Mixing Patterns in Social Networks






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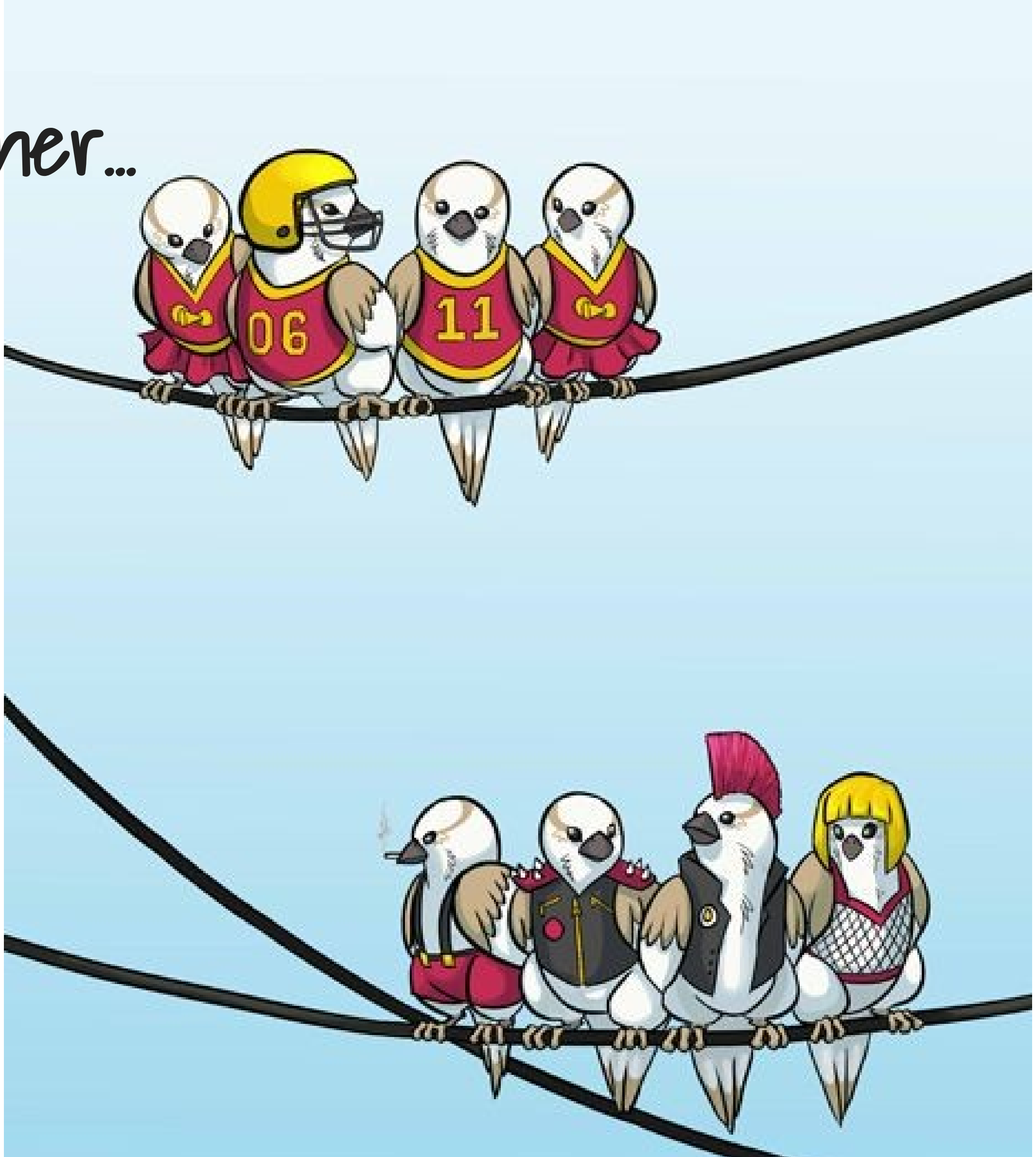
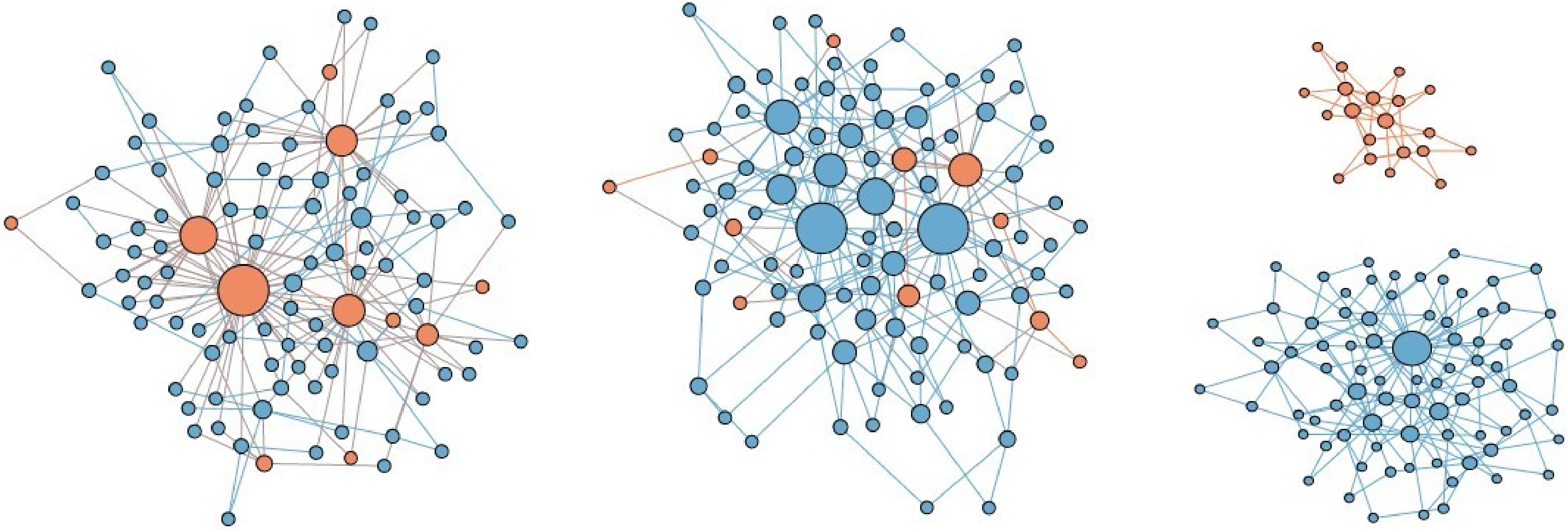
Birds of a feather...



...flock together

Visibility and ranking of minorities

heterophily random homophily



minorities over-represented

minorities under-represented

Karimi et al. “Homophily influences ranking of minorities in social networks” Scientific Reports (2018)

Two questions...

1. Can we detect heterogeneities in mixing within a network?

2. Can we compare mixing patterns between networks?

Mixing in social networks

Generalisation, not rules!

Mixing patterns in networks

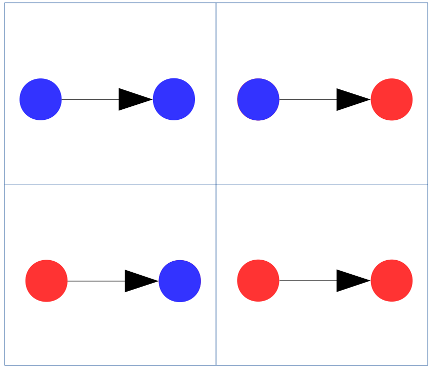
$$r_{\text{global}} = \frac{\sum_g e_{gg} - \sum_g a_g b_g}{1 - \sum_g a_g b_g}$$

Newman "Mixing patterns in networks" Phys. Rev. E (2003)

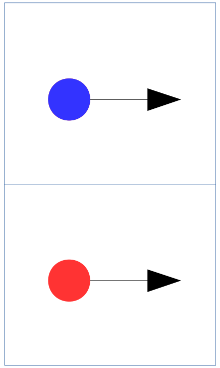
$$r_{\text{global}} = \frac{\sum_g e_{gg} - \sum_g a_g b_g}{1 - \sum_g a_g b_g}$$

Mixing matrix

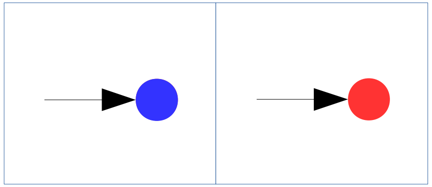
$e_{gh} =$



$a_g =$



$b_g =$

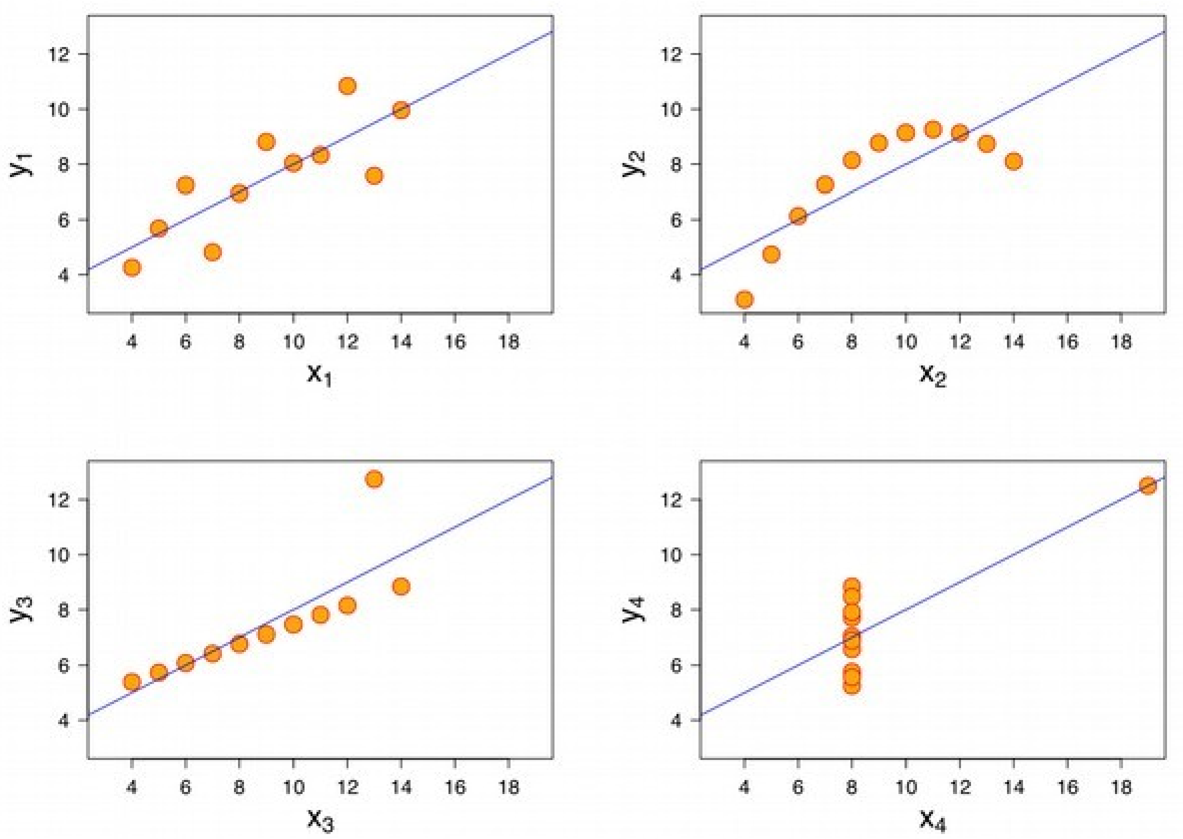


Assume for now, that the network is undirected.

