AP Calculus Summer Packet

There are certain skills that have been taught to you over the past few years that are essential towards your success in AP Calculus AB/BC. If you do not have these skills, you will find that you will consistently get problems incorrect even if you understand the calculus concepts. It is frustrating for students when they make algebraic or careless mistakes, yet get the calculus aspect of the problem. The purpose of this summer packet is to shake off the rust and review these topics.

Don't fake your way through these problems because you will need to understand everything in this very well. Also, don't wait until the last minute to do everything in the packet because you may run out of time or realize that there are concepts you don't understand. Likewise, do not do all the problems right at the beginning of the summer and forget how to do all of them by the time school starts again. You should complete this throughout the summer, doing a few problems each day, like a daily journal. I encourage you to work through this with other students or students that were in AP Calculus in previous years.

Although this summer packet will not be graded, you will have a quiz and a test after the first few days of school that covers all of the topics in this packet. We will go through every topic in class, but we will not go over every single question so do not depend on the in-class review to learn everything. The review should be time to practice things you've relearned (or learned) over the summer. For this packet, you should show all work to help you understand any errors or misunderstandings (if any). It is also very important to complete this packet **without the use of a calculator** (unless otherwise noted). Two-thirds of the AP exam is non-calculator, so it is important to get used to doing mathematics without a calculator.

This summer, Zach Kocan, a rising Senior who successfully completed AP Calculus AB/BC last year, will be hosting review sessions to discuss topics in this packet. Every Tuesday from 7/14 to 8/18 at 2:15pm to 2:45pm, Zach will host a meeting via Google Meet using code **prk-edqi-ggv** . I encourage each of you to attend at least one session and take advantage of this great opportunity. A waiver will be required to share live video to be completed by your guardian, found at https://forms.gle/mttizJ84VqSz5aBA6. You will find a Welcome letter from Zach on the next page with more information.

Below you will find excerpts from letters that last year's students wrote in regards to their thoughts and feelings about this summer packet. We will be reading these letters on a monthly basis throughout the course to get a student's perspective on how to be successful in AP Calculus AB/BC.

"The first few weeks of the course, I was worried that I wasn't prepared for it and didn't have the capacity for this class. The summer packet made me feel more prepared for it. It is really important to do, it gives you a preview of the class. Also, by completing your summer packet, you don't forget what you did in pre calculus. If you think the summer packet and the first test were hard, don't be scared to switch to Calculus AB. It's better to feel comfortable in a class than feel like all the units are rushed."

"The first thing I would say is most important for the class is to do the summer packet, as it covers a lot of things you might've forgotten from pre-calc. It isn't a ton of work and definitely helps for the first week or two of class."

If you have any questions, please email any of us over the summer at Michael.Gough@fcps.org , Mark.Widmeyer@fcps.org (Calculus AB/BC) or Samuel.Mills@fcps.org (Calculus AB). I will get back to you as quickly as I can. I look forward to working with you over the next year!

Mr. Gough, Mr. Widmeyer, Mr. Mills

Greetings everyone,

I hope this message finds you in both good health and spirit, the past few months have been an interesting experience for us all. I am a rising Senior at Oakdale and member of the Leadership Academy, Phi Theta Kappa; I'm reaching out to you because your child has shown interest in taking AP Calculus and/or AP Chemistry. For my leadership project I am holding AP Preparation sessions via Google Meet on Tuesdays and Thursdays throughout the summer to answer any student questions whether they are general to the course or specific to the summer packets, students may join these sessions using **non-fcps google emails**. The sessions will start on 7/14 and occur weekly until 8/20, starting at 2:15pm and lasting around 30 minutes. I successfully passed both classes this past year, however I did not take my summer work and preparation seriously and saw the effects during the first months of school. So in order to help prevent future stress and frustration I highly recommend your child join these sessions to properly prepare themselves for these difficult year long AP courses. In order for the student to use live video on the Google Meet sessions I ask that you please fill out the google form provided below. Thank you for taking the time to read this message and feel free to reach out to me with any questions!

