Calculus Chapter 7 Test Review – #1-9 Noncalculator (30 min), #10-15 (25 min)

1) Determine the area of the region bounded by $y = -x^2 + 2x + 3$ and $y = 3$.

a) $\frac{4}{3}$ b) $\frac{9}{2}$ c) $\frac{20}{3}$ d) $\frac{22}{3}$ e) $\frac{40}{3}$

2) Let R be the region in the first quadrant bounded by the x-axis and the curve $y = 2 - x$. What is the volume produced when R is revolved about the x-axis?

a) $\frac{16\pi}{15} units^3$ b) $\frac{8\pi}{3} units^3$ c) $\frac{16\pi}{3} units^3$ d) $\frac{20\pi}{3}\ units^3$ e) $8\pi\ units^3$

3) What is the volume of the solid when the area between the curve $y = e^x$ and the line $y = 0$, from $x = 0$ to $x = 2$, is revolved around the x-axis?

a) $\frac{\pi}{2}\left(e^4 - e\right)$ b) $\frac{\pi}{2}\left(e^4 - 1\right)$ c) $\pi\left(e^4 - 1\right)$ d) $2\pi\left(e^4 - 1\right)$ e) $\frac{\pi}{2}e^4$

4) Which of the following integrals represents the volume of the solid formed by revolving the region bounded by $y = x^3$, $x = 2$, and $y = 1$ about the y-axis.

a) $\pi\int_1^8\left[(2-0)^2 - \left(\sqrt[3]{y} - 0\right)^2\right]dy$ b) $\pi\int_1^8\left[\left(\sqrt[3]{y} - 0\right)^2 - (2-0)^2\right]dy$ c) $\pi\int_0^2\left[(2-0)^2 - \left(y^3 - 0\right)^2\right]dy$

d) $\pi\int_1^2\left[\left(x^3 - 0\right)^2 - (1-0)^2\right]dx$ e) $\pi\int_1^8\left[\left(2 - \sqrt[3]{y}\right)^2\right]dy$

5) Which of the following represents the volume of the solid that results when the region enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 9$ is revolved about the line $x = 9$.

a) $\pi\int_0^9\left[\left(\sqrt{x}\right)^2\right]dx$ b) $\pi\int_0^3\left[(9-0)^2 - \left(x^2 - 0\right)^2\right]dx$ c) $\pi\int_0^3\left(9 - x^2\right)^2 dx$

d) $\pi\int_0^3\left(9 - y^2\right)^2 dy$ e) $\pi\int_0^9\left(9 - y^2\right)^2 dy$

6) The region in the first quadrant bounded above by the graph of $y = x$ and below by the x-axis on the interval $[0, 4]$ is revolved about the x-axis. If a plane perpendicular to the x-axis at the point where $x = k$ divides the solid into parts of equal volume, then $k =$.

a) 2 b) $\sqrt[3]{16}$ c) $\sqrt{8}$ d) $\sqrt[3]{32}$ e) 4

7) Setup, but do not evaluate, the integral that computes the volume of the solid formed by revolving the region bounded by the graphs of $y = \ln x$, the x-axis, and the line $x = 3$ about the line $x = -1$.

a) $\pi\int_0^{\ln 3}\left[\left(e^y+1\right)^2-16\right]dy$ b) $\pi\int_1^{3}\left[\left(\ln x+1\right)^2-1\right]dx$ c) $\pi\int_1^{3}\left[1-\left(\ln x+1\right)^2\right]dx$

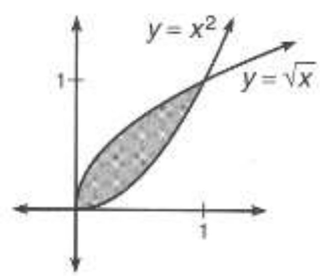
d) $\pi\int_0^{e^3}\left[16-\left(e^y+1\right)^2\right]dy$ e) $\pi\int_0^{\ln 3}\left[16-\left(e^y+1\right)^2\right]dy$

8) Find the volume of the shaded region when it is revolved about the line $y = 1$.

