Practice Chapter 4 Quiz - Noncalculator

1. Find the absolute minimum and maximum value of the function $f(x) = x^3 - 2x^2 - 4x + 16$ on $[1, 4]$.

2. Find the absolute minimum and maximum value of the function $f(x) = \arctan(x^2)$ on $[-1, 1]$.

3. Which of the following functions satisfy the conditions of Rolle's Theorem on the interval $[0, 2]$?

   I. $f(x) = x^2 - 2x$ II. $f(x) = \dfrac{1}{|x-1|}$ III. $f(x) = |x-1|$

4. Find the value c guaranteed by Rolle's Theorem, if it applies, for $f(x) = \ln|x^2 + 3|$ on $[-3, 3]$.

5. Find the value of c that satisfies the Mean Value Theorem for $f(x) = 2\cos(\pi x)$ from $\left(0, \dfrac{1}{2}\right)$.

6. Find the intervals in which the function $f(x) = -x^3 - 6x^2 + 36x$ is increasing/decreasing and identify the $x-$coordinates of the relative extrema.

7. Find the intervals, from $[0, 2\pi]$ in which the function $f(x) = \sqrt{3 + \sin x}$ is increasing/decreasing and identify the $x-$coordinates of the relative extrema.

8. The function f is continuous and differentiable on the closed interval $[3, 7]$. The table below gives selected values of f on this interval. Which of the following statements must be true?

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

   I. The maximum value of f on $[3, 7]$ is 20. II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$.
   III. $f'(x) < 0$ for $4 < x < 5$. IV. There is a relative minimum on $[3, 7]$.

9. A test plane flies in a straight line with positive velocity v(t), in miles per minute, where v is a differentiable function of t. Selected values of v(t) for [0,40] are shown in the table.

t(minutes)	0	5	10	15	20	25	30	35	40
v(t) (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

   Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval (0,40)?

10. Find the $x-$value for the function which the slope of the tangent line is equal to the slope of the secant line from $f(x) = \dfrac{3x+6}{x+1}$ from $[0, 2]$.

Solutions

1. Absolute max at $(4,32)$ and absolute min at $(2,8)$

$f(x)=x^3-2x^2-4x+16 \quad [1,4]$

Candidates are endpoints (1 & 4) and critical values

Critical values: $f'=3x^2-4x-4=0$ * f' isn't und. *

$(3x+2)(x-2)=0 \quad x=-2/3, 2$

out of interval!

$f(1)=1-2-4+16=11$

$f(2)=8-8-8+16=8$

$f(4)=64-32-16+16=32$

$\boxed{\text{Max: } 32 \quad \text{Min: } 8}$

2. Absolute max at $\left(-1,\frac{\pi}{4}\right)$ and $\left(1,\frac{\pi}{4}\right)$ and absolute min at $(0,0)$

$f=\arctan(x^2) \quad [-1,1]$

$f'=\frac{1}{1+(x^2)^2}\cdot 2x=\frac{2x}{1+x^4}$. f' is undefined if $1+x^4=0$

$x^4=-1$ ← NOPE

$f'=0=\frac{2x}{1+x^4}$, $0=2x$, $x=0$

only critical value

$f(-1)=\arctan 1=\pi/4$

$f(0)=\arctan 0=0$

$f(1)=\arctan 1=\pi/4$

$\boxed{\text{Max: } \pi/4 \quad \text{Min: } 0}$

3. I. Yes, II. Not differentiable at $x=1$, III. Not differentiable at $x=1$

I. Rolle's theorem applies if f is differentiable and $f(b)=f(a)$. $f'=2x-2$ which is continuous so f is differentiable. $f(2)=0=f(0)$ so Rolle's Theorem $\boxed{\text{applies}}$

II. f isn't continuous at $x=1$, so f isn't differentiable, so Rolle's $\boxed{\text{does not apply}}$

III. f isn't differentiable at $x=1$ because $\lim_{x\to 1^-} f'(x) \neq \lim_{x\to 1^+} f'(x)$ so Rolle's $\boxed{\text{does not apply}}$

4. $x=0$

$f=\ln|x^2+3|$ on $[-3,3]$. Rolle's applies if $f(3)=f(-3)$ and f is differentiable on $(-3,3)$.

$f(3)=\ln 12=f(-3)$ ✓. If you can't see if f is differentiable, check if f' is continuous...

$f'=\frac{1}{x^2+3}\cdot 2x=\frac{2x}{x^2+3}$. Yep, f' will never be und.