i.e., $a_g == b_g$

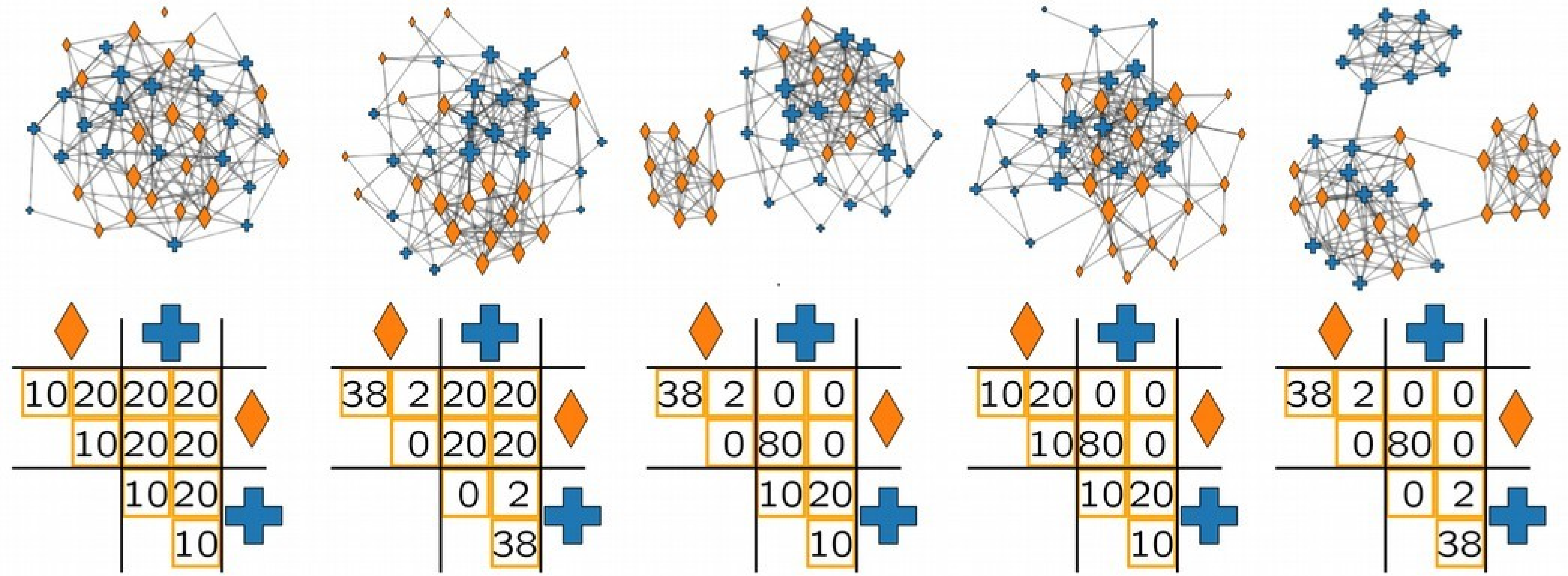
Assortativity is correlation across edges

Assortativity is correlation across edges



Anscombe, "Graphs in Statistical Analysis". American Statistician (1973)

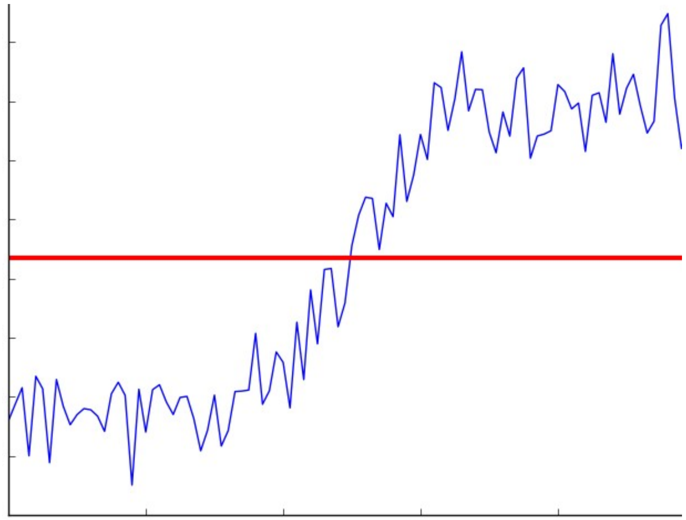
All these networks have assortativity r=0



Peel, Delvenne, Lambiotte, "Multiscale mixing patterns in networks". PNAS (2018)

Can we measure assortativity locally?

Time series analysis

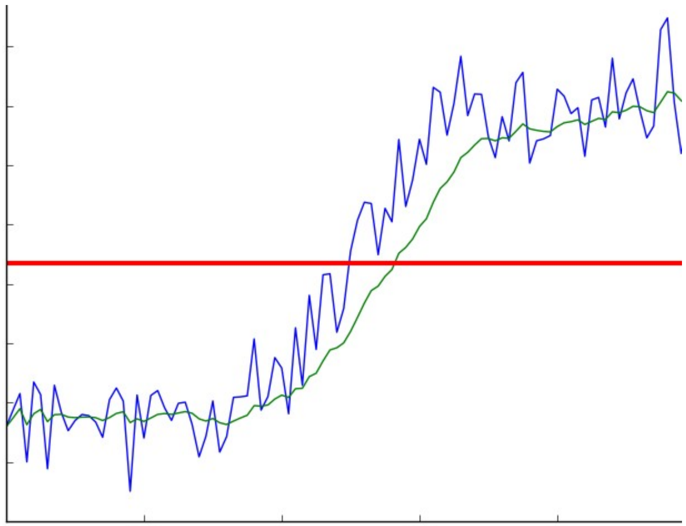


Time series

Mean

The mean is only representative of the data around the middle of the time series

Time series analysis

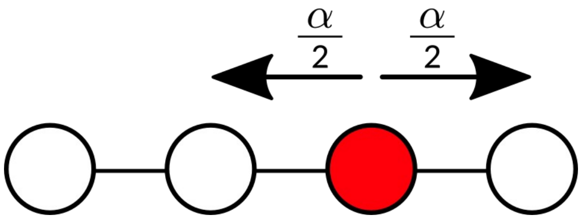


Exponentially weighted mean

Recent points are more relevant

$$S_t = \alpha y_{t-1} + (1-\alpha) S_{t-1} \quad 0 < \alpha \le 1$$

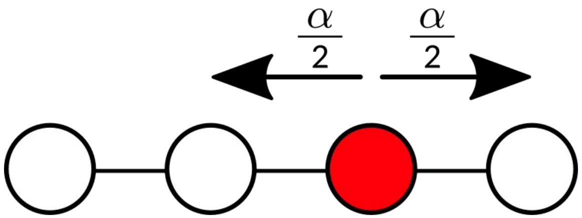
Assortativity is the autocorrelation of a random walk



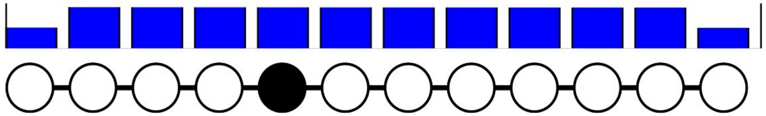
Random walk

Sequence of node attributes

Assortativity is the autocorrelation of a random walk



Random walk

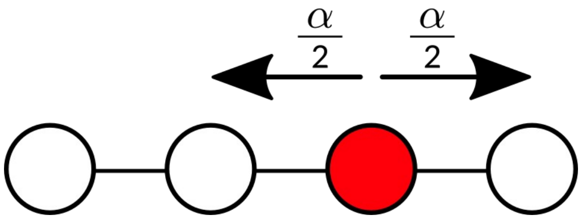


stationary distribution proportional to the degree

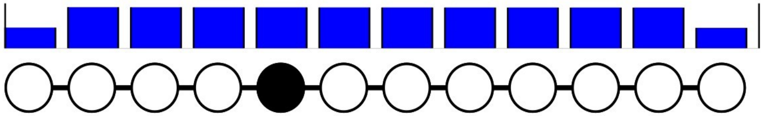
$$w(i) = \frac{k_i}{2m}$$

Sequence of node attributes

Assortativity is the autocorrelation of a random walk



Random walk



stationary distribution proportional to the degree

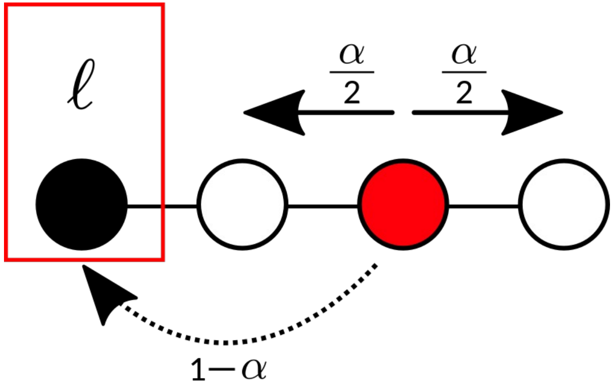
$$w(i) = \frac{k_i}{2m}$$

Sequence of node attributes

Recovers Newman's assortativity

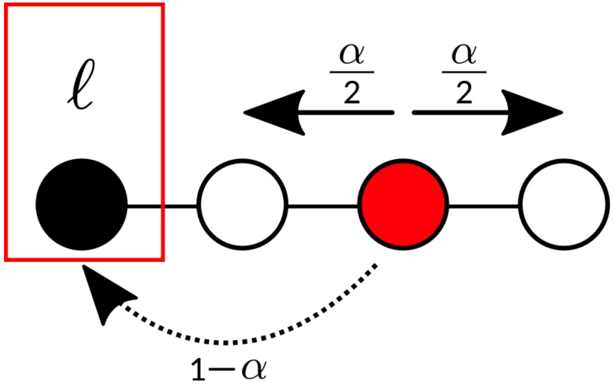
$$e_{gh} = \sum_{ij} w(i) \frac{A_{ij}}{k_i} \delta_{y_i,g}\delta_{y_j,h}$$

"Localise" using random walk with restart

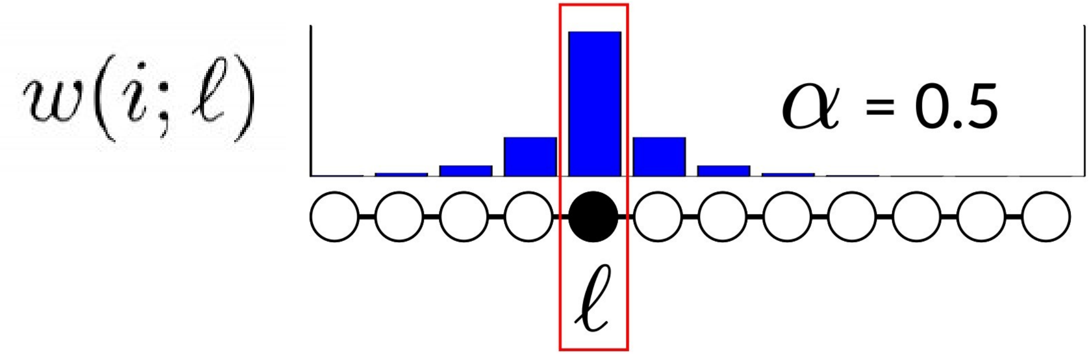


Random walk with restart

"Localise" using random walk with restart

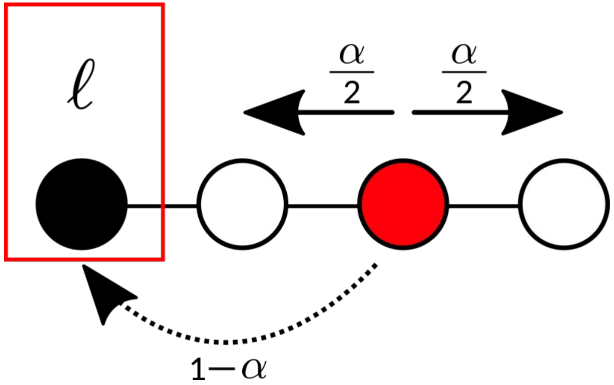


Random walk with restart

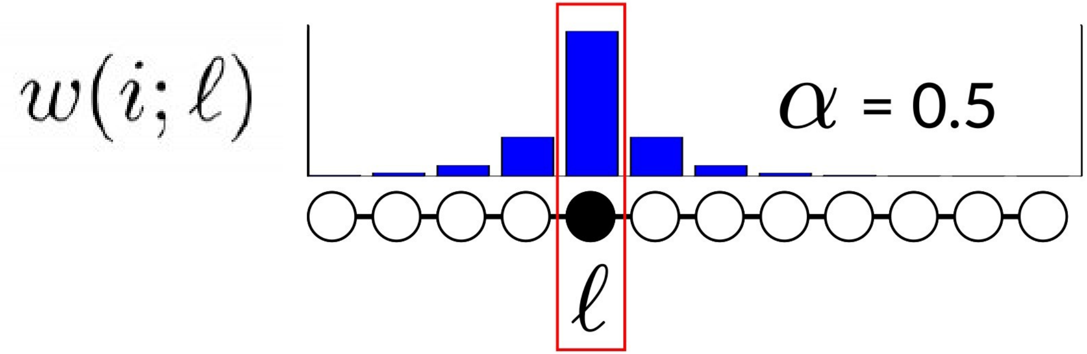


stationary distribution
(Personalised PageRank)