a) $\pi\int_0^1\left[\left(x^2-1\right)^2-\left(\sqrt{x}-1\right)^2\right]dx$ b) $\pi\int_0^1\left[\left(\sqrt{x}-1\right)^2-\left(x^2-1\right)^2\right]dx$

c) $\pi\int_0^1\left[\left(x^2-\sqrt{x}\right)^2\right]dx$ d) $\int_0^1\left[\sqrt{x}-x^2\right]dx$

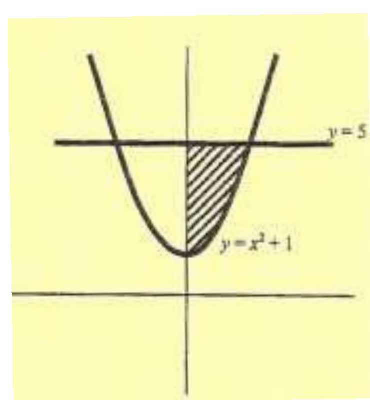
e) $\pi\int_0^1\left[\left(y^2-1\right)^2-\left(\sqrt{y}-1\right)^2\right]dy$



9) Given the figure below, find the area of the shaded region.

a) $\frac{14}{3}$ b) $\frac{16}{3}$ c) $\frac{28}{3}$

d) $\frac{32}{3}$ e) $\frac{65}{3}$



10) Find the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = x^2 + 1$, $y = 4\cos(0.5x)$, and $x = 0$ about the line $y = 5$.

a) 42.779 b) 46.451 c) 76.995 d) 85.558 e) 120.160

11) The area of the first quadrant region bounded by the y-axis, the line $y = 4 - x$ and the graph of $y = x - \cos x$ is approximately…

a) 4.50 units^2 b) 4.54 units^2 c) 4.56 units^2 d) 4.58 units^2 e) 5.00 units^2

12) The integral $\int_a^b \left[(\sin x + 2) - e^{x^2} \right] dx$ computes the area of the region between two curves. Which of the following is the value of b.

a) -1.024 b) -0.601 c) 0 d) 1.024 e) 3.261

13) Find the arc length of the curve $y = x^2 - 2x + 3$ over the interval $[0, 4]$.

a) 11.226 b) 17.929 c) 20.524 d) 35.268 e) 41.667

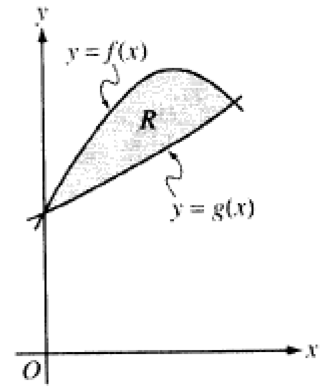
14) Let R be the region in the <u>fourth quadrant</u> enclosed by the x-axis and the curve $y = x^2 - 2kx$, where $k > 0$. If the area of the region R is 36, then the value of k is…

a) 2 b) 3 c) 4 d) 6 e) 9

15) The base of a solid S is the region enclosed by the graph of $4x + 5y = 20$, the x-axis, and the y-axis. If the cross-sections of S perpendicular to the x-axis are semicircles, then the volume of S is…

a) $\frac{5\pi}{3}$ b) $\frac{10\pi}{3}$ c) $\frac{40\pi}{3}$ d) $\frac{25\pi}{6}$ e) $\frac{425\pi}{3}$

FR1: CA



Let *f* and *g* be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{\frac{x}{2}}$. Let *R* be the shaded region in the first quadrant enclosed by the graphs of *f* and *g* shown in the figure above.

a) Find the area of *R*.

b) Find the perimeter of the region *R*.

c) The region *R* is the base of a solid. For this solid, the cross sections perpendicular to the *x*-axis are rectangles whose lengths extend from $y = f(x)$ to $y = g(x)$ and height is half of its length. Find the volume of this solid.

FR2: NC

Let *R* be the region in the first quadrant bounded by the graph of $f(x) = \sqrt{x}$ and by the graph of $g(x) = \frac{x}{3}$.

a) Find the area of the region *R*.

