CIS11032 Logic Designing & Computer Organization

Lesson 03 Boolean Logic

AR Fathima Shafana
Department of ICT
Faculty of Technology
South Eastern University of Sri Lanka

Learning Outcomes

At the completion of this lesson students should be able to,

- Understand the Boolean logic
- Apply Boolean algebra to simplify Boolean equations
- Use K-Map to simplify logic Equations

COURSE OUTLINE

- Boolean Algebra
- Fundamental Laws and Theorems of Boolean Algebra
- Minterm and Maxterm
- Karnaugh Maps
- Don’t Care States

INTRODUCTION

- George Boole, a nineteenth-century English Mathematician, developed a system of logical algebra by which reasoning can be expressed mathematically
- Boole's system of logical algebra was investigated as a tool for analyzing and designing relay switching circuits
- Boolean algebra has turned out to be a convenient tool in two areas:
  I. **Analysis:** It is an economical way of describing the function of digital circuitry.
  II. **Design**: Given a desired function, Boolean algebra can be applied to develop a simplified implementation of that function.
- Today Boolean algebra is the backbone of computer circuit analysis

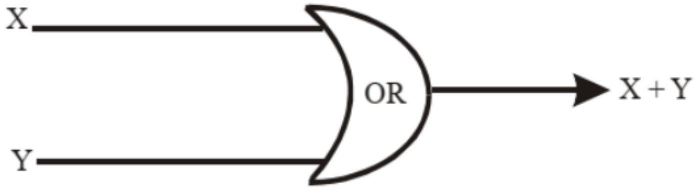
INTRODUCTION contd.

- Boolean algebra works mainly with the following mathematical operations: addition, multiplication and negation.
- These operations are associated with the OR gate, the AND gate and NOT gate respectively.

Logical Addition

- Logical Addition is symbolized with '+'
- However, besides the general addition, it performs the OR operation

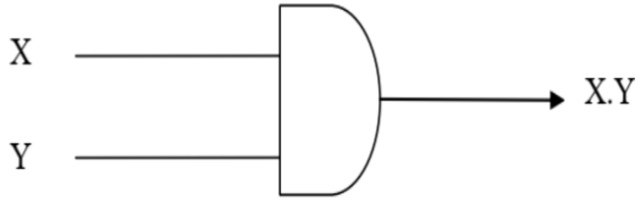
A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1



Logical Multiplication

- Logical Multiplication is symbolized with a '.'
- It performs the AND operation

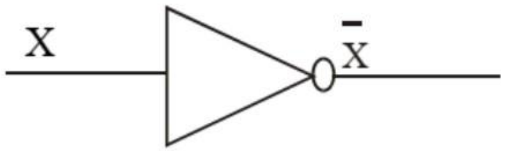
A	B	A . B
0	0	0
0	1	0
1	0	0
1	1	1



Negation

- The symbol used to represent complementation of a variable is a bar (-) above the variable
- The output signal is always the opposite or complement of the input signal
- It takes only one input, hence known as a **Unary Operator**

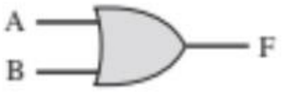
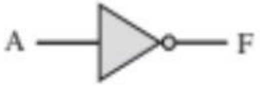
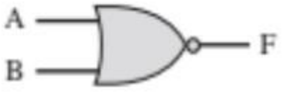
A	$\bar{A}$
0	1
1	0



- Beside the three operators stated above certain other operators are also much prevalently used in practice.