"Localise" using random walk with restart



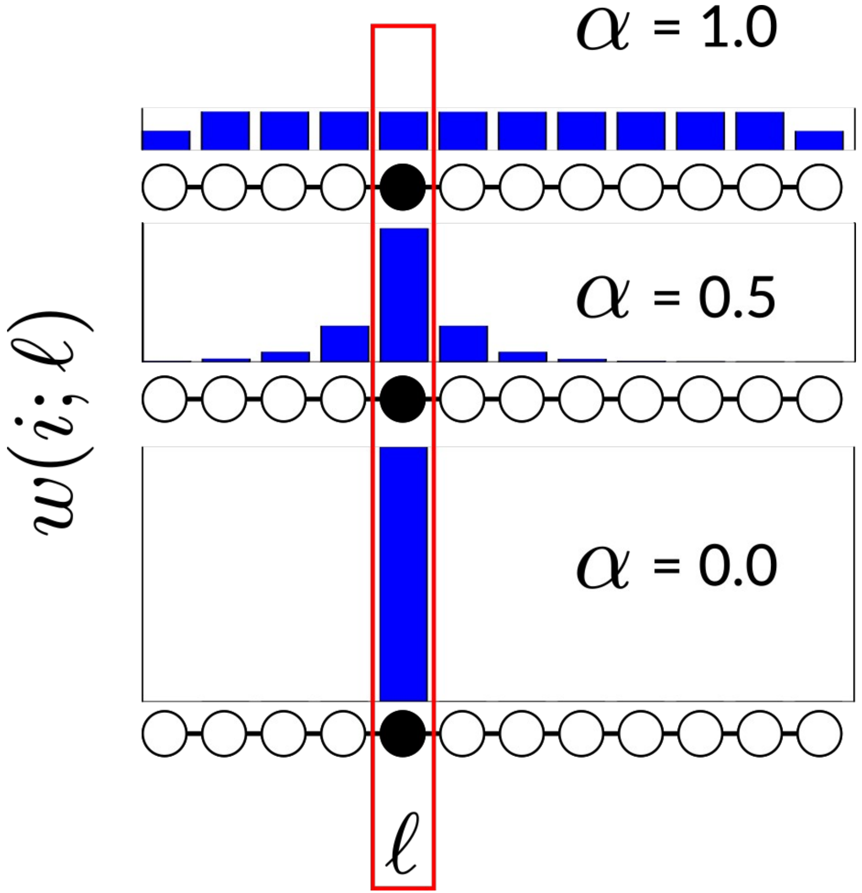
Random walk with restart



stationary distribution
(Personalised PageRank)

Re-weight nodes:

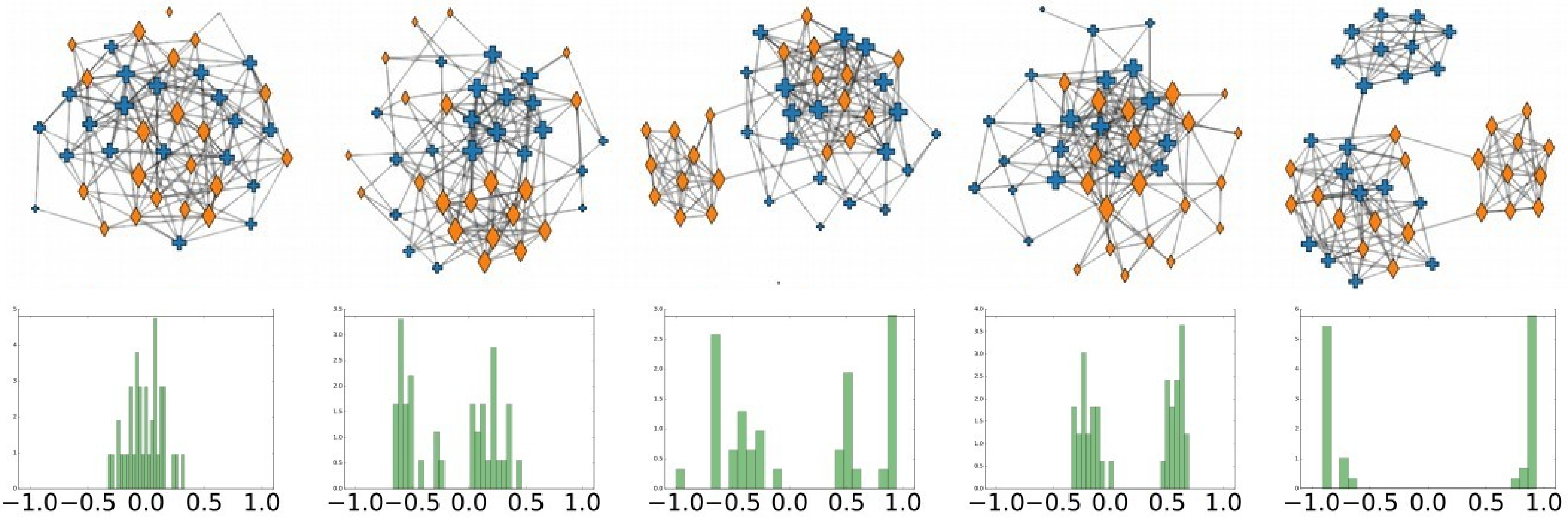
$$e_{gh}(\alpha,\ell) = \sum_{ij} w(i;\ell)\frac{A_{ij}}{k_i}\,\delta_{y_i,g}\delta_{y_j,h}$$



Newman's assortativity (global)

Single node (local)

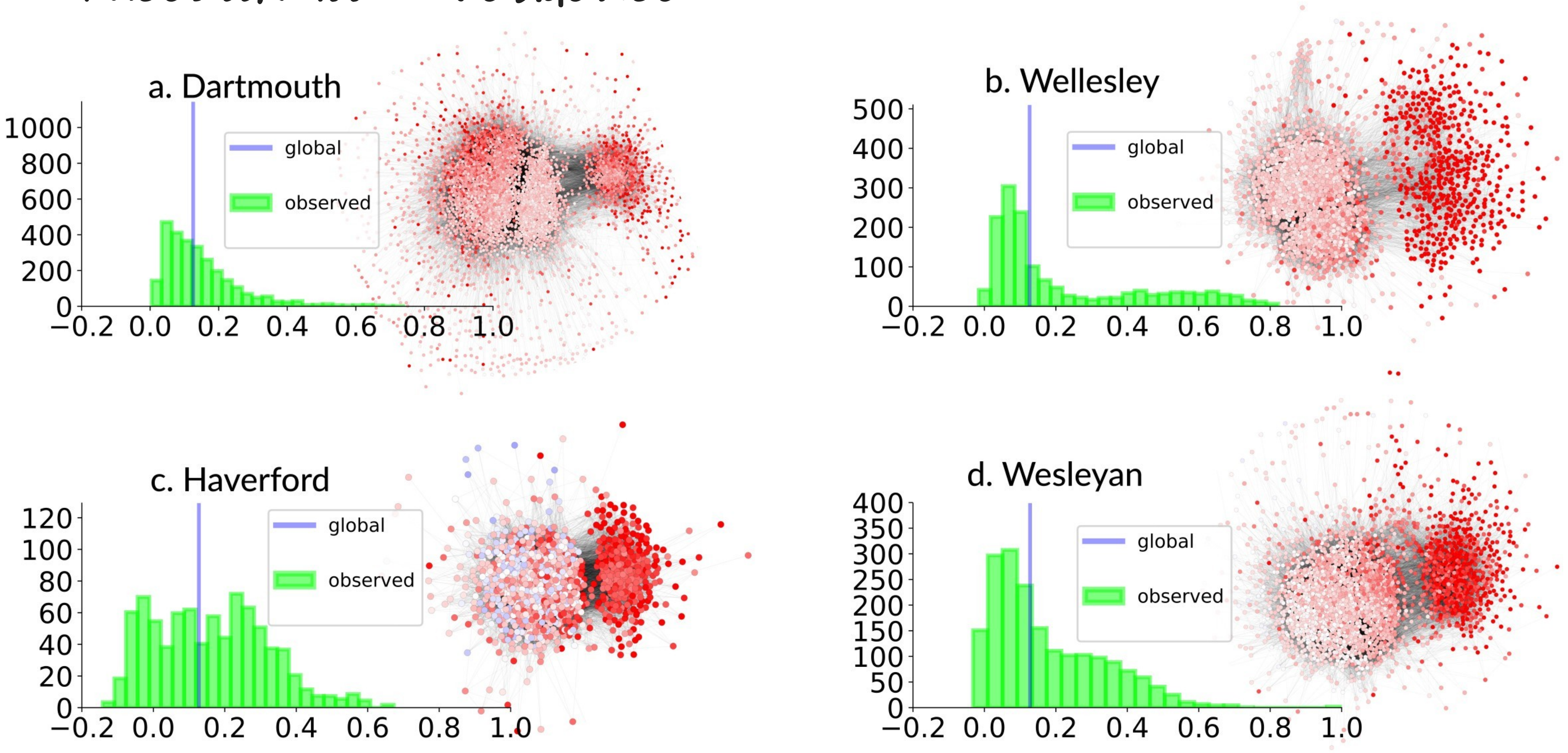
Identify local patterns...



Random mixing

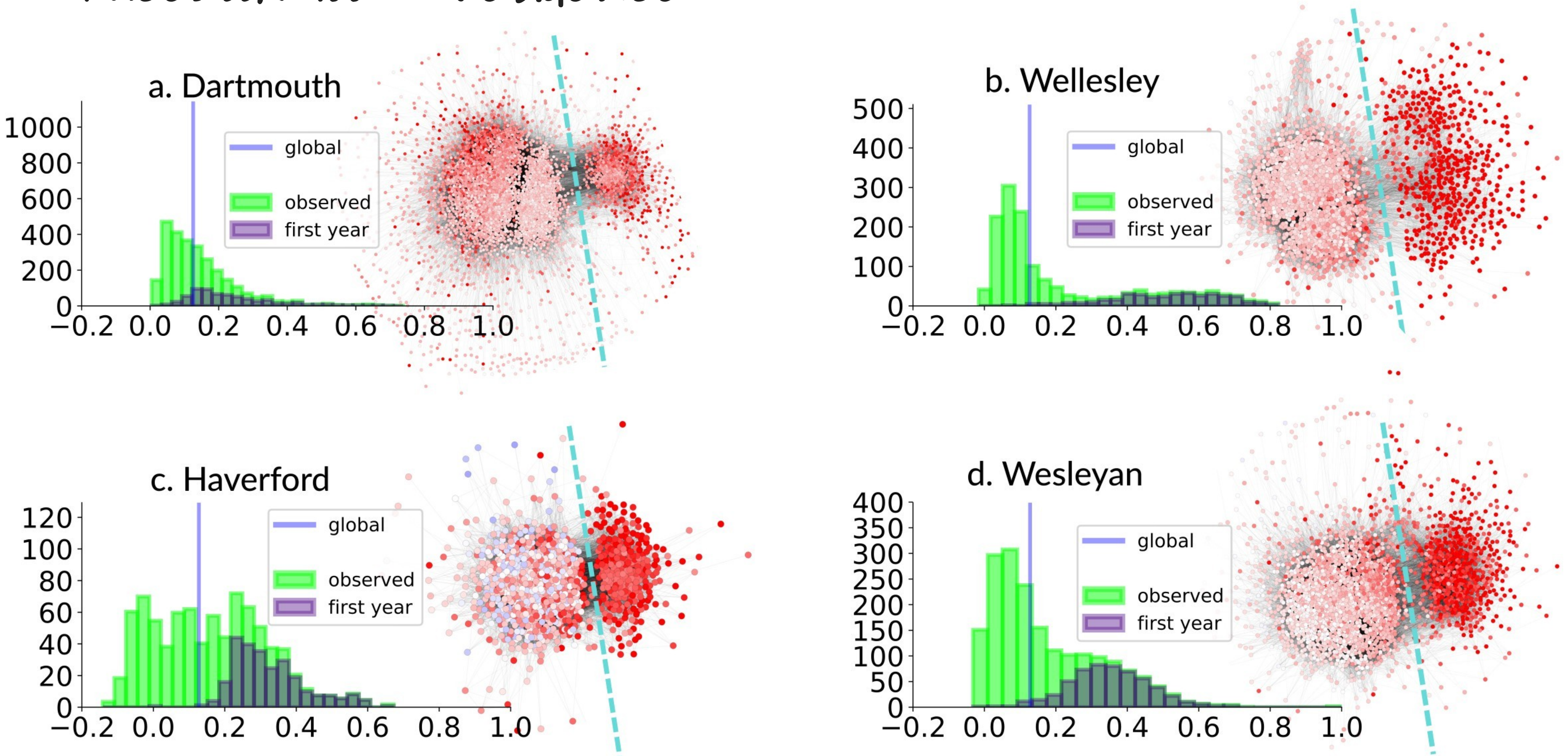
assortative + disassortative

Facebook 100 - residence



Peel, Delvenne, Lambiotte, "Multiscale mixing patterns in networks". PNAS (2018)

