Community detection in networks with unobserved edges

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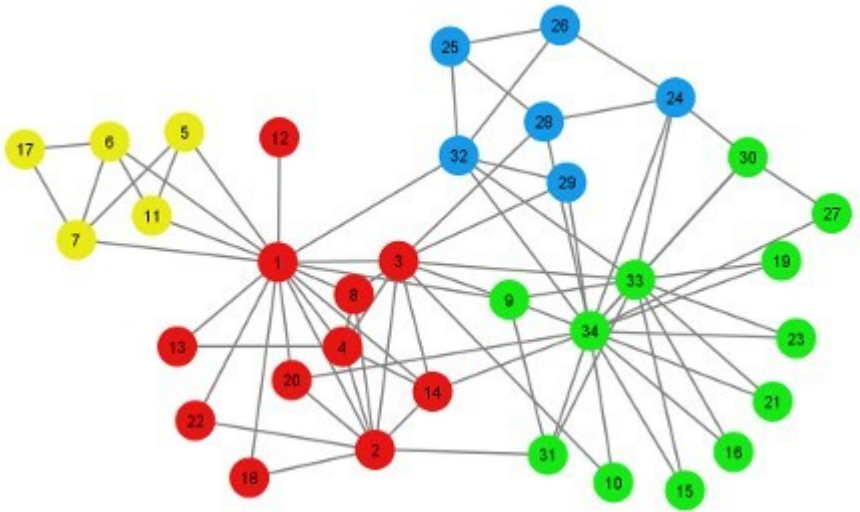
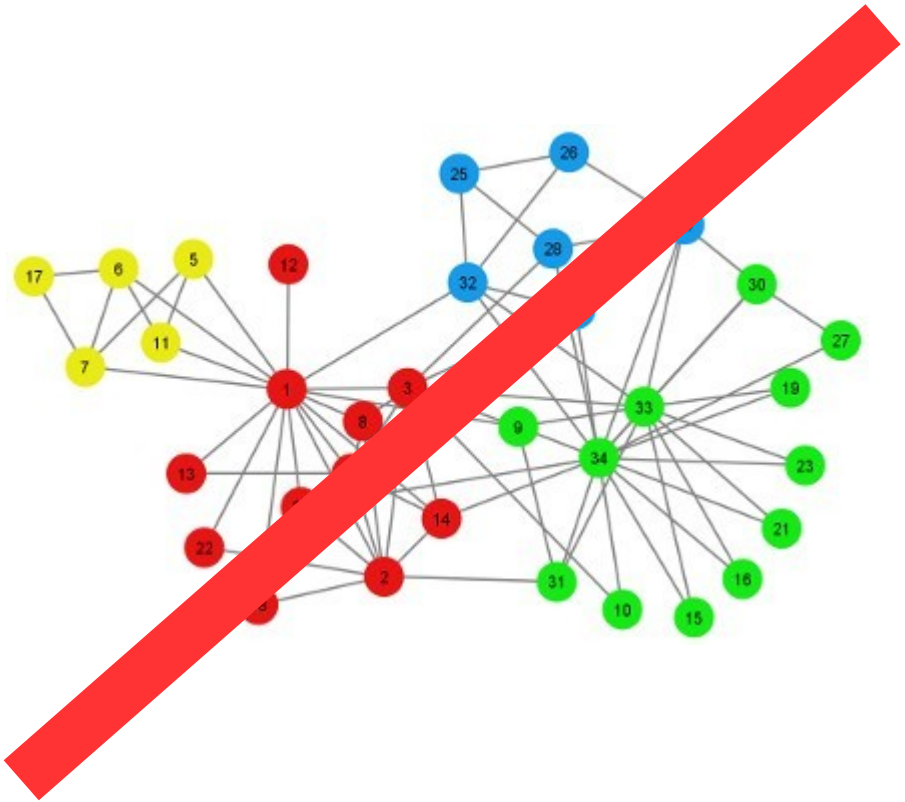
Community detection



Aim: partition the network based on the pattern of connectivity

Community detection



Aim: partition the network based on the pattern of connectivity

But we observe signals on nodes and no links!

