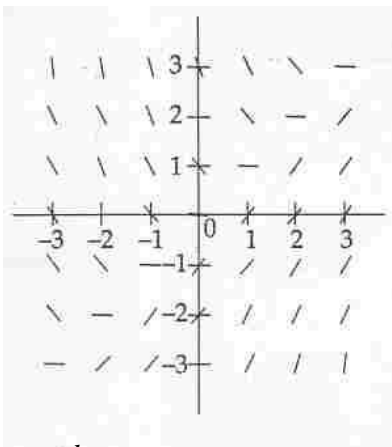


1)



The slope field shown above corresponds to which of the following differential equation?

- a) $\frac{dy}{dx} = \frac{x}{y}$ b) $\frac{dy}{dx} = \frac{y}{x}$ c) $\frac{dy}{dx} = 2 - x$
 d) $\frac{dy}{dx} = x - y$ e) $\frac{dy}{dx} = x + y$

2) If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

- a) $e^{\tan x} + 4$ b) $e^{\tan x} + 5$ c) $5e^{\tan x}$ d) $\tan x + 5$ e) $\tan x + 5e^x$

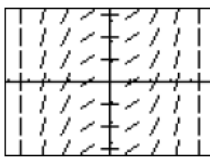
3) Given $f(x) = 3e^{x^2-1} + 1$, find $f(1.2)$ using the equation of the tangent line at $x = 1$.

- a) 2.2 b) 2.8 c) 3.88 d) 4.2 e) 5.2

4) Let $f(1) = -1$ and $y = f(x)$ for the given differential equation $\frac{dy}{dx} = 2y^2$. Then $f(2) =$

- a) $-\frac{2}{3}$ b) $-\frac{1}{3}$ c) 0 d) $\frac{1}{3}$ e) $\frac{2}{3}$

5)



The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

- (A) $y = \sin x$ (B) $y = \cos x$ (C) $y = x^2$ (D) $y = \frac{1}{6}x^3$ (E) $y = \ln x$

6) Which of the following could be a solution to the differential equation $\frac{dw}{dx} = \frac{w^2}{x^2 + 1}$?

- a) $w = e^{\arctan x}$ b) $w = \sqrt{e^{\arctan x}}$ c) $w = \frac{3e^x}{\arctan x}$ d) $w = \frac{-1}{\arctan x} + 3$ e) $w = \frac{-1}{\arctan x + 3}$

- 7) The rate of change of petroleum entering a barrel is proportional to the product of the square root of the amount of petroleum present in the barrel and the amount of empty space in the barrel left to be filled. If p denotes the amount of petroleum present in the barrel and N denotes the volume of the barrel, which of the following differential equations could be used to model this situation with respect to time t , where k is a positive constant?

a) $\frac{dp}{dt} = k\sqrt{p}$ b) $\frac{dp}{dt} = k\sqrt{p}(N - p)$ c) $\frac{dp}{dt} = k\sqrt{p}(p - N)$
d) $\frac{dp}{dt} = k\sqrt{t}(N - t)$ e) $\frac{dp}{dt} = k\sqrt{p}(N - \sqrt{p})$

- 8) If $\frac{dy}{dx} = \frac{x^3 + 1}{y}$ and $y = 2$ when $x = 1$, then, when $x = 2$, $y =$

a) $\sqrt{\frac{27}{2}}$ b) $\sqrt{\frac{27}{8}}$ c) $\pm\sqrt{\frac{27}{8}}$ d) $\pm\frac{3}{2}$ e) $\pm\sqrt{\frac{27}{2}}$

- 9) Given $\frac{dy}{dx} = 2xy$, find $y = f(x)$ given $f(\sqrt{\ln 2}) = 6$

a) $y = 3e^{x^2}$ b) $y = e^{x^2} + 4$ c) $y = e^{x^2} + 6$ d) $y = x^2 + 4$ e) $y = e^{x^2} + \ln 3$

- 10) Given the logistics differential equation $\frac{dP}{dt} = \left(\frac{3P}{20}\right) - \left(\frac{P^2}{1200}\right)$, find the carrying capacity.

a) 15 b) 20 c) 60 d) 180 e) 1200

- 11) Find the value of $\frac{d^2y}{dx^2}$ given $\frac{dy}{dx} = x^2(y - 5)$ at $(2, 4)$.