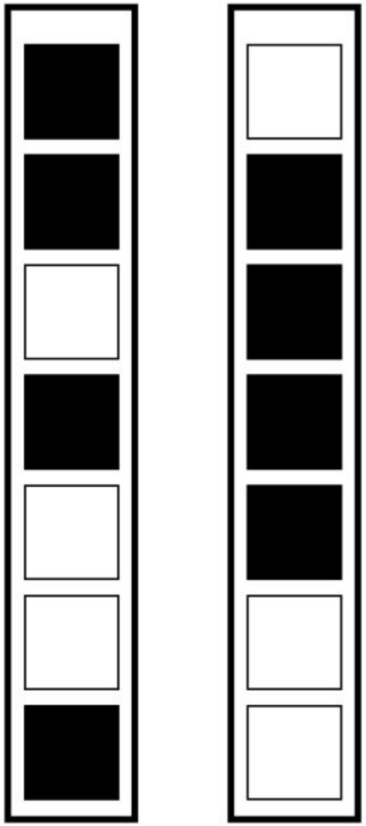
Facebook 100 - residence



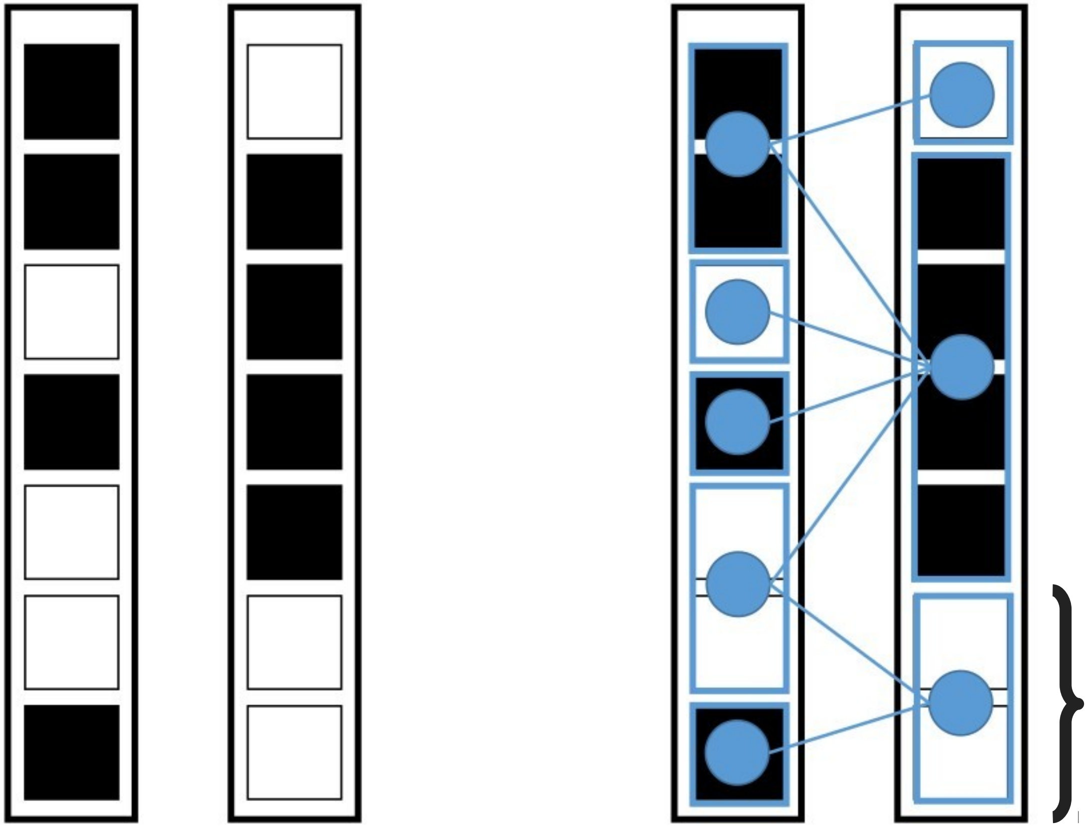
Peel, Delvenne, Lambiotte, "Multiscale mixing patterns in networks". PNAS (2018)

Can we compare assortativity across networks?

Correlation of binary variable (Φ-coefficient)



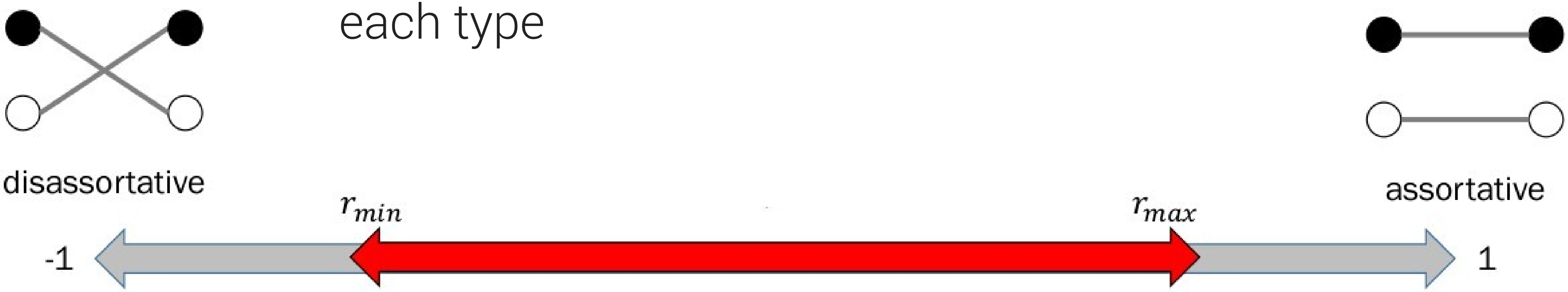
Samples in a network are not independent!



Two samples,
one node

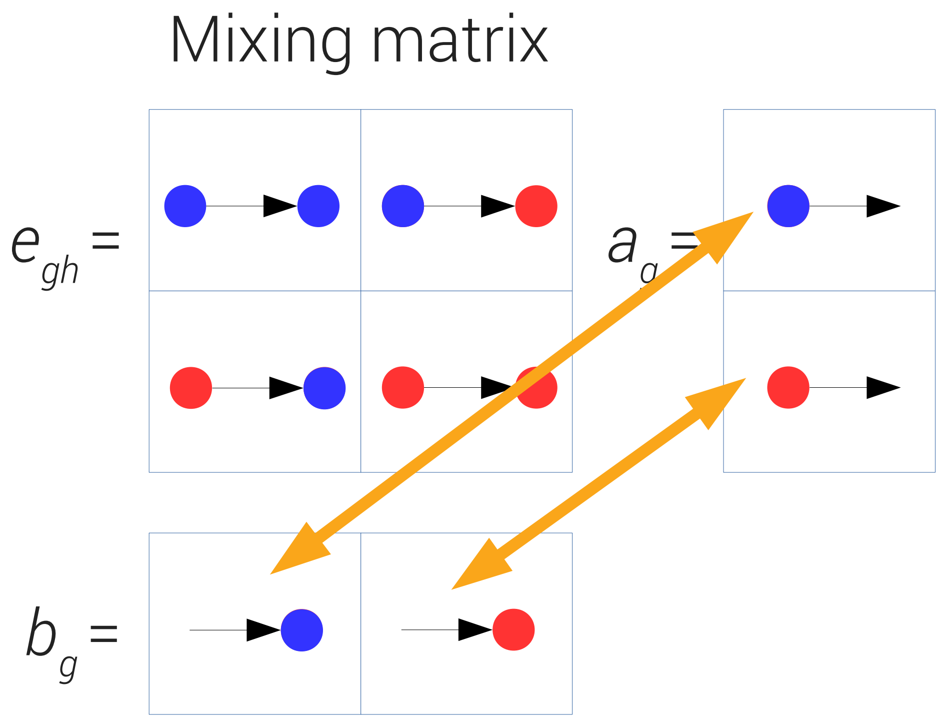
Full range of assortativity is often not attainable

Assortativity is constrained by degree distribution and proportion of nodes of each type



Cinelli, Peel, Iovanella, Delvenne, “Network constraints on the mixing patterns of binary metadata” *in prep.*

We also inherit issues from the Φ-coefficient

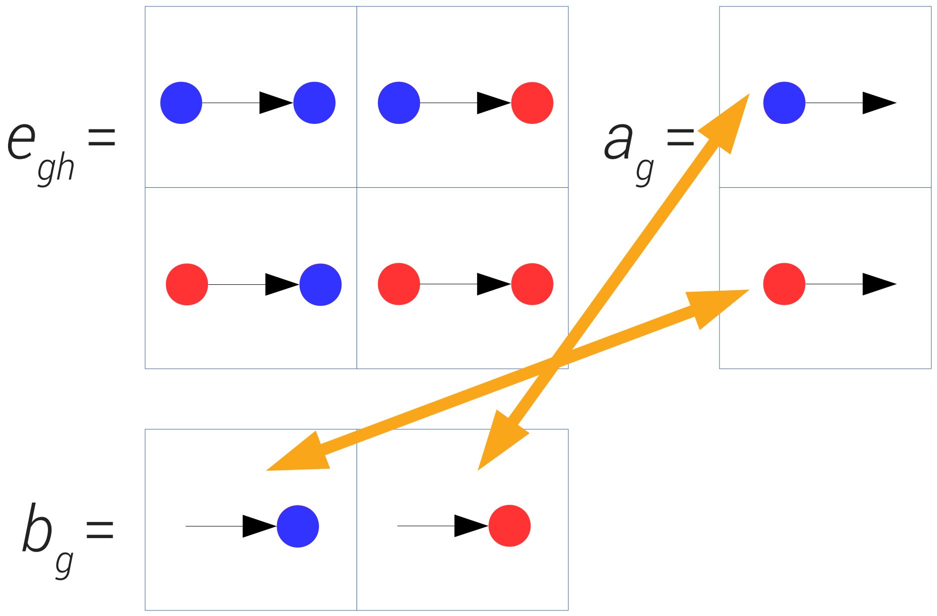


For $r = 1$, we require that
$a_g = b_g = 0.5$

Cureton, "Note on Φ/Φmax". Psychometrika (1959)
Davenport, El-Sanhurry, “Phi/Phimax: Review and Synthesis” Educational and psychological measurement (1991)

