Fintech 545

Week 4 Project

2/23/2024

Yi Wang

Problem 1

According to calculation, the expectation on the expected value and standard deviation of price at time t given each of the 3 types of price returns is shown as following image:

Problem 1. $r_t \sim N(0, \sigma^2) \Rightarrow E[r_t]=0,\ Var[r_t]=\sigma^2$

For classical Brownian Motion, $P_t = P_{t-1} + r_t$

$E[P_t] = E[P_{t-1}] + E[r_t] = P_{t-1}$, $Var[P_t] = Var[P_{t-1}+r_t] = Var[r_t] = \sigma^2$ $sd = \sigma$

For Arithmetic Returns, $P_t = P_{t-1}(1+r_t)$

$E[P_t] = P_{t-1}E[1+r_t] \approx P_{t-1}E[1] = P_{t-1}$.

$Var[P_t] = P_{t-1}^2 Var[1+r_t] = P_{t-1}^2 Var[r_t] = P_{t-1}^2 \sigma^2$

$sd = P_{t-1} \cdot \sigma$

For Log Return, $P_t = P_{t-1} e^{r_t}$

$E[P_t] = E[P_{t-1} e^{r_t}] = P_{t-1} E[e^{r_t}] = P_{t-1} e^{\frac{\sigma^2}{2}}$

$Var[P_t] = P_{t-1}^2 \cdot e^{\sigma^2} (e^{\sigma^2} - 1)$

$sd = P_{t-1} \cdot e^{\frac{\sigma^2}{2}} \sqrt{e^{\sigma^2} - 1}$

Now assuming $P_{t-1} = 100$, $\sigma^2 = 0.01$, the expectation and sd are as follows:

Classical: $E[P_t] = 100$, $sd = \sqrt{0.01} = 0.1$

Arithmetic: $E[P_t] = 100$, $sd = 100 \cdot 0.1 = 10$

Log: $E[P_t] = 100 \cdot e^{0.005} = 100.5$, $sd = 10.08$

I simulated the process in python 1000 times, and the result is shown as following image:

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After 1000 simulations, the mean expected value of the classical model is: 99.99935766889381 , and the mean standard deviation is  0.09969045297153552
After 1000 simulations, the mean expected value of the arithmetic model is: 99.93576688938063 , and the mean standard deviation is  9.969045297153551
After 1000 simulations, the mean expected value of the log model is: 100.43416397261204 , and the mean standard deviation is  10.057954774569003
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By comparison, the simulation results are very close to my expected results.

Problem 2

Using the return_calculate() function, I successfully calculated the arithmetic returns for all prices shown as following image:

	Date	SPY	AAPL	MSFT	AMZN	NVDA	GOOGL	TSLA	GOOG	BRK-B	META	UNH	XOM	LLY
0	2022-09-02	-0.010544	-0.013611	-0.016667	-0.002425	-0.020808	-0.017223	-0.025076	-0.016915	-0.016854	-0.030479	-0.014599	0.018323	-0.025232
1	2022-09-06	-0.003773	-0.008215	-0.010974	-0.010980	-0.013336	-0.009643	0.015581	-0.011042	-0.003890	-0.011103	0.002576	-0.006695	0.018219
2	2022-09-07	0.017965	0.009254	0.019111	0.026723	0.018795	0.024717	0.033817	0.027912	0.016089	0.011669	0.007727	-0.008531	0.015579
3	2022-09-08	0.006536	-0.009618	0.001666	0.002626	0.020126	-0.009776	0.019598	-0.009595	0.008184	0.010412	0.011176	0.008179	0.012516
4	2022-09-09	0.015535	0.018840	0.022977	0.026575	0.028377	0.020945	0.036023	0.021568	0.008576	0.043749	-0.002890	0.016753	0.006973
..	...	...	...	...	...	...	...	...	...	...	...	...	...	...
260	2023-09-18	0.000586	0.016913	-0.003513	-0.002920	0.001503	0.005895	-0.033201	0.004772	0.006986	0.007459	-0.001315	0.008055	-0.007609
261	2023-09-19	-0.002074	0.006181	-0.001246	-0.016788	-0.010144	-0.001230	0.004599	-0.000936	0.000135	0.008329	-0.011110	-0.002635	0.006442
262	2023-09-20	-0.009193	-0.019992	-0.023977	-0.017002	-0.029435	-0.031150	-0.014672	-0.030541	-0.009879	-0.017701	0.023863	-0.007926	-0.009253
263	2023-09-21	-0.016528	-0.008889	-0.003866	-0.044053	-0.028931	-0.024675	-0.026239	-0.023999	-0.009651	-0.013148	0.018308	-0.014089	-0.034250
264	2023-09-22	-0.002249	0.004945	-0.007887	-0.001624	0.014457	-0.001457	-0.042315	-0.000837	-0.008588	0.011328	0.009897	0.001568	0.000745

