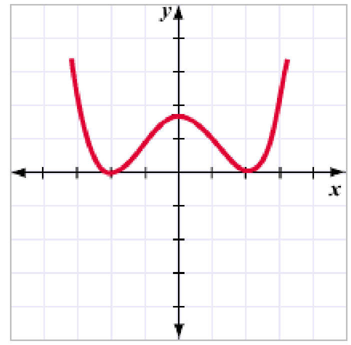
Practice Chapter 4 Test - **Non-Calculator (#1 – 13) and Calculator (#14 – 25)**

_____ 1. The figure shows the graph of the derivative of a function. The derivative is never negative. Which statement is false?



(A) The function has no relative maximum value.
(B) The function has no relative minimum value.
(C) The function is always concave up.
(D) The function has exactly three points of inflection.
(E) The function is increasing for all x.

_____ 2. A function f is differentiable for all values of x, and $f'(x) = 0$ at $x = -3$, $x = 1$, and $x = 2$. If $f''(x) = 3x^2 - 7$, where does a relative minimum occur?

(A) $x = -3$ (B) $x = 1$ (C) $x = 2$ (D) $x = -3, x = 2$ (E) $x = -3, x = 1$

_____ 3. Let f be a function with a derivative $f'(x) = 3x^4 - 27x^3 + 54x^2$. What are the x-coordinates of the relative maxima of the graph of f ?

(A) 0 only (B) 3 only (C) 0 and 6 only (D) 3 and 6 only (E) 0, 3, and 6

_____ 4. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only (B) $t = 3$ only (C) $t = \frac{7}{2}$ only (D) $t = 3$ and $t = \frac{7}{2}$ (E) $t = 3$ and $t = 4$

_____5. The product of two positive numbers is 150. Minimize the sum of twice the first number and three times the second number. What is the sum?

(A) 10 (B) 15 (C) 60 (D) 65 (E) 75

_____ 6. Let f be the function given by $f(x) = 2xe^x - 3x$. The graph of f is concave up when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

_____ 7. Find the maximum value of the function $f(x) = x^3 - 2x^2 - 4x + 16$ on $[1,4]$.

(A) $\frac{2}{3}$ (B) 1 (C) 2 (D) 4 (E) 32

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

_____ 8. The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

I. The maximum value of f on $[3, 7]$ is 20.
II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$.
III. $f'(x) < 0$ for $4 < x < 5$.

(A) I only (B) II only (C) III only (D) II and III (E) I, II, and III

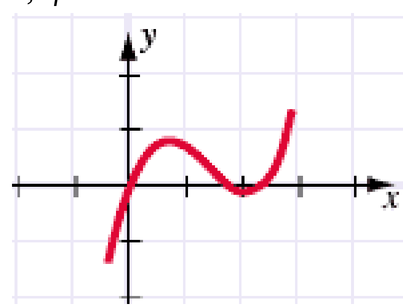
_____ 9. A test plane flies in a straight line with positive velocity v(t), in miles per minute, where v is a differentiable function of t. Selected values of v(t) for [0,40] are shown in the table.

t(minutes)	0	5	10	15	20	25	30	35	40
v(t) (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval (0,40)?

(A) 1 (B) 2 (C) 3 (D) 4 (E) Not enough information

_____ 10. The figure shows the graph of $f'(x)$, the derivative of a function, f. The derivative is continuous and $f'(2) < 0$. Which of the following is true?



I. The function has at least one relative minimum point.
II. The function has at least one relative maximum point.
III. The function has no relative maximum point.

(A) I only (B) II only (C) III only (D) I and II (E) I and III

_____ 11. Let f be the function given by $f(x) = \ln\left|x^2 - 4\right|$. At what x-value is the relative maximum found?