We also inherit issues from the Φ-coefficient

Mixing matrix



For r =1, we require that

$a_g = b_g = 0.5$

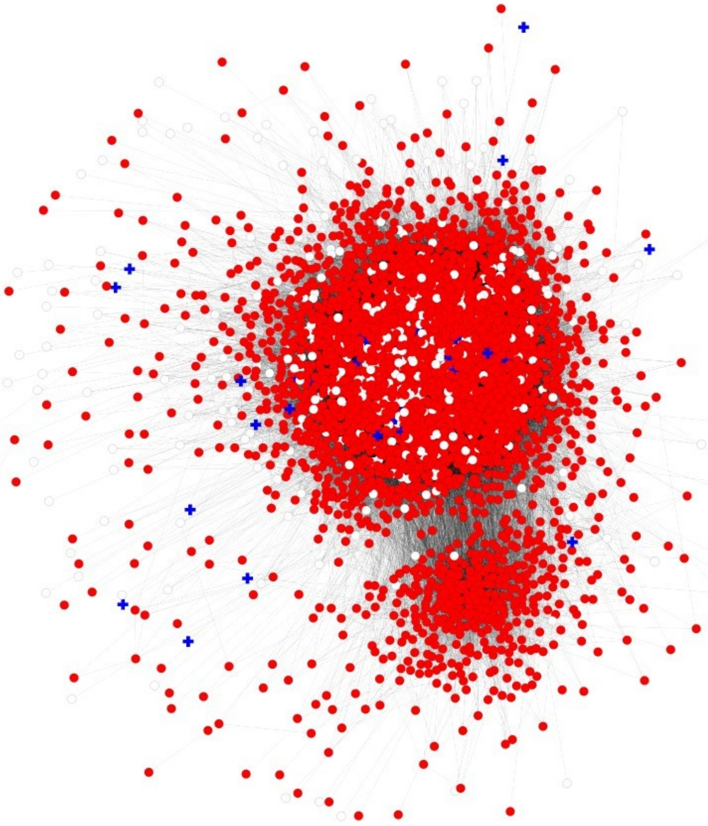
For r =-1, we require that

$a_i = b_j = 0.5$

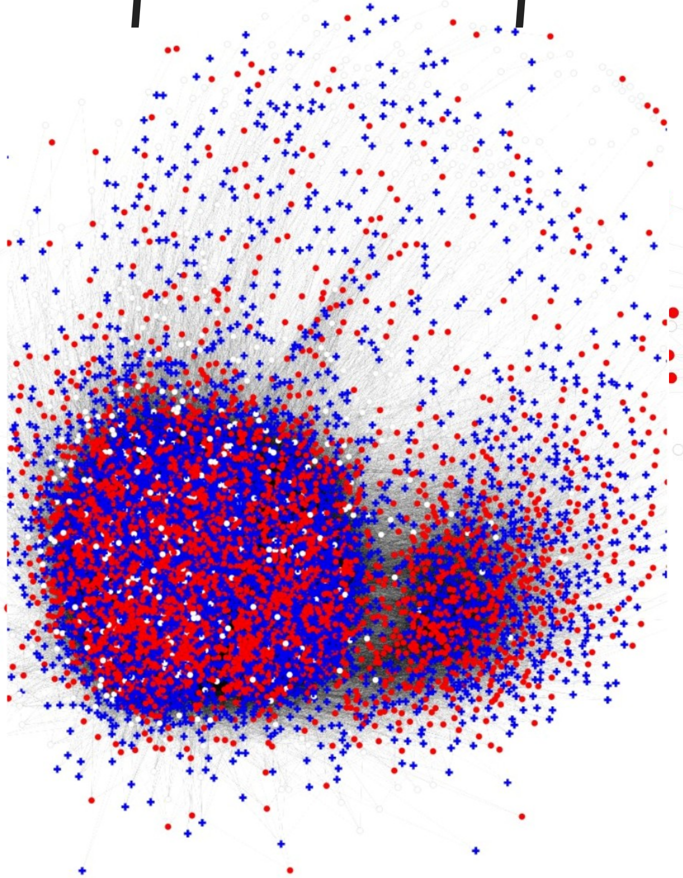
$a_j = b_i = 0.5$

Cureton, "Note on Φ/Φmax". Psychometrika (1959)
Davenport, El-Sanhurry, "Phi/Phimax: Review and Synthesis" Educational and psychological measurement (1991)

