5.4 Quiz Review
Non-Calculator Section

1. Evaluate the following:

a) $\int_{1}^{3} (6x^2 + 4x - 5)dx$

b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\sec\theta \tan\theta + \cos\theta)d\theta$

c) $\int_{2}^{3} \left(\frac{4}{x^2} + \frac{2}{x}\right)dx$

d) $\frac{d}{dx}\left[\int_{4}^{x^2+1} (3\cos t)dt\right]$

2. Find the average value of $f(x) = (3x+1)(x-4)$ from $[2,4]$.

3. Rewrite the following as the equivalent definite integral:

a) $\lim_{n\to\infty} \sum_{k=1}^{n} \ln\left(1 + \frac{3k}{n}\right)\left(\frac{3}{n}\right)$

b) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{5k}{n}\right)^4 \left(\frac{5}{n}\right)$

Calculator Section

4. A tree grows at a rate modeled by $G(t) = 10e^{-0.14t} + 3$ with G measured in feet per year and t in years.

a) What is the growth rate at $t = 3$? Indicate units of measure.

b) How much did the tree grow from $[0,3]$? Indicate units of measure.

c) If the tree is 7 ft tall at $t = 2$, what is the height of the tree at $t = 5$? Indicate units of measure.

d) What is the average growth rate from $[0,3]$? Indicate units of measure.

5. The differentiable function $R(t)$ measures the rate in which snow falls, in in/min, is found in the table:

t (min)	0	2	3	5
R(t)	1.5	3.1	3.9	2.1

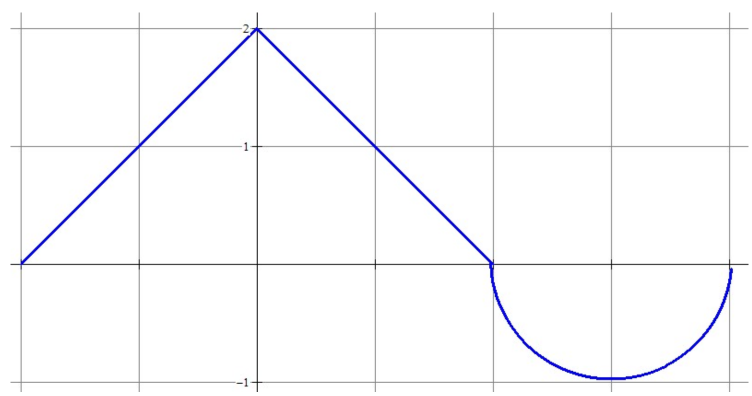
a) Approximate the total amount of snow fallen over the first 5 minutes using a left Riemann sum using 3 intervals. Indicate units of measure.

b) What is the meaning of $\frac{1}{5}\int_0^5 R(t)dt$? Estimate this using a Trapezoidal sum with 3 intervals.

c) What is the least number of times that $R'(t)=0$ from $0<t<5$? Justify your answer.

d) If $E(t)=-0.5(x-3)^2+4$ is used to model $R(t)$, use $E(t)$ to find the exact amount of snow that has fallen from $0<t<5$.

6. The function $f(x)=\int_1^x g(t)dt$ where the graph of g is shown below:



a) Find $f(3)$, $f'(3)$ and $f''(3)$.

b) Find the x-coordinate of all relative extrema of f.

c) When is f concave up? Justify.

d) Given $f(1)=0$, what is the max value of f. Justify.

e) A new function $h(x)$ is defined as $h(x)=5+\int_x^2 g(t)dt$.

What is $h(2)$ and $h'(2)$?

Solutions

1. a) Taking the antiderivative and evaluating gives us $2x^3 + 2x^2 - 5x\Big|_1^3 = (54+18-15)-(2+2-5) = 58$.

   b) Taking the antiderivative and evaluating gives us $\sec\theta + \sin\theta\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \left(2+\frac{\sqrt{3}}{2}\right)-\left(\frac{2}{\sqrt{3}}+\frac{1}{2}\right)$.

   c) Taking the antiderivative and evaluating, while using some log properties, gives us
   $\frac{-4}{x} + 2\ln|x|\Big|_2^3 = \left(-\frac{4}{3}+2\ln 3\right)-\left(-2+2\ln 2\right) = \frac{2}{3}+\ln\frac{9}{4}$.

   d) Since this is taking the derivative of an integral, this is using the Second Fundamental Theorem of Calculus, where $3\cos(x^2+1)\cdot(2x) - 3\cos(4)\cdot(0) = 6x\cos(x^2+1)$.

