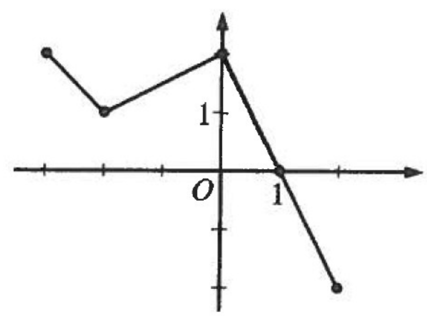
Noncalculator #1-12 (25 minutes) and Calculator #13-20 (25 minutes)

1. A car, starting at rest, moves with an acceleration modeled by $a(t) = 2t - 3\sqrt{t} - 1$ for all $t > 0$. Find the speed of the car at $t = 4$.

a) -4 b) 0 c) 3 d) 4 e) 28

2. Given the graph of f to the right, find $\int_{-2}^{2} f(x)dx$.



Graph of *f*

a) 2 b) 3 c) 4 d) 4.5 e) 5

3. If f and g are continuous functions such that $g'(x) = f(x)$ for all x, then $\int_{2}^{3} f(x)dx =$

a) $g'(2) - g'(3)$ b) $g'(3) - g'(2)$ c) $g(3) - g(2)$ d) $f(3) - f(2)$ e) $f'(3) - f'(2)$

4. Find $f'(2)$ given $f(x) = \int_{3}^{x^2} \sqrt{5t - \cos 3t}\,dt$.

a) $\sqrt{10 - \sin 6}$ b) $\sqrt{20 - \cos 12}$ c) $\sqrt{5 + 3\sin 12}$ d) $4\sqrt{20 - \cos 12}$ e) $\sqrt{20 + 3\sin 12}$

5. What is the average value of $2t^3 - 3t^2 + 4$ over the interval $-1 \le t \le 1$?

a) 0 b) $\frac{7}{4}$ c) 3 d) 4 e) 6

6. $\int \cos(3 - 2x)dx =$

a) $\sin(3-2x) + C$ b) $-\sin(3-2x) + C$ c) $\frac{1}{2}\sin(3-2x) + C$ d) $-\frac{1}{2}\sin(3-2x) + C$ e) $-\frac{1}{5}\sin(3-2x) + C$

7. $\int \frac{x-2}{x-1} dx =$

a) $-\ln|x-1|+C$ b) $x+\ln|x-1|+C$ c) $x-\ln|x-1|+C$ d) $x+\sqrt{x-1}+C$ e) $x-\sqrt{x-1}+C$

8. $\int_0^2 \frac{x}{16+x^4} dx =$

a) $\frac{\pi}{32}$ b) $\frac{\pi}{16}$ c) $\frac{\pi}{8}$ d) $\frac{\pi}{4}$ e) $\frac{\pi}{2}$

9. $\int \left(\cot 2\theta + \cos\theta e^{\sin\theta}\right) d\theta =$

a) $\frac{1}{2}\ln|\sin 2\theta| + e^{\sin\theta} + C$ b) $-\frac{1}{2}\ln|\sin 2\theta| + e^{\sin\theta} + C$ c) $\frac{1}{2}\ln|\cos 2\theta| + e^{\sin\theta} + C$
d) $-2\ln|\sin 2\theta| + e^{\sin\theta} + C$ e) $2\ln|\sin 2\theta| + e^{\sin\theta} + C$

10. $\int_0^1 5x(1-2x^2)^3 dx =$

a) $\frac{-5}{8}$ b) $\frac{-5}{16}$ c) 0 d) $\frac{5}{16}$ e) $\frac{5}{4}$

11. $\int_0^4 \frac{2x-3}{x^2+16} dx =$

a) $\ln 2 - \frac{3\pi}{16}$ b) $\ln 2 - \frac{3\pi}{4}$ c) $\ln 2 - 3\pi$ d) $\ln 16 - \frac{3\pi}{16}$ e) $\ln 16 - \frac{3\pi}{4}$

12. A particle starts at (5, 0) when $t = 0$ and moves along the x-axis in such a way that at time $t > 0$ its velocity is given by $v(t) = \frac{1}{1+t}$. Determine the position of the particle at $t = 3$.