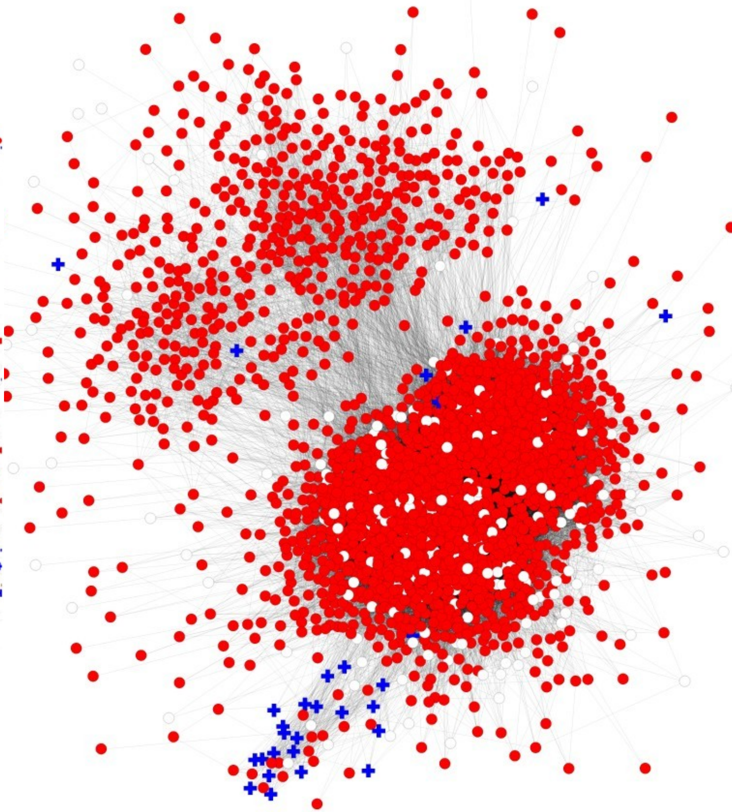
Order these networks by assortativity



Smith

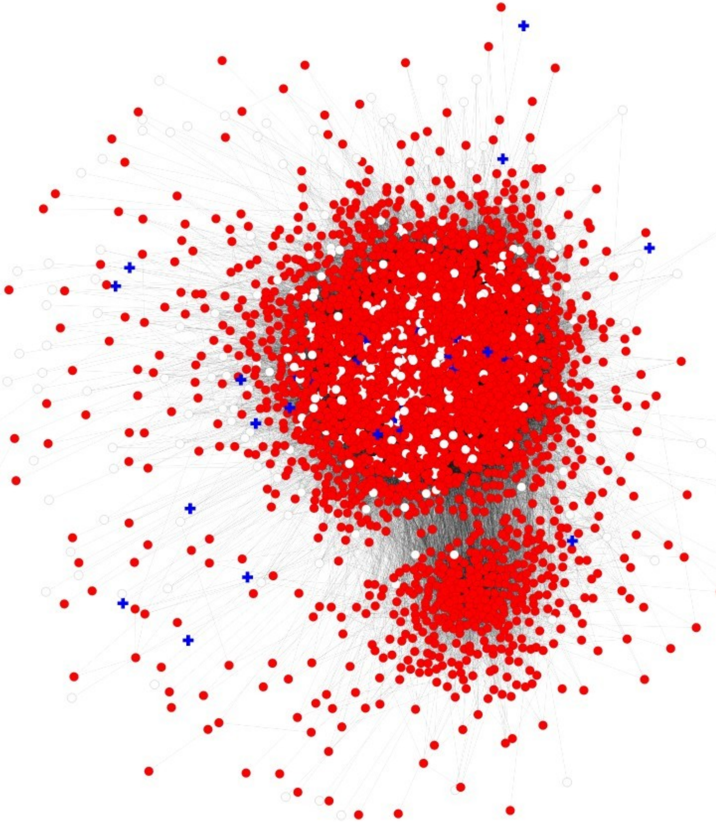


Stanford

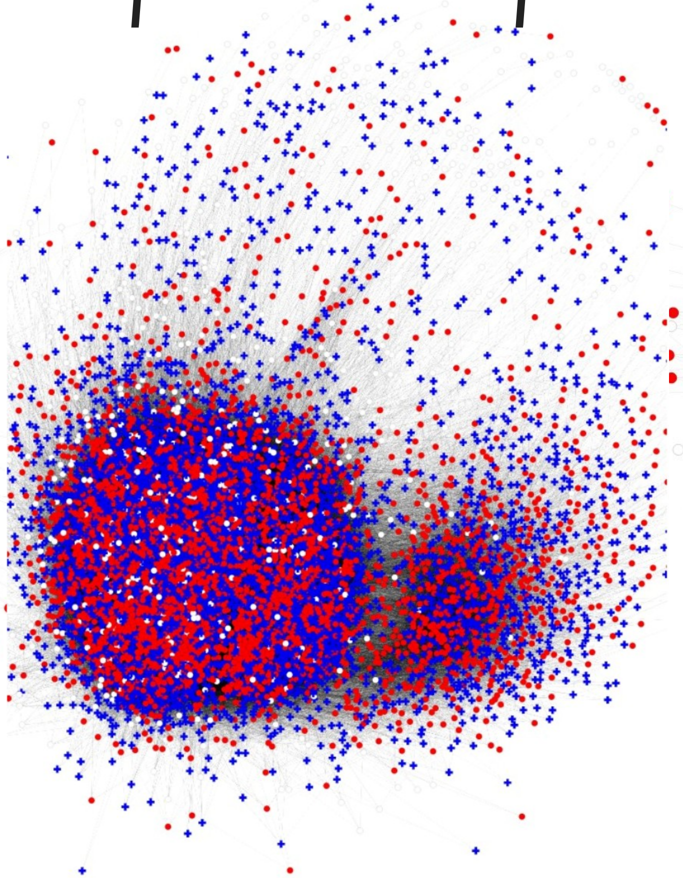


Wellesley

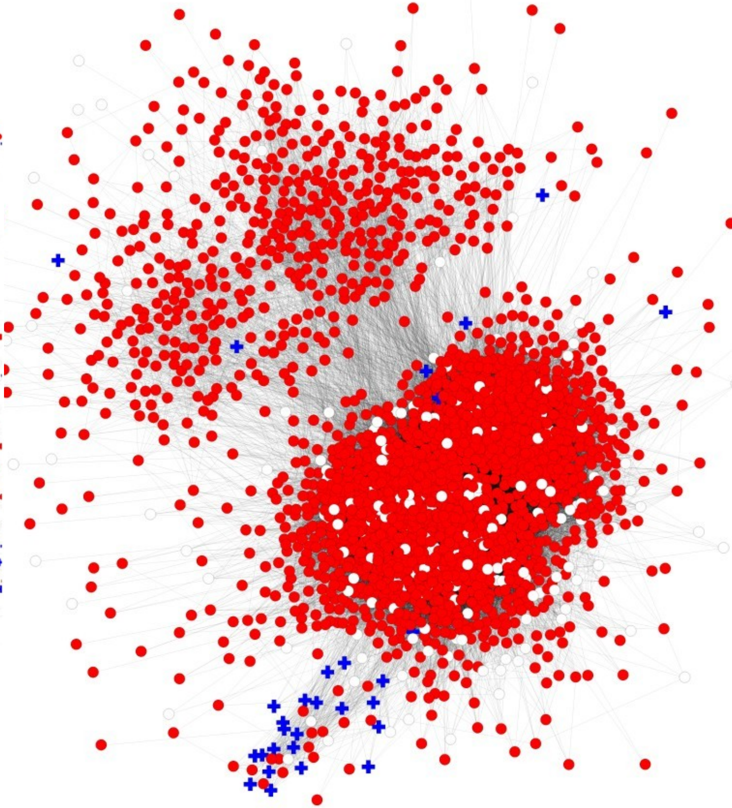
Order these networks by assortativity



Smith r=0.025

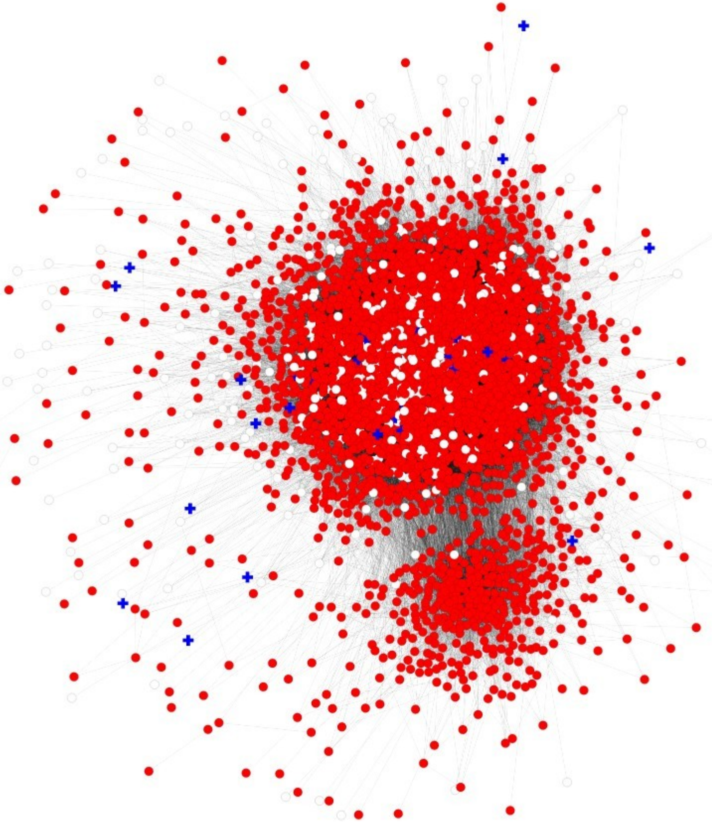


Stanford r=0.057

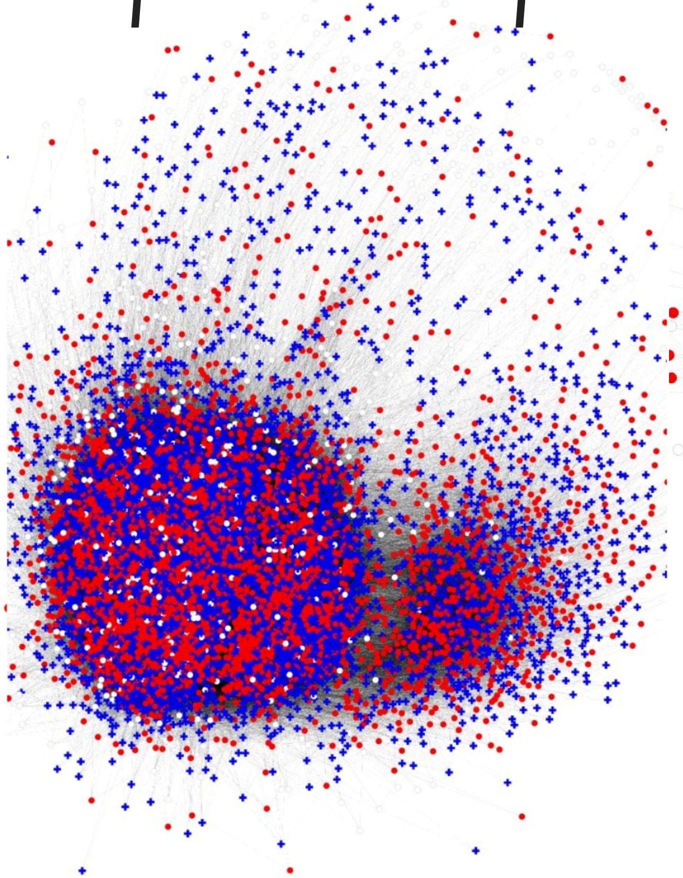


Wellesley r=0.246

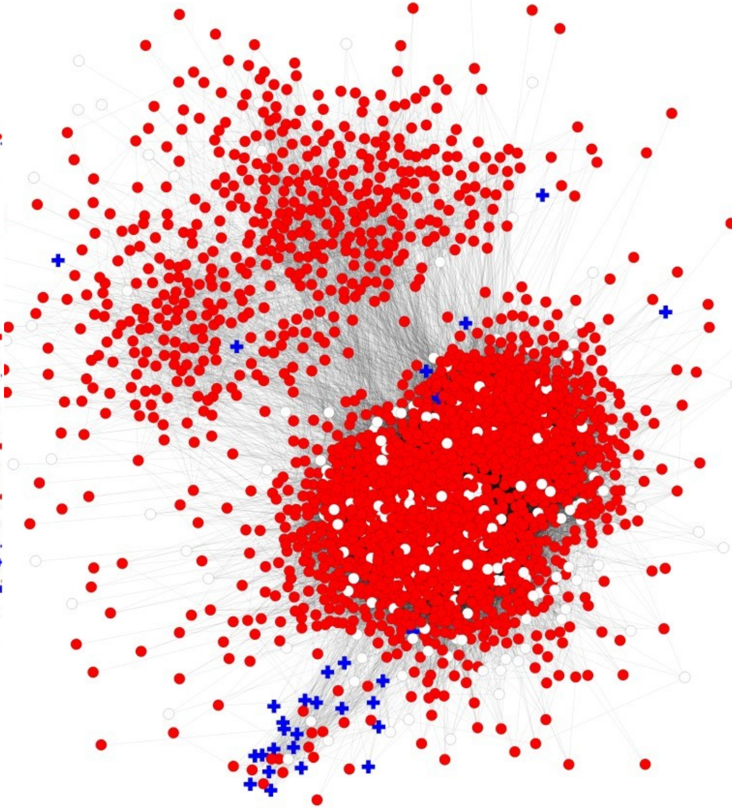
Order these networks by assortativity



Smith r=0.025
(-0.006, 0.811)



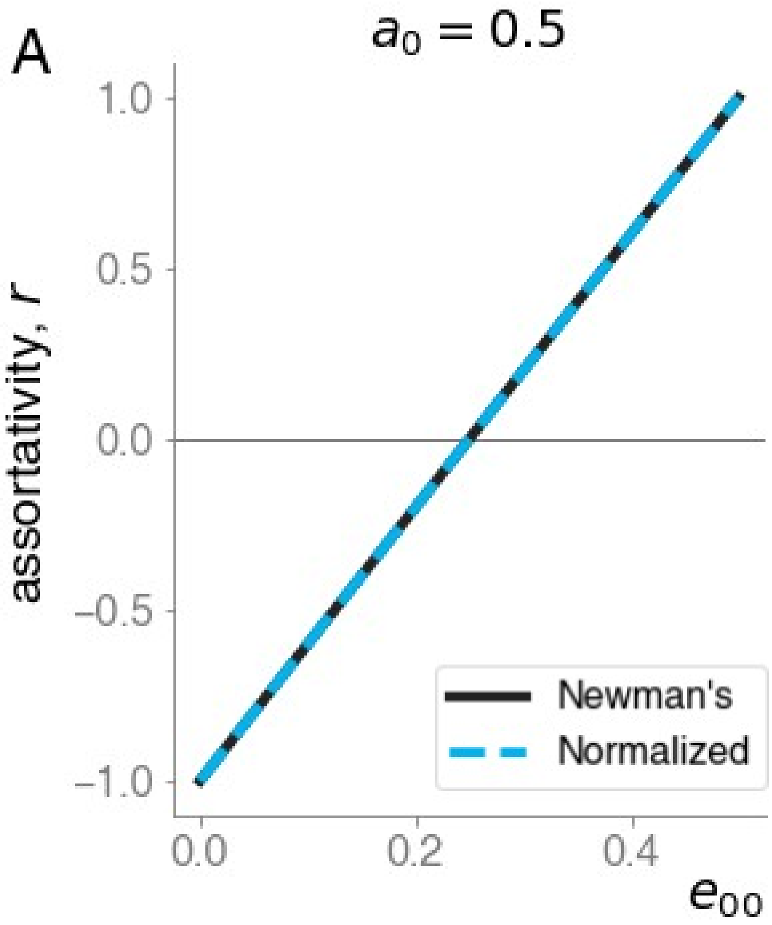
Stanford r=0.057
(-0.988, 1.000)

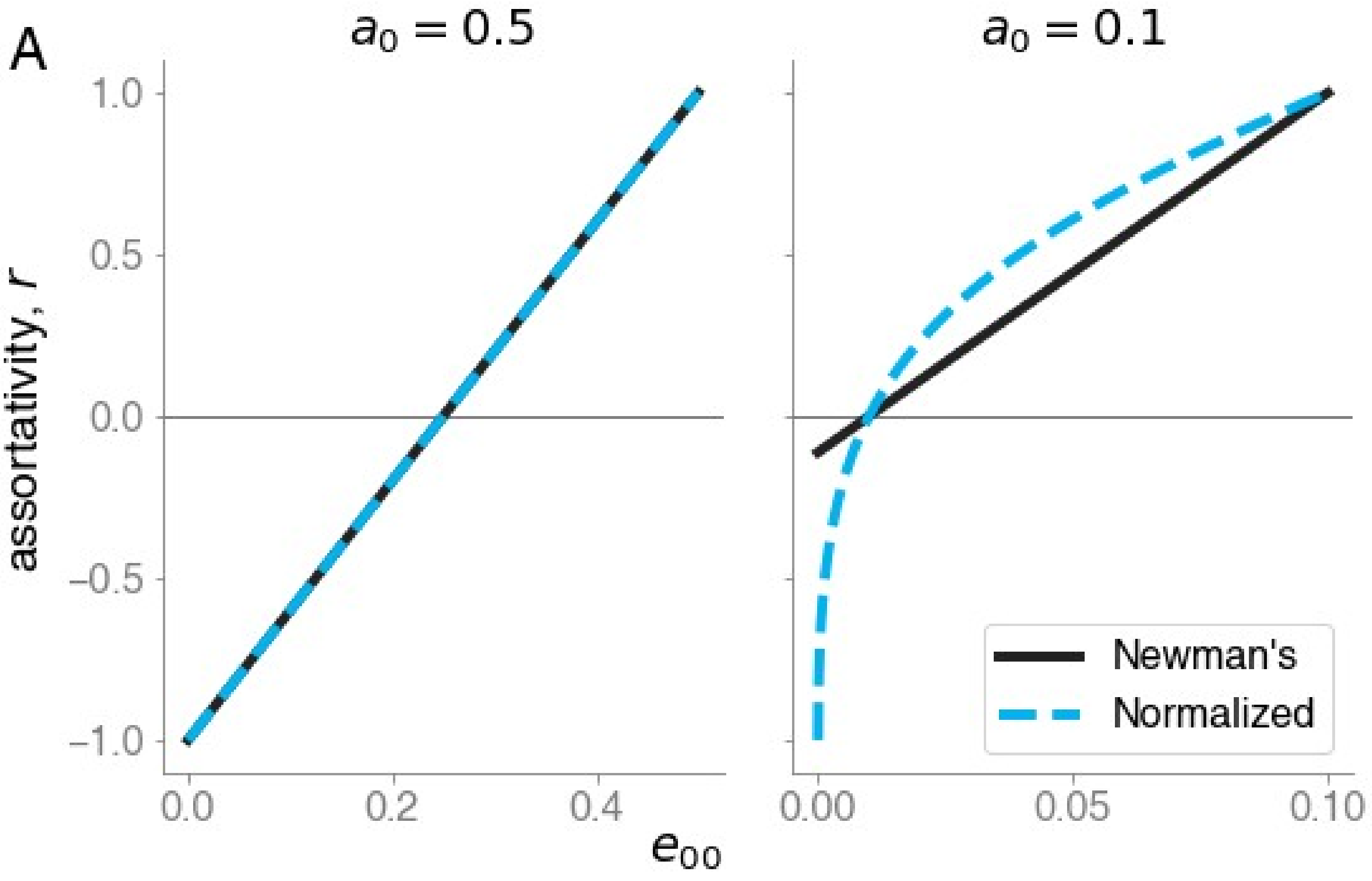


Wellesley r=0.246
(-0.009, 0.368)


WHEN YOU'RE TOLD THE RANGE OF ASSORTATIVITY IS THIS BIG
BUT REALLY IT'S ONLY THIS BIG

Can we standardise assortativity?

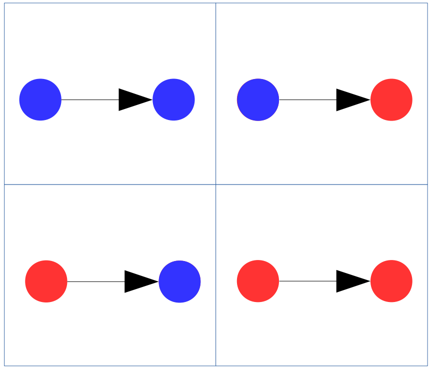

A
$a_0 = 0.5$
assortativity, r
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
e_{00}
Newman's
Normalized


A
$a_0 = 0.5$
$a_0 = 0.1$
assortativity, r
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
e_{00}
0.00
0.05
0.10
Newman's
Normalized

What does the normalisation mean?

