Problem Chosen	2024	Team Control Number
B	MCM/ICM Summary Sheet	2423196

Deep Blue Rover: Explore Deep Sea Safely

Summary

Discover the mesmerizing wonders of the ocean and dive into a world of unknown beauty. Deep sea journey is very attractive but also extremely dangerous, thus the exploration must ensure safety. We have developed a location and search and rescue model to ensure the safety of the MCMS company and to obtain permission from the Greek government for the great trip.

For Task I, we build a state prediction model to predict the area of the lost submersible. Firstly, we rasterize the ocean and establish coordinate system, then we derive the physical motion equations based on dynamical analysis. Secondly, according to a large number of variables for the motion state, we introduce the state-space equations to describe the motion and predict the location. Then, we divide the uncertainty analysis into two aspects, which are data uncertainty and model uncertainty analysis. As for the first aspect, we introduce data range to determine the uncertainty. For the second, we determine the model uncertainty on the basis of Richardson extrapolation. It is ultimately determined that the submersible should send information such as seawater density and velocity, and submersible velocity to narrow the prediction uncertainty, and based on this information, some required equipment for the submersible such as gyroscopes were recommended.

For Task II: we construct a preparation model based on the idea of decision tree. The target of selecting the equipment abstracted as a process of continuous judgment and screening. The node of the decision tree represents the categories of equipment, and the layers represent the process of judgment considering the availability, maintenance, readiness, and usage costs as the factors. As a result, we select the 3D Multibeam Sonar, which is represented by the $7130 - MkII$ model.

For Task III, The search-deploy model is launched based on the prediction model. The search route planning is considered in a two-dimensional plane. Then, the search task of the host ship is divided into two stages. In the first stage, considering the searching outside the prediction area, we build a catch-up model to solve the traveling distance to get the prediction area, then we build a multi-objective optimization model to describe it and find the deployment of host ship. In the second stage, we consider path planning within the prediction area. We model the area by a Gaussian distribution, and the Deployment of the main ship is dependent on the first stage of path planning, then we get that the probability of finding the missing submersible is 96.87 $\%$ when the host ship search time is 30 minutes.

For Task IV, we evaluated the changes of the first three models in the Caribbean Sea and found that the range of the prediction range became larger, the sonar system in Model II outputs the same, and the rescue vessel needed to be equipped with more equipment. Based on the probability distribution model of Task III, we optimise the search strategy in the search-deploy model and got that the optimized model can shorten 32.39$\%$ to 51.72$\%$ compared to the original model.

After modelling, we also conduct sensitivity analysis, which reveals our model's robustness to some parameters. The memo addressed to the Government of Greece is presented in the paper.

Keywords: State-Space, Probability Distribution Model, Uncertainty Analysis, Shortest Routh Planning

Contents

1 Introduction

1.1 Background

Sea exploration is a journey to discover the mesmerizing wonders of the ocean and dive into a world of unknown beauty. In recent years, a very large number of scientists have arrived at the bottom of the sea aboard deep-sea manned submersibles to observe and study the phenomena of the deep sea most directly. With the continuous development of deep-sea technology, ordinary people can also go on voyages with professional submersibles.

However, the mysterious and fascinating ocean is also extremely dangerous, and we must learn, understand, explore and protect the ocean with a sense of awe. Before the trip to search for undersea shipwrecks, a series of safety measures must be prepared, such as personnel: safety training of trip personnel, skills training. In terms of equipment, the safety of submersible equipment is fully guaranteed, Last but not least, emergency rescue equipment and measures are planned and organized in advance.

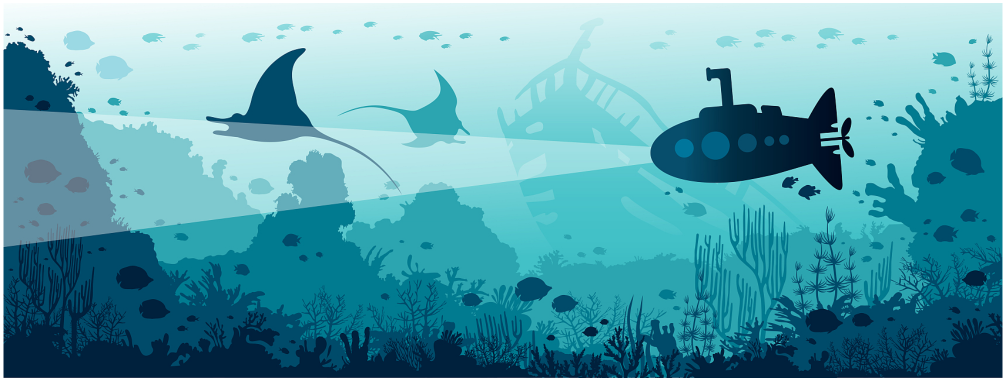


Figure 1: Sea Exploration into the Deep Blue

1.2 Restatement of the Problem

Unlike in a typical search and rescue on the land or on the surface of a sea, searching for submersibles is under much more complex environment. Through in-depth analysis and resarch on the background of the problem, combined with the restricted conditions given, we need to solve the following tasks:

1. Task 1: Locate: Build a model to predict the position of the submersible over time and give the degree of accuracy. Determine what equipment the submersible will need to carry and what information should be sent to the host ship in advance to improve the accuracy above and ensure safety.
2. Task 2: Prepare: Determine the search and rescue equipment to be deployed and carried by the host ship and the rescue vessel, taking into account the type of equipment and the balance of several factors.

3. Task 3: Search: Establish a search model based on the localization model to determine the detailed search and rescue pattern of the equipment to ensure the shortest possible time to locate the lost submersible.

4. Task 4: Extrapolate: Explore the scalability of models for different search and rescue locations and multi-objective submersibles.

5. Task 5: Considering the results obtained above, prepare two pages of memo including adequate search and rescue plans and submit to the Greece Government.

1.3 Our Work

In this paper, we firstly rasterise the ocean information and establish a three-dimensional coordinate system, and then establish the state-space equations based on the initial state of the submersible in the ocean and the force analysis, and then analyse the sources of prediction uncertainty on the basis of this analysis. Based on this, the sources of prediction uncertainty are analysed, and the sensor equipment that the submersible needs to carry and the data that it needs to send to the host ship are determined with the goal of reducing the uncertainty.