(A) None (B) -2 (C) 0 (D) 2 (E) -2 and 2

_____ 12. Let f be defined as $f(x) = 4\cos x - x^2$. When is the graph of f concave down from $[0, 2\pi)$?

(A) $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$ (B) $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$ (C) $\left(0, \frac{2\pi}{3}\right)$ (D) $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$ (E) $\left(0, \frac{2\pi}{3}\right), \left(\frac{4\pi}{3}, 2\pi\right)$

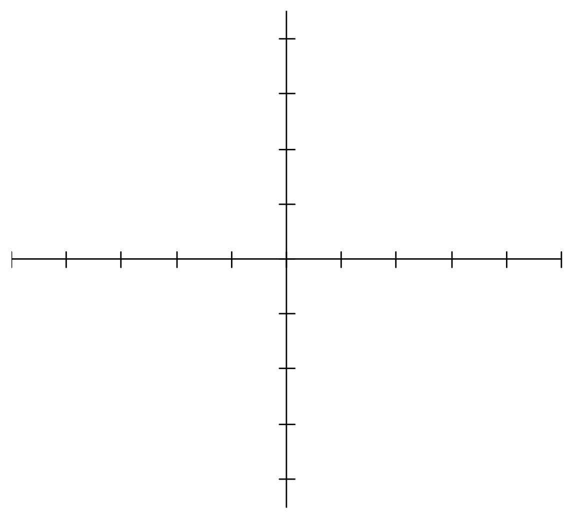
13. A function f is continuous on the interval [-5, 5] and its first and second derivatives have the values given in the following table:

x	**(-5, -1)**	**-1**	**(-1, 2)**	**2**	**(2, 4)**	**4**	**(4, 5)**
f'(x)	Negative	0	Positive	Positive	Positive	0	Positive
f''(x)	Positive	Positive	Positive	0	Negative	0	Positive

a. Find the x-coordinates of the relative maxima and minima of f on [-5, 5], if any. Justify your answer.

b. Find the x-coordinates of all points of inflection of f on the interval [-5, 5], if any Justify your answer.

c. Given $f(-3) = -1$, sketch a possible graph for f which satisfies all of the given properties.



______ 14. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = t^2 \ln(t+2)$. What is the acceleration of the particle at time $t = 6$?

(A) 1.500 (B) 20.453 (C) 29.453 (D) 74.860 (E) 133.417

______ 15. Find the value c guaranteed by Rolle's Theorem for $f(x) = 2\arctan x - \frac{\pi}{2}x + \frac{\pi}{4}$ on $[0, 1]$.

(A) 0.068 (B) 0.273 (C) 0.363 (D) 0.523 (E) It does not apply.

______ 16. The function f has the first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1+x+x^3}$. What is the x-coordinate of the inflection point of the graph of f ?

(A) 1.008 (B) 0.473 (C) 0 (D) -0.278 (E) f has no inflection point.

_____ 17. Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

(A) 15 (B) 18 (C) 21 (D) 24 (E) 27

_____ 18. Let f be the function with the derivative given by $f'(x) = e^{0.143x}\cos(1.6x - 1)$. How many relative extrema does f have on the interval $-1 < x < 6$?

(A) One (B) Two (C) Three (D) Four (E) Five

_____ 19. Let f be the function with the derivative given by $f'(x) = e^{0.143x}\cos(1.6x - 1)$. How many points of inflection does f have on the interval $-1 < x < 6$?

(A) One (B) Two (C) Three (D) Four (E) Five

_____ 20. A can of mountain dew has a volume of 21.65 cubic inches. What is the radius of the cylindrical can that will use the minimum material? (Formulas for a cylinder: $V = \pi r^2 h$ and $SA = 2\pi r^2 + 2\pi rh$)

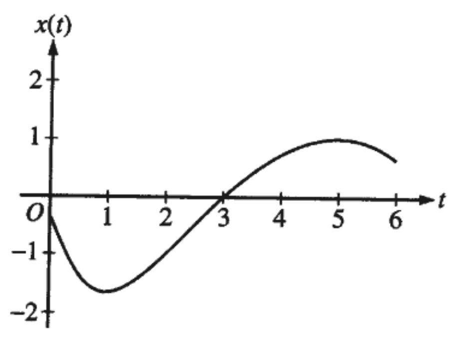
(A) 1.199in (B) 1.273in (C) 1.362in (D) 1.510in (E) 1.620in