Google Form for Video Consent - https://forms.gle/mttizJ84VqSz5aBA6

Google Meet Code: **prk-edqi-ggv**

Thanks,
Zachary Kocan

Email: zachary.kocan@gmail.com
Phone: 301-856-8208

A. Factor and simplify completely

1) $\frac{x^2+x-12}{2x^3-16x^2+30x}$

$\frac{(x+4)(x-3)}{2x(x-3)(x-5)} = \boxed{\frac{x+4}{2x(x-5)}}$

2) $\frac{x^3-64}{x^2-16}$ $= \boxed{\frac{x^2+4x+16}{x+4}}$

$\frac{(x-4)(x^2+4x+16)}{(x-4)(x+4)}$

3) $2x^2-13x-15$

$\boxed{(2x-15)(x+1)}$

4) t^4-13t^2+36

$(t^2-9)(t^2-4)$

$\boxed{(t+3)(t-3)(t+2)(t-2)}$

5) $x^{\frac{5}{2}}-5x^{\frac{3}{2}}-6x^{\frac{1}{2}}$

$x^{1/2}(x^2-5x-6)$

$\boxed{x^{1/2}(x-6)(x+1)}$

6) $(x-3)^2(2x+1)^3+(x-3)^3(2x+1)^2$

$(x-3)^2(2x+1)^2[2x+1+x-3]$

$\boxed{(x-3)^2(2x+1)^2(3x-2)}$

B. Simplify each expression in order to obtain a single fraction. Show all work.

1) $\frac{1}{x+h}-\frac{1}{x}$

$\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}$

$\boxed{\frac{-h}{x(x+h)}}$

2) $\frac{\frac{4}{x+1}}{1-\frac{2}{x}} \cdot \frac{x(x+1)}{x(x+1)}$

$\frac{4x}{x(x+1)-2(x+1)}$

$\boxed{\frac{4x}{x^2-x-2}}$

3) $(x+1)^{\frac{1}{2}}+\frac{1}{2}x(x+1)^{-\frac{1}{2}}$

$\sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$

$\frac{2(x+1)+x}{2\sqrt{x+1}}$

$= \boxed{\frac{3x+2}{2\sqrt{x+1}}}$

C. If $f(x)=1-x^2$, $g(x)=2x+1$ and $m(x)=\begin{cases}\sqrt{5-4x} & , x<3\\ |x-7| & , x\geq 3\end{cases}$, find:

1) $f(3)$

-8

2) $g(-2)$

-3

3) $m(-11)$

7

4) $g(f(0))$

$g(1)=3$

5) $f(m(5))$

$f(2)=-3$

6) $m(m(-1))$

$m(3)=4$

7) $f(x+h)$

$1-(x+h)^2$

$1-(x^2+2xh+h^2)$

$=1-x^2-2xh-h^2$

8) $f(g(a))$

$f(2a+1)$

$1-(2a+1)^2$

$-4a^2-4a$

9) $m(t^2+4)$ since $t^2+4\geq 4$

$|t^2+4-7|$

$|t^2-3|$

10) $g(x)-f(x)$

$(2x+1)-(1-x^2)$

x^2+2x

11) $f(t)g(t)$ **let t=x**

$(1-x^2)(2x+1)$

$-2x^3-x^2+2x+1$

12) $\frac{g(x+h)-g(x)}{h}$

$\frac{2(x+h)+1-(2x+1)}{h}$

$\frac{2x+2h+1-2x-1}{h}$

$\frac{2h}{h}=2$

D. Using point-slope form $y-y_1=m(x-x_1)$, write an equation for the line…

1) containing the points (1, -3) and (-5, 2)

$m=\frac{2-(-3)}{-5-1}=\frac{5}{-6}=-5/6$

$y+3=-5/6(x-1)$

or

$y-2=-5/6(x+5)$

2) perpendicular to the line in #1, containing the point (3, 4)

$m_\perp=6/5$

$y-4=\frac{6}{5}(x-3)$

E. Identify the mistakes in each of the following mathematical statements.

1) $(3x+4)^2 = 9x^2+16$

Can't distribute power

$= 9x^2+24x+16$

2) $3x^{-2} = \frac{1}{3x^2}$

3 isn't raised to negative power

$= 3/x^2$

3) $\sqrt{x^2+9} = x+3$

Can't distribute radical

$\sqrt{x^2+9}$

4) $\frac{x^2+15x}{x+2} = \frac{x+15}{2}$

$\frac{x(x+15)}{x+2}$ ← no x as factor in denominator, so can't cancel

F. Determine the exact value of each expression. Remember NO CALCULATORS!

1) $\sin 0 =$ 0

2) $\cos \pi =$ -1

3) $\tan 2\pi =$ 0

4) $\sin \frac{3\pi}{4} =$ $1/\sqrt{2} = \sqrt{2}/2$

5) $\cos \frac{7\pi}{6} =$ $-\sqrt{3}/2$

6) $\tan \frac{\pi}{3} =$ $\sqrt{3}$

7) $\sec \frac{3\pi}{2} =$ $\frac{1}{\cos 3\pi/2} \rightarrow$ und.