For the equipment needed by the host ship, we determine the search equipment deployment based on the decision tree logic model in terms of availability, maintenance and cost considerations. For the host ship deployment point and search path model, we developed a probabilistic model for the prediction range, which describes the relationship between the probability of successful search in the prediction area and time. Also based on the inverse solution of the path planning problem, we determined the deployment point. Then we optimised the path planning strategy based on the probabilistic model to obtain the gain ratio relative to the original model.

In summarize, our models are as follows:

- State-space equations relating the state of motion and position of the submersible (11).
- Multi-objective optimisation equations for host ship (17).
- Two-Dimensional Normal Distribution of Predictive Probabilities for Submersible .
- Gain proportionality equation for the optimised path strategy .

2 Assumptions and Notations

2.1 Assumptions

Through the full analysis of the problem, in order to simplify our model, we make the following reasonable assumptions.

Assumption 1 *The submersible loses its signal and at the same time loses power.*
If the submersible still has power after losing the signal, the prediction of the position of the submersible has a subjective error caused by the direction and size of the power, and it is not possible to establish a reasonable and effective prediction model.

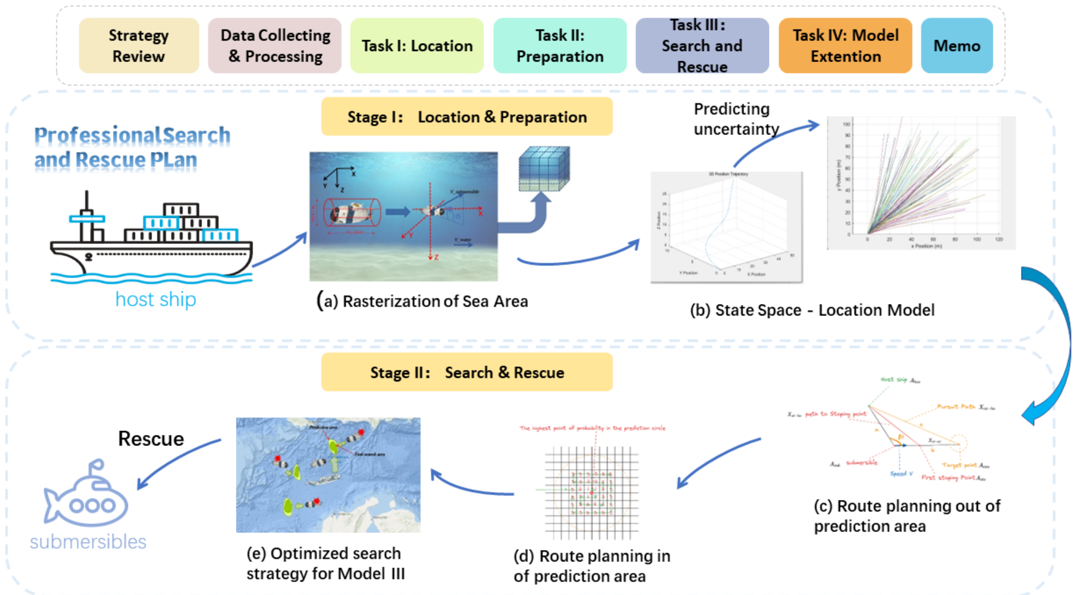


Figure 2: Our Work

Assumption 2 *The velocity of the ocean currents is considered as the velocity of the sea water, without considering the variation of the magnitude of the currents with the depth and density of the ocean.*

Ocean currents are the only data that can directly represent the speed of seawater, and because the direction and magnitude of ocean current speeds can also be subject to error in measurements, and the role of density and ocean depth in influencing the speed of ocean currents is complex and the influencing factors are not large within the range of motion of the submersible.

Assumption 3 *The direction of seawater velocity exists only in the horizontal plane.*

Since we use current data instead of seawater velocity, the current velocity data itself lacks the significance of a record of the vertical direction, and the vertical velocity of currents in the deep ocean can be considered as a random error.

Assumption 4 *It is assumed that the depth and planimetric coordinates of the submersible can be detected by the search equipment of the main vessel during the search. The effects of seabed fish and geomorphology on the search equipment are not considered during the search.*

Under this assumption, we are able to translate the main ship's search for submariners into a two-dimensional plane, and idealizing the range of the search device allows the model to focus more on planning the search path. **Assumption 5** *Errors arising from the transformation of latitude and longitude coordinates to the Cartesian coordinate system are not considered.*

2.2 Notations

Primary notations used in this paper are listed in Table 1. In addition, a set containing element $\cdot$ is expressed as $\cdot$. Different types of fungi are distinguished by subscripts i, e.g., F_i, V_{E_i}. Generally, symbols with the same capital letter but different subscripts represent the variables with similar properties, e.g., V_E and V_D both represent the rate, while V_E denotes the extension rate and V_D

denotes the decomposition rate.

Table 1: Notations

Symbol	Definitions
t	Time (in s)
$L_x(t), L_y(t), L_z(t)/L(x, y, z, t)$	The most suitable temperature of a type of fungi

3 Model I: Linear state-space localization model

3.1 Data Preparation and Model Analysis

3.1.1 Ocean Data Collection

The question did not provide us with ocean data in Inoian Sea and Caribben Sea which the MCMS Submersible is going to explore. Through the analysis of the task, we need to collect the relevant information of Ocean such as density of sea water, the topography and depth and the speed of ocean currents. We have got our data from Authoritative marine data collection agencies. The data sources are shown in Table **??**