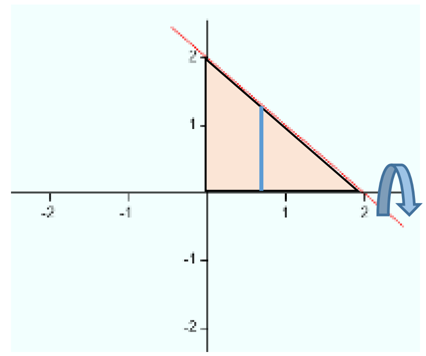
b) Set up, but **do not integrate** an integral expression in terms of a single variable for the volume of the solid generated when *R* is revolved about the line $x = -2$.

c) Let the base of a solid be the region *R*. If all cross sections perpendicular to the *x*-axis are squares, find the volume of the solid.

<u>**Answers**</u>

1) To determine which function is on top, set the two functions equal. $-x^2+2x+3=3$, $-x^2+2x=0$ so $-x(x-2)=0$, $x=0,2$. Use $x=1$ to determine which is greater so $y_1=4$, $y_2=3$. So, the area is $\int_0^2 [(-x^2+2x+3)-3]dx=\int_0^2(-x^2+2x)dx$. $-\frac{1}{3}x^3+x^2\Big|_0^2=-\frac{8}{3}+4=\frac{4}{3}$ or A.

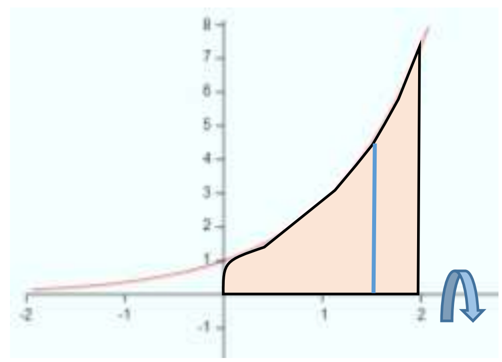
2) Doing a quick sketch shows the region being rotated.
Since this is revolved around the *x*-axis, everything is in terms of *x*.



So, $\pi\int_0^2(2-x-0)^2\,dx=\pi\int_0^2(4-4x+x^2)dx$.

These can all be integrated as is, so $\pi\left(4x-2x^2+\frac{1}{3}x^3\right)\Big|_0^2=\frac{8}{3}\pi$ or B.

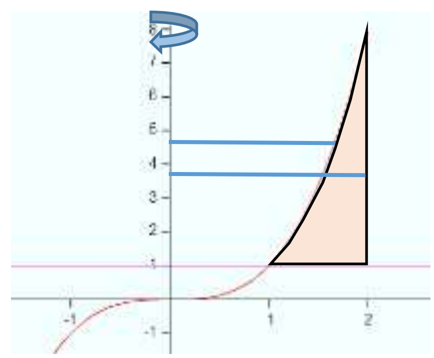
3) Doing a quick sketch shows the region being rotated.
Since this is revolved around the *x*-axis, everything is in terms of *x*.



So $\pi\int_0^2(e^x-0)^2\,dx=\pi\int_0^2 e^{2x}dx$. This requires a u-substitution where

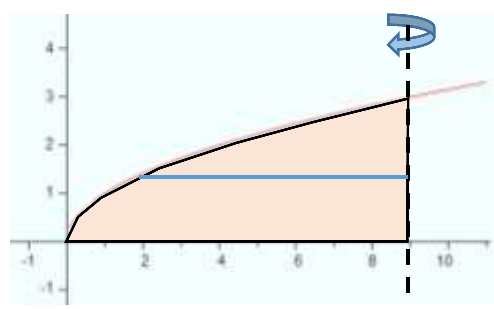
$u=2x, du=2dx$ so $\frac{\pi}{2}\int_0^2 e^u du=\frac{\pi}{2}e^{2x}\Big|_0^2=\frac{\pi}{2}(e^4-1)$ or B.

4) Doing a quick sketch shows the region graphed.
Since this is revolved around the *y*-axis, everything is in terms of *y*.



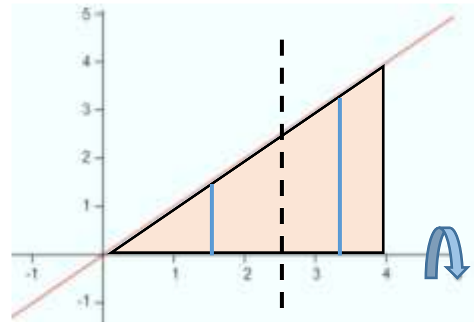
So $y=x^3\rightarrow x=\sqrt[3]{y}$ and $y(2)=8$. This has an outer radius from $x=2$ and an inner radius from $x=\sqrt[3]{y}$. So, $\pi\int_1^8\left[(2-0)^2-(\sqrt[3]{y}-0)^2\right]dy$ or A.