a) $\frac{97}{16}$ b) $\frac{95}{16}$ c) $\frac{79}{16}$ d) $1+\ln 4$ e) $5+\ln 4$

13. Which of the following limit expressions is equivalent to $\int_0^1 \sin(3+4x)dx$?

a) $\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(3+\frac{4k}{n}\right)\left(\frac{1}{n}\right)$ b) $\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(\frac{k}{n}\right)\left(\frac{1}{n}\right)$ c) $\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(3+\frac{k}{n}\right)\left(\frac{4}{n}\right)$

d) $\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(\frac{3k}{n}\right)\left(\frac{4}{n}\right)$ e) $\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(3+\frac{4k}{n}\right)\left(\frac{4}{n}\right)$

14. Find the distance traveled in the first four seconds for a particle whose velocity is given by $v(t)=7e^{-t^2}$, where t stands for time in seconds, given $t \geq 0$.

a) 0.976 b) 6.204 c) 6.359 d) 8.046 e) 12.720

15. $\int \frac{2x+5}{\sqrt{4-x^2}}dx =$

a) $-2\sqrt{4-x^2}+5\arcsin x+C$ b) $-\sqrt{4-x^2}+5\arcsin x+C$ c) $-2\sqrt{4-x^2}+5\arctan x+C$

d) $-2\sqrt{4-x^2}+5\arcsin\frac{x}{2}+C$ e) $-2\sqrt{4-x^2}+\frac{5}{2}\arcsin\frac{x}{2}+C$

16. A particle is moving along the y-axis such that the position is changing at a rate that can be modeled by the equation $R(t)=\frac{2}{t^2+9}$, where t is measured as time. Given that the particle is at the value $y=2$ at $t=5$, find the position of the particle at $t=2$.

a) -0.921 b) 1.115 c) 1.705 d) 1.853 e) 2

17. Oil is leaking from a tanker at a rate of $R(t)=500e^{-0.2t}$ gallons per hour, where t is measured in hours. The amount of oil that has leaked out after 10 hours is closest to…

a) 2140 gallons b) 2150 gallons c) 2160 gallons d) 2170 gallons e) 2180 gallons

18. Find $f(x)$ if $f''(x)=3x^2$ and $f'(1)=0$ and $f(2)=0$.

a) x^3-1 b) x^3-8 c) $\frac{x^4}{4}-4$ d) $\frac{x^4}{4}-x-2$ e) $\frac{x^4}{4}-x$

19. Calculate the average value of $f(\theta) = \arcsin 2\theta^2$ on the interval $\left[\frac{-\pi}{6}, \frac{\pi}{6}\right]$.

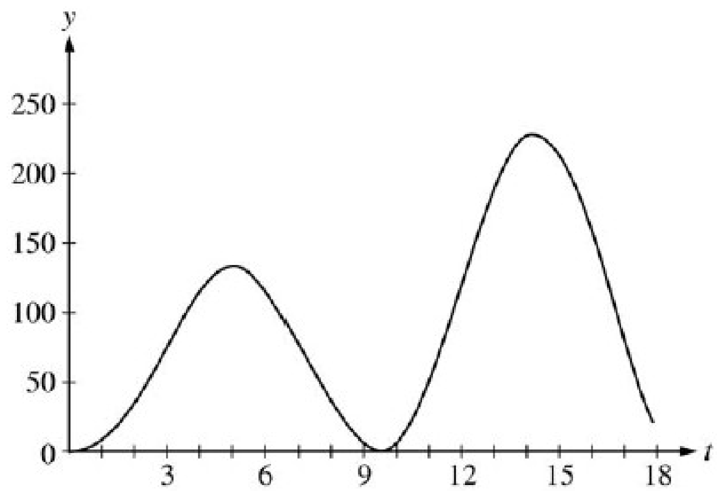
a) 0.094 b) 0.187 c) 0.196 d) 0.205 e) 0.374

20. The rate at which ice is melting in a pond is given by $\frac{dV}{dt} = \sqrt{1+2^t}$ where V is the volume of ice in cubic feet and t is the time in minutes. What amount of ice has melted in the first 5 minutes?

a) 14.49 ft^3 b) 14.51 ft^3 c) 14.53 ft^3 d) 14.55 ft^3 e) 14.57 ft^3

FREE RESPONSE – CA #1,2 NC #3

1)