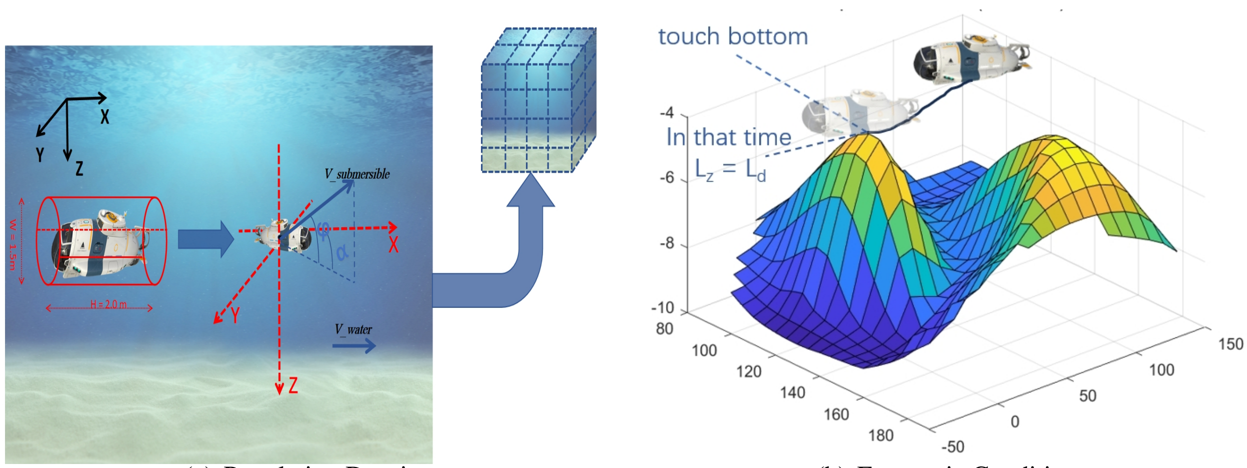
Table 2: **Data and Database Websites**

Data Description	**Database Websites**
Density of Sea Water in Ionian and Caribbean	http://msdc.qdio.ac.cn/data/
Seafloor Topography and Depth	https://download.gebco.net/
Speed and Direction of ocean currents	https://www.oceanmotion.org/html/resources/oscar.html

3.1.2 The Abstraction of Submersible and Sea Model

- Firstly, we determined the relevant data of a representative civilian submersible [1], as well as the density of seawater in the Ionian Sea in relation to the ideal model and the dynamic viscosity of seawater can be not considered in the model. In order to further simplify the problem, the submersible is regarded as a standard cylindrical object during the modeling process, but the material of the surface is the same as the normal submersible [2].

- Next, we set up a three-dimensional coordinate system and rasterize the marine areas, as shown in the figure. 3(b) In order to better describe and determine the position of the submersible, we introduce the Cartesian three-dimensional coordinate system, and since the coordinate transformation of latitude and longitude is not considered in the assumptions, we do not take into account the errors caused by the coordinate transformation. Considering that the degree of density change and seafloor topography in the seafloor is not obvious in a certain area, in order to facilitate the judgment in the subsequent model, and in order to simplify the complexity of the algorithm of the localization model, we decided to rasterize the space of the sea area where the submersible is located, and each cell of the raster after rasterization has the same density, the same depth of seawater. Each raster cell range is 5*5*5 m^3. The determination of this range relies on our analysis of the submarine data and the density of the ocean, as well as the degree of topographic variability. A simplified schematic figure 3(a) is as follows.



(a) Population Density (b) Economic Conditions

Figure 3: The 3D Abstract Schematic of the submersible and the sea water.

$$\begin{cases} v_{H,x} = \vec{v_H} \cdot \cos\varphi \cdot \cos\alpha \\ v_{H,y} = \vec{v_H} \cdot \cos\varphi \cdot \sin\alpha \end{cases} \tag{1}$$

3.2 Location Model Construction

3.2.1 Submarine Dynamics Analysis

A force analysis is first performed to determine the next change in the state of motion of the submersible. The force of gravity received by the submersible and the buoyancy in the water obtained according to Archimedes' principle is expressed as:

$$\begin{cases} F_b = G_w = \rho_w g V_w \\ \vec{G} = mg \end{cases} \tag{2}$$

For the friction force on the submersible, we can simplify it as Equation 3,where S denotes

$$f = \frac{1}{2}\rho_w v^2 S C_d \tag{3}$$

According to Warsi [3]. ,the total drag force on submersible is generaed by the friction and pressure forces acting on a body immersed in a flowing fluid. The drag on a body is usually expressed in terms of a dimensionless drag coefficient[4]. Thus is done calculated the drag coefficient in the flow.Then the total drag coefficient, C_d, is calculated by:

$$C_d = C_{df} + C_{dp} \tag{4}$$

For our hypothetical cylinder model, reference [4] gives specific reference values. Therefore, we obtain the parameters of the drag coefficient,therefore, the submarine is subjected to a total of forces:

$$\begin{cases} \vec{F}_b = \rho_w g h_d \\ \vec{f} = \frac{1}{2}\rho_w v^2 S C_d \\ C_d = 0.85 \end{cases} \tag{5}$$

For our hypothetical cylinder model, reference [**?**] gives specific reference values. Therefore, we obtain the parameters of Equation **??**. The force synthesis and decomposition of the submersible is performed to obtain the following Equation 10:

$$\begin{cases} \vec{F} = \vec{F}_b + \vec{f} + \vec{G} \\ F_x = G - F_b = mg - \rho_H g h_d \\ F_y = f \sin\alpha = \frac{1}{2}\rho_w v^2 S C_d \sin\alpha \\ F_z = f \cos\alpha = \frac{1}{2}\rho_w v^2 S C_d \cos\alpha \end{cases} \tag{6}$$

According to the Assumption , we consider that the submersible has only three degrees of freedom in space. After we coordinate the position, gas pedal, and velocity of the submersible , we get Equation 7:

$$\begin{cases} \vec{L}_t = \begin{pmatrix} L_{x,t}, & L_{y,t}, & L_{z,t} \end{pmatrix} \\ \vec{v}_t = \begin{pmatrix} v_{x,t}, & v_{y,t}, & v_{z,t} \end{pmatrix} \\ \vec{a}_t = \begin{pmatrix} a_{x,t}, & a_{y,t}, & a_{z,t} \end{pmatrix} \end{cases} \tag{7}$$

Based on the previous analysis, we analyze the motion of the submersible. Combining Newton's second law and the laws of motion, there is the following equation:

$$\begin{cases} v_t = v_{t_0} + a_t \\ L_t = L_{0,t} + v_{0,t} + \frac{1}{2}at^2 \end{cases} \tag{8}$$

Since we discretized the model in section 3.1.2, we got discrete data. And to simplify the Location predicting model, we take the motion of the submersible in steps of T_0, and we treat the motion of the submersible as a uniform linear motion in each unit. Then we get the kinematics equations for uniform linear motion: Equation 8.

$$\begin{cases} \vec{F} = m\vec{a} \\ v_(t) = \int a(t)dt + c_1 \\ L_(t) = \int v(t)dt + c_2 \end{cases} \tag{9}$$