5) Doing a quick sketch shows the region graphed.
Since this is revolved around a vertical line like the *y*-axis, everything is in terms of *y*. So $y=\sqrt{x}\rightarrow x=y^2$ and $y(9)=3$.

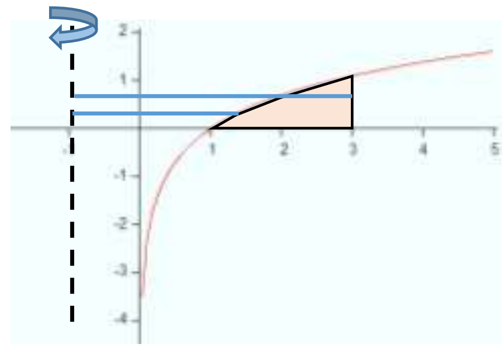


So, $\pi\int_0^3(y^2-9)^2\,dy$ or D.

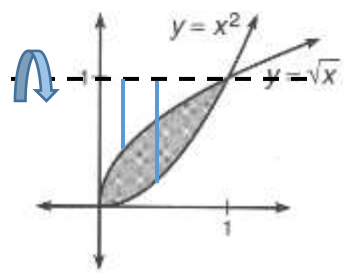
6) Doing a quick sketch shows the region graphed. There is some value $x = k$, as shown as the dashed line, that splits the volume into 2 equal sections. So, $\pi\int_0^k (x-0)^2\,dx = \pi\int_k^4 (x-0)^2\,dx$. These can be integrated as is, so $\pi\left[\frac{1}{3}x^3\right]_0^k = \pi\left[\frac{1}{3}x^3\right]_k^4$ or $\frac{1\pi}{3}k^3 = \frac{64\pi}{3} - \frac{1\pi}{3}k^3$, $\frac{2\pi}{3}k^3 = \frac{64\pi}{3}$. So, $k^3 = 32$, $k = \sqrt[3]{32}$ or D.



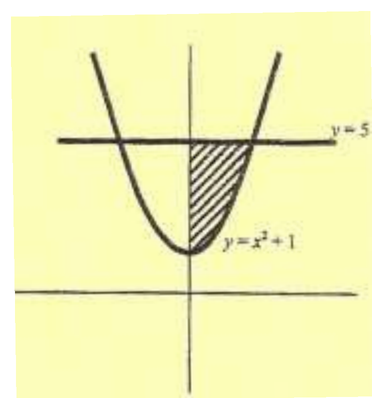
7) Doing a quick sketch shows the region graphed.
Since this is revolved around a vertical line like the *y*-axis, everything is in terms of *y*. So $y = \ln x \rightarrow x = e^y$ and $y(3) = \ln 3$. This has an outer radius from $x = 3$ and an inner radius from $x = e^y$.
So, $\pi\int_0^{\ln 3}\left[(3+1)^2 - (e^y+1)^2\right]dy$ or E.



8) Since this is revolved around a horizontal line like the *x*-axis, everything is in terms of *x*. This has an outer radius from $y = x^2$ and an inner radius from $y = \sqrt{x}$. So $\pi\int_0^1\left[\left(x^2-1\right)^2 - \left(\sqrt{x}-1\right)^2\right]dx$ or A.

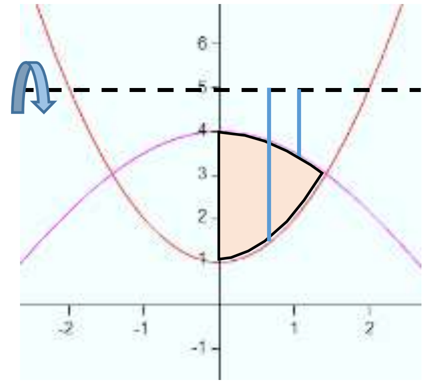


9) The figure shows that $y = 5$ is always on top and the left bound is $x = 0$. To find the right bound, set $x^2 + 1 = 5$, $x^2 = 4$, $x = 2$.
So, $\int_0^2\left[5-(x^2+1)\right]dx = \int_0^2\left(-x^2+4\right)dx$. This can be integrated as is, so $-\frac{1}{3}x^3 + 4x\Big|_0^2 = -\frac{8}{3} + 8 = \frac{16}{3}$ or B.



