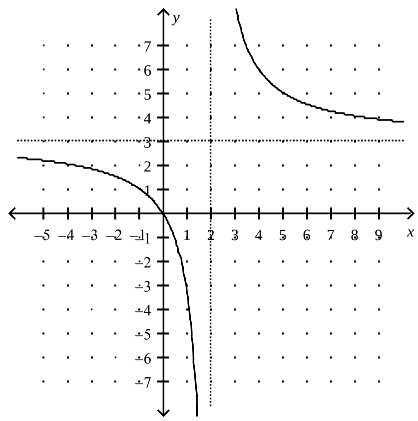
#1-8 (Non-Calc), #9-13 (Calc)

Chapter 2 Review Sheet

1) Use the graph to find each limit:

a) $\lim\limits_{x\to 2^+} g(x)$ b) $\lim\limits_{x\to 0} g(x)$



2) Find the limit: a) $\lim\limits_{x\to 3}\dfrac{x^2+1}{x}$ b) $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{x}$ c) $\lim\limits_{x\to -2}\dfrac{x^2-4}{x^3+8}$ d) $\lim\limits_{x\to 2}\begin{cases}\dfrac{x^2-5x+6}{x-2} & x\neq 2\\ x+7 & x=2\end{cases}$

e) $\lim\limits_{x\to -1^-}\dfrac{x^2-2x+1}{x+1}$ f) $\lim\limits_{\Delta x\to 0}\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$ for $f(x)=3x^2+x$ g) $\lim\limits_{x\to 0^+}\left(x-\dfrac{1}{x^3}\right)$

3) Determine the intervals on which the function is continuous: $f(x)=\dfrac{3x^2-x-2}{x-1}$

4) Determine the intervals on which the function is continuous: $g(x)=\left.\begin{matrix}5-\sqrt{x} & x\le 4\\ 2x-3 & x>4\end{matrix}\right\}$

5) Find the horizontal and vertical asymptotes: a) $h(x)=\dfrac{4x}{4-x^2}$ b) $f(x)=1+\dfrac{2}{x}$ c) $f(x)=\dfrac{2x-4}{3x^2-12}$

6) Find the limit: a) $\lim\limits_{x\to\infty}\dfrac{2x-1}{3x+2}$ b) $\lim\limits_{x\to\infty}\dfrac{2x+1}{\sqrt{x^2-x}}$ c) $\lim\limits_{x\to\infty}\dfrac{\sin 2x}{x}$ d) $\lim\limits_{x\to -\infty}\dfrac{4x^2}{x+2}$

7) If $f(x)$ is continuous for all real numbers and if $f(x)=\dfrac{x^2+5x+4}{x+1}$ when $x\neq -1$, then $f(-1)$ equals?

8) Find the values of B and C so that the function is continuous. Justify your answer using the definition of continuity.

$$f(x)=\begin{cases}x+6 & x<2\\ Bx & 2\le x<4\\ 2x+C & x\ge 4\end{cases}$$

9) Does the function $f(x) = x^2 - 4x + 3$ have a zero in the interval $[2,4]$? Justify your response.

10) A utility company burns coal to generate electricity. The cost C in dollars of removing p % of the air pollutants in the stack emissions is :

$$C = \frac{80,000p}{100-p}, \quad 0 \le p < 100$$

Find the cost of the removing:

a) 15% b) 90% c) Find the limit of C as $p \to 100$

11) If $\lim_{x\to 3} f(x) = 5$, decide which of the following must be true. If it is false, give a reason why:

a) $f(3) = 5$

b) $f(3)$ must be defined

c) $\lim_{x\to 3^-} f(x) = 5$

d) There is a removable discontinuity at $x = 3$

12) Sketch a possible graph for a function $f(x)$ that satisfies the following 5 properties:

a) $\lim_{x\to 2^-} f(x) = \infty$ b) $\lim_{x\to 2^+} f(x) = -1$ c) $\lim_{x\to \infty} f(x) = 3$

d) $\lim_{x\to -\infty} f(x) = -3$ e) $\lim_{x\to -2} f(x) = 4$

13) Evaluate and round to 3 decimals: a) $\lim_{x\to 0} \frac{e^x - 1}{x}$ b) $\lim_{x\to 0} \frac{2^x - (x+1)}{\arctan 2x}$ c) $\lim_{x\to 3^-} \begin{cases} 3 - \cos x & x \le 3 \\ 0.23e^{0.1x} + 1 & x > 3 \end{cases}$

Solutions:

1) a) We want to know where $g(x)$ goes as x approaches ∞, which goes to ∞.

b) As goes as x approaches 0, the graph of $g(x)$ approaches 0.

