Calculus Prereq. Practice Test – Non Calculator

1. Evaluate the following trigonometric expressions:

a) $\sin\frac{3\pi}{4}$ b) $\cos\frac{-\pi}{6}$ c) $\sec\frac{7\pi}{3}$ d) $\cot\frac{5\pi}{6}$ e) $\csc\frac{5\pi}{3}$ f) $\tan\frac{\pi}{2}$

2. Solve the following equations for θ from $[0, 2\pi)$:

a) $2\tan^2\theta - 5 = 1$ b) $2\cos^3\theta - \cos\theta = 0$ c) $\ln(2\sec\theta - 3) = 0$

3. Solve the following equations:

a) $5x^3 - 3x^2 + 7x = 0$ b) $\dfrac{2-3e^x}{-1+5e^x} = -4$ c) $2\sqrt{3x+1} - \dfrac{5}{3\sqrt{3x+1}} = 0$

4. Graph the following functions, then find the domain and range:

a) $f(x) = \sqrt{x-5} + 2$ b) $g(x) = 2\sin x - 3$ c) $h(x) = \dfrac{1}{x+3} + 1$

5. Determine which of the following lines are perpendicular and which are parallel, if any:

Line 1: $2x + 3y = 9$ Line 2: $y + 2 = \frac{3}{2}(x-4)$ Line 3: $y = \frac{2}{3}x - 7$

6. Factor and simplify the following:

a) $\dfrac{4b^3+13b^2+3b}{2a^2-4a-30}\bullet\dfrac{10a^2-90}{b^6+27b^3}$

b) $2(x^2+1)^2(3x-4)^2+5(x^2+1)^3(3x-4)$

7. Given $a(x)=10-x^2$ and $b(x)=\sqrt{2x+1}$, determine:

a) $a(b(7))$

b) $3b(x)-\dfrac{1}{b(x)}$

c) $b(a(2t))$

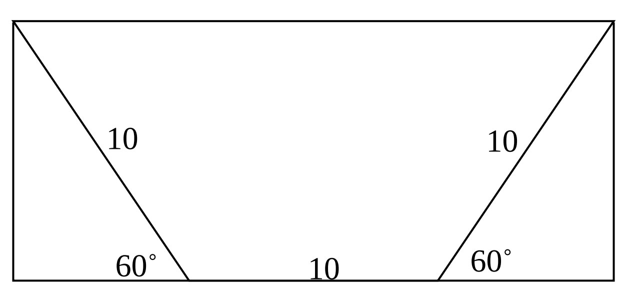
d) $\dfrac{a(x+h)-a(x)}{h}$

e) Domain of $b(x)$

f) Range of $a(x)$

8. Solve the following for x. Write your solution in interval notation. $3x^3+6x^2\geq 45x$

9. Find the area of the trapezoid within the rectangle.



Solutions

1. a) We can use the reference angle to find $\sin\frac{\pi}{4}$. Using the triangle and knowing that this angle is in the 2nd quadrant and sine is positive, we get $\frac{\sqrt{2}}{2}$.

   b) We can use the reference angle to find $\cos\frac{\pi}{6}$. Using the triangle and knowing that this angle is in the 4th quadrant and cosine is positive, we get $\frac{\sqrt{3}}{2}$.

   c) We can use the reference angle and the reciprocal of secant to find $\cos\frac{\pi}{3}$. Using the triangle and knowing that this angle is outside the unit circle, we can subtract 2π to get this is in the 1st quadrant. Since cosine is positive, we get $\frac{1}{2}$ and taking the reciprocal gives us 2.

   d) We can use the reference angle and the reciprocal of cot to find $\tan\frac{\pi}{6}$. Using the triangle and knowing this angle is in the 2nd quadrant and tan is negative, we get $-\frac{1}{\sqrt{3}}$ and the reciprocal is $-\sqrt{3}$.

   e) We can use the reference angle and the reciprocal of cosecant to find $\sin\frac{\pi}{3}$. Using the triangle and knowing this angle is in the 4th quadrant and sin is negative, we get $-\frac{\sqrt{3}}{2}$ and the reciprocal is $-\frac{2}{\sqrt{3}}$.

   f) Since this is a quadrantal angle, we can use the fact that $\tan = \frac{\sin}{\cos}$ to evaluate this. At $\frac{\pi}{2}$, we know $\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$ so the $\tan\frac{\pi}{2}$ is *und*.