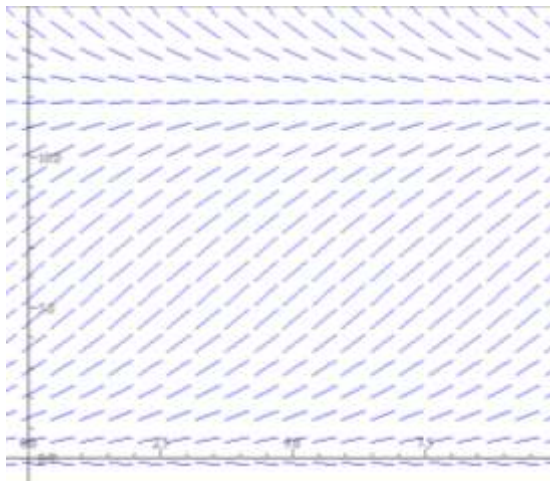
a) -20 b) -16 c) 0 d) 4 e) 8

- 12) At each point (x, y) on a certain curve, the slope of the curve is the product of three times the square of x and y . If the curve contains the point $(0, 8)$, then its equation is...
- a) $y = 8e^{x^3}$ b) $y = x^3 + 8$ c) $y = e^{x^3} + 7$ d) $y = \ln(x+1) + 8$ e) $y^2 = x^3 + 8$
- 13) Given that $y(2) = 3$ and $\frac{dy}{dx} = 1.7e^{2x-5}\sqrt{y}$, what is the approximation for $y(2.6)$ if Euler's method is used with 3 steps of equal size, starting at $x = 2$?
- a) 3.217 b) 3.325 c) 3.551 d) 3.948 e) 4.076
- 14) In 1990 the population of a town was 21,000 and in 2000 it was 20,000. Assuming the population changes continuously at a rate proportional to the existing population, estimate the population in the year 2020.
- a) 17,619 b) 18,000 c) 19,048 d) 18,141 e) None of these
- 15) If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$, when $x = \frac{\pi}{2}$ what is the value of y when $x = 0$?
- a) -1 b) $-\frac{1}{3}$ c) 0 d) $\frac{1}{3}$ e) 1
- 16) Given $\frac{dy}{dx} = \frac{y+4}{\sqrt{4-x^2}}$ and $y = f(x)$, find $f(x)$.
- a) $Ce^{\arcsin\left(\frac{x}{2}\right)} - 4$ b) $Ce^{2\arcsin\left(\frac{x}{2}\right)} - 4$ c) $Ce^{\frac{1}{2}\arcsin\left(\frac{x}{2}\right)} - 4$ d) $\arcsin\frac{x}{2} + C$ e) $Ce^{\sqrt{4-x^2}} - 4$
- 17) Let $y = f(x)$ represent the solution to the differential equation $\frac{dy}{dx} = \frac{-2\cos x + 1}{2\sin y - 1}$. Which of the following values results in vertical tangent lines to the graph of $f(x)$?
- a) $x = \frac{\pi}{6}$ b) $x = \frac{\pi}{3}$ c) $y = \frac{\pi}{6}$ d) $y = \frac{\pi}{3}$ e) $y = \frac{\pi}{2}$

CA 1) The population, P , of a culture of the bacteria *Clostridium perfringens* is found to grow at a rate proportional to the square root of its population. Let t be the time in days, with $t \geq 0$ and k be a constant of proportionality.

- Write an equation modeling the population of colonies, $P(t)$, given that $P(0) = 1600$ and the culture contains 3600 colonies after 3 days.
- How many days from the beginning will the population be 5000?
- After 10 days, the population of the culture changes to a new growth rate, which is modeled by $S(t) = 600e^{-0.12t}$. What is the total population of the culture of *Clostridium perfringens* after 14 days (from the beginning)?

NC 2) Consider the logistic differential equation $\frac{dy}{dt} = \frac{1}{12}y(24 - 2y)$. Let $y = P(t)$ be the particular solution to the differential equation with $P(0) = 3$.



- A slope field for this differential equation is given. Sketch 3 different possible solution curves through the points $(0,3)$, $(6,12)$ and $(1,14)$.
- Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $P(2)$.
- Find $\lim_{t \rightarrow \infty} P(t)$ and find the P -value in which the graph is increasing most rapidly.
- Describe the values of k , with $k \geq 0$, for which $P(0) = k$ will have 1 horizontal asymptotes and which values will have 2 horizontal asymptotes.

