Welford's algorithm in batches

Recursive Batch Mean

$$N\ \bar{x}_N - (N-k)\bar{x}_{N-k} = \sum_{i=1}^{N} x_i - \sum_{i=1}^{N-k} x_i = \sum_{i=N-k-1}^{N} x_i =$$

$$\bar{x}_N = \frac{\sum_{i=N-k-1}^{N} x_i + (N-k)\bar{x}_{N-k}}{N} = \bar{x}_{N-k} + \frac{\sum_{i=N-k-1}^{N} (x_i - \bar{x}_{N-k})}{N}$$

Recursive Variance

$$\begin{aligned}
N\sigma_N^2 - (N-k)\sigma_{N-k} &= \sum_{i=1}^{N} (x_i - \bar{x}_{N-k})^2 - \sum_{i=1}^{N-k} (x_i - \bar{x}_{N-K})^2 = \\
&\sum_{i=1}^{N-k} (x_i - \bar{x}_N)^2 - \sum_{i=1}^{N-k} (x_i - \bar{x}_N)^2 - (x_i - \bar{x}_{N-k})^2 \\
&\sum_{i=1}^{N-k} (x_i - \bar{x}_N)^2 - \sum_{i=1}^{N-k} (x_i + x_i - \bar{x}_{N-k} - \bar{x}_N)(\bar{x}_{N-k} - \bar{x}_N) \\
&\sum_{i=1}^{N-k} (x_i - \bar{x}_N)^2 - (\bar{x}_{N-k} - \bar{x}_N) \sum_{i=1}^{N-k} (x_i - \bar{x}_N) = \\
&\sum_{i=1}^{N-k} (x_i - \bar{x}_N)(x_i - \bar{x}_N + \bar{x}_N - \bar{x}_{N-k}) = \sum_{i=1}^{N-k} (x_i - \bar{x}_N)(x_i - \bar{x}_{N-k}) \\
\sigma_N^2 &= \frac{\sum_{i=1}^{N-k} (x_i - \bar{x}_N)(x_i - \bar{x}_{N-k}) + (N-k)\sigma_{N-k}^2}{N} = \\
&\sigma_{N-k}^2 + \frac{\sum_{i=1}^{N-k} (x_i - \bar{x}_N)(x_i - \bar{x}_{N-k}) + k\,\sigma_{N-k}^2}{N}
\end{aligned}$$