2. a) Since there is a single variable, we can isolate it, giving us $\tan^2\theta = 3$ and taking the square root gives us $\tan\theta = \pm\sqrt{3}$. We will use all quadrants since this is positive and negative, so looking at the triangle gives us $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

   b) We can start off factoring out a GCF, giving us $\cos\theta\left(2\cos^2\theta - 1\right) = 0$ where each part can be set equal to 0. So, we get $\cos\theta = 0$ and $2\cos^2\theta - 1 = 0$ or $\cos\theta = \pm\frac{1}{\sqrt{2}}$ meaning this has angles in all quadrants. So, we get $\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$.

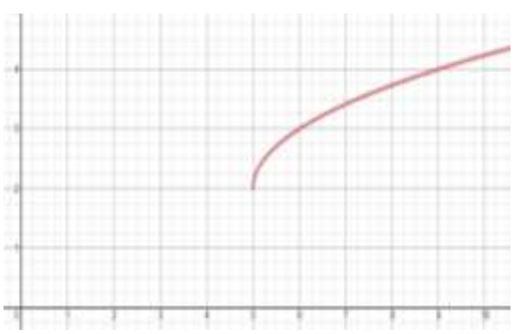
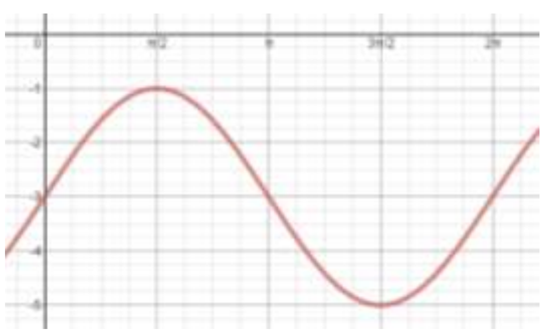
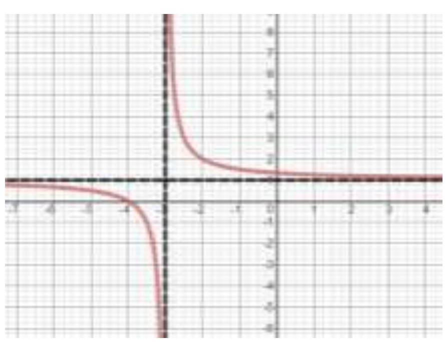
c) Since there is a single variable, we can isolate it by exponentiating both sides, giving us $2\sec\theta - 3 = e^0 = 1$ and $\sec\theta = 2$. Taking the reciprocal gives us $\cos\theta = \frac{1}{2}$ and using the triangle and knowing cosine is positive in the 1st and 4th quadrant, we get $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$.

3. a) We can start off factoring out a GCF, giving us $x(5x^2 - 3x + 7) = 0$. Looking at the quadratic inside, we see that factoring doesn't work nicely, so we use the quadratic formula. This gives us $x = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(7)}}{10}$ or $x = \frac{3 \pm \sqrt{9 - 140}}{10}$ and since this will gives us the square root of a negative, the quadratic does not contribute a solution, making the only solution coming from the GCF or $x = 0$.

b) Since there is a variable in the denominator, we should multiply it to both sides to get it out, or $2 - 3e^x = -4(-1 + 5e^x)$ or $2 - 3e^x = 4 - 20e^x$. Getting a single variable gives us $17e^x = 2$ or $e^x = \frac{2}{17}$ and taking the ln of both sides gives us $x = \ln\left(\frac{2}{17}\right)$.

c) Since there is a variable in the denominator, we should multiply it to both sides to get it out, or $3\sqrt{3x+1}\left(2\sqrt{3x+1}\right) - 5 = 0$ or $6(3x+1) - 5 = 0$. Isolating this single variable gives us $18x + 6 - 5 = 0$ or $x = -\frac{1}{18}$. Since this does involve square roots, we can check for extraneous solutions, but it's OK.

4. a) Since this has $\sqrt{x}$ as the parent function, the domain would be $[0, \infty)$ and the range would be $[0, \infty)$. The -5 on the inside would cause the graph to shift 5 to the right, making the domain $[5, \infty)$. The $+2$ on the outside shifts the graph up 2, making the range $[2, \infty)$. The graph is below on the left.

b) This would have $\sin x$ as the parent function, so the domain would be $(-\infty, \infty)$ and the range would be $[-1, 1]$. There is no transformations made to the domain, keeping the domain $(-\infty, \infty)$. The coefficient of 2 causes the graph to be stretched vertically and the -3 on the outside shifts the graph down 3, making the range $[-5, -1]$. The graph is above in the middle.

c) This would have $\frac{1}{x}$ as the parent function, so the domain would be $(-\infty, 0), (0, \infty)$ and the range would be $(-\infty, 0), (0, \infty)$. The $+3$ in the inside would cause the graph to shift left 3, making the domain $(-\infty, -3), (-3, \infty)$. The $+1$ on the outside shifts the graph up 1, making the range $(-\infty, 1), (1, \infty)$. The graph is above to the right.