According to Eq.**??** , we obtain the relationship between the state quantities of the spatial motion case of the submersible:Equation 10

$$\begin{cases} a_{x,t} = \frac{\rho_w v^2 S C_d \cos\alpha}{2m} \\ a_{y,t} = \frac{\rho_w v^2 S C_d \sin\alpha}{2m} \\ a_{z,t} = \frac{\rho_w v^2 S C_d \cos\alpha}{2m} \\ v_t = v_{0,t} + a_t \\ L_t = L_{0,t} + v_{0,t} + \frac{1}{2}at^2 \end{cases} \tag{10}$$

3.2.2 Location Predicting Based on State-Space Simulation Model

After analyzing the dynamics state of the submersible in 3D coordinate system, we build a model to predict the position of the submersible over a period of time. The possible states motion of the submersible are analyzed. The each positional state of a submersible is affected by many physical parameters e.g. at each moment, the 3D position of the submersible, its velocity and acceleration which will all have an effect on the next state. This is a result of the fact that the description of the problem requires a large number of variables to describe every state. Therefore, we are inspired to choose a state-space representation to simulate the possible positional states of the submersible in three dimensions for simplicity of description. This is a common form of describing dynamic systems in modern control theory. In state space, the dynamic behavior of a system can be described by a set of first-order differential equations, i.e., continuous-time system or difference equations. On this basis, according to the characteristics of submersible information communication on the seabed, we simplify the state of the submersible by rasterization of the marine area, as a result of discretizing the continuously changing positional state data.Thus, we get the data of discrete time system to describe the state information of the submarine.And the state equation11 is below to describe the system.There are 9 state variables in the state space equation under this linear time-varying system, which are:

$$\begin{cases} \mathbf{x}(t+1) = \mathbf{\Phi}\mathbf{x}(t) + \mathbf{G}u(t) \\ y(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{J}u(t) \end{cases} \tag{11}$$

where $x(t)$ is the state vector, representing the state information of the submarine in the space state at moment t.$x(t+1)$ is also the state vector representing the state information of the submersible in its spatial state at the next moment, i.e., t+T.$u(t)$ is the input vector representing the information about the external influence on the spatial state of the submersible motion.$y(t)$ is the output vector, which represents the output of the system, this model focuses on the three position variables l_x, l_y, l_z in the output vector. Matrix Describes the dependency between the vectors, according to the kinematic relationship described by Eq**??**, we get the information of the four matrices as:

$$\mathbf{\Phi} = \begin{bmatrix} 0 & 0 & 0 & 0.1 & 0 & 0 & (1/2)*0.1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & (1/2)*0.1^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & (1/2)*0.1^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{G} = \mathbf{I}$$

$$\mathbf{H}=\begin{bmatrix}0&0&0&0.1&0&0&(1/2)*0.1^2&0&0\\0&0&0&0&0.1&0&0&(1/2)*0.1^2&0\\0&0&0&0&0&0.1&0&0&(1/2)*0.1^2\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\end{bmatrix}$$

$$\mathbf{J}=0$$

Finally, we use MATLAB to simulate the change position state of the submersible at the point of getting lost and losing kinetic energy based on the model established .in MATLAB to get the predicted path of the submarine trajectory. The predicted path results of the submersible trajectory is shown in Fig **??**.

3.3 Uncertainty Analysis for Prediction

For a model to predict an unpredictable future state in this problem, the characterization of its uncertainty cannot rely on a confidence analysis. The uncertainty of the model this ultimately leads to the extent of the range of the position in which the submersible is located.Therefore, we introduce a random error σ to simulate the range of the submersible trajectory for being subjected to the random error. The fig4 is shown below:

The uncertainties in this prediction model come from two sources, one from the data, i.e. the uncertainty due to the possible ranges of the current inputs to the model.For example, the sea water density data we obtained is a range value, so we need to analyze the change in the predicted range due to the size of the range in order to assess the uncertainty of the prediction in a real situation.We assessed the extent to which the density range and the direction and size of the current affect the model separately.

The other aspect is the solution algorithm of the model itself since we discrete the submersible motion state in a certain step size to compute the state information at the next moment. Thus we can analyze the effect of the step size by Richardson extrapolation. The analysis equation 12 is below:where x and y are the input and output respectively.Besides, the model uncertainty also comes from the simplification of the parameters and the idealization of the model, such as the change of sea water velocity , the sea depth, the submersible model is fitted to a cylinder,as well as the degree of discretization.

$$\frac{\partial y}{\partial x}\approx\frac{y(x+\triangle x)-y(x)}{\triangle x} \tag{12}$$

Submersible information delivery and Equipment Carry:
Therefore, we recommended that the submersible should carry gyroscopes, water velocity sensors, and density sensors. The submersible should send information about the position, seawater flow rate, and seawater density to the host ship to minimize the prediction error with the help of these equipment.

4 Model II:Equipment Preparation Decision Model

This section describes the main idea of logistic decision model which determines the devices of the host ship and the vessel equipped. For Model II, the commonly used decision analysis model will be complicated and difficult to analyze here because of the similar technology used for underwater detection equipment and the costs of the related equipment in terms of availability, maintenance and usage are difficult to be determined and lack of data volume. We are inspired by the idea of decision tree which is commonly used in machine learning.We use decision tree as a logic model for the selection of related equipment here. Specifically, the judgment conditions of the decision tree based on the related data of the Ionian Sea and the requirements of the topic. We finally determine the type of equipment shown in table /ref4.1 which gives a detailed information about the searching equipment that need to be carried by the host ship. Considering the possible depth of the moving area of a submersible in the sea, such as the maximum depth of the Ionia Sea is about 5,152 meters and the average depth is more than 3,000 meters,so we choose the search equipment that meet this requirement to keep communication security. The related judgment process is shown in Fig 4.