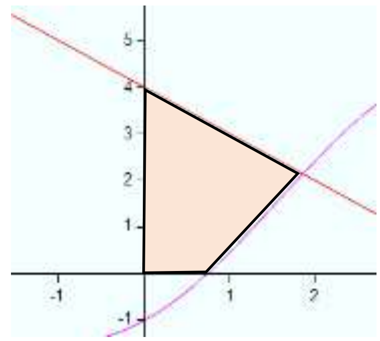
10) Doing a quick sketch shows the region graphed.
Since this is revolved around a horizontal line like the x-axis, everything is in terms of x. Set the two functions equal and let the calculator find the bounds, which are at $x = 1.42411$. This has an outer radius from $y = x^2 + 1$ and an inner radius from $y = 4\cos(0.5x)$.

So $\pi \int_0^{1.42411} \left[\left(x^2 + 1 - 5\right)^2 - \left(4\cos(0.5x) - 5\right)^2 \right] dx = 42.779$ or A.



11) Doing a quick sketch shows the region graphed. The area in the first quadrant has $y = 4 - x$ on top, but has the x-axis and $y = x - \cos x$, so two integrals are needed. The first integral has an upper bound at the zero of $y = x - \cos x$ at $x = 0.739085$ and the second integral has an upper bound at the intersection at $x = 1.85825$. So the total area is the sum of the two

$\int_0^{0.739085} [(4 - x) - 0] dx + \int_{0.739085}^{1.85825} [(4 - x) - (x - \cos x)] dx = 4.538$ or B.



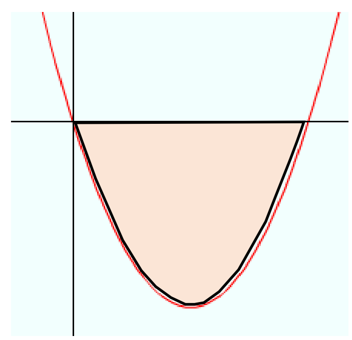
12) The bounds of the integral will be where the graphs intersect. This happens at $x = -0.601$, $x = 1.024$. The value of b would be the greater of these values since $\sin x + 2$ is on top, so D.

13) Arc length is $\int_a^b \sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2} \, dx$ and $\frac{dy}{dx} = 2x - 2$. So $\int_0^4 \sqrt{(1)^2 + (2x - 2)^2} \, dx = 11.226$ or A.

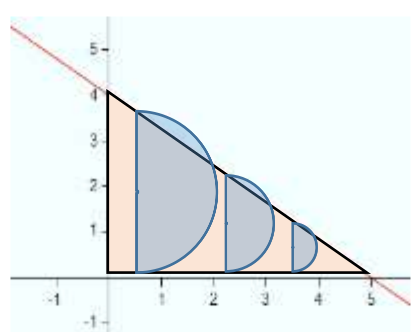
14) The graph is a parabola, facing up, with zeros at $0 = x^2 - 2kx$, $0 = x(x - 2k)$.

So, $x = 0, 2k$. Therefore, the area is $\int_0^{2k} \left[0 - \left(x^2 - 2kx\right)\right] dx = 36$ which can be integrated as it is after distributing the negative. So, $\left. -\frac{1}{3}x^3 + kx^2 \right|_0^{2k} = 36$ or

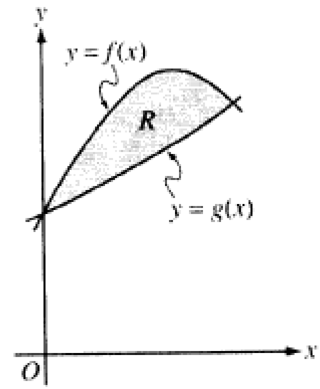
$-\frac{8}{3}k^3 + 4k^3 - 0 = 36$, $\frac{4}{3}k^3 = 36$, $k^3 = 27$. So $k = 3$ or B.