Answers

- 1) This slope field shows us slopes of 0 at $(-1, -1)$, $(1, 1)$, $(2, 2)$. Since we are asked to find the differential equation, we find which gives us 0 when plugging in the coordinates. So **D**.
- 2) To find y given $\frac{dy}{dx}$, we must separate the variables and integrate. So, $\frac{dy}{y} = \sec^2 x dx$ and $\int \frac{dy}{y} = \int \sec^2 x dx$. This gives us $\ln|y| = \tan x + C$. We can either solve for y or find the $+C$. Solving for y gives us $y = Ce^{\tan x}$ and plugging in $(0, 5)$ is $5 = Ce^{\tan 0}$, $5 = C$ or $y = 5e^{\tan x}$, so **C**.
- 3) The tangent line requires a point $f(1)$ and a slope $f'(1)$. We find $f(1) = 4$ and $f' = 6xe^{x^2-1}$, so $f'(1) = 6$. Therefore the tangent line is $y - 4 = 6(x - 1)$. So, approximating $f(1.2)$ is $y - 4 = 6(1.2 - 1)$, $y = 6(0.2) + 4$, $y = 5.2$ so **E**.
- 4) To find y given $\frac{dy}{dx}$, we must separate the variables and integrate. So, $\frac{dy}{y^2} = 2dx$ and $\int \frac{dy}{y^2} = \int 2dx$. This gives us $-\frac{1}{y} = 2x + C$. Solving for C by plugging in $(1, -1)$ gives us $1 = 2 + C$, $C = -1$. Solving for y gives us $y = -\frac{1}{2x-1}$. So $f(2) = -\frac{1}{3}$ or **B**.
- 5) Since we are asked to find the solution from a slope field, we should look at what graph the line segments are making. The graph doesn't look oscillating, so that isn't A or B. The graph appears cubic, so **D**.
- 6) To find a possible w given $\frac{dw}{dx}$, we must separate the variables and integrate. So, $\frac{dw}{w^2} = \frac{dx}{x^2 + 1}$ and $\int \frac{dw}{w^2} = \int \frac{dx}{x^2 + 1}$. This gives us $-\frac{1}{w} = \frac{1}{1} \arctan x + C$. Solving for w gives $w = -\frac{1}{\arctan x + C}$ so **E**.
- 7) To write the differential equation, we need to read all of the problem and understand what each variable represents. "Proportional" means there is a k . The "square root of the amount of petroleum present in the barrel" means \sqrt{p} and "the amount of empty space in the barrel left to be filled" means $N - p$ since the empty space would be the total volume N minus the amount present p . So **B**.

8) To find y given $\frac{dy}{dx}$, we must separate the variables and integrate. So, $ydy = (x^3 + 1)dx$ and $\int ydy = \int (x^3 + 1)dx$. This gives us $\frac{1}{2}y^2 = \frac{1}{4}x^4 + x + C$. Solving for C by plugging in $(1, 2)$ gives us $2 = \frac{5}{4} + C$, $C = \frac{3}{4}$. Solving for y gives us $y = \sqrt{\frac{1}{2}x^4 + 2x + \frac{3}{2}}$ (it is not \pm because the given y is positive). So $f(2) = \sqrt{8 + 4 + \frac{3}{2}}$ or **A**.

9) To find y given $\frac{dy}{dx}$, we must separate the variables and integrate. So, $\frac{dy}{y} = 2xdx$ and $\int \frac{dy}{y} = \int 2xdx$. This gives us $\ln|y| = x^2 + C$. Solving for y gives us $y = Ce^{x^2}$. Solving for C by plugging in $f(\sqrt{\ln 2}) = 6$ gives us $6 = Ce^{(\sqrt{\ln 2})^2}$, $6 = Ce^{\ln 2}$, $6 = 2C$, $C = 3$, so $y = 3e^{x^2}$ or **A**.

10) To write the logistic solution, we need to get “1-”. This means we should factor out a $\frac{3P}{20}$, giving us $\frac{dP}{dt} = \frac{3}{20}P\left(1 - \frac{P}{180}\right)$. The carrying capacity would therefore be the denominator, so **D**.

11) To find $\frac{d^2y}{dx^2}$, we must take the derivative of $\frac{dy}{dx} = x^2(y - 5)$, with respect to x , meaning that we must use implicit differentiation. So, using the product rule, we get $\frac{d^2y}{dx^2} = 2x(y - 5) + x^2\left(\frac{dy}{dx}\right)$ which is $2x(y - 5) + x^2(x^2(y - 5))$. So, at $(2, 4)$, $\frac{d^2y}{dx^2} = 2(2)(-1) + (4)(4)(-1) = -20$ or **A**.

12) Given the slope, to find the equation means we must integrate. So, $\frac{dy}{dx} = 3x^2y$ or $\frac{dy}{y} = 3x^2dx$. Integrating gives us $\ln|y| = x^3 + C$ and solving for y gives us $y = Ce^{x^3}$. Finding C using $(0, 8)$ gives us $8 = Ce^0$, $C = 8$ so $y = 8e^{x^3}$ or **A**.