8) $\csc \frac{2\pi}{3} =$ $\frac{1}{\sin 2\pi/3} = \frac{2}{\sqrt{3}}$

9) $\cot \frac{11\pi}{6} =$ $\frac{1}{\tan 11\frac{\pi}{6}} = -\sqrt{3}$

10) $\sin \frac{-\pi}{6} =$ $-1/2$

11) $\tan \frac{-\pi}{2} =$ und

12) $\arcsin \frac{\sqrt{3}}{2} =$ $\pi/3$

13) $\tan^{-1}(-1) =$ $-\pi/4$

14) $\arccos 0 =$ $\pi/2$

15) $\arcsin\left(-\frac{1}{2}\right) =$ $-\pi/6$

16) $e^{\ln 4} =$ 4

17) $\ln 1 =$ 0

18) $\ln \sqrt{e} =$ $\ln e^{1/2} = 1/2$

G. Simplify the following using identities.

1) $\sin^2 x + \cos^2 x =$ 1

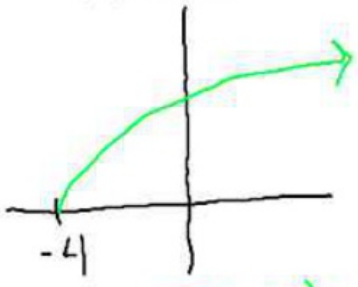
2) $2 + 2\tan^2 x =$ $2\sec^2 x$

3) $\sin\left(\arccos\left(\frac{1}{2}\right)\right) =$ $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

4) $\tan\left(\arcsin\left(\frac{x}{3}\right)\right) =$ $\frac{x}{\sqrt{9-x^2}}$

H. For each of the following, sketch the function and then determine its domain and range.

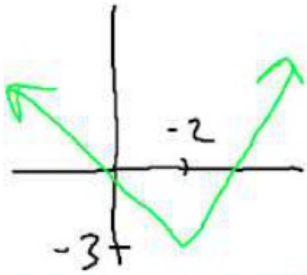
1) $y = \sqrt{x+4}$



D: $[-4, \infty)$

R: $[0, \infty)$

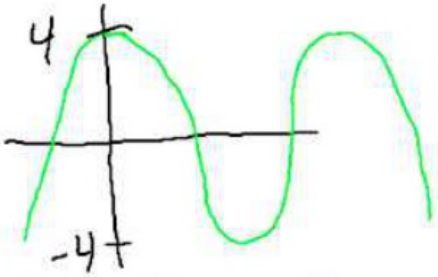
2) $y = |x-2| - 3$



D: $(-\infty, \infty)$

R: $[-3, \infty)$

3) $y = 4\cos x$



D: $(-\infty, \infty)$

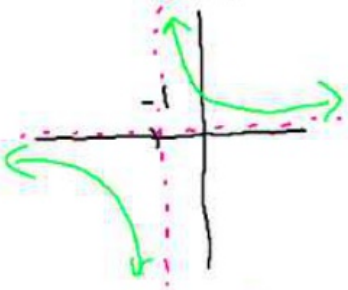
R: $[-4, 4]$

4) $y = 2 + e^x$

D: $(-\infty, \infty)$

R: $(2, \infty)$

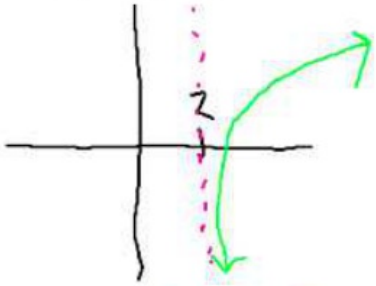
5) $y = \frac{1}{x+1}$



D: $(-\infty, -1), (-1, \infty)$

R: $(-\infty, 0), (0, \infty)$

6) $y = \ln(x-2)$

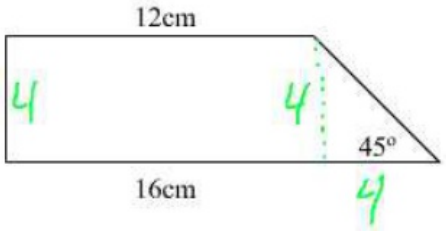


D: $(2, \infty)$

R: $(-\infty, \infty)$

I. Geometry

1) Find the area of the right trapezoid below.