	...	AXP	REGN	CB	ADI	CI	ETN	SLB	PGR	SCHW	LRCX	ZTS	C	BSX	AMT
0	...	-0.008735	-0.033315	-0.006558	-0.013670	-0.001180	-0.010593	0.033107	-0.010428	-0.019242	-0.004236	-0.015244	0.001846	-0.012198	-0.026355
1	...	-0.007944	-0.011453	-0.002358	-0.002691	-0.004641	0.008449	-0.014118	0.000572	0.001848	-0.008019	-0.000892	-0.012695	-0.002717	0.013275
2	...	0.024562	0.039148	0.017067	0.016663	0.016652	0.020295	-0.008030	0.038537	0.018731	0.012279	0.022698	0.008503	0.026994	0.020930
3	...	0.008342	0.188468	0.007383	0.009489	0.002448	0.013945	0.029951	0.015880	0.019083	0.016574	-0.011908	0.026116	0.029901	0.008362
4	...	0.032048	0.021824	-0.000564	0.013475	0.007327	0.017244	0.038774	-0.004179	0.018863	0.026460	0.036721	0.015431	0.005385	-0.000306
..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
260	...	-0.025160	0.001348	0.009001	0.008869	0.007485	0.006938	0.010399	0.013118	-0.006183	0.020125	-0.003329	-0.001639	0.001890	-0.003386
261	...	-0.001754	0.014857	0.000747	-0.005397	-0.002453	-0.013644	-0.012743	0.013589	-0.002247	-0.016519	0.012970	0.000938	0.000566	-0.012087
262	...	-0.013304	-0.006143	-0.002380	-0.014937	0.009450	-0.006986	-0.010591	0.001544	-0.018361	-0.010062	-0.002748	-0.008903	0.020177	0.000282
263	...	-0.016027	-0.009908	-0.005941	-0.012722	0.012216	-0.018635	-0.016223	-0.002032	-0.011646	-0.013686	-0.026725	-0.013948	-0.002403	-0.045601
264	...	-0.010536	0.001773	0.002777	0.009434	-0.004814	0.009542	0.003740	0.006039	-0.015354	0.014286	0.000283	-0.018940	-0.006856	-0.018368

After removing the mean from the series to make the mean(META) = 0, I calculated the VaR using different models.

VaR using a normal distribution: 0.05465390107572427

VaR using a normal distribution with an Exponentially Weighted Variance (lambda = 0.94): 0.030137068179582536

VaR using a MLE fitted T distribution: 0.043134714950376095

VaR using a fitted AR(1) model: 0.05422696780310033

VaR using a Historic Simulation: 0.03939050784430346

Problem 3

I wrote a python function which calculates the VaR using an exponentially weighted covariance with lambda = 0.94. In this function, it calculates the portfolio value and asset weights to get the EWMA covariance matrix. Using the matrix, the function calculates the variance and thus gets the portfolio standard deviation. Finally, using the inverse cumulative distribution function and std, the function calculates and returns the value of VaR. The value of VaR of the aggregated portfolio could not directly calculated by adding the three VaR values. I aggregated the portfolio data frames first and I use my function to generate the total VaR.

VaR for Portfolio A: 15206.390964355216

VaR for Portfolio B: 7741.250980957811

VaR for Portfolio C: 17877.73305925083

Total VaR: 37972.294549340135

I chose historic simulation as my model as the past data is a good predictor for the future risk. My VaR results are shown as following:

VaR for Portfolio A: 17065.300954189384

VaR for Portfolio B: 10983.463846970466

VaR for Portfolio C: 22186.519225797383

Total VaR: 47618.778375631664