2) a) Using direct substitution, we see this limit goes to $\frac{10}{3}$.

b) Direct substitution gives us the indeterminate $\frac{0}{0}$ and since there are square roots, we should look to multiply by the conjugate. So, we should multiply the numerator and denominator by $\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$, giving us $\lim_{x\to 0}\frac{4+x-4}{x\left(\sqrt{4+x}+2\right)} = \lim_{x\to 0}\frac{x}{x\left(\sqrt{4+x}+2\right)}$ and dividing out the x gives us $\lim_{x\to 0}\frac{1}{\sqrt{4+x}+2}$. Plugging 0 back in gives us $\frac{1}{4}$.

c) Direct substitution gives us the indeterminate $\frac{0}{0}$ so we should factor both. So, this gives us $\lim_{x\to -2}\frac{(x+2)(x-2)}{(x+2)(x^2-2x+4)}$ and dividing out the $(x+2)$ gives us $\lim_{x\to -2}\frac{(x-2)}{(x^2-2x+4)}$. Plugging -2 back in gives us $-\frac{1}{3}$.

d) Since we are looking at the limit as x approaches 2, we should use the first equation since the second equation only applies when x is equal to 2 (so therefore the first equation is every other $x-$value). So, evaluating $\lim_{x\to 2}\frac{x^2-5x+6}{x-2}$ gives us the indeterminate form $\frac{0}{0}$, so we must factor. This gives us $\lim_{x\to 2}\frac{(x-2)(x-3)}{x-2}$ and dividing out the $(x-2)$ gives us $\lim_{x\to 2} x-3$, which is -1.

e) One-sided limits require us to approach things a little more conceptually. Direct substitution gives us $\frac{4}{0}$, which does not mean that factoring will help. So, we should instead think of numbers slightly less than -1 since this is a one-sided limit from the left. Plugging in numbers slightly smaller (or more negative) than -1 gives us $\frac{\sim 4}{small \quad negative \quad number}$ and dividing by a small number is the same as multiplying by a large number. Additionally, since this is a positive divided by a negative, we are approaching a large negative number. Therefore, the limit goes to $-\infty$.

f) Substituting $f(x)$ into this expression gives us $\lim_{\Delta x \to 0} \frac{3(x+\Delta x)^2 + (x+\Delta x) - (3x^2 + x)}{\Delta x}$. Direct substitution will lead to the indeterminate form, so we should multiply out and distribute the negatives, giving us $\lim_{\Delta x \to 0} \frac{3x^2 + 6x\Delta x + 3\Delta x^2 + x + \Delta x - 3x^2 - x}{\Delta x}$ and simplifying the numerator gives us $\lim_{\Delta x \to 0} \frac{6x\Delta x + 3\Delta x^2 + \Delta x}{\Delta x}$. Every term has a Δx, so we cancel it out, giving us $\lim_{\Delta x \to 0} 6x + 3\Delta x + 1$. Plugging $\Delta x = 0$ into this expression gives us $6x+1$.

g) Direct substitution gives us $0 - \frac{1}{0}$, which not indeterminate, so factoring will not help. So, we should instead think of numbers slightly greater than 0 since this is a one-sided limit from the right. Plugging in numbers slightly greater than 0 gives us $\sim 0 - \frac{1}{small\ \ positive\ \ number}$ and dividing by a small number is the same as multiplying by a large number. Additionally, since this is a positive divided by a positive, we are approaching a large positive number. However, since this is zero minus a large positive number, this will beome a large negative number, or the limit goes to $-\infty$.

3) A function is not continuous where there is a domain restriction, so where the denominator is equal to 0. This happens at $x = 1$, meaning this is continuous everywhere except at $x = 1$, so $(-\infty, 1), (1, \infty)$.

4) A piecewise function is not continuous where there is a domain restriction and possibly at the break in domain. There are possible domain restrictions due to the square root, so there would be restrictions when $x \le 0$. However, since we only use this equation when $x \le 4$, there are domain restrictions when $x \le 0$. The only other place where this function might not be continuous is at the change in domain at $x = 4$. To determine if we are continuous at $x = 4$, we must verify that $g(4) = \lim_{x \to 4^-} g(x) = \lim_{x \to 4^+} g(x)$. We can use the top equation to show that $g(4) = 3$. To find the limit from the left, we need to use values less than 4, so we would use the top equation. Direct substitution gives us $\lim_{x \to 4^-} g(x) = 3$. To find the limit from the right, we need to use values greater than 4, so we would use the bottom equation. Direct substitution gives us $\lim_{x \to 4^+} g(x) = 5$. So, since the limits from the left and right are not equal, this function is not continuous at $x = 4$. So $g(x)$ is continuous from $[0, 4), (4, \infty)$.

