CIS11032 Logic Designing & Computer Organization

Lesson 02 Arithmetic with Binary

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Learning Outcomes

At the completion of this lesson students should be able to,

- Perform arithmetic calculations using binary digits

COURSE OUTLINE

- Arithmetic Operations
  - Single Bit Binary Addition with Carry
  - Multiple Bit Addition
  - Single Bit Subtraction with Borrow
  - Multiple Bit Subtraction
  - Multiplication
  - *BCD Addition*

Single Bit Binary Addition with Carry

Given two binary digits (X,Y), a carry in (Z) we get the following sum (S) and carry (C):

Carry in (Z) of 0:

Z	**0**	**0**	**0**	**0**
X	**0**	**0**	**1**	**1**
+ Y	**+ 0**	**+ 1**	**+ 0**	**+ 1**
C S	**0 0**	**0 1**	**0 1**	**1 0**

Carry in (Z) of 1:

Z	**1**	**1**	**1**	**1**
X	**0**	**0**	**1**	**1**
+ Y	**+ 0**	**+ 1**	**+ 0**	**+ 1**
C S	**0 1**	**1 0**	**1 0**	**1 1**

Multiple Bit Binary Addition

- **Extending this to two multiple bit examples:**

Carries	**0**	**0**
Augend	**01100**	**10110**
Addend	**+10001**	**+10111**
Sum		

- **Note: The 0 is the default Carry-In to the least significant bit.**

Single Bit Subtraction with Borrow

Given two binary digits (X,Y), a borrow in (Z) we get the following difference (S) and borrow (B):

Borrow in (Z) of 0:	**Z**	**0**	**0**	**0**	**0**
	X	**0**	**0**	**1**	**1**
	- Y	**-0**	**-1**	**-0**	**-1**
	BS	**0 0**	**1 1**	**0 1**	**0 0**
Borrow in (Z) of 1:	**Z**	**1**	**1**	**1**	**1**
	X	**0**	**0**	**1**	**1**
	- Y	**-0**	**-1**	**-0**	**-1**
	BS	**11**	**1 0**	**0 0**	**1 1**

Single Bit Subtraction with Borrow contd.

This can be tabulated as follows:

Inputs			Outputs	
x	y	b_{in}	d	b_{out}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Multiple Bit Subtraction

- **Extending this to two multiple bit examples:**

Borrows	**0**	**0**
Minuend	**10110**	**10110**
Subtrahend	**- 10010**	**- 10011**
Difference		

- **Notes: The 0 is a Borrow-In to the least significant bit. If the Subtrahend > the Minuend, interchange and append a – to the result.**

Multiplication

The binary multiplication table is simple:

0 * 0 = 0 | 1 * 0 = 0 | 0 * 1 = 0 | 1 * 1 = 1

Extending multiplication to multiple digits:

Multiplicand	**1011**
Multiplier	**x 101**
Partial Products	**1011**
	0000 -
	1011 - -
Product	**110111**

Multiplication

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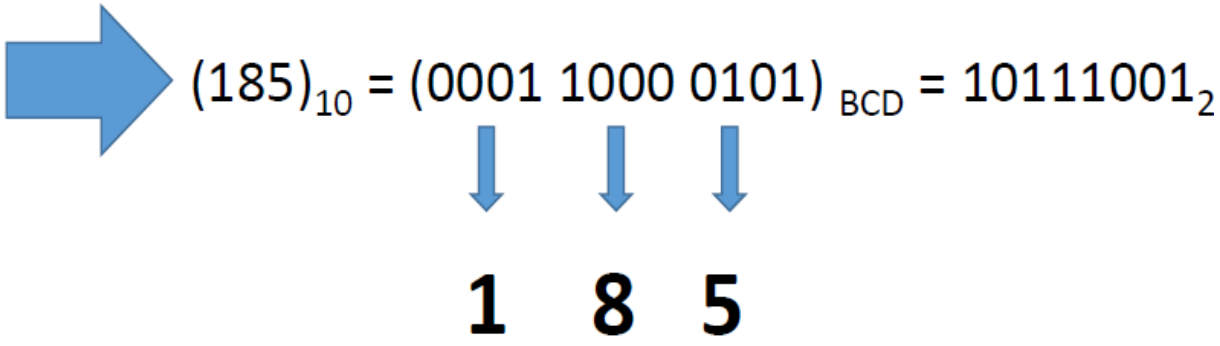
Binary Coded Decimal (BCD)

- A set of four elements can be coded with two bits, with each element assigned one of the following bit combinations: **00, 01, 10, 11.**
- Bit combination of an n-bit code = 0 to 2^n-1
- The code most commonly used for the decimal digits is the straight binary assignment
- This scheme is called Binary Coded Decimal (BCD)

Binary Coded Decimal (BCD) *contd.*

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001



Binary Coded Decimal (BCD) *contd.*

- Although BCD use bits in their representation,
  BCD numbers are decimal numbers and not binary numbers.

- The only difference between a decimal number and BCD,
  Decimals are written with the symbols 0,1,2,…,9 and BCD numbers use the binary code 0000,0001,0010,…,1001

BCD Addition

4	0100	4	0100	8	1000
+5	+0101	+8	+1000	+9	1001
9	1001	12	1100	17	10001
			+0110		+0110
			10010		10111

Why 6?

Rules for BCD Addition

1.Given numbers are to be added using the rule of binary.
2.Judge the result of addition.
If the result of addition of two binary number is greater than 9, It is not valid for BCD number.

If the four bit result of addition is greater than 9 and if a carry bit is present in the result then it is invalid and we have to add 6 whose binary equivalent is $(0110)_2$ to the result of addition.

BCD Addition: Example

184 + 576

BCD	1	1		
	0001	1000	0100	184
	+0101	0111	0110	+576
Binary sum	?	?	?	
Add 6	?	0110	0110	
BCD sum	0111	0110	0000	760

Rules for BCD Subtraction

1. Find the 9's complement of the subtrahend.
2. Add the minuend and the 9's complement of subtrahend.
3. Given numbers are to be added using the rule of binary.

If the result of addition of two binary number is greater than 9, It is not valid for BCD number.

Note: If a carry is generated at the end, add it to the result

If the four bit result of addition is greater than 9 and if a carry bit is present in the result then it is invalid and we have to add 6 whose binary equivalent is $(0110)_2$ to the result of addition.

BCD Subtraction: Example

255 – 63 ➔ 255 + (-63)

BCD code for 255 :	0010	0101	0101
BCD code for 936 :	1001	0011	0110
Addition :	1011	1000	1011
If Invalid BCD then add 6 :	0110		0110
Addition :	10001	1000	10001
Remaining bits except carry :	10001	1000	0001
Carry :		1	
Addition :	10001	1001	0001
BCD value :	11	9	1

The left most bit of the result is 1, called carry and This will be added to 191.

191 + 1 = 192

So final answer of BCD Subtraction is 192

BCD Subtraction: Example

984 – 599 ➔ 984 + (-599)

BCD code for 984 :	1001	1000	0100
BCD code for 400 :	0100	0000	0000
Addition :	1101	1000	0100
If Invalid BCD then add 6 :	0110		
Addition :	10011	1000	0100
BCD value :	13	8	4

The left most bit of the result is 1, called carry and This will be added to 384.

384 + 1 = 385

So final answer of BCD Subtraction is 385

Thank You