2. After multiplying out the function, the average value is $\frac{1}{4-2}\int_2^4 (3x^2 - 11x - 4)dx$. Taking the antiderivative and evaluating gives us $\frac{1}{2}\left[x^3 - \frac{11}{2}x^2 - 4x\Big|_2^4\right] = \frac{1}{2}\left[{}^-40 - {}^-22\right] = -9$.

3. a) The formal definition is $\lim_{n\to\infty}\sum_{k=1}^{n} f(a+k\Delta x)(\Delta x) = \int_a^b f(x)dx$. We can identify that the width $\Delta x = \frac{3}{n}$, meaning that the width of the interval is 3. We can also see that the lower limit of integration is 1 from the $f(a+k\Delta x)$ and since the width is 3, then the upper limit is 4. We can then identify the function $f(x) = \ln x$, so the definite integral is $\int_1^4 \ln x dx$.

   b) We can identify that the width $\Delta x = \frac{5}{n}$, meaning that the width of the interval is 5. We can also see that the lower limit of integration is 2 from the $f(a+k\Delta x)$ and since the width is 5, the upper limit is 7. We can then identify the function $f(x) = x^4$, so the definite integral is $\int_2^7 x^4 dx$.

4. a) Since G is the growth rate and we want rate, we simply have to plug 3 in, so $G(3) = 9.57\ ft/yr$.

   b) Since G is the growth rate and we want how much the tree has grown, we must integrate the rate. So, $\int_0^3 G(t)dt = 33.497\ ft$.

   c) Since G is the growth rate and we want to know a final amount, we must integrate the rate and add on the initial condition. This gives us $7 + \int_2^5 G(t)dt = 34.514\ ft$.

d) Since G is the growth rate and we want average growth rate, we must find the average value. This gives us $\frac{1}{3-0}\int_0^3 G(t)dt = 11.166\ ft/yr$.

5. a) A Left Riemann sum uses the area of rectangles, where the widths are not equal, so we must find each individual rectangles width and height, by using the left value. So, $2(1.5)+1(3.1)+2(3.9)=13.9in$.
   b) This is setup perfectly to be the average snowfall rate from $0<t<5$. Finding the area of the 3 trapezoids gives us $\frac{1}{5}\left[\frac{1}{2}(2)(1.5+3.1)+\frac{1}{2}(1)(3.1+3.9)+\frac{1}{2}(2)(3.9+2.1)\right]=2.82in/\min$.
   c) Looking at the values of R, we see that they increase and then decrease. Since R is a differentiable function, we know that there must be at least one instance where $R'(t)=0$.
   d) Since $E(t)$ is a rate and we want the final amount, we must integrate, giving us $\int_0^5 E(t)dt = 14.167in$.

6. a) $f(3)=\int_1^3 g(t)dt$ which represents the area under the curve, which can split up into a triangle, which is above the axis, and a quarter of a circle below the axis, making the area $\frac{1}{2}-\frac{\pi}{4}$. $f'(x)=g(x)$ by the 2nd Fundamental Theorem of Calculus, so $f'(3)=g(3)$, which is the $y-$value, so $f'(3)=g(3)=-1$. $f''(x)=g'(x)$ by taking the derivative of both sides, so $f''(3)=g'(3)$, which represents the slope of the graph of $g(x)$ at $x=3$. Looking at the graph, we can see that $f''(3)=g'(3)=0$.
   b) There is a relative max at $x=2$ because $f'=g$ changes from positive to negative.
   c) f is concave up when $f''=g'>0$, meaning the graph of g is increasing, which is $(-2,0),(3,4)$.
   d) An absolute max occurs at either endpoints or critical values, where $f'(x)=g(x)$ are 0 or undefined. This means the candidates are $x=-2,2,4$, which we would have to plug into $f(x)$ to determine which has the highest value. So, $x=2$ $f(-2)=\int_1^{-2} g(t)dt=-3.5$, $f(2)=\int_1^2 g(t)dt = 0.5$, $f(4)=\int_1^4 g(t)dt = 0.5-\frac{\pi}{2}$. So, the maximum value is 0.5.
   e) Plugging in $x=2$, we get $h(2)=5+\int_2^2 g(t)dt = 5$. Taking the derivative gives us $h'(x)=-g(x)$ by the 2nd Fundamental Theorem of Calculus, so $h'(2)=-g(2)=0$.