5.1 – 5.2, 5.6 Quiz Review

For #1-4, find the area and decide whether it is an over/underestimate

1. $y = 2x - x^2$ from $[1,2]$ with $n = 4$ using LEA.

2. $y = \sqrt{x}$ from $[0,8]$ with $n = 4$ using REA.

3. $y = x^3 + 2$ from $[2,10]$ with $n = 4$ using MPA.

4. $y = \sqrt{16 - x^2}$ from $[2,4]$ with $n = 5$ using TZA.

Evaluate the following

5. $\int (6x^2 - 7x + 5e^x)dx$

6. $\int \sqrt{x}(x - 3\sqrt{x})dx$

7. $\int (3\sec x \tan x + 4\sin x)dx$

8. $\int \frac{9x^4 - x^2 + 5x}{3x} dx$

9. $\int \frac{\sin x + 3\csc x}{\tan x} dx$

10. $\int 4x\left(\frac{5}{x^2} - 3\sqrt[4]{x^5}\right)dx$

11. Find the original function $f(x)$ given $f'(x) = \frac{4}{\sqrt{x}} - 2$ and the condition $f(4) = -1$.

12. A cannonball is shot up from the ground with a velocity of $30 m/s$. The acceleration is $-9.8 m/s^2$.

a) What is the height and velocity function of the cannonball?

b) What is the maximum height of the cannonball?

c) What is the velocity of the cannonball when it hits the ground?

13. A region's beverage consumption $C(t)$, in L/month, over various months, t , where $t = 0$ is the beginning of the first month can be modeled by the following table:

t	0	3	6	8	12
C(t)	15	25	30	25	10

a) Approximate the area under the curve of $C(t)$ from $[0,12]$ using the Trapezoidal Rule with 4 subintervals. Describe the meaning of this answer.

b) Assuming $C(t)$ is a function that is concave down everywhere, will the answer from a) be an over or under estimate?

Solutions

1. The width of each subinterval is $\frac{b-a}{n}=\frac{1}{4}$. A LEA starts with the area of a rectangle, whose $y-$value (height) is evaluated at the left endpoint. So the area is the sum of the areas of the 4 rectangles expressed by $0.25*f(1)+0.25*f(1.25)+0.25*f(1.5)+0.25*f(1.75)$ or 0.781. A LEA being an over or underestimate is dependent on whether the function is increasing or decreasing. Since $y'=2-2x$, which has a critical value at $x=1$. Using the line test, we see that the function is decreasing from $[1,2]$, therefore this is an overestimate.

2. The width of each subinterval is $\frac{b-a}{n}=2$. A REA starts with the area of a rectangle, whose $y-$value (height) is evaluated at the right endpoint. So the area is the sum of the areas of the 4 rectangles expressed by $2*f(2)+2*f(4)+2*f(6)+2*f(8)$ or 17.384. A REA being an over or underestimate is dependent on whether the function is increasing or decreasing. Since $y=\sqrt{x}$ is always increasing, this is an overestimate.

3. The width of each subinterval is $\frac{b-a}{n}=2$. A MPA starts with the area of a rectangle, whose $y-$value (height) is evaluated midway between an interval. So the area is the sum of the areas of the 4 rectangles expressed by $2*f(3)+2*f(5)+2*f(7)+2*f(9)$ or 2464. A MPA being an over or underestimate is dependent on whether the function is concave up or down. Since $y''=6x$, which has a possible point of inflection at $x=0$. Using the line test, we see that the function is concave up from $[2,10]$, therefore this is an underestimate.

4. The width of each subinterval is $\frac{b-a}{n}=0.4$. A TZA starts with the area of a trapezoid, whose $y-$value (height) is evaluated at each values within an interval. So the area is the sum of the areas of the 5 trapezoids expressed by
$\frac{1}{2}(0.4)(f(2)+f(2.4))+\frac{1}{2}(0.4)(f(2.4)+f(2.8))+\frac{1}{2}(0.4)(f(2.8)+f(3.2))+\frac{1}{2}(0.4)(f(3.2)+f(3.6))+\frac{1}{2}(0.4)(f(3.6)+f(4))$ or 4.773. A TZA being an over or underestimate is dependent on whether the function is concave up or down. Since this is a semicircle, the function is always concave down. Therefore, this is an underestimate.