Motivating examples...

Identify assets whose prices vary coherently to better manage risk

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Identify regions of the brain to predict the onset of psychosis and learn about the ageing of the brain

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Identify regions of the brain to predict the onset of psychosis and learn about the ageing of the brain

Identify assets whose prices vary coherently to better manage risk

Identify climate zones to better understand factors affecting our climate

Is there really a network?

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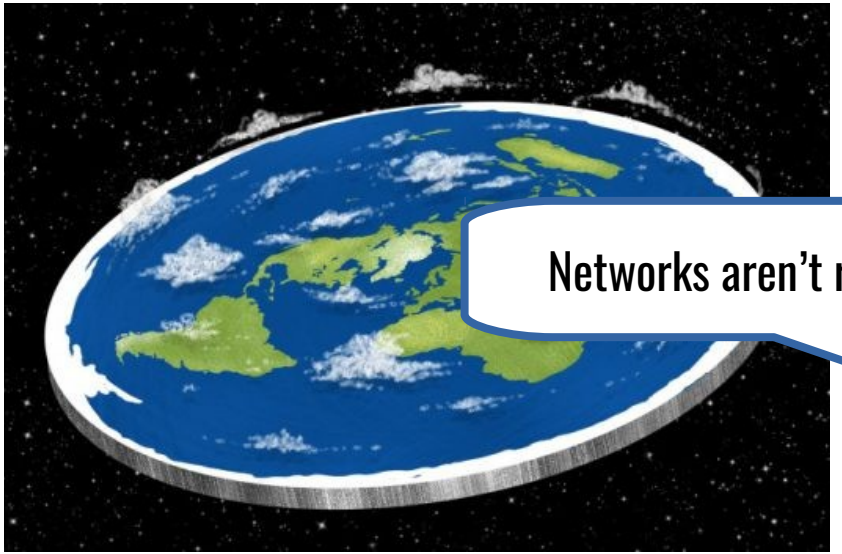

We don't have to directly observe something to believe it is true

Common practise

- Calculate pairwise correlations between signals (e.g. Pearson's).
- Threshold (and Binarize) the matrix of correlations.
- Perform community detection on this (notional) network

Problems

- This procedure commonly invokes point-estimates at each step
  - Does not capture the uncertainty of individual links

