

Name _____

Problem Set 08

"DO THE PROBLEM SETS. It doesn't seem important earlier in the year, but the problem sets are truly pages out of the AP exam. Failing to complete them just puts you off in a worse spot for when the time comes to prepare for the AP exam." A 2019 Nerd

Noncalculator Multiple Choice Section

NC1) $\int_{-1}^2 (x^2 - 2x) dx =$

- A) $-\frac{8}{3}$ B) $-\frac{4}{3}$ C) 0 D) $\frac{4}{3}$

NC2) Let g be the function defined by $g(x) = x^3 + 3x^2$. How many relative extrema does g have?

- A) Zero B) One C) Two D) Three

NC3)

t (hours)	0	2	7	9
$R(t)$ (tons per hour)	15	9	5	4

On a certain day, the rate at which material is deposited at a recycling center is modeled by the function R , where $R(t)$ is measured in tons per hour and t is the number of hours since the center opened.

Using a Right Riemann sum with three subintervals indicated by the data in the table, what is the approximate number of tons of material deposited in the first 9 hours since the center opened?

- A) 49 B) 51 C) 68 D) 85

NC4) Let g be a continuous function. Using the substitution $u = 2x - 1$, the integral $\int_3^4 g(2x-1)dx$ is equal to which of the following?

A) $\frac{1}{2} \int_3^4 g(u)du$

B) $\int_3^4 g(u)du$

C) $\frac{1}{2} \int_5^7 g(u)du$

D) $\int_5^7 g(u)du$

NC5) Let $y = \sin x - \ln(2x)$, then $\frac{d^3y}{dx^3} =$

A) $\cos x - \frac{2}{x^3}$

B) $-\cos x - \frac{2}{x^3}$

C) $-\cos x - \frac{1}{x^3}$

D) $\cos x - \frac{1}{x^3}$

NC6) Let f be the function defined below, where c is a constant. If f is continuous at $x=1$, what is the value of c ?

$$f(x) = \begin{cases} 7 + cx & , x < 1 \\ 9 + 2 \ln x & , x \geq 1 \end{cases}$$

A) 2

B) 3

C) 4

D) 5

NC7) Let $y = f(x)$ be a twice differentiable function such that $f(1) = 2$ and $\frac{dy}{dx} = y^3 + 2$. What is the value of $\frac{d^2y}{dx^2}$ at $x=1$?

- A) 12 B) 60 C) 120 D) 150

NC8) Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$ with the initial condition $f(1) = -2$. Which of the following is an expression for $f(x)$?

- A) $-\sqrt{x^2 + 2x + 1}$ B) $\sqrt{x^2 + 2x + 1}$ C) $-2 + \sqrt{x^2 + 2x + 1}$ D) $\sqrt{x^2 + x + 2}$

NC9) Let f be a function with first derivative defined by $f'(x) = \frac{4x^2 - 6}{x^2}$ for $x > 0$. It is known that $f(1) = 2$ and $f(4) = 8$. What value of x in the open interval $(1, 4)$ satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1, 4]$?

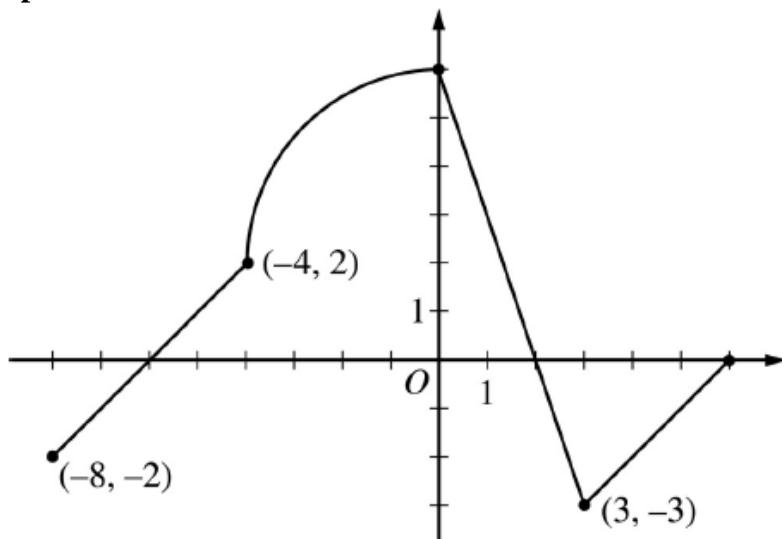
- A) $\sqrt{6}$ B) $\sqrt{3}$ C) $\sqrt{2}$ D) 1

NC10) Which of the following limits is equal to $\int_2^6 x^2 dx$?