_____ 21. Which of the following functions satisfy the conditions of Rolle's Theorem on the interval $[0, 2]$?

I. $f(x) = x^2 - 2x$ II. $f(x) = \dfrac{1}{|x-1|}$ III. $f(x) = |x-1|$

(A) I only (B) II only (C) III only (D) I and III (E) I, II, and III

_____ 22.



A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

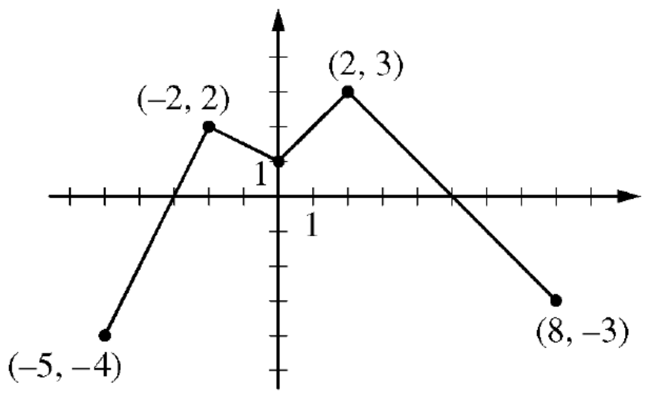
_____ 23. Find the value of c that satisfies the Mean Value Theorem for $f(x) = -1.5\sin(\pi x)$ from $(0.3, 1.2)$.

(A) 0.4712 (B) 0.664 (C) 0.702 (D) 0.727 (E) 0.750

_____ 24. Over which of the following intervals is f both increasing and concave down given $f'(x) = \ln(2x+3) - e^{\cos x}$ from $[0,7]$.

(A) $(0, 1.059)$ (B) $(1.059, 7)$ (C) $(3.632, 6.236)$ (D) $(1.059, 6.236)$ (E) $(1.059, 3.632)$

25.



The graph of a differentiable function f', consisting of four line segments, is shown above. Given that $f(2) = -4$, answer the following questions:

a) Find the $x-$values in which the graph of $f(x)$ has a relative maximum. Justify your answer.

b) Find the $x-$values in which the graph of $f(x)$ has a point of inflection. Justify your answer.

c) Find the $x-$values in which f'' is undefined. Justify your answer.

d) Write the equation of the tangent line to $f(x)$ at $x = 2$.

e) A new function g is defined such that $g(x) = xf(x)$. Is the graph of g concave up or down at $x = 5$? Explain your reasoning.

Solutions

1. Since f' never changes from above to below (or vice versa) the $x-$axis, A and B are true. Since f' increases and decreases, then f is concave up and concave down so C is false.

2. This is the second derivative test. Since $x=-3,1,2$ are critical values, find f'' at each. So, $f''(-3)=20$, $f''(1)=-4$, and $f''(2)=5$. A relative min would be when f is concave up so $f''>0$. Therefore, D.

3. Relative maxima occur when f' changes from positive to negative. So the critical numbers are $3x^4-27x^3+54x^2=0$, $3x^2(x-3)(x-6)=0$ so $x=0,3,6$. So creating the number line gives you B.

4. A particle at rest is when $v(t)=x'(t)=0$. So $v(t)=6t^2-42t+72=0$, $6(t-3)(t-4)=0$ or $t=3,4$ so E.

5. $xy=150$, $S=2x+3y$ where you are trying to find the minimum of S. So S must be in terms of 1 variable. So, $x=\frac{150}{y}\rightarrow S=\frac{300}{y}+3y$. So, $S'=-\frac{300}{y^2}+3$ and when $S'=0$, $\frac{300}{y^2}=3$, $3y^2=300$, $y=10$. We can eliminate $y=0,-10$ since both numbers are positive. So, $x=\frac{150}{10}=15$. Use the line test to show that $y=10$ is a minimum and the sum is $S=2(15)+3(10)=60$ so C.