5. Since each of these can be integrated as it is, we get $2x^3-\frac{7}{2}x^2+5e^x+C$.

6. Since there is no product rule, we must multiply it to get $\int \sqrt{x}\left(x-3\sqrt{x}\right)dx=\int\left(x^{\frac{3}{2}}-3x\right)dx$. Since both of these are power rules, so the answer is $\frac{2}{5}x^{\frac{5}{2}}-\frac{3}{2}x^2+C$.

7. Both of these are antiderivative rules, so we get $3\sec x-4\cos x+C$.

8. Since there is no quotient rule, we must divide each of the terms in the numerator by $3x$. So, $\int\left(\frac{9x^4}{3x}-\frac{x^2}{3x}+\frac{5x}{3x}\right)dx=\int\left(3x^3-\frac{1}{3}x+\frac{5}{3}\right)dx$. This new integrand is all power rules, so the answer is $\frac{3}{4}x^4-\frac{1}{6}x^2+\frac{5}{3}x+C$.

9. Since there is no quotient rule, we can divide these and try to find a trig antiderivative rule that applies. Dividing by $\tan x$ is the same as multiplying by $\cot x$, so that makes the integral $\int\left(\sin x\cot x+3\csc x\cot x\right)dx$. The second term fits an antiderivative, whereas the first does not, so we should rewrite $\cot x$ as $\frac{\cos x}{\sin x}$. So, $\int\left(\sin x\frac{\cos x}{\sin x}+3\csc x\cot x\right)dx=\int\left(\cos x+3\csc x\cot x\right)dx$. These both can be integrated, so $\int\left(\cos x+3\csc x\cot x\right)dx=\sin x-3\csc x+C$.

10. Since there is no product rule, we need to distribute and rewrite the radical as an exponent. This gives us $\int 4x\left(\frac{5}{x^2}-3\sqrt[4]{x^5}\right)dx=\int\left(\frac{20}{x}-12x^{\frac{9}{4}}\right)dx$. So, $20\ln|x|-\frac{48}{13}x^{\frac{13}{4}}+C$.

11. If we need to find $f(x)$, given $f'(x)$, we must integrate $f'(x)$. So, integrating both sides gives us $\int f'(x)dx=\int\left(\frac{4}{\sqrt{x}}-2\right)dx=\int\left(4x^{-\frac{1}{2}}-2\right)dx$. This is $f(x)=8\sqrt{x}-2x+C$ and to find C, we must use the initial condition $f(4)=-1$. So, $-1=8\sqrt{4}-2(4)+C$ and solving gives us $-1=16-8+C$ or $C=-9$. This gives us $f(x)=8\sqrt{x}-2x-9$.

12. a) Given acceleration, we must integrate to get to velocity. So, if $a(t) = -9.8$, then integrating both sides gives us $\int a(t)dt = \int -9.8dt$ or $v(t) = -9.8t + C$. Since the initial velocity is $v(0) = 30$, then $30 = -9.8(0) + C$ or $C = 30$. This gives us $v(t) = -9.8t + 30$. Then, to find the height function from velocity, we must integrate. So $\int v(t)dt = \int (-9.8 + 30)dt$ or $h(t) = -4.9t^2 + 30t + C$. Then since the cannonball was shot from the ground, the initial height is $h(0) = 0$. So, $0 = -4.9(0)^2 + 30(0) + C$ and $C = 0$. Therefore the height equations is $h(t) = -4.9t^2 + 30t$.

b) After graphing the height function, you can let the calculator find the maximum height of 45.918.

c) The cannonball hitting the ground is when the height is 0. So, letting the calculator solve the equation $0 = -4.9t^2 + 30t$ gives us $t = 6.12244898$. Then, the velocity is $v(6.122448898) = -30$.

13. a) Since there are not equal widths, then we must take each trapezoid individually. A TZA starts with the area of a trapezoid, whose $y-$value (height) is evaluated at each values within an interval. So the area is the sum of the areas of the 4 trapezoids expressed by

$\frac{1}{2}(3-0)(C(0)+C(3))+\frac{1}{2}(6-3)(C(3)+C(6))+\frac{1}{2}(8-6)(C(6)+C(8))+\frac{1}{2}(12-8)(C(8)+C(12))$ or

$\frac{1}{2}(3)(15+25)+\frac{1}{2}(3)(25+30)+\frac{1}{2}(2)(30+25)+\frac{1}{2}(4)(25+10)=$ 267.5. Since we have integrated a rate, this will tell us that approximately 267.5L of beverage was consumed over 12 months.

b) Since the function is concave down, then a trapezoidal approximation is an underestimate.