- A) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{k}{n}\right)^2 \frac{1}{n}$
 B) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{4k}{n}\right)^2 \frac{1}{n}$
 C) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{k}{n}\right)^2 \frac{4}{n}$
 D) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{4k}{n}\right)^2 \frac{4}{n}$

Noncalculator Free Response Section

FR1)



Graph of g

A continuous function g is defined on the closed interval $-8 \leq x \leq 6$. The graph of g , shown above, consists of three line segments and a quarter circle centered at the point $(0, 2)$. Let f be the function

$$\text{given by } f(x) = \int_{-8}^x g(t) dt.$$

- a) Find all values of x in the interval $-8 < x < 6$ at which f has a critical point. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

b) Find $f(2)$.

c) Find the values of x in which f has a point of inflection. Justify your answer.

d) Let h be the function defined by $h(x) = \frac{g(x)}{3-x^2}$. Find $h'(1)$.

FR2) The function f is defined by

$$f(x) = \begin{cases} 3x^2 + 4x & , x \leq 0 \\ e^{4x} + 2 & , x > 0 \end{cases}$$

a) Is f continuous at $x=0$? Justify your answer.

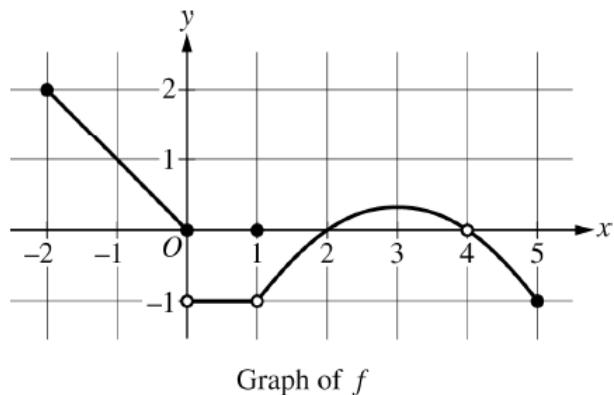
b) Find $f'(-3)$ and $f'(2)$.

c) Explain why $f'(0)$ does not exist.

d) Let g be the function given by $g(x) = \int_{-1}^x f(t)dt$. Find $g(2)$.

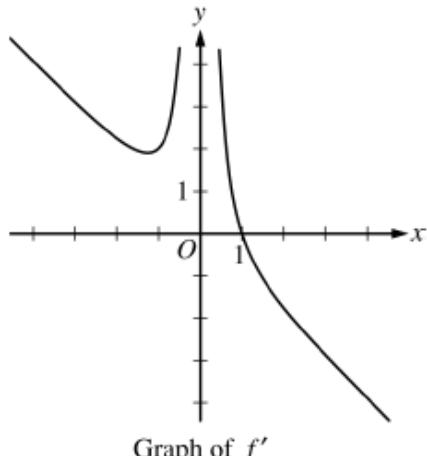
Calculator Multiple Choice Section

CA1) The graph of the function f is shown below. For what values of c does $\lim_{x \rightarrow c} f(x) = 0$?

