Limits

The concept of limit which is basic to the study of calculus, helps us to describe the behaviour of a function f when the independent variable x takes values very close to a particular value a.

Example:

Consider the function $f(x) = \dfrac{2x^2 - 2}{x - 1}$, $(x \neq 1)$. Let us consider what happens to the function as x approaches the value 1.

$x < 1$	$f(x)$
0.5	3
0.9	3.8
0.99	3.98
0.999	3.998
0.9999	3.9998
0.99999	3.99998
0.999999	3.999998

$x > 1$	$f(x)$
1.5	5
1.1	4.2
1.01	4.02
1.001	4.002
1.0001	4.0002
1.00001	4.00002
1.000001	4.000002

We see that as x approaches 1 (from values less than 1 as well as from values greater than 1), $f(x)$ approaches 4. We say that the **limit** of the function $f(x) = \dfrac{2x^2 - 2}{x - 1}$ as x approaches 1 is 4 and we write this as $\lim\limits_{x \to 1} \dfrac{2x^2 - 2}{x - 1} = 4$.

We note that the limit of the function as x approaches 1 exists, although the function itself is not defined at the point $x = 1$.

Let f be a function of x. We say that the limit of $f(x)$ as x approaches a is L if $f(x)$ gets arbitrarily close to L as x gets arbitrarily close to a, and we write this as $\lim\limits_{x \to a} f(x) = L$.