15) Doing a quick sketch shows the region graphed. Since the cross sections are perpendicular to the x-axis, this is in terms of x. So, $y = -\frac{4}{5}x + 4$, with bounds from $x = 0, 5$. So, $\frac{\pi}{8} \int_0^5 \left(-\frac{4}{5}x + 4\right)^2 dx = 10.472$ or B.



FR1



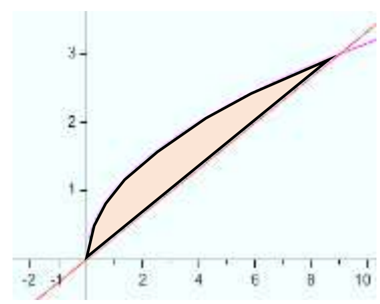
a) The area is found by finding the intersection points, which occurs at $x = 0, 1.13569$. So $\int_0^{1.13569} \left(f(x) - g(x)\right)dx = 0.429$.

b) The perimeter uses 2 arc lengths where $f' = 2\cos 2x$ and $g' = \frac{1}{2}e^{\frac{x}{2}}$.

So, the perimeter is $\int_0^{1.13569} \sqrt{1+[f'(x)]^2}\,dx + \int_0^{1.13569} \sqrt{1+[g'(x)]^2}\,dx = 3.127$.

c) Since the cross sections are perpendicular to the *x*-axis, we are in terms of x. The cross sections are rectangles whose area is length*height. The length is $f(x) - g(x)$ and, since the height is half of its length, the height is $\frac{1}{2}[f(x) - g(x)]$. Therefore, the area is $\frac{1}{2}[f(x) - g(x)]^2$, and the volume would be $\int_0^{1.13569} \frac{1}{2}[f(x) - g(x)]^2\,dx = 0.099$

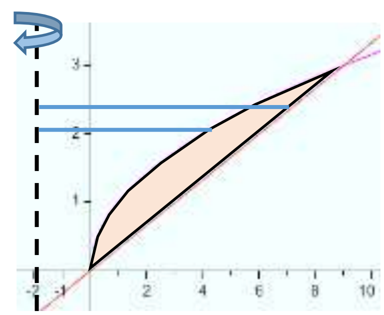
FR2



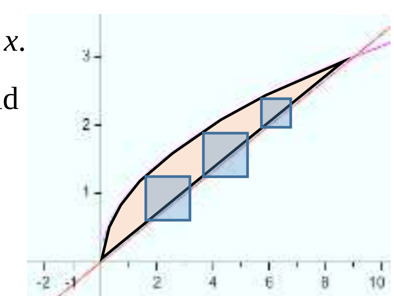
a) The figure shows that $f(x) = \sqrt{x}$ is always on top of $g(x) = \frac{x}{3}$.

They intersect when $\sqrt{x} = \frac{x}{3}$, $x = \frac{x^2}{9}$, $9x = x^2$, $x^2 - 9x = 0$ at $x = 0, 9$.

So, $\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right)dx = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{6}x^2\Big|_0^9 = \left(18 - \frac{27}{2}\right) - 0$ or $\frac{9}{2}$.



b) Since this is revolved around a vertical line like the *y*-axis, everything is in terms of *y*. So $f(x) = \sqrt{x} \to x = y^2$ and $g(x) = \frac{x}{3} \to x = 3y$ with $f(0) = 0$, $f(9) = 3$. This has an outer radius from $x = 3y$ and an inner radius from $x = y^2$. So, $\pi\int_0^3 \left[(3y+2)^2 - (y^2+2)^2\right]dy$.



c) Since the cross sections are perpendicular to the *x*-axis, this is in terms of *x*. So, the length of each square is $f(x) - g(x) = \sqrt{x} - \frac{x}{3}$. The volume would be $\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right)^2 dx$ which can be integrated by multiplying it out. So,

$$\int_0^9 \left(x - \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{9}x^2\right)dx = \frac{1}{2}x^2 - \frac{4}{15}x^{\frac{5}{2}} + \frac{1}{27}x^3\Big|_0^9 \text{ or } \frac{81}{2} - \frac{4}{15}(9)^{\frac{5}{2}} + 27 = 2.7.$$