

$$1. \textcircled{1} G(s) = C(sI - A)^{-1}B$$

$$= [0 \ 1] \left( \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix} \right)^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \frac{s-2}{s^2+s-6} = \frac{s-2}{(s-2)(s+3)} = \frac{1}{s+3}$$

极点  $s = -3 < 0$

系统 BIBO 稳定

$$\textcircled{2} A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix} \quad |\lambda I - A| = 0$$

$$\Rightarrow \lambda_1 = -3, \lambda_2 = 2 > 0$$

系统不是内部稳定的。

$$2. \quad A^T P + P A = -Q = - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -3 \end{bmatrix}, \quad P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4p_{11} + p_{31} + p_{13} & -3p_{12} + p_{32} & -2p_{13} + p_{33} - 3p_{13} \\ -3p_{21} + p_{23} & -2p_{22} & -4p_{23} \\ -5p_{31} + p_{33} & -4p_{32} & -6p_{33} \end{bmatrix}$$



$$\Rightarrow P_{22} = -\frac{1}{2}$$

$$P_{33} = -\frac{1}{6}$$

$$\Rightarrow P = \begin{bmatrix} P_{11} & 0 & -\frac{1}{18} \\ 0 & -\frac{1}{2} & 0 \\ -\frac{1}{30} & 0 & -\frac{1}{6} \end{bmatrix}$$

$$P_{32} = 0$$

$$P_{23} = 0$$

$$P_{12} = 0$$

$$P_{13} = -\frac{1}{18}$$

$$P_{31} = -\frac{1}{30}$$

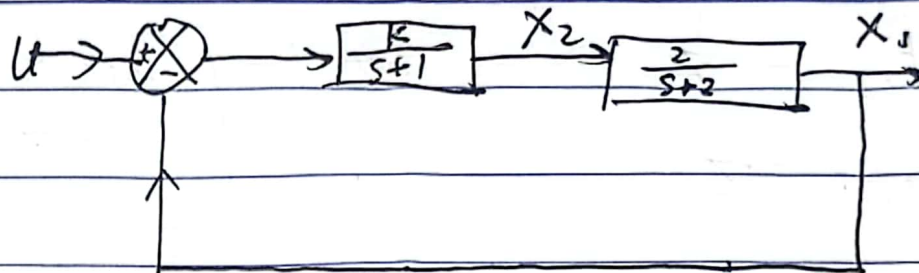
$$P_{21} = 0$$

$$P_{11} < 0$$

$\Rightarrow$  系统不是渐近稳定的



3.



$$\begin{cases} \dot{X} = \begin{bmatrix} -2 & 2 \\ -k & -1 \end{bmatrix} X + \begin{bmatrix} 0 \\ k \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} X \end{cases}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{令 } Q = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^T P + P A = -Q$$

$$\Rightarrow P = \begin{bmatrix} \frac{k^2 + 2k + 3}{2(k+1)} & \frac{1-k}{6(k+1)} \\ \frac{1-k}{6(k+1)} & \frac{k+5}{6(k+1)} \end{bmatrix}$$

$$\Rightarrow \frac{k^2 + 2k + 3}{2(k+1)} > 0 \quad \Rightarrow \quad \begin{matrix} k \neq 0 \text{ 且} \\ k > -1 \end{matrix}$$

$$\left| \frac{k^2 + 2k + 3}{2(k+1)} - \frac{k+5}{6(k+1)} - \left[ \frac{1-k}{6(k+1)} \right]^2 \right| > 0$$