At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

2) During a recent snowfall, students monitored the accumulation of snow on the roof of their school. The table shows the data they collected for the 12-hour period of the snowfall. There was no snow on the roof at the beginning of the snowfall ($t = 0$).

t (hours)	**S(t) (rate of snowfall in./hr)**
0	0
2	1.6
3	2.1
4.5	2.4
6.5	2.8
8	2.2
10.5	1.8
12	1.6

a) Use a REA to approximate $\int_0^8 S(t)dt$ using 5 subintervals. Be sure to include correct units.

b) Explain the meaning of $\frac{1}{8}\int_0^8 S(t)dt$ and approximate this value using the trapezoidal rule with 5 subintervals. Be sure to include units.

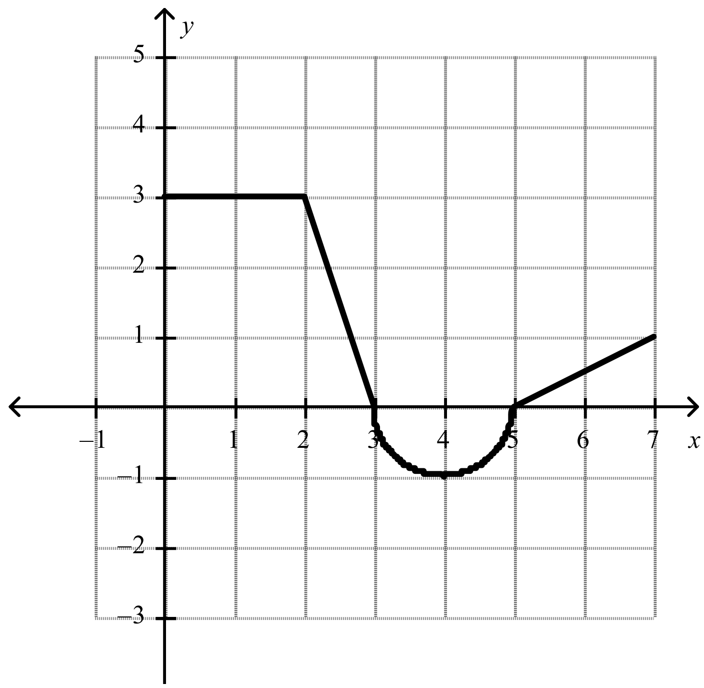
c) The rate $S(t)$ can be modeled by the function $P(t) = 1.3\sin\left(\frac{\pi}{11}t - 0.34\right) + 1.5$ for $t \geq 0$. Find the actual amount of snow that has accumulated over the first 12 hours.

d) The rate in which snow is removed after 3 hours from the roof is modeled by the function $R(t) = e^{0.13t} - 0.174$ for $t \geq 3$. Write an equation with an integral expression that models the amount of snow present on the roof, $A(t)$, at any time $t \geq 3$ given no snow was removed before $t = 3$.

e) At what time t is the amount of snow on the roof a maximum? How much snow is present at that time?

FREE RESPONSE – NONCALCULATOR

3) A particle moves along a coordinate axis. Its position at time t (sec) is $s(t) = 2t + \int_2^t f(x)\,dx$ feet, where f is the function whose graph is shown.



a) What is the particle's position at $t = 0$ and $t = 3$?

b) What is the particle's velocity at $t = 4$?

c) Approximately when is the acceleration of the particle zero? Justify your answer.

d) At what time during the 1st 7 seconds does s have its smallest value? Justify your answer.

e) What is the total distance travelled by the particle over the 7 seconds?

f) When is the particle decelerating? Justify your answer.

Answers

1. Given acceleration and that the car starts at rest, meaning $v(0)=0$, we must integrate. So $v(t)=\int\left(2t-3\sqrt{t}-1\right)dt=t^2-2t^{\frac{3}{2}}-t+C$. Using $v(0)=0$, $0=0+C$, $C=0$. This means $v(t)=t^2-2t^{\frac{3}{2}}-t+C$. So speed $=|v(4)|=|-4|=4$, D.

2. The graphical representation of integration is area under the curve of f from $[-2,2]$. So, this would be the area of a trapezoid and 2 triangles (whose areas both cancel out). The area is 3, so B.