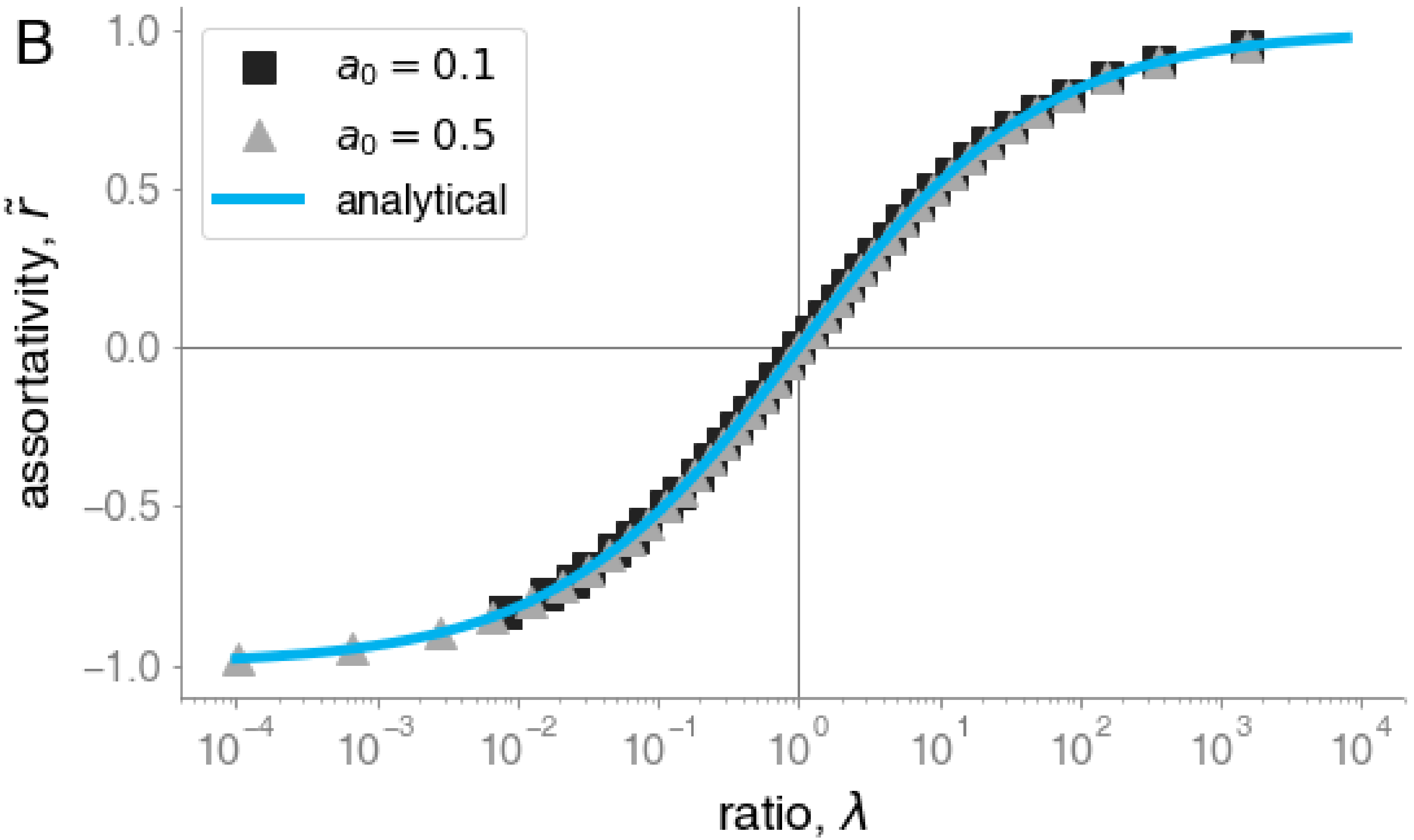
Mixing matrix

e_{gh} =



$$\lambda = \frac{e_{00}e_{11}}{e_{01}e_{10}}$$

Maintains the ratio between diagonal and off-diagonal elements



B
$a_0 = 0.1$
$a_0 = 0.5$
analytical
1.0
0.5
0.0
−0.5
−1.0
assortativity, $\tilde{r}$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
ratio, λ

How does this compare to Newman's assortativity?

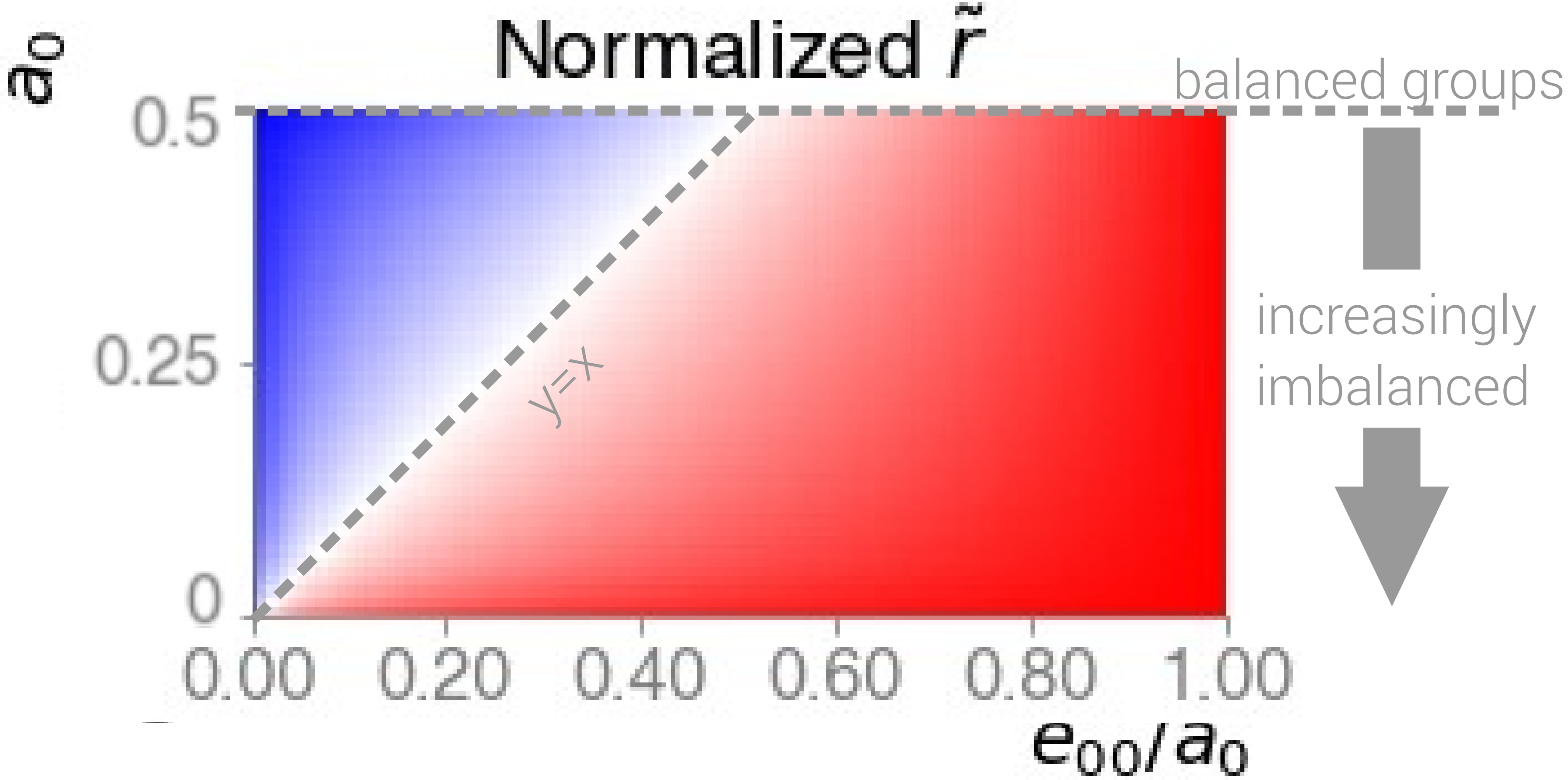
$$e_{ij}$$

$$\begin{array}{c|c} e_{00} & a_0 - e_{00} \\ \hline a_0 - e_{00} & e_{11} \end{array}$$

Mixing matrix

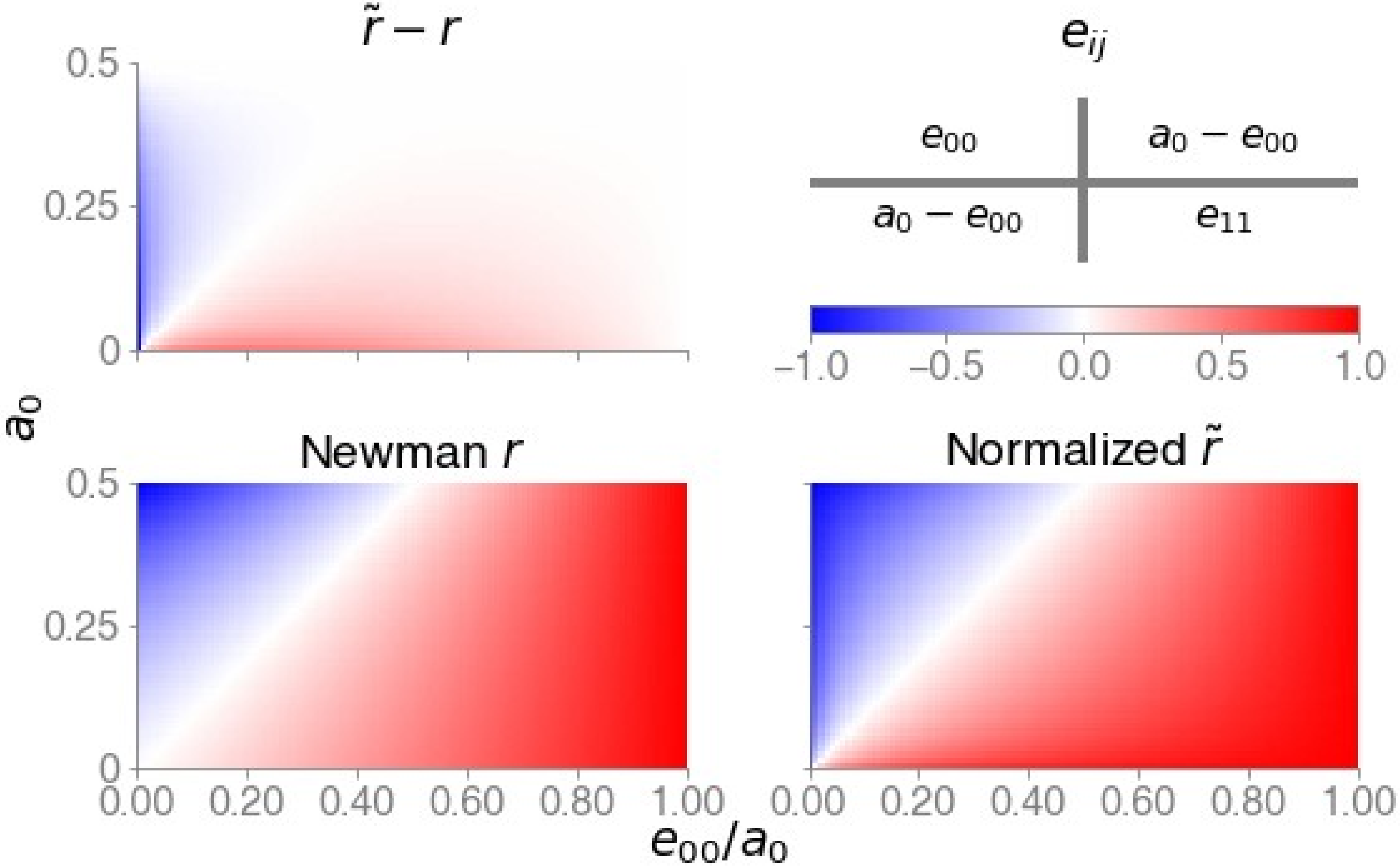
Vary two parameters:

- a0 : proportion of edges incident on minority group
- e00 : proportion of minority in-group edges