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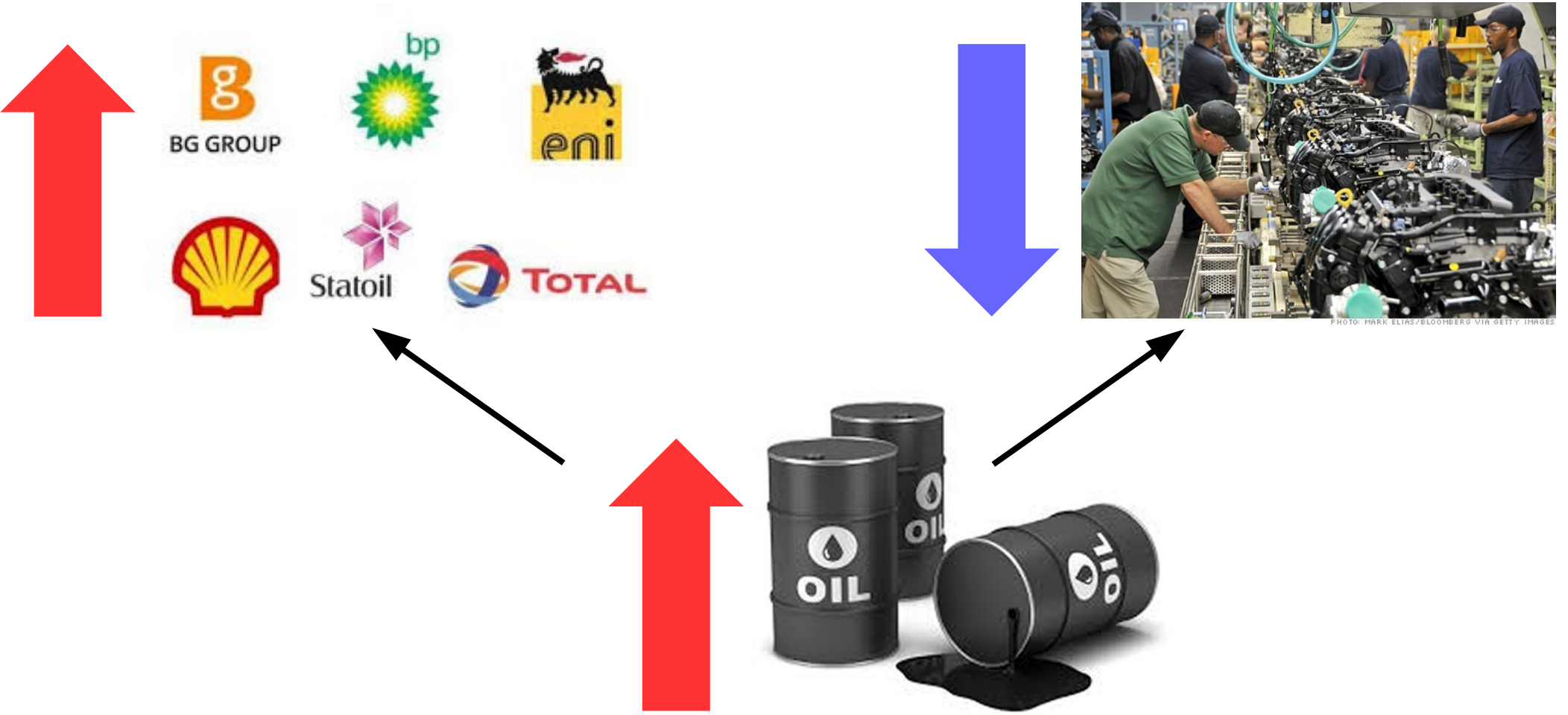
  - Does not capture the uncertainty of individual links

- Unclear how to include missing data.

- No intrinsic/clear notion of the right number of communities.