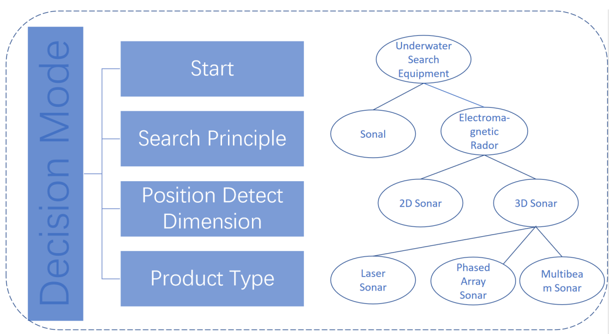


Figure 4: The decision process of the equipment choosing

Common EM waves cannot travel deep underwater due to high attenuation. Extremely low frequency (ELF) EM waves can be used for deep water propagation [138][**?**]. However, ELF requires extremely large antennas. Compared to EM waves, the sound waves (acoustic signals) travel deeper and faster underwater. In addition, the intensity of the sound waves is higher underwater than in the air. Therefore, sound waves are well suited for underwater propagation [**?**]. Sonar is based on the transmission and reception of sound signals and is used for two major purposes. One is underwater environment mapping and imaging [**?**], and the other is underwater vehicle detection, tracking, and classification. The detection of an underwater vehicle can be performed using sonar in the active or passive mode [**?**]. Active sonar sends out particular frequency sound pulses into the water. The reflected sound waves from different objects are collected to determine the presence of an underwater vehicle. A signal processing diagram of active sonar is shown in Fig**??**. The selection of the frequency of the active sonar depends on the requirement. For example, low-frequency sound pulses can be used for long-range detection, whereas high frequency is used for high-resolution imaging.

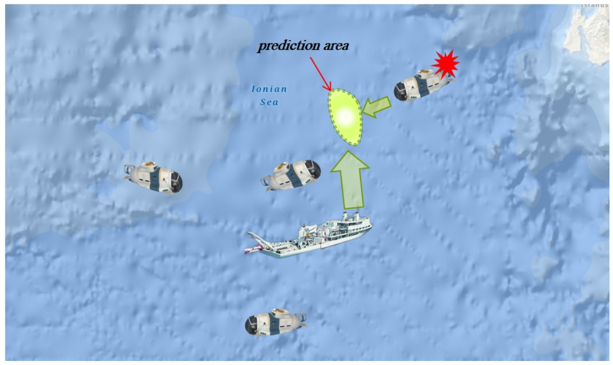


Figure 5: Search Path Diagram

5 Model III: Search and Rescue Model

5.1 Model Preparation

There are some additional assumptions to simplify analysis of the question.

- Hypothesis 1: The whole search and rescue process is modeled as a constant speed. From the perspective of the whole search process, the total length of the rescue must be greater than the time of the acceleration and deceleration process. The host ship search process acceleration and deceleration tend to be fixed and cannot be reduced. Based on the optimization purpose, this fixed part can be removed.
- Hypothesis 2: For all submersible, they are within a certain safety range. If a submersible moves beyond a certain safety range from the host ship, we should be able to alert it at the first time to keep it moving within the safety range, only to ensure that it can be successfully rescued, so that we have a basis for optimizing the path only to shorten the rescue time.
- The Explanation of the Prediction Area: Our uncertainty analysis of localization model in Section 1 gives us a roughly spherical interval for the predicted position of the submersible. It maps to a roughly circular region in the x, y plane where the search needs to be performed.

5.2 Search and Deploy Model Based of Dynamic Programming Ideas

This problem is essentially an optimal route planning problem. The problem is divided into two sub-problems of traveling outside the prediction circle and traveling inside the prediction circle based on the idea of dynamic programming. Next we will analyze these two sub-problems in turn below.

5.2.1 Route planning out of prediction area

We analyze the problem is route planning . Due to the shortest straight line between two points, we need to determine the coordinate point where the host ship goes to first. We consider two factors here, the point A_{sto} where the center buoyancy pause and sinking occurs outside of the first prediction circle versus the point A_{exa} with the maximum probability of the model's prediction circle that happens to be pursued. Since the time of occurrence of the point A_{sto} is determined by the localization model, there exists t_{sto} when A_{sto} happens to exist. In the following, the process of host ship A_{hos} arriving at the point A_{exa} and meeting with it is solved as the chasing problem, and the encounter time t_{arr} is solved. The chase problem is schematized in Figure **??** below. Since the coordinates of the points A_{sto}, A_{exa} and A_{sub} are determined with the velocity $\vec{v}$, so that

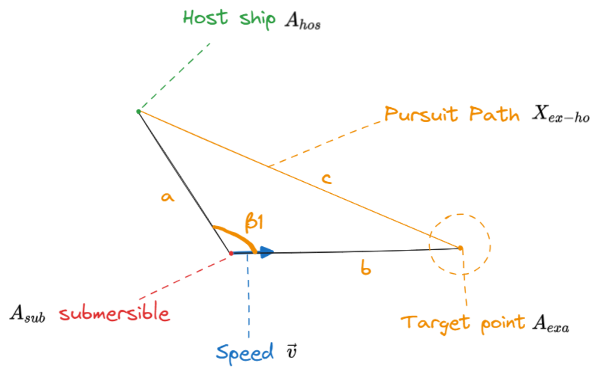


Figure 6: Simplified diagram of the chasing problem

a, b, β_1 are known quantities (a,b,c denotes the distance as shown in figure7), we have the following equation13 solve for the quantity c.

$$c = \sqrt{a^2 + b^2 - 2ab\cos\beta_1} \tag{13}$$

Host ship travels at v_{run}, so we can find t_{arr} by the following equation 14

$$t_{arr} = \frac{c}{v_{run}} \tag{14}$$

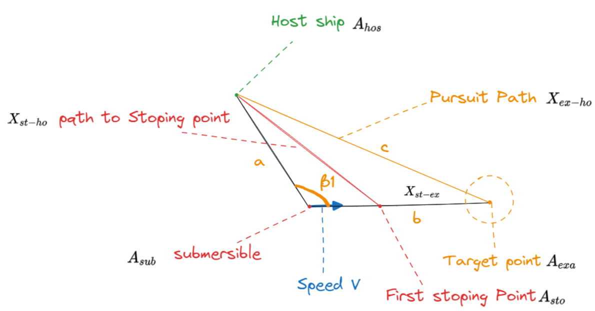


Figure 7: Schematic diagram of route planning outside the prediction area

Afterwards, we compare the encounter time t_{arr} with the time t_{sto} of the first appearance of the pause point based on the above schematic 7.

- When $t_{arr} \leq t_{sto}$, we choose to chase to reach the corresponding point
- When $t_{arr} > t_{sto}$, we compare the size of the initial pause point A_{sto} and the distance ratio of A_{exa} to A_{hos}.

If $X_{ex-ho} > X_{st-ho}$, the initial pause point A_{sto} going to A_{exa}, the vast majority of the other stopping points are on the path of and can all be detected by micro-alteration of direction to can be disregarded. When $X_{ex-ho} < X_{st-ho}$, the initial pause point A_{sto} is set off again after proceeding to the neighborhood of point 3, and this departure is considered in the pursuit problem. The overall main consideration is path planning outside the prediction circle.