$\therefore$ Rolle's Theorem applies!

So, $f'=0=\frac{2x}{x^2+3}$, $0=2x$ $\boxed{x=0}$

5. $x=\frac{1}{\pi}\arcsin\left(\frac{2}{\pi}\right)$

$f=2\cos(\pi x)$ on $(0,\frac{1}{2})$. Since f is differentiable, MVT applies! So, $f'(c)=\frac{f(b)-f(a)}{b-a}$

$f'=-2\sin(\pi x)\cdot\pi$ So, $-2\pi\sin(\pi x)=\frac{0-2}{\frac{1}{2}-0}=-4$

$f(\frac{1}{2})=2\cos\frac{\pi}{2}=0$

$f(0)=2\cos 0=2$

$\sin\pi x=\frac{2}{\pi}$

$\pi x=\arcsin(\frac{2}{\pi})$

$\boxed{x=\frac{\arcsin(\frac{2}{\pi})}{\pi}}$

6. Increasing: $(-6,2)$, Decreasing: $(-\infty,-6)(2,\infty)$, Relative Min: $x=-6$, Relative Max: $x=2$

$f'=-3x^2-12x+36$. critical points (f' is 0 / und)

$f'=0=-3(x^2+4x-12)$

$f'=0=-3(x+6)(x-2)$ $x=2,-6$

f': $-$ | $+$ | $-$

-6, 2

(-10) (0) (4)

$f'(-10)=-3\cdot-\cdot-=-$

$f'(0)=-3\cdot+\cdot-=+$

$f'(4)=-3\cdot+\cdot+=-$

f increases $\rightarrow f'>0$ $(-6,2)$

f decreases $\rightarrow f'<0$ $(-\infty,-6)$ + $(2,\infty)$

f has rel max when f' changes from pos to neg $x=2$

f has rel min when f' changes from neg to pos $x=-6$

7. Increasing: $\left(0,\frac{\pi}{2}\right),\left(\frac{3\pi}{2},2\pi\right)$, Decreasing: $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$, Relative Min: $x=\frac{3\pi}{2}$, Relative Max: $x=\frac{\pi}{2}$

$f' = \frac{1}{2}(3+\sin x)^{-1/2}\cdot\cos x = \frac{\cos x}{2\sqrt{3+\sin x}}$

only goes $[-1,1]$

f' is undefined if $2\sqrt{3+\sin x}=0$, $3+\sin x=0$, $\sin x \neq -3$

$f'=0$ when $\cos x=0$, so $x=\pi/2, 3\pi/2$

f': + , − , + at (0), $\pi/2$, (π), $3\pi/2$, (2π)

Inc: $f'>0$ $(0,\pi/2),(3\pi/2,2\pi)$
Dec: $f'<0$ $(\pi/2,3\pi/2)$
Rel Max: $x=\pi/2$
Rel Min: $x=3\pi/2$

8. II and IV

I. Not necessarily true since we don't know what f does in between. so No

II. Since f is differentiable and $f(3)=f(7)$, Rolle's Theorem applies so $\boxed{\text{TRUE}}$

III. $f'<0$ means f decreases during the <u>whole interval</u>, but we don't know what's happening so f may increase in there.

IV. f changes from decreasing to increasing so it must happen $\boxed{\text{TRUE}}$

9. 2 times as guaranteed by Rolle's Theorem, between $(0,15)$ and $(25,30)$

Since we are given $v(t)$ and asked if $a(t)$ is over 0, and $a(t)=v'(t)$, this is Rolle's theorem. It's given that $v(t)$ is differentiable, so we need to know how many times $v(a)=v(b)$. This happens $\boxed{\text{twice}}$

10. $x=-1+\sqrt{3}$

slope of tangent line means f'
slope of secant line means average rate of change

So, when does $f' = \frac{f(b)-f(a)}{b-a}$ on $[0,2]$ MVT!

Since f is differentiable on $(0,2)$ * $x=-1$ is outside *

$f' = \frac{(x+1)(3)-(3x+6)(1)}{(x+1)^2} = \frac{-3}{(x+1)^2} \to \frac{-3}{(x+1)^2} = \frac{-1}{1}$

(Note: $-1-\sqrt{3}$ is outside)

$\frac{f(2)-f(0)}{2-0} = \frac{4-6}{2-0} = -1$

$-3 = -1(x+1)^2$

$3=(x+1)^2$

$\pm\sqrt{3} = x+1$

$\sqrt{3} = |x+1| \Rightarrow \boxed{x=-1+\sqrt{3}}$