5) a) Horizontal asymptotes can be found by taking $\lim_{x \to \infty} \frac{4x}{4 - x^2}$, and since the power is greater in the denominator, this will approach 0, so there is a horizontal asymptote at $y = 0$. Vertical asymptotes occur at nonremovable discontinuities and we must set the denominator equal to 0 to find those discontinuities. So $4 - x^2 = 0$ or $x = -2, 2$ and to determine if this is a nonremovable discontinuity, we must figure out if the limit exists. So, using direct substitution for $\lim_{x \to -2} \frac{4x}{4 - x^2}$ gives us $\frac{-8}{0}$ and since nothing cancels out, we know this limit does not exist. Using direct substitution for $\lim_{x \to 2} \frac{4x}{4 - x^2}$

gives us $\frac{8}{0}$ and since nothing cancels out, we know this limit does not exist. Therefore, there are vertical asymptotes at $x=-2$, $x=2$.

b) Horizontal asymptotes can be found by taking $\lim_{x\to\infty} 1+\frac{2}{x}$, and since the power is greater in the denominator, this will approach $1+0$, so there is a horizontal asymptote at $y=1$. Vertical asymptotes occur at nonremovable discontinuities and we must set the denominator equal to 0 to find those discontinuities, so $x=0$. To determine if this is a nonremovable discontinuity, we must figure out if the limit exists. Using direct substitution for $\lim_{x\to 0} 1+\frac{2}{x}$ gives us $1+\frac{2}{0}$ and since nothing cancels out, we know this limit does not exist. Therefore, there is a vertical asymptote at $x=0$.

c) Horizontal asymptotes can be found by taking $\lim_{x\to\infty} \frac{2x-4}{3x^2-12}$, and since the power is greater in the denominator, this will approach 0, so there is a horizontal asymptote at $y=0$. Vertical asymptotes occur at nonremovable discontinuities and we must set the denominator equal to 0 to find those discontinuities, so $3(x^2-4)=0$ or $x=-2, 2$. To determine if this is a nonremovable discontinuity, we must figure out if the limit exists. So, using direct substitution for $\lim_{x\to 2} \frac{2x-4}{3x^2-12}$ gives us $\frac{0}{0}$, so we should factor to see what cancels out. So, $\lim_{x\to 2} \frac{2(x-2)}{3(x-2)(x+2)}$ or $\lim_{x\to 2} \frac{2}{3(x+2)}$ which gives us $\frac{2}{12}=\frac{1}{6}$ after we substitute again. This means that the limit exists, so there is no vertical asymptote here (Note that it is a hole). Using direct substitution for $\lim_{x\to -2} \frac{2x-4}{3x^2-12}$ gives us $\frac{-8}{0}$ and even plugging this into the simplified-factored version gives us $\frac{2}{0}$, meaning that the limit does not exist. Therefore, there is a vertical asymptote at $x=-2$.

6) a) We can evaluate limits to infinity by focusing on the highest power in both sides of the fraction. Since the highest power in each is 1, we can focus on the ratio of the coefficients, so $\frac{2x}{3x} \to \frac{2}{3}$.

b) Focusing on the highest power, we see the numerator has a power of 1 and we can simplify the denominator to be $\sqrt{x^2}=|x|$, meaning the powers are the same. Since this is going to positive infinity, we know $\frac{2x}{|x|} \to \frac{2x}{x} \to 2$.

c) Since this is not all polynomials, we have to think about what the numerator and denominator are doing. So as $x\to\infty$, $\sin 2x$ will constantly be oscillating between $[-1,1]$ and as $x\to\infty$, x will be approaching ∞. So combining these responses gives us $\frac{[-1,1]}{\infty}$ which will approach 0.

d) Focusing on the highest power, we see the numerator has a power of 2 and the denominator has a power of 1. Therefore, the numerator is growing much faster, so $\frac{4x^2}{x} \to 4x \to -\infty$ (or DNE).