Summarizing the above, we introduce the path determination function: the Objective Function

$$f\left(t_{arr}, t_{sto}, X_{st-ho}, X_{ex-ho}\right) = \begin{cases} X_{ex-ho} & , t_{arr} \leq t_{sto} \\ \min\left(X_{ex-ho}, X_{st-ho}\right) + X_{st-ex} & , t_{arr} > t_{sto} \end{cases} \tag{15}$$

Once all the route planning considerations outside the prediction circle are complete, the combined toute determination function values for each point apply the same weight to each submersible to determine the host ship A_{hos} whose predicted encounter average time to all submersible points is the shortest to do so. We use a multi-objective planning model for modeling with the following equation **??**:

$$\min z = \frac{\left(\sum_{i=1}^{n} \gamma_i \sqrt{|X_{hos}^2 - X_i^2| + |Y_{hos}^2 - Y_i^2|}\right)}{n} \tag{16}$$

where X_i represents the determined virtual coordinates of the ith submersible after the introduction of the path determination function, Y_i represents the Y-coordinate of the determined virtual coordinates of the ith submersible after the introduction of the path determination function, n is the number of the total number of submersibles, and γ_i is the weight coefficient for the ith vessel, which has value 1 in this model.

We introduce the path decision function formula in Equation 15 with the distance between two points formula mapping it to formula :

$$\min z = \frac{\left(\sum_{i=1}^{n} \gamma_i f\left(t_{arr}, t_{hap}, X_{ha-ho}, X_{ex-ho}\right)\right)}{n} \tag{17}$$

We finally solved the model to obtain the optimal main ship deployment coordinates (X,Y), which ensures that it has a reasonable minimum time to each diver point. At this point we determine the optimal planning path for a range of probabilities centered in A_{exa} in a single master ship.

5.2.2 Route planning in prediction area

- First we determine the search mode in prediction area after reaching the search position:

  since all the values corresponding to the individual covariates of uncertainty are equally probabilistic with respect to the overall uncertainty, their impact on the final result conforms to a normal distribution, we use a Gaussian formula to represent the probability of detecting a diver at each of our prediction points, and we'll start from the standard Gaussian formula below to our normal Gaussian distribution:

$$p\left(x, y\right) = p\left(x\right) p\left(y\right) = \frac{1}{2\pi} exp(-\frac{x^2 + y^2}{2}) \tag{18}$$

We use the variables $\vec{q} = \begin{bmatrix} x & y \end{bmatrix}^T$ to vectorize this formula

$$p\left(\vec{q}\right) = \frac{1}{2\pi} exp(-\frac{1}{2} V^T V) \tag{19}$$

This time, with $\vec{q} = \vec{A}(\vec{p} - \vec{\mu})$, where $\vec{A}$is the coefficient of the linear combination of each component in $\vec{p}$, i.e., $\vec{A}$ denotes the linear relationship of each variable. There are:

$$p\left(\vec{q}\right) = \frac{\left|\vec{A}\right|}{2\pi} exp(-\frac{1}{2}\left(\vec{p} - \vec{\mu}\right)^T V^T V \left(\vec{p} - \ vec\mu\right)) \tag{20}$$

The final expression for its covariance is $\Sigma = \left(A^T A\right)^{-1}$ where $\left|\vec{A}\right|$ is the determinant to obtain an ordinary normal distribution:

$$p\left(x, y\right) = \frac{1}{2\pi \left|\Sigma\right|^{n/2}} exp\left(-\frac{1}{2}(\vec{p} - \vec{\mu})^T \Sigma^{-1} (\vec{p} - \ \vec{m}u)\right) \tag{21}$$

where the data solved by bringing in the localization model gives $\vec{\mu} = \left(x_{exa}, y_{exa}\right), \Sigma = 1.02$. The final probability distribution is displayed as a plot of the probability distribution with the precise prediction point as the (0,0) point as follows in fig10.

For this Gaussian distribution of predicted points, it is obvious to conclude that the shortest time to find the lost submersible will be obtained when the larger the probability $p\left(x, y\right)$ and there will be no repetition of accumulating the same region to fit the smaller the search time t. We need to determine the path to maximize this form, from which we determine the search path such as the following formula.

- Rasterization of Prediction Area Since each dx and dy in a Gaussian distribution triggers a change in the data, the amount of data is too dense to plan paths, so we determine the amount of probability in each region by rasterizing the prediction range with a Gaussian distribution to make sure that path planning does not probe the same region over and over again. This is shown in the following figure: Since each dx and dy in a Gaussian distribution triggers a change in the data, the amount of data is too dense to allow for path planning, so we determine the amount of probability within each region by rasterizing the prediction range with a Gaussian distribution to make certain that path planning does not repeatedly probe the same region. This is shown in the figure below:

  (1) The figure above. The probability in extends outward from the point of highest original probability, and eventually to the middle point of the grid as the occupied probability of this grid, and passing through this point represents the detection of the complete grid. The grid probability formula is as follows:

$$F\left(X = n\right) = \frac{1}{\sqrt{2\pi}\sigma} exp(-\frac{n}{2\sigma^2 N})$$

Path determination based on rasterization We determine the probability points of each grid based on the probability distribution circle and determine the priority, from which we make

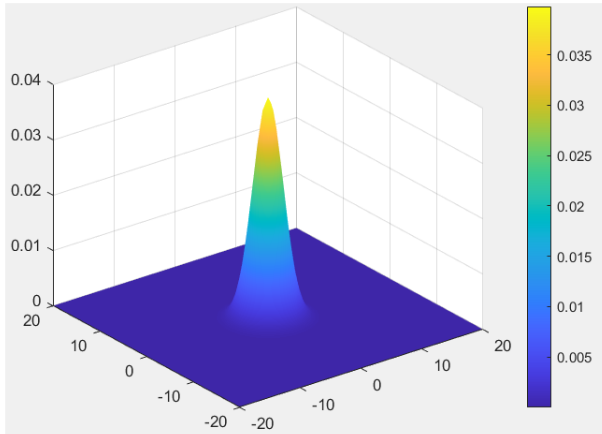


Figure 8: The Gaussian distribution of predicted points

the following route based on our grid, we can see that its in line with the principle of not repeatedly detecting the same area, and after passing through the periphery of the small probability of having to pass through once, the smaller the time t, are in line with the probability of being equipped with the larger the grid. From this we determine the path that minimizes the time to find the submersible for our region theory as follows .... , and set the following equation to represent its path planning

$$\vec{X} = \varphi\left(t\right)$$

where $\varphi\left(t\right)$ is the path planning function of the following figure .