13)

x	y	dx	$dy = 1.7e^{2x-5}\sqrt{y}dx$	$(x + dx, y + dy)$
2	3	0.2	$dy = 1.7e^{2(2)-5}\sqrt{3}(0.2) = 0.2166432$	$(2.2, 3.2166432)$
2.2	3.2166432	0.2	$dy = 1.7e^{2(2.2)-5}\sqrt{3.2166432}(0.2) = 0.3346599$	$(2.4, 3.5513031)$
2.4	3.5513031	0.2	$dy = 1.7e^{2(2.4)-5}\sqrt{3.5513031}(0.2) = 0.5245826$	$(2.6, 4.0758857)$

So **E**.

14) "A rate proportional to the existing population" means that the population follows $P = Ce^{kt}$ with ordered pairs at $(0, 21000)$ and $(10, 20000)$. So $21000 = Ce^{k \cdot 0}$, $21000 = C$ which gives us

$$P = 21000e^{kt}. \text{ To find } k, 20000 = 21000e^{k(10)}, \ln\left(\frac{20}{21}\right) = 10k, k = -0.004879, \text{ so}$$

$$P = 21000e^{-0.004879t}. \text{ The population in the year 2020 } (t = 30) \text{ is } P = 21000e^{-0.004879(30)} = 18141 \text{ so } \mathbf{D}.$$

15) To find y given $\frac{dy}{dx}$, we must separate the variables and integrate. So, $dy = \sin x \cos^2 x dx$ and

$$\int dy = \int \sin x \cos^2 x dx. \text{ This requires a u-substitution, so } u = \cos x, du = -\sin x dx \text{ meaning we need a}$$

$$-1. \text{ So } \int dy = -\int u^2 du, \text{ giving us } y = -\frac{1}{3}\cos^3 x + C. \text{ Solving for } C \text{ by plugging in } f\left(\frac{\pi}{2}\right) = 0 \text{ gives}$$

$$\text{us } 0 = 0 + C, C = 0, \text{ so } y = -\frac{1}{3}\cos^3 x. \text{ Therefore } y(0) = -\frac{1}{3}\cos^3(0) = -\frac{1}{3} \text{ or } \mathbf{B}.$$

16) To find y given $\frac{dy}{dx}$, we must separate the variables and integrate. So, $\frac{dy}{y+4} = \frac{dx}{\sqrt{4-x^2}}$ and

$$\int \frac{dy}{y+4} = \int \frac{dx}{\sqrt{4-x^2}}. \text{ The left requires a u-substitution, so } u = y+4, du = dy \text{ and the right is set up}$$

$$\text{to be } \arcsin x \text{ with } u = x, du = dx, a = 2. \text{ So } \int \frac{du}{u} = \int \frac{du}{\sqrt{a^2-u^2}}, \text{ giving us } \ln|y+4| = \arcsin \frac{x}{2} + C.$$

$$\text{Solving for } y \text{ gives us } y+4 = Ce^{\arcsin \frac{x}{2}}, y = Ce^{\arcsin \frac{x}{2}} - 4, \text{ so } \mathbf{A}.$$

17) Given the derivative, a vertical tangent line exists when the denominator is 0. So

$$2 \sin y - 1 = 0, \sin y = \frac{1}{2}, y = \frac{\pi}{6}, \text{ so } \mathbf{C}.$$

CA 1) a) To find P given $\frac{dP}{dt}$, we must separate the variables and integrate. $\int \frac{dP}{\sqrt{P}} = \int k dt$. This gives

$$\text{us } 2\sqrt{P} = kt + C \text{ with points at } (0, 1600) \text{ and } (3, 3600). \text{ So } 2\sqrt{1600} = 0 + C, C = 80. \text{ Then,}$$

$$2\sqrt{3600} = 3k + 80, 120 = 3k + 80, k = \frac{40}{3}. \text{ So } 2\sqrt{P} = \frac{40}{3}t + 80 \text{ or } \mathbf{P = \left(\frac{20}{3}t + 40\right)^2}.$$