3. Since $g'(x)=f(x)$, then we can replace the $f(x)$ with $g'(x)$. So, $\int_2^3 f(x)dx=\int_2^3 g'(x)dx$. Integrating the derivative cancels out and you get $\int_2^3 g'(x)dx=g(x)\big|_2^3=g(3)-g(2)$, so C.

4. Taking the derivative of the integral is the Second FTC, so $f'(x)=\sqrt{5x^2-\cos 3x^2}\bullet 2x-\sqrt{15-\cos 9}\bullet 0$ or $f'(x)=2x\sqrt{5x^2-\cos 3x^2}$. So $f'(2)=4\sqrt{20-\cos 12}$ or D.

5. Average value is $\frac{1}{1-(-1)}\int_{-1}^{1}\left(2t^3-3t^2+4\right)dt$, so $\frac{1}{2}\left[\frac{1}{2}t^4-t^3+4t\right]_{-1}^{1}=\frac{1}{2}\left[\left(\frac{1}{2}-1+4\right)-\left(\frac{1}{2}+1-4\right)\right]$. So, C.

6. This requires a u-substitution with $u=3-2x$, $du=-2dx$, so $\int\cos(3-2x)dx=-\frac{1}{2}\int\cos u\,du$. So, $-\frac{1}{2}\sin u+C$ or $-\frac{1}{2}\sin(3-2x)+C$ so D.

7. Since the powers are the same, we should do long division. $\frac{x-2}{x-1}=1-\frac{1}{x-1}$, so $\int\frac{x-2}{x-1}dx=\int 1dx-\int\frac{1}{x-1}dx$. The first antiderivative is x and the second requires a u-substitution with $u=x-1$, $du=dx$. This results in $\int\frac{1}{u}du=\ln|u|+C$, so $\int\frac{x-2}{x-1}dx=x-\ln|x-1|+C$ or C.

8. This is arctanx with $u=x^2$, $du=2xdx$, $a=4$, so $\int_0^2\frac{x}{16+x^4}dx=\frac{1}{2}\int_0^4\frac{1}{a^2+u^2}du=\frac{1}{2}\left(\frac{1}{a}\arctan\left(\frac{u}{a}\right)\right)\Big|_0^4$. (I used a complete u-sub to get the bounds). So, $\frac{1}{8}\arctan\left(\frac{x^2}{4}\right)\Big|_0^2=\frac{1}{8}\left(\arctan 1-\arctan 0\right)=\frac{\pi}{32}$, so A.

9. We should split this into 2 integrals that will both require u-substitutions. So, $\int \cot 2\theta d\theta + \int \cos\theta e^{\sin\theta} d\theta$. For the first, let $u = 2\theta$, $du = 2d\theta$ and for the second, let $u = \sin\theta$, $du = \cos\theta d\theta$. This means $\frac{1}{2}\int \cot u du + \int e^u du$, so $\frac{1}{2}\ln|\sin u| + e^u + C$ or $\frac{1}{2}\ln|\sin 2\theta| + e^{\sin\theta} + C$ so A.

10. This requires a u-substitution where $u = 1-2x^2$, $du = -4xdx$. After pulling the constant 5 out front, we get $-\frac{5}{4}\int_0^1 u^3 du$. So $-\frac{5}{4}\left(\frac{u^4}{4}\right)\Big|_1^{-1} = -\frac{5}{16}\left(1-2x^2\right)^4\Big|_0^1 = \left(-\frac{5}{16} - \left(-\frac{5}{16}\right)\right) = 0$ or C.

11. Separate the numerator to get $\int_0^4 \frac{2x}{x^2+16}dx - \int_0^4 \frac{3}{x^2+16}dx$. The first integral is a natural log where the second is an arctan. So for the first, $u = x^2+16$, $du = 2xdx$ and for the second, $u = x$, $du = dx$, $a = 4$. So, $\int_0^4 \frac{1}{u}du - 3\int_0^4 \frac{1}{a^2+u^2}du = \ln\left|x^2+16\right| - \frac{3}{4}\arctan\left(\frac{x}{4}\right)\Big|_0^4 = \left(\ln 32 - \frac{3}{4}\arctan 1\right) - \left(\ln 16 - \frac{3}{4}\arctan 0\right)$. This can simplify to $\ln 32 - \ln 16 - \frac{3}{4}\left(\frac{\pi}{4}\right) = \ln\left(\frac{32}{16}\right) - \frac{3\pi}{16} = \ln 2 - \frac{3\pi}{16}$ or A.