6. f is concave up is when $f''>0$, so $f'=2e^x+2xe^x-3$ and $f''=2e^x+2e^x+2xe^x=e^x(2x+4)$. The "critical" numbers would be $f''=0=e^x(2x+4)$ or $x=-2$. Using the line test gives B.

7. Absolute max occurs at endpoints or critical values. The critical values are $f'=0=3x^2-4x-4$ (never und.) , $0=(3x+2)(x-2)$, $x=2$ ($x=-\frac{2}{3}$ is out of the interval). So $f(1)=11$, $f(2)=8$, $f(4)=32$, so E.

8. I. Not guaranteed to be the max. II. Since f is differentiable and $f(3)=f(7)=20$, then by Rolle's Theorem, this is true. III. If $f'<0$, then f is decreasing. f is not guaranteed to always decrease, so B.

9. Given velocity, the acceleration is the derivative, so we want to find when $v'=0$. We see v increases, decreases, then increases by the table of values. Since v is differentiable, $v'=0$ must be all the times in between the intervals, so B.

10. I. f has a relative min when f' changes from negative to positive, which does happen (twice). II. f has a relative max when f' changes from positive to negative, which does happen. III. See II. So D.

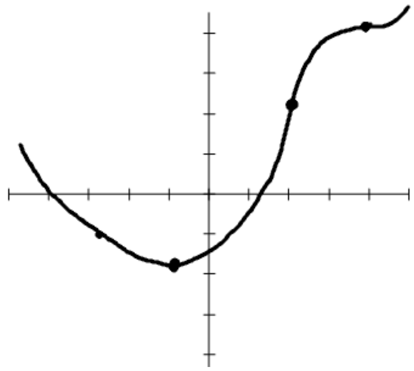
11. A relative max is when f' changes from positive to negative. So $f'=\frac{2x}{x^2-4}$. The critical values are when $f'=0$ (at $x=0$) and f' is undefined (at $x=\pm 2$). Doing the line test gives C.

12. f is concave down when $f''<0$. So $f'=-4\sin x-2x$, $f''=-4\cos x-2$. $f''=0$ and undefined when $0=-4\cos x-2$ or $\cos x=-\frac{1}{2}$, at $x=\frac{2\pi}{3},\frac{4\pi}{3}$. Using the line test gives E.

13. a. Relative mins occur when f' changes from negative to positive, so $x=-1$. Relative maxs occur when f' changes from positive to negative, so none.

b. Points of inflection occur when f'' changes sign, so $x=2,4$.

c.



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14. To find acceleration given velocity, we must differentiate. Since it's the derivative at a point, we will use the calculator so $v'(6)=29.453$, so C.

15. Since f is differentiable from $[0,1]$ and $f(0)=f(1)=\frac{\pi}{4}$, there must be a $c-$value where $f'(c)=0$. So,

$$f'=\frac{2}{1+x^2}-\frac{\pi}{2}=0 \text{ or } \frac{2}{1+x^2}=\frac{\pi}{2} \text{ or } 1+x^2=\frac{4}{\pi} \text{ or } x=0.523\text{, so D.}$$

16. A point of inflection occurs when f' changes from increasing to decreasing or vice versa, so using the calculator shows that f' changes from increasing to decreasing at $x=0.473$, so B.

17. If $g'>0$, then g is increasing and if $g''>0$, then g is concave up. Concave up means that g is increasing more than it was before. So if g is increasing by 6, then g must increase by more than 6, so E.