5. To determine if the lines are parallel or perpendicular, we must focus on comparing the slopes. So, we must get Line 1 into slope-intercept form by solving for y. This gives us $3y = 9 - 2x$ or $y = -\frac{2}{3}x + 3$, making the slope $-\frac{2}{3}$. Line 2 is in point-slope form, meaning the slope is $\frac{3}{2}$. Line 3 is already in slope-intercept form, meaning the slope is $\frac{2}{3}$. Parallel lines have the same slope, which does not happen here and perpendicular lines have opposite reciprocal slopes, so Lines 1 & 2 are perpendicular.

6. a) We can start out by factoring out a GCF from each of the polynomials, giving us $\frac{b(4b^2 + 13b + 3)}{2(a^2 - 2a - 15)} \bullet \frac{10(a^2 - 9)}{b^3(b^3 + 27)}$. We can then factor each of the 3 quadratics and the 4th is a sum of cubes, which can all be factored to $\frac{b(4b+1)(b+3)}{2(a-5)(a+3)} \bullet \frac{10(a+3)(a-3)}{b^3(b+3)(b^2 - 3b + 9)}$. We can then cancel out all the terms in a numerator and a denominator, giving us $\frac{5(a-3)(4b+1)}{b^2(a-5)(b^2 - 3b + 9)}$.

b) We can factor out a GCF, giving us $(x^2+1)^2(3x-4)\left[2(3x-4) + 5(x^2+1)\right]$ and then we can distribute the inside, giving us $(x^2+1)^2(3x-4)\left[6x - 8 + 5x^2 + 5\right]$. Combining like terms within the brackets gives us $(x^2+1)^2(3x-4)\left[5x^2 + 6x - 3\right]$.

7. a) This can be found by first finding $b(7)$ and then plugging that into $a(x)$. So, $b(7) = \sqrt{15}$ and then $a(\sqrt{15}) = -5$.

b) Substituting $b(x) = \sqrt{2x+1}$ into this expression gives us $3\sqrt{2x+1} - \frac{1}{\sqrt{2x+1}}$ and then finding a common denominator gives us $\frac{3(2x+1)}{\sqrt{2x+1}} - \frac{1}{\sqrt{2x+1}}$. Simplifying gives us $\frac{6x+2}{\sqrt{2x+1}}$.

c) This can be found by first finding $a(2t)$ and then plugging that into $b(x)$. So, $a(2t)=10-(2t)^2$ which can simplify to $a(2t)=10-4t^2$ and then $b(10-4t^2)=\sqrt{2(10-4t^2)+1}$ or $\sqrt{20-8t^2+1}=\sqrt{21-8t^2}$.

d) Substituting $a(x)=10-x^2$ into this expression gives us $\dfrac{10-(x+h)^2-(10-x^2)}{h}$. Multiplying the squared out and distributing the negative gives us $\dfrac{10-x^2-2xh-h^2-10+x^2}{h}$. Simplifying gives us $\dfrac{-2xh-h^2}{h}$ and since every term has an h, we can cancel one out, giving us $-2x-h$.

e) Since we have a square root, we want to make sure $2x+1\geq 0$ or $\left[-\frac{1}{2},\infty\right)$.

f) Knowing that this is a parabola, we know the range of the parent function would be $[0,\infty)$. This is then shifted up 10 and reflected about the $x-$axis, making the range $(-\infty,10]$.

8. We can start off by setting this "equal" to 0 and factoring out a GCF, giving us $3x(x^2+2x-15)\geq 0$. Factoring this quadratic then gives us $3x(x+5)(x-3)\geq 0$ and finding the zeros gives us $x=-5,0,3$. We can then create a number line and plug in values in between the zeros to determine whether the function is positive or negative in between the intervals. We then want the value to be positive and will include the endpoints, since this is greater than or equal to 0, giving us $[-5,0],[3,\infty)$.

9. The area of the trapezoid could be found by finding the dimensions of the trapezoid itself or by subtracting the area of the two triangles from the rectangle. Since the triangles are 30-60-90 with a hypotenuse of 10, then the horizontal leg is 5 and the vertical leg is $5\sqrt{3}$. This means the height of the rectangle and the base of the trapezoid are both $5\sqrt{3}$. The heights of the trapezoid (the 2 parallel sides) are 10, which is given, and 20, which is found by adding the two legs of the triangles of 5 each with the given 10. So, the area of the trapezoid is $\frac{1}{2}(5\sqrt{3})(10+20)$ or $75\sqrt{3}$.