7) $f(x)$ is continuous when $f(-1) = \lim_{x\to -1} f(x)$, so we must find $\lim_{x\to -1} \frac{x^2+5x+4}{x+1}$. Direct substitution gives us the indeterminate $\frac{0}{0}$, so we should factor, giving us $\lim_{x\to -1} \frac{(x+1)(x+4)}{x+1}$. The $x+1$ can cancel out, giving us $\lim_{x\to -1} x+4$, which is 3. Therefore, in order for $f(x)$ to be continuous, $f(-1)=3$.

8) Since this is piecewise, we want the one – sided limits at each $x = 2, 4$ to be equal from the left and right. So, we want $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ and $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x)$. So, if $\lim_{x\to 2^-} f(x)$ uses the top equation, we get $\lim_{x\to 2^-} f(x) = 8$ and if $\lim_{x\to 2^+} f(x)$ uses the middle equation, we get $\lim_{x\to 2^+} f(x) = 2B$. In order to be continuous, $2B = 8$ or $B = 4$. Next, if $\lim_{x\to 4^-} f(x)$ uses the middle equation, we get $\lim_{x\to 4^-} f(x) = 4B$ and since $B = 4$, $\lim_{x\to 4^-} f(x) = 16$. Then, if $\lim_{x\to 4^+} f(x)$ uses the bottom equation, we get $\lim_{x\to 4^+} f(x) = 8 + C$. In order to be continuous, $16 = 8 + C$ or $C = 8$.

9) Since there are no domain restrictions, we know that $f(x)$ is continuous everywhere. To see if there is a zero from $[2, 4]$, we should find $f(2)$ and $f(4)$. So, $f(2) = -1$ and $f(4) = 3$ and according to the Intermediate Value Theorem, since $f(x)$ is continuous from $[2, 4]$, there must be a value c, such that $2 \le c \le 4$, where $f(2) \le f(c) \le f(4)$. Or, since $f(x)$ goes from negative to positive, it must pass through 0 at some point in between 2 and 4, so yes.

10) a) Since p is a percent, we want to simply plug in $p = 15$, so $14,117.65.

b) Plugging in $p = 90$ gives us $720,000.00.

c) Since we are approaching 100, we could try direct substitution, which gives us $\frac{8{,}000{,}000}{0}$. We have to take a one-sided limit from the left, because it says that $0 \le p < 100$, meaning we should think of numbers slightly less than 100. Plugging these in gives us $\frac{\sim 8{,}000{,}000}{small\ \ positive\ \ number}$ and dividing by a small number is the same as multiplying by a large number. Therefore, the limit goes to ∞.

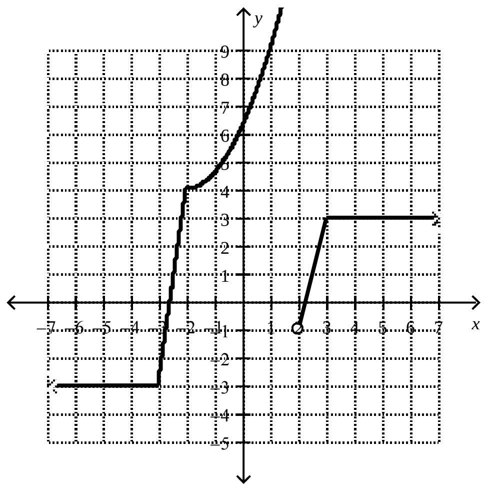
11) a) The limit describes what the graph is doing around $x = 3$, not what is happening at exactly $x = 3$. So there doesn't have to be a defined point, so this is not necessarily true, or False.

b) Same explanation as part a), so False.

c) If the overall limit exists, then the one sided limit must also exist at the same value, so this is True.

d) A removable discontinuity would be a hole on, which could happen if the limit exists. However, there could be a defined point at $(3,5)$, so False.

12) Part a) states that as x approaches 2 from the left, the graph goes up to ∞, making a vertical asymptote. Part b) says that as x approaches 2 from the right, the graph goes to -1. Part c) and d) show horizontal asymptotes at both $y=3$ to the right and $y=-3$ to the left. Finally, part e) shows that there is either a hole or defined point at $(-2,4)$. A possible graph is shown below, but there are many other options.



13) a) We can use the calculator to plug in values approaching 0 from both sides and we see that they are approaching 1.

b) Using the calculator, we see the values are approaching -0.153.

c) Since this a one-sided limit coming in from the left, we should choose values less than 3. This means that we will plug those values into the top equation, giving us 3.990.