From the above figure we have the optimal path time function for the rasterized circle:

$$t_{avg} = \sum_{i=1}^{n} p_i t_i$$

where p_i is the probability of reaching the point at the moment t_i in the path planning along $varphi\left(t\right)$. From this we determine the optimal path planning in the uncertainty circle, as shown in the following figure ().

Routh planning graphs in probability circles We modeled through MATLAB to the prediction circle with this path planning to find the total probability of change over time image, from the figure can be clearly seen with the increase in time when the time change dt dp is also decreasing, indicating that we this path planning in line with the smaller the time t, are in line with the larger the probability of being equipped with a grid

Overall Search Pattern Description Through the idea of dynamic programming, we decompose the overall search process into two processes, the path planning outside the circle of uncertainty (i.e., the path planning to chase the circle of uncertainty) and the path planning inside the circle of uncertainty, as long as the two converge to the optimal, we can achieve the global convergence of the optimal. Ultimately, we plan the two processes to achieve the purpose of minimizing the time to find the lost diver, and finally obtain the overall search

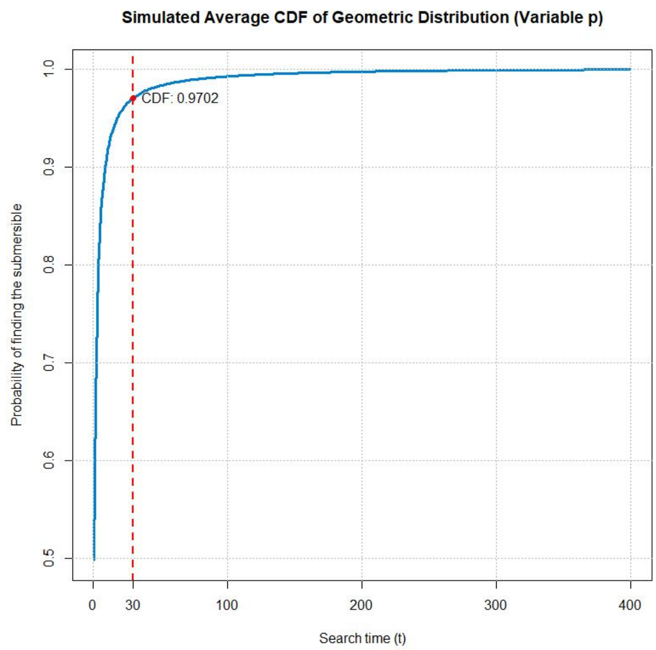


Figure 9: Simulated Average CDF

pattern, as follows... denotes,and the path planning formula () for the search pattern can be obtained.

$$X(x_{op}) = \begin{cases} f\left(t_{arr}, t_{hap}, X_{ha-ho}, X_{ex-ho}\right) & , x_{op} = 1 \\ \varphi\left(t\right) & , x_{op} = 2 \end{cases}$$

$$x_{op}$$

which represents the progress of our planning.

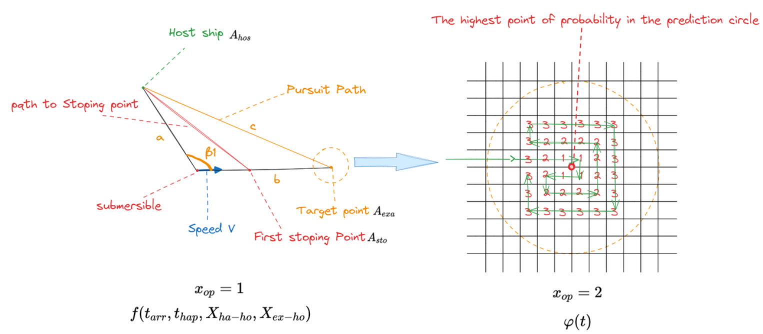


Figure 10: Overall path planning for the search model

6 Model IV: Extension of Model

6.0.1 Improvement of Model I II

For the prediction model, we analyse the sensitivity of some models to the input data to predict outcomes in the Caribbean Sea. The results of these predictions are shown in Table. For Model II, we recommend keeping the sonar equipped on the main ship unchanged, as the average depth of the Caribbean Sea, although deeper than the Ionian Sea, is still within the detection range of the 7130-MkII sonar. This is despite the fact that the depth of the Caribbean Sea is greater than 7686 meters, which is the depth of the Ionian Sea.

6.0.2 Improvement of Model III

Task IV involves solving the case of multiple submersibles operating in the same nearby sea. This is viewed as a search problem for multiple submarines lost at the same time. Different from Model III, the multi-submarine rescue is decomposed into determining the search and rescue sequence, single-submarine search path planning, and sub-search using the probability model. We will begin with a single submersible path search for optimizing the search path. The CDF image of the geometric distribution of the successful search of a certain submersible in Model III shows that the probability of success remains almost constant. The probability of success is nearly higher when the event searched by the main ship reaches a certain threshold, which is 30 minutes in the figure. It is evident that after a certain amount of time, the rate of change of probability, represented by the equation $k = racdpdt$, decreases significantly, and the cost of time increases. Based on this observation, it is decided that when the host ship has explored a certain area and the probability reaches P_{go}, regardless of the search outcome, other lost divers must be explored. The threshold for the search time of the main ship is set at 80 $\%$. After the main ship has searched a certain area for 24.8 minutes, it moves on to the next search area. The process is illustrated in Figure x.

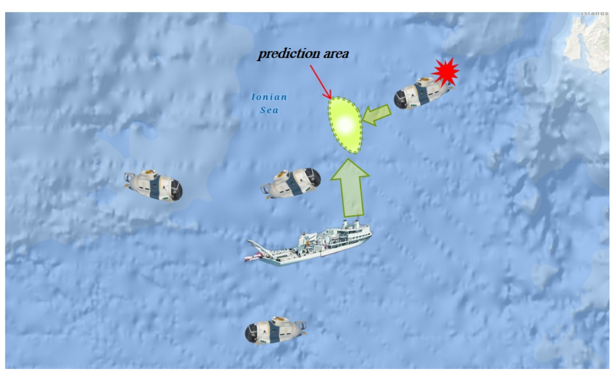


Figure 11: Search Path Diagram

To show the extent of the gain of this decision, we ratio the search time of not executing the strategy to the time that would have been saved by executing the strategy to get its gain ratio.