12. Given the initial condition that at $t = 0, x = 5$ and velocity, we must integrate to find position. So, $x(3) = x(0) + \int_0^3 v(t)dt$ or $x(3) = 5 + \int_0^3 \frac{1}{1+t}dt$. Using a u-substitution with $u = 1+t$, $du = dt$, we get $x(3) = 5 + \int_0^3 \frac{1}{u}du$ and $x(3) = 5 + \ln|1+t|\Big|_0^3 = 5 + \ln 4 - \ln 1 = 5 + \ln 4$ or E.

13. Since the bounds go from $[0,1]$, we know that the width is $\Delta x = \frac{1}{n}$. Also, we can see that the function starts at $a = 0$, so $\lim_{n\to\infty}\sum_{k=1}^{n} f(a+k\Delta x)(\Delta x) = \lim_{n\to\infty}\sum_{k=1}^{n} f\left(0+\frac{k}{n}\right)\left(\frac{1}{n}\right) = \lim_{n\to\infty}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)\left(\frac{1}{n}\right)$. Since $f(x) = \sin(3+4x)$, we can determine that $f\left(\frac{k}{n}\right) = \sin\left(3+\frac{4k}{n}\right)$ and so $\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(3+\frac{4k}{n}\right)\left(\frac{1}{n}\right)$ or A.

14. Total distance travelled is found by $\int |v(t)|dt = \int_0^4 \left|7e^{-t^2}\right|dt$, which can be found on the calculator, so B.

15. Separate the numerator to get $\int \frac{2x}{\sqrt{4-x^2}}dx + \int \frac{5}{\sqrt{4-x^2}}dx$. The first integral is a power rule where the second is an arcsin. So for the first, $u = 4-x^2$, $du = -2xdx$ and for the second, $u = x$, $du = dx$, $a = 2$. So, $-\int u^{-\frac{1}{2}}du + 5\int \frac{1}{\sqrt{a^2-u^2}}du = -2u^{\frac{1}{2}} + 5\arcsin\left(\frac{u}{a}\right) + C = -2\sqrt{4-x^2} + 5\arcsin\left(\frac{x}{2}\right) + C$. So, D.

16. Given the rate of change of position and the position at $t = 2$, we should integrate and add the initial condition. So, $y(2) = y(5) + \int_5^2 \frac{2}{t^2+9}dt$ or $\int_2^5 \frac{2}{t^2+9}dt = y(5) - y(2)$. Both show that $y(2) = y(5) - \int_2^5 \frac{2}{t^2+9}dt$ and knowing $y(5) = 2$, we can use the calculator to give us C.

17. Given the rate, we can integrate to find the total amount. So $\int_0^{10} R(t)dt = 2161.66$ or C.

18. To find $f(x)$ from $f''(x)$, we need to integrate twice. So integrating once gives us $\int f''(x)dx = \int 3x^2 dx = x^3 + C$. Knowing $f'(1) = 0$, we get $0 = (1)^3 + C$, $0 = 1 + C$, $C = -1$. So, $f'(x) = x^3 - 1$. Integrating a second time gives us $f(x) = \int (x^3 - 1)dx = \frac{1}{4}x^4 - x + C$. Given $f(2) = 0$, we get $0 = \frac{1}{4}(2)^4 - 2 + C$, $0 = 2 + C$, $C = -2$, so $f(x) = \frac{1}{4}x^4 - x - 2$ or D.

19. Average value is $\frac{1}{\frac{\pi}{6} - \left(-\frac{\pi}{6}\right)} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (\arcsin 2\theta^2)d\theta = 0.187$, so B.

20. Given the rate, we can integrate to find the total amount. So $\int_0^5 \frac{dV}{dt}dt = 14.53$ or C.

FR1. a) $\int_0^{18} L(t)dt = 1658$

b) Set $L(t) = 150$ which occurs when $t = 12.42831$, 16.12166, so $L(t) \geq 150$ from $[12.428, 16.122]$.