The signals we observe from many nodes are driven by a few latent factors



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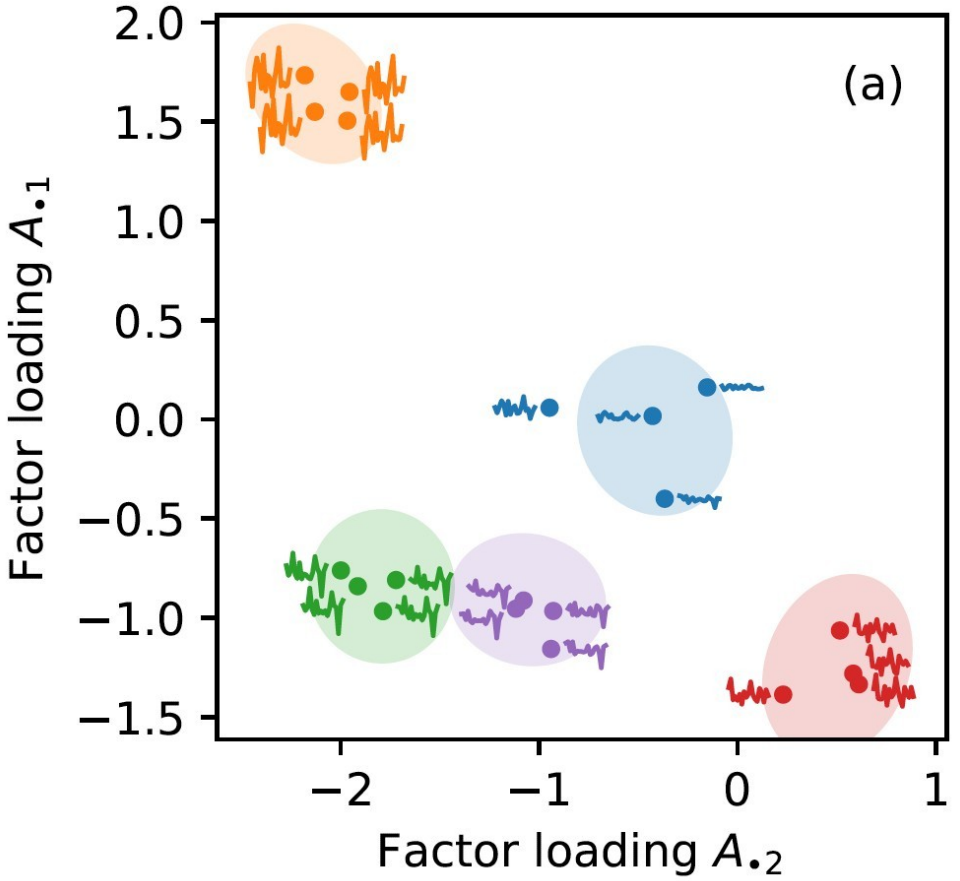
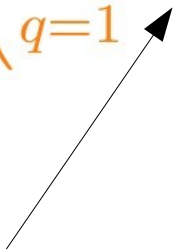


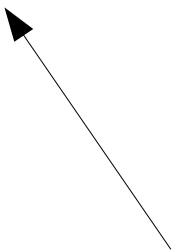
Notion of a community is: a group of nodes that influenced similarly by the latent factors

$$y_{ti}|A, x, \tau \sim \text{Normal}\left(\sum_{q=1}^{p} x_{tq} A_{iq}, \tau_i^{-1}\right)$$

Observed time series



Latent factor time series



Factor loadings

$$y_{ti}|A, x, \tau \sim \text{Normal}\left(\sum_{q=1}^{p} x_{tq} A_{iq}, \tau_i^{-1}\right)$$