The first is an expression for the search time performed without the strategy, where D denotes the search range of a single diver. N denotes N lost divers. S denotes the length of the path between two divers. To simplify the process, we consider both D and S equal. The obtained equation is as follows: $t = \frac{D+NS}{v_1}$ We then carved out the time saved by executing this strategy again, with 1/K denoting the proportion of single ranges searched when executing this strategy, and p being the Gaussian-distributed probability function of the predicted ranges in Model III, yielding the following equation:

$$\triangle t = \frac{\frac{D}{k} - \sum_{i=1}^{N}\left[N \cdot S \cdot (1-p)^{i} \cdot p^{N-i}\right]}{v_1} \tag{22}$$

The time saved is equal to the difference between the time that the primary vessel spends less time probing within the predicted range and the increase in the search distance travelled because exploration was not completed. We compare Eq. 1 with Eq. 2 to obtain the gain factor:

7 Evaluate of the Model

7.1 Strengths

- **Applies widely**
  In the prediction model and the path planning model, we rasterise the ocean in three dimensions and two dimensions respectively, which can simplify the model and reduce the spatial complexity of the algorithm.

- **Improve the quality of the airport service**
  In the prediction of the motion state and the description of the probability of the predicted range of the submersible, we discretise the continuous process and the continuous distribution, which can better meet the requirements of the model and satisfy the requirements of the planning solution in the graphical problem.

- **stable and robust**
  In the prediction model, we creatively introduce the state space equations commonly used in modern control theory, which can more intuitively understand the motion and output state of the system, and the matrixed representation substantially reduces the algorithm complexity. unexpected changes.

- **high generalizability**
  We use a probabilistic model to portray the relationship between successful exploration and time during the search process, and use this to optimise the planning strategy, with a high degree of coherence between the models.

7.2 Weaknesses

- For the analysis of question 2, we lacked data sources, so we did not carry out the mathematical evaluation of the model, and the final conclusions drawn may lack the support of data.

- In carrying out the ocean data processing, the influence of ocean topography on the detection range was ignored, and the interaction between density and other factors was not considered, so this is the main source of error that the model is not statistically analysable.

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MEMORANDUM

To: Greek Government

From: MCMS Company

Subject: Safety Report on Manned Submersible Deep Sea Expeditions

Date: February 6, 2024

In order to ensure the safety and smoothness of this activity, we have conducted in-depth research, carefully planned and compiled a detailed safety report for the *Doving Deep Blue* deep-sea expedition. The report includes detailed information about the expedition including project overview, detailed information, safety assessment, personnel training, first aid and emergency response measures and so on. Through this report, we hope to demonstrate that our deep sea exploration activity is under tight safety control in order to gain your support and permission.

- I. Project Overview
  This project is a deep-sea expedition of our self-designed, self-integrated and developed manned submersible. Taking advantage of the design of the submersible, we will venture into the unknown world of the deep sea and explore the secrets of the depths of the seabed. The expedition will be carried out in the Ionian Sea and even other areas in Caribbean Sea.

- II. The Detail of the Submersible
  The submersible type we use is "Dive Dover" which is Independent research and development.We insure that the all What's more, we will strictly follow the process regulations and safety operation manual of the submersible to ensure that every dive is a safe dive.

- III. Safety
  Eric Forsythe, Associate Professor of the School of Electrical and Mechanical Engineering at the University of Adelaide, Australia, speaks highly of our deep-sea submersibles, and believes that this safety report can provide you with more confidence. Seaworthiness: All the submersibles have met the seaworthiness requirements when they are shipped from the factory and are suitable for underwater operations under various weather conditions. The submersibles will be used in strict compliance with environmental regulations to protect the marine ecosystem. Pressure resistance: The submersibles are designed as spheres or cylinders, which are geometrically more resistant to crushing pressure. "The Striker has successfully withstood 10,909 meters of pressure in actual operation, proving the advancement of our deep-sea exploration technology. Oxygen and Power Reserves: The submersible has sufficient internal oxygen reserves, a reliable mains power supply and back-up system, and ensures that alternate power sources (e.g. hydraulics) are also available in the event of a power outage. Information security system: Our submersible adopts advanced information receiving technology, which allows unobstructed communication with the mother ship and guarantees the exchange of information during the expedition.

- IV. Personnel Composition
  We have selected a team of skilled and experienced divers and commanders, each of whom has undergone rigorous training and testing to ensure that they will be able to accurately and skillfully complete their tasks during the mission. In addition, the researchers also have solid professional knowledge and rich working experience, and are able to quickly analyze and process marine data.

- V. Safety systems and safeguards
  We will use a series of personnel and equipment safety and security systems, including first aid equipment, backup oxygen supply, communication equipment, and escape systems, to ensure that the lives of personnel can be secured in the event of a submersible malfunction or an emergency. The main vessel is also computerized to ensure that the personnel on board are kept informed of the status of the submersible.

- VI. Positioning and Search and Rescue Plan
  We have formulated a detailed emergency plan, including equipment failure and unexpected weather. In case of problems, we can carry out rescue immediately to ensure the life safety of personnel.

- VII. Training and Memorandum
  Each of us has undergone rigorous training in deep-sea exploration and has in-depth knowledge of the use of submersibles, which allows us to deal with all kinds of emergencies. At the same time, we have developed a detailed memorandum, so that personnel can know how to operate in an emergency to ensure safety.

- VIII. Insurance
  We have signed a detailed insurance contract with the insurance company to ensure that this deep-sea adventure is covered by insurance. We have always been committed to ensuring the safety of personnel and equipment, which is the principle that our company has always insisted on.

In summary, after investing a lot of manpower, material resources and technology with sophisticated equipment, high standards of training, strict security systems and detailed plans to deal with unexpected situations, we are confidently ready to go on deep-sea expeditions. Although the deep sea is full of unknowns and challenges, we have reason to believe that through scientific and rigorous RD thinking, efficient and accurate operation procedures and personnel spirit, we can overcome many difficulties and carry out successful deep-sea exploration. Every expedition, every dive, shows us the mystery and charm of the ocean, and everything we do is to minimize the risk and improve the safety of the exploration angle. We look forward to unraveling the mysteries of the deep sea with submersibles in the near future, learning more about the ocean, and contributing to the progress of mankind.