18. Relative extrema occur when f' changes from positive to negative or vice versa. This occurs 4 times so D.

19. Points of inflection occur when f' changes from increasing to decreasing or vice versa. This occurs 3 times so C.

20. $21.65=\pi r^2h$ and we want to minimize $SA=2\pi r^2+2\pi rh$. So we must get surface area to be in terms of r. So, $h=\frac{21.65}{\pi r^2}$ and $SA=2\pi r^2+2\pi r\left(\frac{21.65}{\pi r^2}\right)$ or $SA=2\pi r^2+\frac{43.3}{r}$. Differentiating gives us $SA'=4\pi r-\frac{43.3}{r^2}$. This will give us critical values of $0=4\pi r-\frac{43.3}{r^2}$ or $4\pi r=\frac{43.3}{r^2}$ or $43.3=4\pi r^3$. This gives us $r=1.51$. Checking this on the number line shows us $r=1.51$ is a minimum, so D.

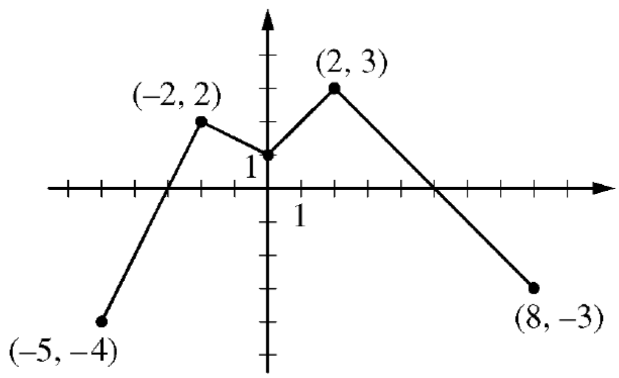
21. I. Since f is differentiable and f', then Rolle's Theorem applies. II. Since f is not differentiable at $x=1$, Rolle's Theorem does not apply. III. Since f is not differentiable at $x=1$, Rolle's Theorem does not apply. So, A.

22. Velocity increasing means that acceleration is positive. Since we are given information about the position, then positive acceleration represents a positive second derivative. Graphically, a positive second derivative means the position function is concave up. This happens from $0<t<2$, so A.

23. Since f is differentiable, then the MVT applies and there is guaranteed a point where $f'(c)=\dfrac{f(b)-f(a)}{b-a}$. So, $-1.5\pi\cos(\pi x)=\dfrac{f(1.2)-f(0.3)}{1.2-0.3}$ or $-1.5\pi\cos(\pi x)=2.328$. Setting this equal to 0 and solving it on the calculator gives us $x=0.664$, so B.

24. f is both increasing and concave down when f' is positive and decreasing, respectively. So we want to look for where the graph of f' is above the $x-$axis and decreasing, which is $(3.632, 6.236)$ so C.

25.



The graph of a differentiable function f', consisting of four line segments, is shown above. Given that $f(2)=-4$, answer the following questions:

a) $f(x)$ has a relative maximum when f' changes from positive to negative. So, $x=5$.

b) $f(x)$ has a point of inflection when f' changes from increasing to decreasing or vice versa. So, $x=-2, 0, 2$.

c) f'' is undefined when the derivative, or slope, of f' is undefined. This would be a sharp turn on the graph of f', so $x=-2, 0, 2$.

d) The equation of the tangent line to $f(x)$ requires a point $f(2)$ and a slope $f'(2)$. It is given that $f(2)=-4$ and according to the graph, $f'(2)=3$. Therefore, the equation is $y+4=3(x-2)$.

e) For g to be concave up and down, $g''>0$ and $g''<0$ respectively. So $g'(x)=f(x)+xf'(x)$ and $g''(x)=f'(x)+f'(x)+xf''(x)$. So, $g''(5)=f'(5)+f'(5)+5f''(5)$ and according to the graph $f'(5)=0$. Also, $f''(5)$ represents the slope of the graph of f' at $x=5$. Looking at the given ordered pairs, the slope of the line going through $x = 5$ is -1. So, $g''(5) = 0 + 0 + 5(-1) = -5$ and since $g''(5) < 0$, g is concave down.