2}{g}\dot{y}^2+\frac{1}{4}\frac{W_2}{g}(\dot{x}_1+\dot{y})^2+W_1x_1\sin\alpha+W_2y$

$$\begin{cases}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right)-\frac{\partial L}{\partial x_1}=0\\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right)-\frac{\partial L}{\partial y}=0\end{cases} \Rightarrow \begin{cases}\frac{3}{2}\frac{W_1}{g}\ddot{x}_1+\frac{1}{2}\frac{W_2}{g}(\ddot{x}_1+\ddot{y})-W_1\sin\alpha=0\\ \frac{W_2}{g}\ddot{y}+\frac{1}{2}\frac{W_2}{g}(\ddot{x}_1+\ddot{y})-W_2=0\end{cases}$$

解得 轮心C加速度 $\ddot{x}_1=\frac{6W_1\sin\alpha-2W_2}{9W_1+2W_2}g$

圆柱B的中心E的加速度 $\ddot{y}=\frac{2}{3}g-\frac{6W_1\sin\alpha-2W_2}{3(9W_1+2W_2)}g$

3.11.9

解：系统动能 $T=\frac{1}{2}m_1\dot{x}_1^2+\frac{1}{2}m_2 v_{Ba}^2$

$=\frac{1}{2}m_1\dot{x}_1^2+\frac{1}{2}m_2[(\dot{x}_1+\dot{x}_2\cos\alpha)^2+(\dot{x}_2\sin\alpha)^2]$

$=\frac{1}{2}(m_1+m_2)\dot{x}_1^2+m_2\dot{x}_1\dot{x}_2\cos\alpha+\frac{1}{2}m_2\dot{x}_2^2$

系统位能 $U=-m_2gx_2\sin\alpha+\frac{1}{2}k(x_2+\frac{m_2g\sin\alpha}{k})^2-\frac{1}{2}k\cdot(\frac{m_2g\sin\alpha}{k})^2$

$L=T-U=\frac{1}{2}(m_1+m_2)\dot{x}_1^2+m_2\dot{x}_1\dot{x}_2\cos\alpha+\frac{1}{2}m_2\dot{x}_2^2+m_2gx_2\sin\alpha$

$-\frac{1}{2}k(x_2+\frac{m_2g\sin\alpha}{k})^2+\frac{1}{2}k(\frac{m_2g\sin\alpha}{k})^2$

$$\begin{cases}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right)-\frac{\partial L}{\partial x_1}=0\\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right)-\frac{\partial L}{\partial x_2}=0\end{cases} \Rightarrow \begin{cases}(m_1+m_2)\ddot{x}_1+m_2\ddot{x}_2\cos\alpha=0\\ m_2\ddot{x}_1\cos\alpha+m_2\ddot{x}_2-m_2g\sin\alpha+k(x_2+\frac{m_2g\sin\alpha}{k})=0\end{cases}$$

综上，系统运动微分方程为：

$$\begin{cases}(m_1+m_2)\ddot{x}_1+m_2\ddot{x}_2\cos\alpha=0\\ m_2\ddot{x}_1\cos\alpha+m_2\ddot{x}_2+kx_2=0\end{cases}$$

3.11.10

解：牵连速度 $\vec{v}_e=W_0R\sin\theta$　相对速度 $\vec{v}_r=R\cdot\dot{\theta}$

绝对速度 $\vec{v}=\vec{v}_e+\vec{v}_r$　又 $\vec{v}_e\perp\vec{v}_r$

系统动能 $T=\frac{1}{2}Mv^2=\frac{1}{2}M(v_e^2+v_r^2)=\frac{1}{2}MR^2(W_0^2\sin^2\theta+\dot{\theta}^2)$

系统位能 $U=MgR(1-\cos\theta)$

质点的拉格朗日函数 $L=T-U$

$=\frac{1}{2}MR^2(W_0^2\sin^2\theta+\dot{\theta}^2)-MgR(1-\cos\theta)$

由拉格朗日方程式

$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right)-\frac{\partial L}{\partial\theta}=0$ 得

质点运动方程为：$MR^2\ddot{\theta}-MR^2W_0^2\sin\theta\cos\theta+MgR\sin\theta=0$

3.11.11

解：系统有两个自由度，以 α, β 为广义坐标

$\delta W=F\delta x+P\delta y$

$x_D=b\cos\alpha+l\cos\beta$　$\delta x=-b\sin\alpha\,\delta\alpha-l\sin\beta\,\delta\beta$

$y_C=a\sin\alpha+l\sin\beta$　$\delta y=a\cos\alpha\,\delta\alpha+l\cos\beta\,\delta\beta$

平衡时 $\delta W=F\delta x+P\delta y$

$=(-b\sin\alpha F+a\cos\alpha P)\delta\alpha+(-l\sin\beta F+l\cos\beta P)\delta\beta$

令 $\delta\alpha\neq0$, $\delta\beta=0$ 得 $\frac{\delta W}{\delta\alpha}=-b\sin\alpha F+a\cos\alpha P=0$

令 $\delta\beta\neq0$, $\delta\alpha=0$ 得 $\frac{\delta W}{\delta\beta}=-l\sin\beta F+l\cos\beta P=0$

综上，平衡时　$\alpha=\arctan\frac{aP}{bF}$　$\beta=\arctan\frac{P}{F}$

解：系统只有一个自由度 取广义坐标 θ

$$\delta W = M\delta\theta - k r_r \delta r_r$$

又 $\delta r_r = \delta r_e \tan\theta = \left(\frac{0.3}{\cos\theta}\delta\theta\right)\cdot\tan\theta$

$$= 0.3\sec\theta\cdot\tan\theta\,\delta\theta$$

$$r_r = \frac{0.3}{\cos\theta} - 0.3$$

令 $\delta W = M\delta\theta - k\left(\frac{0.3}{\cos\theta} - 0.3\right)0.3\sec\theta\tan\theta\,\delta\theta = 0$

即 $M = 450(\sec\theta - 1)\sec\theta\tan\theta$

综上 $M = 450(\sec\theta - 1)\sec\theta\tan\theta$ $(N\cdot m)$ 时系统平衡

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11.9