4. 证明: $E[X(t)] = E(\sin(At+\theta)) = 0$

$R_X(\tau) = E[X(t)X(t+\tau)]$

$= E(\sin(At+\theta)\sin(At+\theta+A\tau))$

$= E\left(-\frac{1}{2}(\cos(2At+2\theta+A\tau) - \cos(A\tau))\right)$

$= -\frac{1}{2}(E(\cos(2At+2\theta+A\tau)) - E(\cos A\tau))$

只与τ有关

$\therefore X(t) = \sin(At+\theta)$ $-\infty < t < -\infty$ 是平稳过程

5. (1) $E[Y_T] = E(aX(t)+b) = b + aE(X(t)) = b + a m_X$

$R_Y(\tau) = E((aX(t)+b)(aX(t+\tau)+b)) = E(a^2X(t)X(t-\tau) + abX(t) + abX(t+b^2))$

$= a^2E(X(t)X(t-\tau)) + 2ab m_X + b^2 = a^2R_X(\tau) + 2ab m_X + b^2$

$\therefore$ 是平稳过程

12. (1) $R_{XY}(-\tau) = E[X(t+\tau)Y(t)] = E[Y(t)X(t+\tau)] = R_{YX}(\tau)$

(2) $|R_{XY}(\tau)|^2 = \{E[X(t)Y(t+\tau)]\}^2 \le E[X^2(t)]\,E[Y^2(t+\tau)] = R_X(0)R_Y(0)$

$R_{XY}(\tau) \le \sqrt{R_X(0)}\cdot\sqrt{R_Y(0)}$