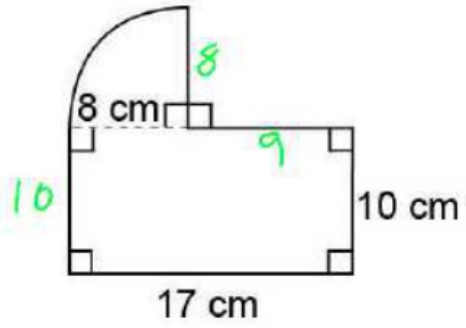


$A = \frac{1}{2} b(h_1 + h_2)$

$= \frac{1}{2}(4)(12+16)$

$= 56\ cm^2$

2) Find the perimeter of the figure below



$P = 8+9+10+17+10+\frac{1}{4}(2\pi \cdot 8)$

$= 54 + 4\pi\ cm$

J. Solve each inequality, expressing the solution in interval notation.

1) $x^2 - 5x \geq 14$

$x^2 - 5x - 14 \geq 0$

$(x-7)(x+2) \geq 0$

$x = 7, -2$

$(-\infty, -2], [7, \infty)$

2) $(x+3)(x-1)^2 < 0$

$x = -3, 1$

$(-\infty, -3)$

K. Solve the equation for x, where x is a real number.

1) $x^2+3x-4=14$

$x^2+3x-18=0$

$(x+6)(x-3)=0$

$x=-6,3$

2) $\frac{x^4-1}{x^3}=0$

$x^4-1=0$

$(x^2-1)(x^2+1)=0$

$x=\pm 1$

3) $\sqrt{x-1}-\frac{5}{\sqrt{x-1}}=0$

$\frac{x-1-5}{\sqrt{x-1}\ \sqrt{x-1}}=0$

$\frac{x-6}{\sqrt{x-1}}=0$

$x-6=0$ $x=6$

4) $2x^2+5x=8$

$2x^2+5x-8=0$

$x=\frac{-5\pm\sqrt{25-4(-16)}}{4}$

$x=\frac{-5\pm\sqrt{89}}{4}$

5) $\frac{20}{e^{2x}+1}=2$

$20=2(e^{2x}+1)$

$10=e^{2x}+1$

$9=e^{2x}$

$\ln 9=2x$

$x=\frac{1}{2}\ln 9=\ln 3$

6) $5\ln(2x+1)-3=6$

$5\ln(2x+1)=9$

$\ln(2x+1)=9/5$

$2x+1=e^{9/5}$

$2x=e^{9/5}-1$

$x=\frac{e^{9/5}-1}{2}$

Solve each equation on the interval $[0,2\pi)$.

7) $\sin x+1=0$

$\sin x=-1$

$x=3\pi/2$

8) $2\tan x\cos x-\tan x=0$

$\tan x(2\cos x-1)=0$

$\tan x=0$ $\quad 2\cos x-1=0$

$x=0,\pi$ $\quad \cos x=1/2$

$x=\pi/3, \frac{5\pi}{3}$

9) $4\sin^2 x-1=0$

$4\sin^2 x=1$

$\sin^2 x=1/4$

$\sin x=\pm 1/2$

$x=\pi/6, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

10) $\tan 2x=1$

$2x=\frac{\pi}{4}, \frac{5\pi}{4}+2n\pi$

$x=\frac{\pi}{8}, \frac{5\pi}{8}+n\pi$

$x=\pi/8, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

Solve for z.

11) $3xz=-10yz+19$

$3xz+10yz=19$

$z(3x+10y)=19$

$z=\frac{19}{3x+10y}$

12) $h=\sqrt[3]{\frac{2x^4}{z}}$

$h^3=\frac{2x^4}{z}$

$zh^3=2x^4$

$z=\frac{2x^4}{h^3}$

L. The number of elk after t, given $0 \le t \le 300$ and t is measured in months, in a state park is modeled by the function $P(t) = \frac{1216}{1+73e^{-0.04t}}$. **You may use a calculator for this problem.**

1) What was the initial population?

$P(0) \approx 16.432$

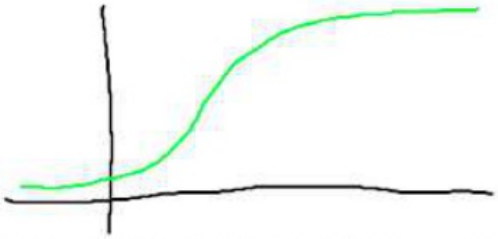
2) When will the number of elk be 750?

$750 = \frac{1216}{1+73e^{-0.04t}}$

put into Y_1 put into Y_2 find intersection

$t = 119.159$

3) What is the maximum number of elk possible in the park?



approaches 1216

4) The number of prairie dogs in the same state park can be modeled by the function $D(t) = 694 + 317\cos\left(\frac{161}{t+1}\right)$. After how many months will the population of elk be the same as the prairie dog population?

How does this sound?

$750 = \frac{1216}{1+73e^{-0.04t}}$

Y_1 Y_2 intersection

$t = 121.225$