Community mean

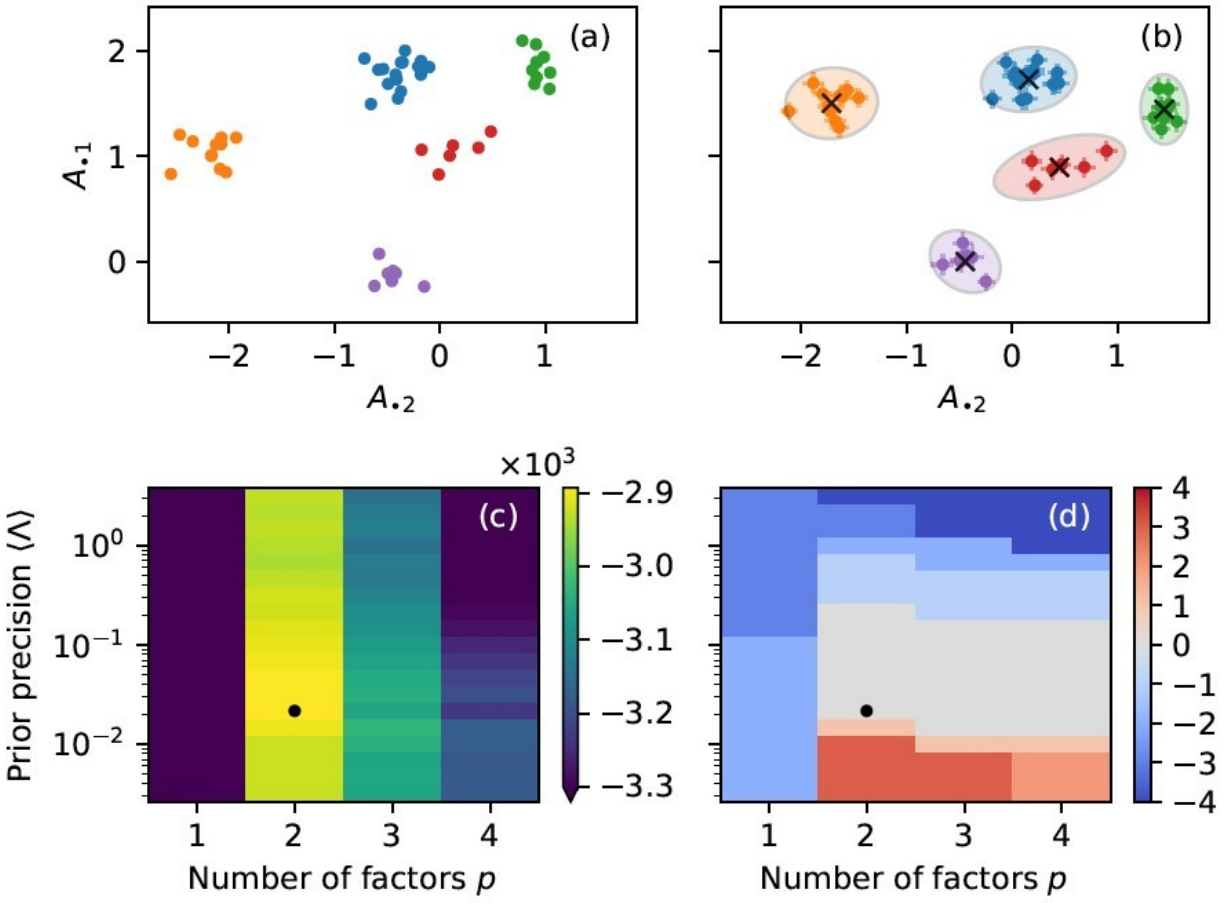
Community precision

$$A_i \sim \sum_{k=1}^{K} z_{ik} \text{Normal}\left(\mu_k, \Lambda_k^{-1}\right),$$

$$\text{where } z_{ik} = \begin{cases} 1 & \text{if } g_i = k \\ 0 & \text{otherwise} \end{cases}.$$