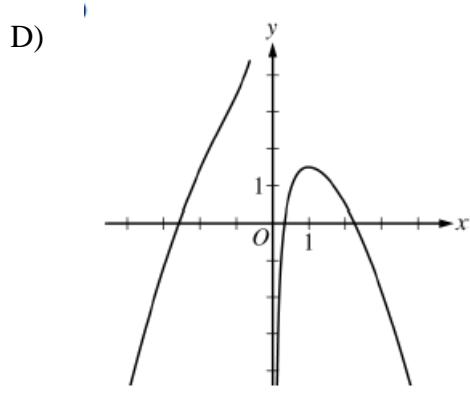
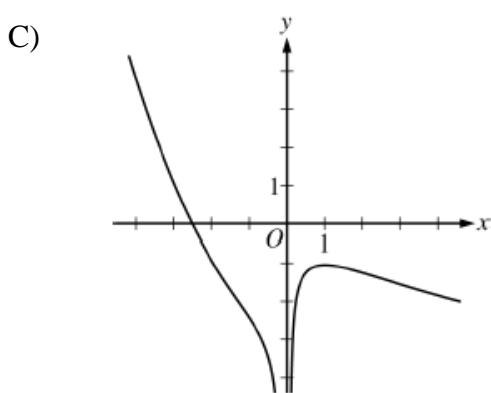
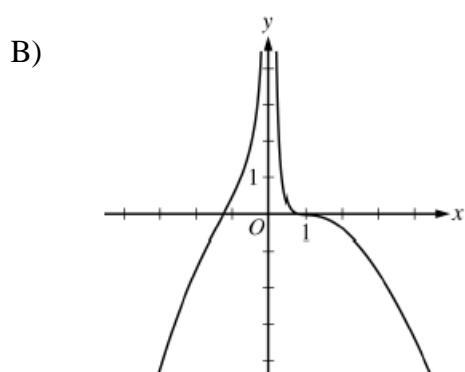
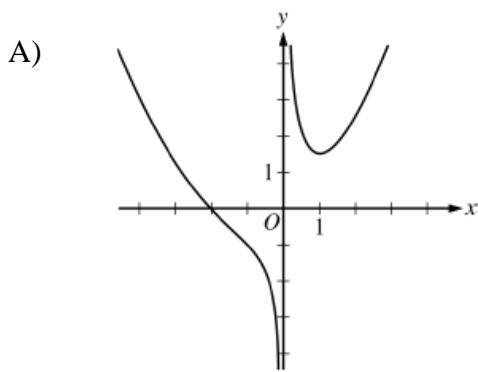


- A) 2 only B) 0, 1 and 2 C) 2 and 4 only D) 1, 2 and 4

CA2)



The graph of f' , the derivative of the function f , is shown above. Which could be the graph of f ?



CA3) Let f be a differentiable function such that $f(1) = \pi$ and $f'(x) = \sqrt{x^3 + 5}$. What is the value of $f(6)$?

- A) 14.866 B) 37.376 C) 40.518 D) 42.808

CA4) The velocity of a particle moving along the x -axis is given by $v(t) = \sqrt{t} + \sin(e^t)$ for $t \geq 0$. Which of the following statements describes the motion of the particle at $t = 1$?

- A) The particle is moving to the left with positive acceleration.
B) The particle is moving to the right with positive acceleration.
C) The particle is moving to the left with negative acceleration.
D) The particle is moving to the right with negative acceleration.

CA5) The first derivative of the function f is defined by $f'(x) = \frac{x+3e^{-x}}{x^2+0.6}$. On what intervals is f increasing?

- A) $-1.524 < x < -0.269$ only B) $-\infty < x < \infty$
C) $x > 1.370$ only D) There are no intervals in which f is increasing

Calculator Free Response Section

FR1)

t (minutes)	0	2	5	7	10
$h(t)$ (inches)	3.5	10.0	15.5	18.5	20.0

The depth of water in tank A , in inches, is modeled by a differentiable and increasing function h for $0 \leq t \leq 10$, where t is measured in minutes. Values of $h(t)$ for selected values of t are given in the table above.

- a) Use the data in the table to find an approximation for $h'(1)$. Show the computations that lead to your answer. Indicate units of measure.

- b) Approximate the value of $\int_0^{10} h(t) dt$ using a Left Riemann sum with the four subintervals indicated by the data in the table. Is this approximation greater than or less than $\int_0^{10} h(t) dt$? Give a reason for your answer.

- c) The depth of water in tank B , in inches, is modeled by the function $g(t) = 3.2 + 11.5\sqrt{\sin(0.16t)}$ for $0 \leq t \leq 10$, where t is measured in minutes. Find the average depth of water in tank B over the interval $0 \leq t \leq 10$. Is this value greater than or less than the average depth of water in tank A over the interval $0 \leq t \leq 10$? Give a reason for your answer.
- d) According to the model given in part c), is the depth of the water in tank B increasing or decreasing at time $t = 7$? Give a reason for your answer.

FR2) A hive contains 1400 bees at time $t = 0$. During the time interval $0 \leq t \leq 4$ hours, bees enter the hive at a rate modeled by $E(t) = 95 \arctan(t) - 7t\sqrt{9+t}$, where $E(t)$ is measured in bees per hour. During the same time interval, bees leave the hive at a rate modeled by $L(t) = 67 \cos(0.3t) - 5t$, where $L(t)$ is measured in bees per hour.

a) How many bees leave the hive during the time interval $0 \leq t \leq 2$?

b) Write an expression involving one or more integrals for the total number of bees in the hive at time t for $0 \leq t \leq 4$. Find the total number of bees in the hive at $t = 4$.

c) Find the minimum number of bees in the hive for $0 \leq t \leq 4$. Justify your answer.