Name ________________________________ **Problem Set 4**

"You're gonna get weekly problem sets. They are such a great thing so please do them! If you have trouble procrastinating or not doing them weekly, find a friend to review answers with. Some weeks I found myself doing the problem sets on Sunday night and let me tell you the oceans of tears I cried out in frustration." A 2019 Nerd

FR1. 1980 – AB 7 **(No Calculator)**

Let p and q be real numbers and let $f(x)$ be the function defines by

$$f(x)=\begin{cases} 1+3p(x-1)+(x-1)^2 & x\le 1 \\ qx+p & x>1 \end{cases}$$

a) Find the value of q, in terms of p, for which $f(x)$ is continuous at $x=1$. Justify your answer.

b) Using the results from a), find the values of p and q for which $f(x)$ is differentiable at $x=1$.

c) If p and q have the values determined in part b), is f'' a continuous function? Justify your answer.

FR2. 1993 – AB 1 **(No Calculator)**

Let $f(x)$ be the function given by $f(x)=x^3-5x^2+3x+k$, where k is a constant.

a) On what interval(s) is $f(x)$ decreasing? Justify your answer.

b) On what interval(s) is the graph of $f(x)$ concave up? Justify your answer.

c) Find the value of k for which $f(c)=17$ is its relative minimum at $x=c$.

MC1. The temperature of a room, in degrees Fahrenheit, is modeled by H, a differentiable function of the number of minutes after the thermostat is adjusted. Of the following, which is the best interpretation of $H'(5)=2$?

(A) The temperature of the room is 2 degrees Fahrenheit, 5 minutes after the thermostat is adjusted.

(B) The temperature of the room increases by 2 degrees Fahrenheit during the first 5 minutes after the thermostat is adjusted.

(C) The temperature of the room is increasing at a constant rate of $\frac{2}{5}$ degree Fahrenheit per minute.

(D) The temperature of the room is increasing at a rate of 2 degrees Fahrenheit per minute, 5 minutes after the thermostat is adjusted.

MC2. Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2)=-5,\ f(5)=5,$ and $f(9)=-5$, which of the following must be true?

I. f has at least 2 zeros.

II. The graph of f has at least one horizontal tangent.

III. For some $c,\ 2<c<5,\ f(c)=3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

MC3. The local linear approximation to the function g at $x=\frac{1}{2}$ is $y=4x+1$. What is the value of $g\left(\frac{1}{2}\right)+g'\left(\frac{1}{2}\right)$?

(A) 4

(B) 5

(C) 6

(D) 7

MC4. CA The temperature, in degrees Fahrenheit $(°F)$, of water in a pond is modeled by the function H given by $H(t)=55-9\cos\left(\frac{2\pi}{365}(t+10)\right)$, where t is the number of days since January 1 $(t=0)$. What is the instantaneous rate of change of the temperature of the water at time $t=90$ days?

(A) 0.114°F/day

(B) 0.153°F/day

(C) 50.252°F/day

(D) 56.350°F/day