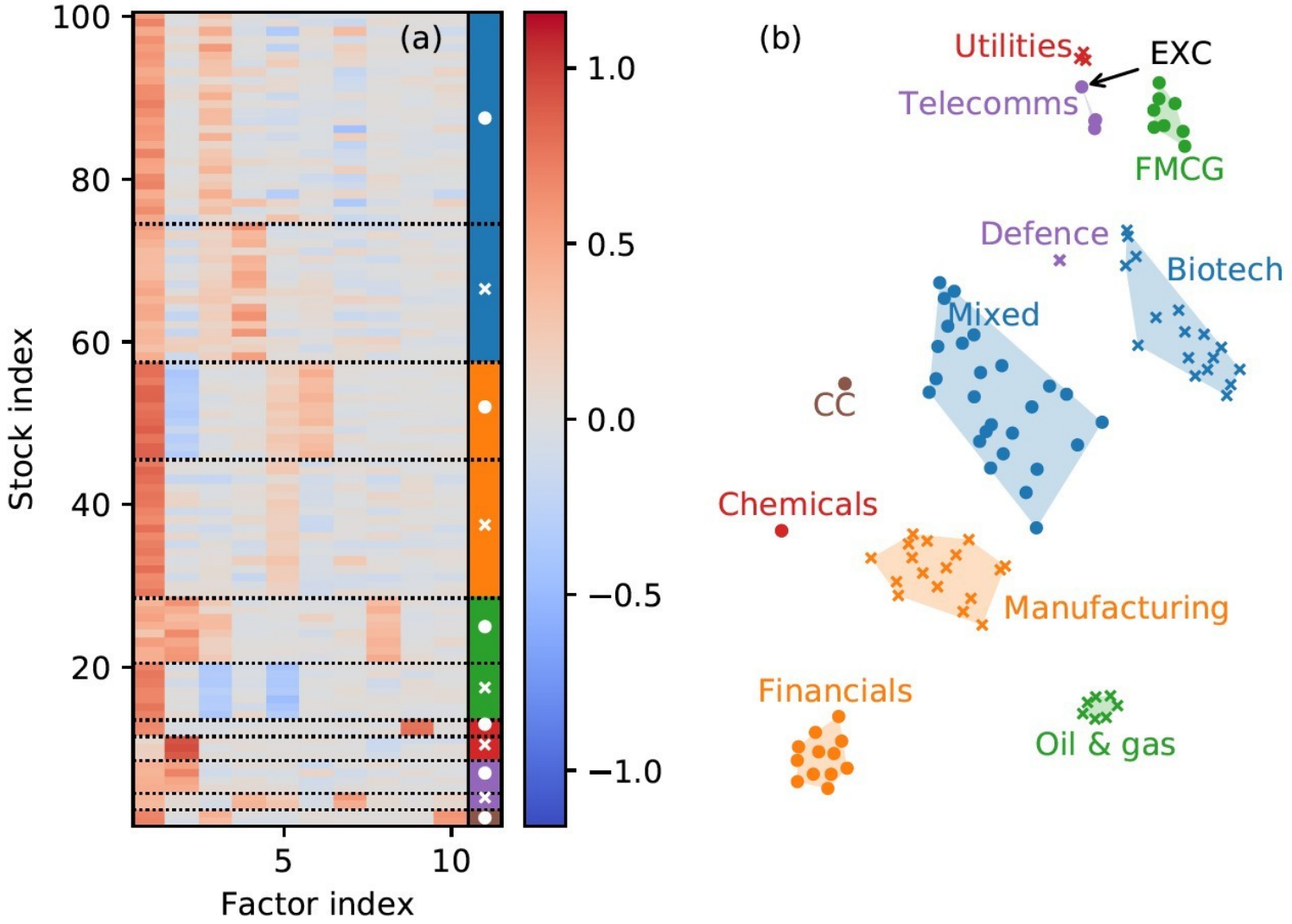
Generated

Inferred

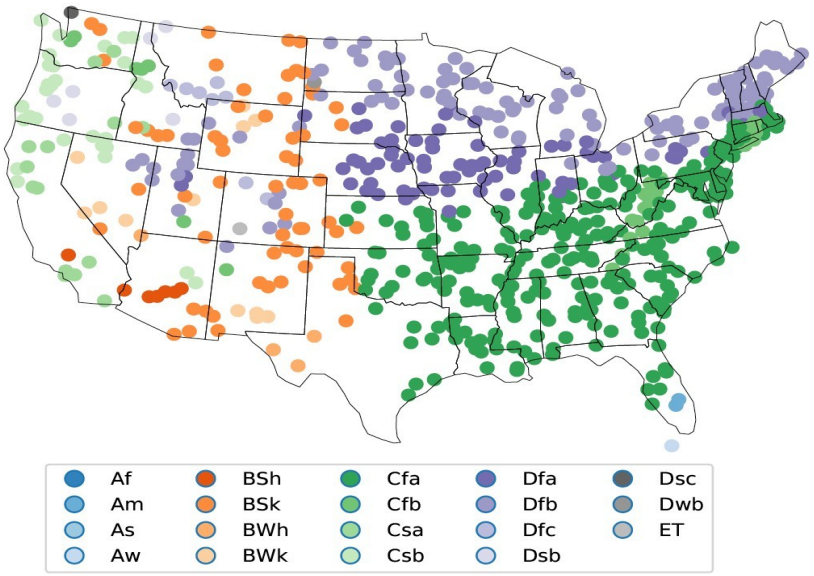


Lower bound on the marginal likelihood (ELBO)