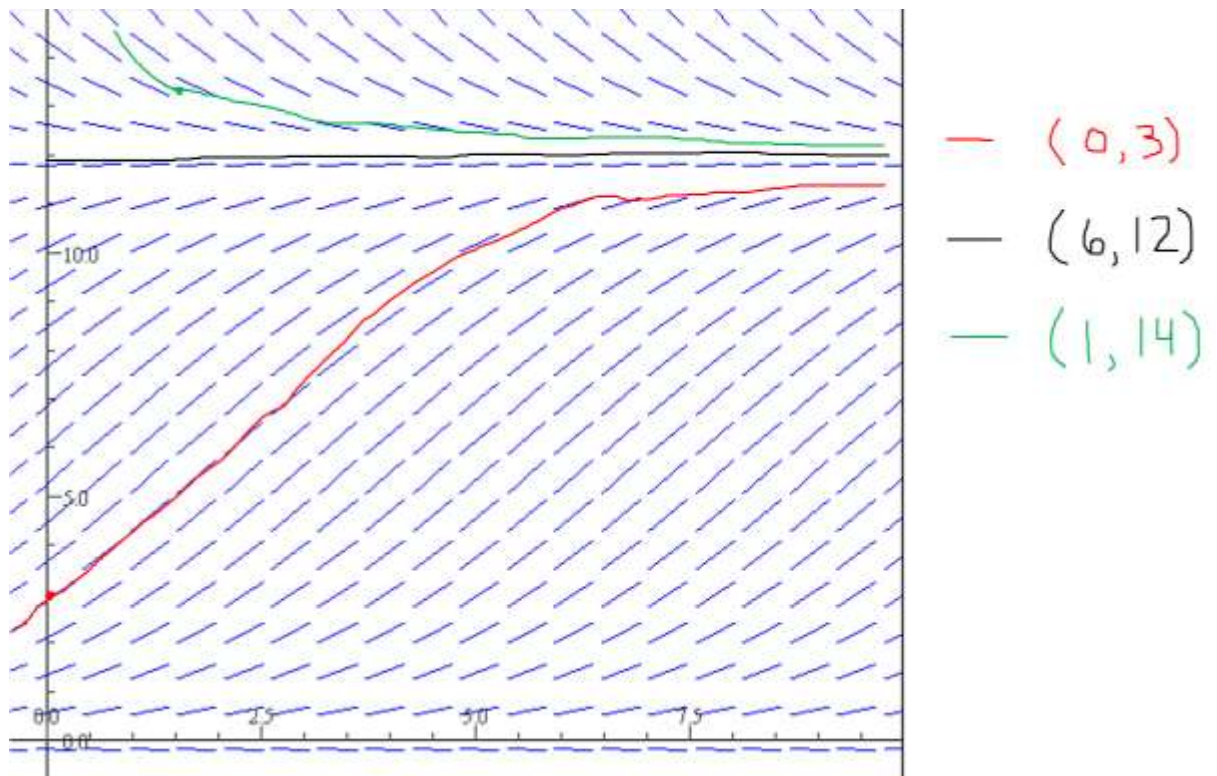
$$\text{b) } 5000 = \left(\frac{20}{3}t + 40\right)^2 \text{ or } \sqrt{5000} = \frac{20}{3}t + 40, t = \frac{3}{20}(\sqrt{5000} - 40) \text{ or } \mathbf{t = 4.607 \text{ days.}}$$

$$\text{c) We can use } P \text{ to find the population until 10 days. So } P(10) = \left(\frac{20}{3}(10) + 40\right)^2 = 11377.777.$$

$$\text{Since } S \text{ is a rate, we integrate and add on } P(10). \text{ So } P(14) = P(10) + \int_{10}^{14} S(t) dt \text{ or } \mathbf{11951.878}.$$

NC 2)

a)



b)

t	y	dt	$dy = \frac{1}{12} y(24 - 2y)dt$	$(t + dt, y + dy)$
0	3	1	$dy = \frac{1}{12} (3)(24 - 6)(1) = 4.5$	(1, 7.5)
1	7.5	1	$dy = \frac{1}{12} (7.5)(24 - 15)(1) = 5.625$	(2, 13.125)

c) $\frac{dy}{dt} = \frac{1}{12} y(24 - 2y)$ can be rewritten into the differential equation $\frac{dy}{dt} = 2y \left(1 - \frac{y}{12} \right)$. Since this is logistic, the $\lim_{t \rightarrow \infty} P(t)$ is the carrying capacity, which means that $L = 12$. The function is increasing the fastest at half its' carrying capacity, so $P = 6$.

d) If $P(0)$ is in between 0 and the carrying capacity, the solution will have 2 horizontal asymptotes. If $P(0)$ is above the carrying capacity, then there will be 1 horizontal asymptote. At 0 and the carrying capacity, the curve will be just a horizontal line. So:

2 Horizontal Asymptotes – $0 < k < 12$

1 Horizontal Asymptote – $k = 0, k \geq 12$