P	Q	NOT P $(\overline{P})$	P AND Q $(P \cdot Q)$	P OR Q $(P + Q)$	P NAND Q $(\overline{P \cdot Q})$	P NOR Q $(\overline{P + Q})$	P XOR Q $(P \oplus Q)$
0	0	1	0	0	1	1	0
0	1	1	0	1	1	0	1
1	0	0	0	1	1	0	1
1	1	0	1	1	0	0	0

Logic Gates for Boolean Operators

Name	Graphical Symbol	Algebraic Function	Truth Table
AND		$F = A \bullet B$ or $F = AB$	A B | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = A + B$	A B | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
NOT		$F = \overline{A}$ or $F = A'$	A | F 0 | 1 1 | 0
NAND		$F = \overline{AB}$	A B | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$F = \overline{A + B}$	A B | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
XOR		$F = A \oplus B$	A B | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0

Fundamental Laws and Theorems of Boolean Algebra

OR Operations

1. $X + 0 = X$
2. $X + 1 = 1$
3. $X + X = X$
4. $X + \bar{x} = 1$

AND Operations

5. $X . 0 = 0$
6. $X . 1 = X$
7. $X . X = X$
8. $X . \bar{x} = 0$

3rd and 7th are called "Idempotent"

Fundamental Laws and Theorems of BooleanAlgebra

Double Complement

9. $\bar{\bar{x}} = X$

Commutative Laws

10. X + Y = Y + X

11. XY = YX

Associative Laws

12. (X + Y) +Z = X +(Y + Z)

13. (X . Y). Z =X. (Y. Z)

Distribution Law

14. X (Y + Z) = XY + XZ

Dual of Distributive Law

15. X + Y .Z = (X + Y) . (X + Z)

Fundamental Laws and Theorems of BooleanAlgebra

Laws of Absorption

16. $X + XZ = X$

17. $X (X + Z) = X$

Identity Theorem

18.$X+ \bar{x}\ Y = X+Y$

19. $X (\bar{x} + Y) = XY$

De' Morgan's Theorem

20. $\overline{X + Y} = \bar{X}\bar{Y}$

21 $\overline{XY} = \bar{X} + \bar{Y}$

EXAMPLE 1

$$\mathbf{XY + YZ +}\overline{Y}\ \mathbf{Z = XY + Z}$$

$$
\begin{aligned}
L.H.S. = \quad & XY + YZ + \overline{Y}\ Z \\
= \quad & XY + Z(Y + \overline{Y}) \\
= \quad & XY + Z.1 \\
= \quad & XY + Z \\
L.H.S = \quad & R.H.S.
\end{aligned}
$$

Exercises: Simplify using Boolean Algebraic Laws

1. AB + AB’ + A’B
2. AB + A’B + ABC’ + AB’C
3. (A+B).(A’+B)
4. AB + AB + ABC

Show that A+AB+ABC = A

Question 01

$AB + AB' + A'B$

$A(B+B') + A'B$ $\qquad B+B' = 1$

$A.1 + A'B$ $\qquad A+A'B = A+B$

$A+B$

Question 02 $AB + A'B + ABC' + AB'C$

$B(A+A') + ABC' + AB'C$ $\qquad A+A' = 1$

$B.1 + ABC' + AB'C$

$B + ABC' + AB'C$

$B + BAC' + AB'C$ $\qquad X+XY=X$

$B + AB'C$

$B + B'AC$ $\qquad X+X'Y=X+Y$

$B + AC$

Question 03 $(A+B).(A'+B)$

$A.A' + AB + A'B + B.B$

$A.A' = 0;\ B.B = B$

$0 + AB + A'B + B$

$B + AB + A'B$

$B + AB = B$

$B + A'B$

$X + XY = X$

B

Question 04 $AB + AB + ABC$

$AB + AB + ABC$

$AB + ABC$

AB

$X+X = X$

$X + XY = X$

Show that $A+AB+ABC = A$

LHS : A + AB + ABC	$X+XY=X$
A + ABC	$X+XY=X$
A	
RHS	
Hence, Proved.	

Example 2

Find the complement of the expression

I. $X + YZ$

II. $\bar{A}\,B + C\,\bar{D}$

$(X + YZ)'$

$X' . (YZ)'$

$X' . (Y'+Z')$

$(A+B)' = A'.B'$

$(AB)' = A'+B'$

Example 2

Find the complement of the expression

I. $X + YZ$

II. $\bar{A}\,B + C\,\overline{D}$

(A’B+CD’)’

(A+B’).(C’+D)