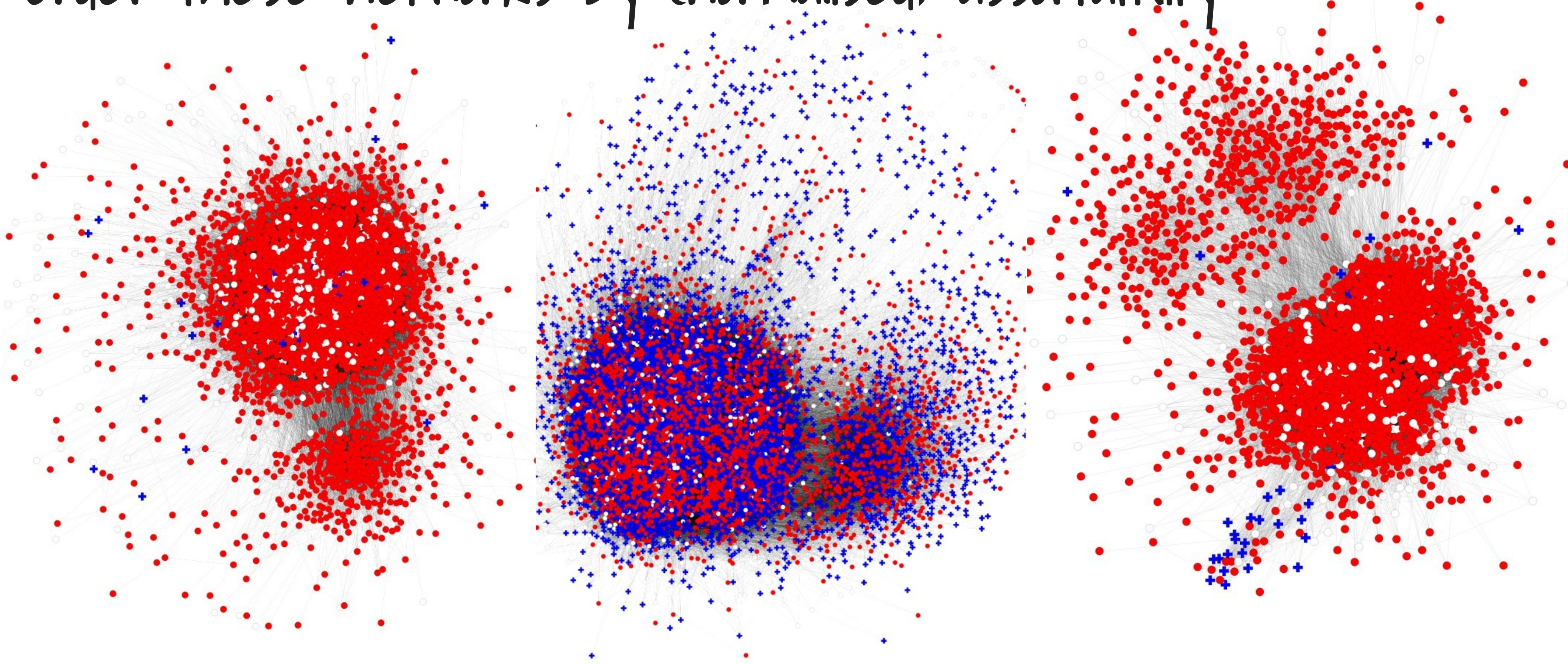


Normalized $\tilde{r}$
a_0
0.5
0.25
0
0.00
0.20
0.40
0.60
0.80
1.00
e_{00}/a_0
y=x
balanced groups
increasingly
imbalanced

How does this compare to Newman's assortativity?

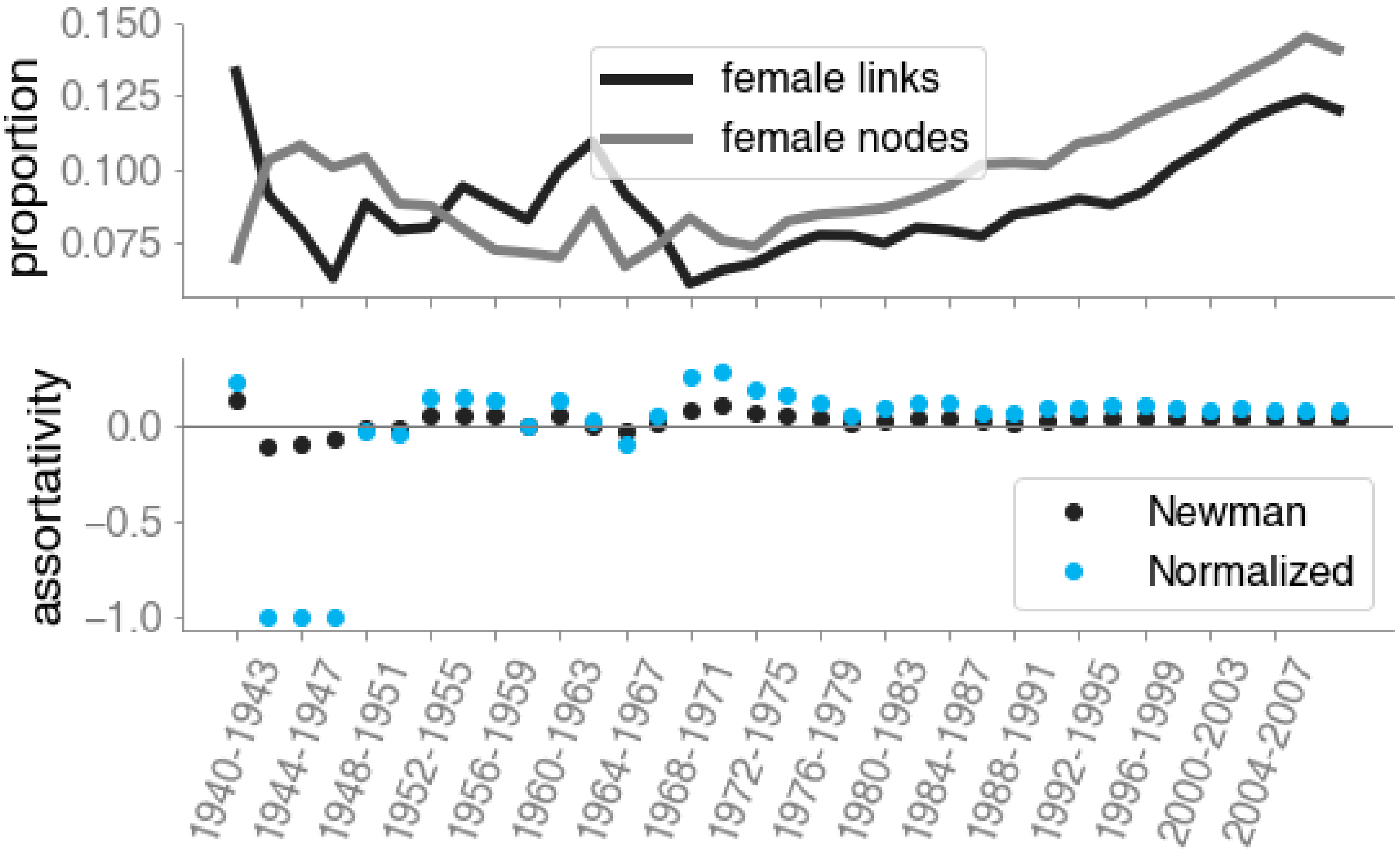


Order these networks by (normalised) assortativity

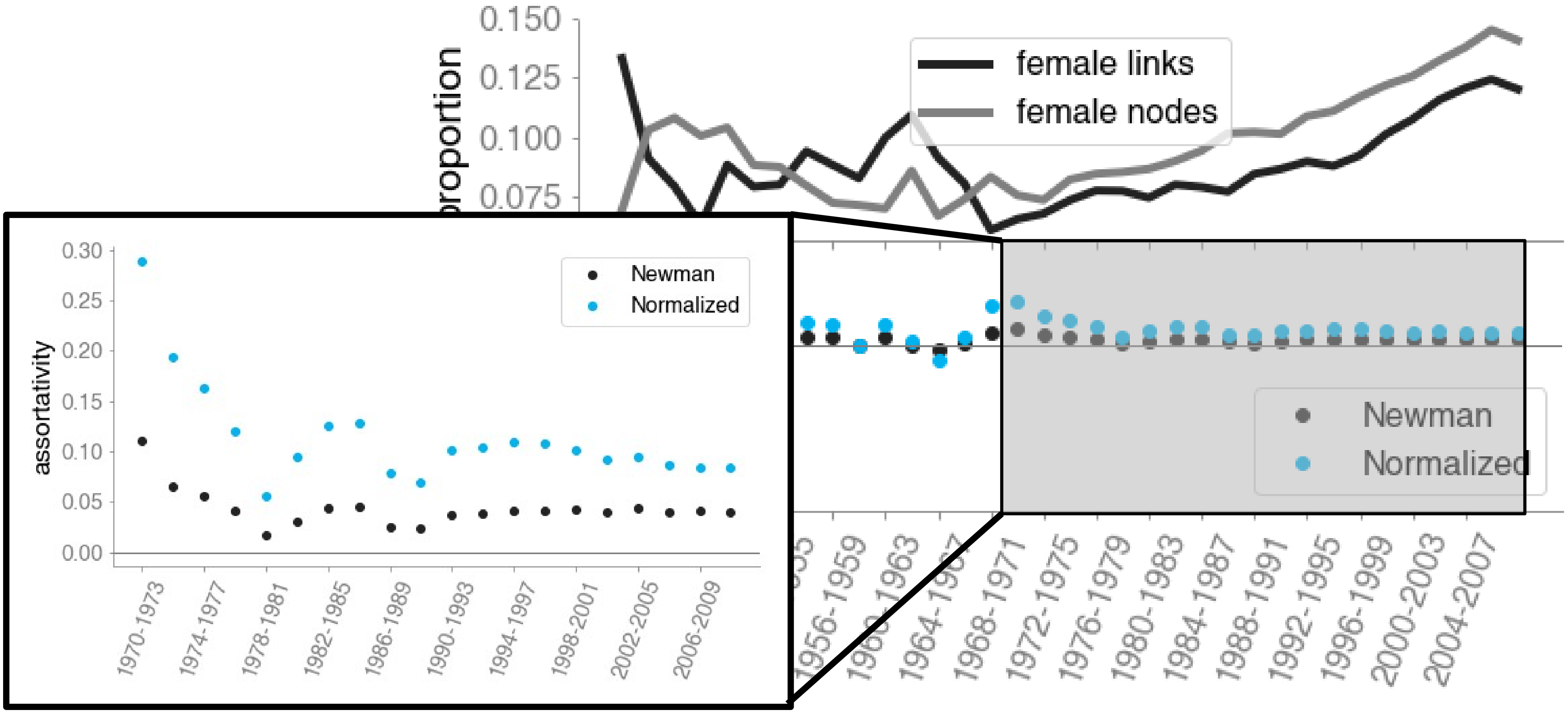


Smith r=0.325 Stanford r=0.057 Wellesley r=0.789

Physics Collaboration Network



Physics Collaboration Network



Summary

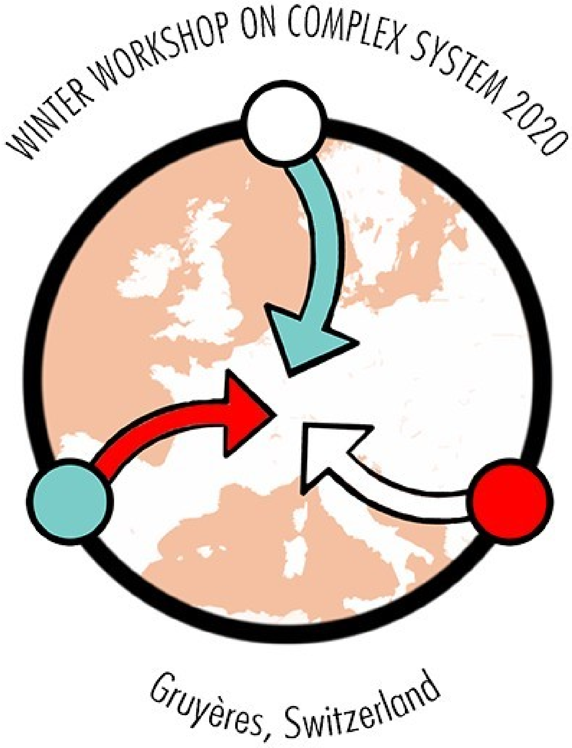
Assortativity plays an important role in understanding the organisation of complex networks

Multiscale mixing: detect heterogeneous mixing patterns in a network

Normalised assortativity: compare mixing patterns across networks?

#methodsmatter

Advertisement



https://wwcs2020.github.io/

#swisscheesesaremadeofthese

In collaboration with...

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Fariba Karimi

Mauro Faccin

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@PiratePeel

Peel, Delvenne, Lambiotte, "Multiscale mixing patterns in networks". PNAS (2018)
Cinelli, Peel, Iovanella, Delvenne, “Network constraints on the mixing patterns of binary metadata” *in prep.*
Cinelli, Faccin, Karimi, Peel, “Gender mixing preferences across networks” *in prep.*