For each of the following, set up the integral (including the limits of integration) for the area between the curve, but ***do not evaluate***. NO CALCULATOR!!

1) Find the area of the region bounded by $y = x^2$ and the line $y = x$ and the lines $x = 1$ and $x = 4$.

2) Find the area of the region enclosed by $y = x^2 - 4x$ and $y = 16 - x^2$.

3) Find the area of the region enclosed by $x = y^2 - 4y$ and $x = y$.

4) Find the area of the region bounded by the line $y = 4$ and $y = x^2$ over the interval $-3 \le x \le 4$.

5) Find the value of x on $[0, 6]$ that divides the region bounded by $y = x^2$, $y = 0$, $x = 0$, and $x = 6$ into two parts of equal area.

For each of the following, set up the integral (including the limits of integration) for the volume of revolution described, but ***do not evaluate***.

6) Find the volume of the solid obtained by rotating the region bounded by $y = 1 - x^2$ and $y = 0$ about the x-axis.

7) Find volume of the solid obtained by rotating the region bounded by $y = x^{\frac{2}{3}}$, $x = 1$, and $y = 0$ about the y-axis.

8) Find volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = \sqrt{x}$ about the line $y = 2$.

9) Find the volume of the solid obtained by rotating the region bounded by $y = x^2 + 1$, $y = 3$, and to the right of $x = 0$ about the line $y = 5$.

10) Find the volume of the solid of revolution by rotating the region bounded by $y = x^2$ and $y = 2x$ when revolved around the line $x = 2$.

Answers

1) $\displaystyle \int_1^4 (x^2 - x)\,dx$

2) $\displaystyle \int_{-2}^4 \left[(16 - x^2) - (x^2 - 4x)\right]dx$

3) $\displaystyle \int_0^5 \left[y - (y^2 - 4y)\right]dy$

4) $\displaystyle \int_{-3}^{-2} (x^2 - 4)\,dx + \int_{-2}^2 (4 - x^2)\,dx + \int_2^4 (x^2 - 4)\,dx$

5) $\displaystyle \int_0^c x^2\,dx = \int_c^6 x^2\,dx \rightarrow \left.\frac{1}{3}x^3\right|_0^c = \left.\frac{1}{3}x^3\right|_c^6$ so $\frac{1}{3}c^3 = 72 - \frac{1}{3}c^3 \rightarrow \frac{2}{3}c^3 = 72$ or $c = \sqrt[3]{108}$

6) $\displaystyle \pi\int_{-1}^1 (1 - x^2)^2\,dx$

7) $\displaystyle \pi\int_0^1 \left[(1 - 0)^2 - \left(y^{\frac{3}{2}} - 0\right)^2\right]dy$

8) $\displaystyle \pi\int_0^1 \left[(x^2 - 2)^2 - (\sqrt{x} - 2)^2\right]dx$

9) $\displaystyle \pi\int_0^{\sqrt{2}} \left[(x^2 + 1 - 5)^2 - (3 - 5)^2\right]dx$

10) $\displaystyle \pi\int_0^4 \left[\left(\frac{y}{2} - 2\right)^2 - (\sqrt{y} - 2)^2\right]dy$