The average value over this interval would be $\frac{1}{16.12166 - 12.42831} \int_{12.42831}^{16.12166} L(t)dt = 199.426$.

c) Since there are 500 oncoming cars in every 2 hour interval, in order for the product to be 200,000, the total number of cars turning left must be 400. Given $L(t)$ is the rate, we need to find a 2 hour time interval in which $\int_a^b L(t)dt \geq 400$. Looking at a graph, the maximum rates occur around 13 to 15, giving this the best chance of being above 400. So, $\int_{13}^{15} L(t)dt = 431.931 \geq 400$, so yes.

FR2. a) A REA uses area of rectangles, so $\int_0^8 S(t)dt \approx 2(1.6) + 1(2.1) + 1.5(2.4) + 2(2.8) + 1.5(2.2) = 17.8 in.$

b) This is set up to be the average value of $S(t)$ from $[0,8]$, so this is the average snowfall rate from $[0,8]$. Approximating this is

$$\frac{1}{8}\int_0^8 S(t)dt \approx \frac{1}{8}\left[\frac{1}{2}(2)(0+1.6)+\frac{1}{2}(1)(1.6+2.1)+\frac{1}{2}(1.5)(2.1+2.4)+\frac{1}{2}(2)(2.4+2.8)+\frac{1}{2}(1.5)(2.8+2.2)\right]$$

, which gives us $1.972\, in/hr$.

c) $\int_0^{12}\left(1.3\sin\left(\frac{\pi}{11}t-0.34\right)+1.5\right)dt = 26.836$

d) $A(t)=\int_0^3 P(t)dt+\int_3^t[P(x)-R(x)]dx$ or $A(t)=\int_0^t P(x)dx-\int_3^t R(x)dx$ or $A(t)=4.833+\int_3^t[P(x)-R(x)]dx$.

e) An absolute maximum occurs at critical points, which is when $A'(t)=0$ *or und.*, or at the endpoints, which is $t=0,12$. Using the 2nd FTC, we get $A'(t)=P(t)-R(t)=0$ or when $P(t)=R(t)$. This occurs at $t=8.1074108$. So, we have to substitute $t=8.1074108$, 12 back into $A(t)$. We know the amount of snow at 0 is 0. So $A(8.10741)=\int_0^3 P(t)dt+\int_3^{8.10741}[P(t)-R(t)]dt=8.425$ and

$A(12)=\int_0^3 P(t)dt+\int_3^{12}[P(t)-R(t)]dt=$ 3.157. So the max amount of snow is $A(8.10741)=8.425$.

FR3. a) $s(0)=2(0)+\int_2^0 f(x)dx$ or $s(0)=-\int_0^2 f(x)dx=-6$. $s(3)=2(3)+\int_2^3 f(x)dx$ or $s(3)=6+\int_2^3 f(x)dx=\frac{15}{2}$.

b) Given position, we must differentiate to find velocity. So using the 2nd FTC, $v(t)=s'(t)=2+f(t)$. So $v(4)=2+f(4)$ and given a graph of $f(x)$, we just use the $y-$value. So $v(4)=2+-1=1$.

c) Given velocity, we can differentiate to find acceleration. So, $a(t)=v'(t)=f'(t)$ and this is 0 when $f(t)$ has a slope of 0, or a horizontal tangent line. So, $0<t<2, t=4$.

d) The smallest value means that we are finding the absolute minimum of s. This means we look at the endpoints, which are 0 and 7, and the critical points, which are when $s'(t)=2+f(t)=0$ or $f(t)=-2$. The graph of $f(t)$ never reaches -2, so it could only be the endpoints. $s(0)=-6$ and $s(7)=2(7)+\int_2^7 f(x)dx=14+\frac{3}{2}-\frac{\pi}{2}+1$. So $s(0)<s(7)$ and the minimum is at $t=0$.

e) Distance travelled is $\int_0^7 |2+f(x)|dx$. Since there is no function to integrate using rules, we have to use the graphical approach. The $2+$ means the graph is shifted up 2. So, making a new graph would give an area of $22.5-0.5\pi$.

f) The particle is decelerating when $a(t)=f'(t)<0$. So this is where the graph of $f(t)$ is decreasing. So, $(2,3),(3,4)$.