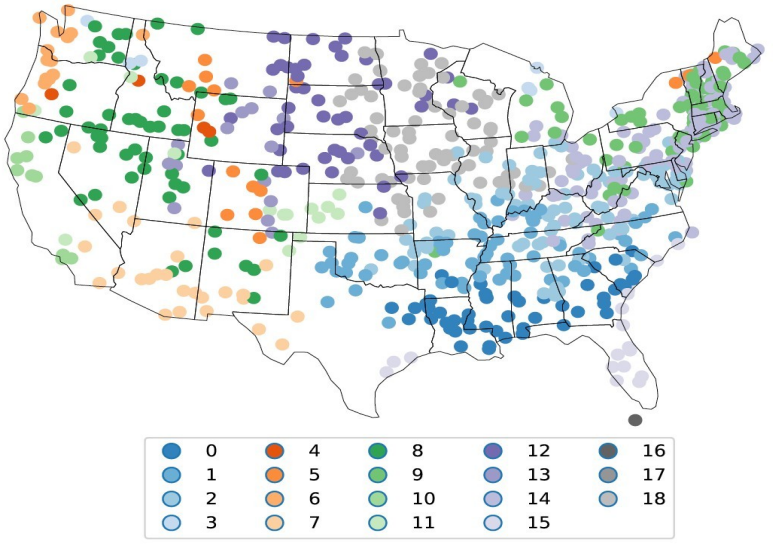
Difference between $K_{generated}$ and $K_{inferred}$


(a)
100
80
60
40
20
Stock index
5
10
Factor index
1.0
0.5
0.0
−0.5
−1.0
(b)
Utilities
EXC
Telecomms
FMCG
Defence
Biotech
Mixed
CC
Chemicals
Manufacturing
Financials
Oil & gas

US cities climate data



Koppen climate zones



inferred climate zones

Predicting missing data

Our method $A_i^T x$
0.301

Köppen-Geiger (*51*)
0.706

RMSE predicting held-out climate signals

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Köppen-Geiger (*51*)	Fenn *et al.* (*2*)
0.706	0.727

RMSE predicting held-out climate signals

Predicting missing data

Node-level prediction

Our method $A_i^T x$
0.301

Community-level prediction

Köppen-Geiger (*51*)	Fenn *et al.* (*2*)
0.706	0.727

Community-level prediction

RMSE predicting held-out climate signals

Predicting missing data

Node-level prediction	Community-level prediction	Community-level prediction	Community-level prediction
Our method $A_i^T x$	**Our method $\mu_{g_i}^T x$**	**Köppen-Geiger (*51*)**	**Fenn *et al.* (*2*)**
0.301	0.578	0.706	0.727

RMSE predicting held-out climate signals

What happened to the network?

- Since we skip explicit interpretation of A our inference framework is basically a Bayesian (time-series) clustering.

- One can re-interpret AA^T as a network, or interpret distances between time-series in the latent-space as links in a network, but this is optional.

Is a network appropriate?

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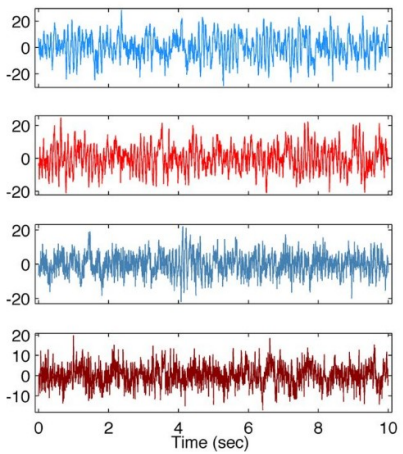

Problem solved?

In collaboration with...

Till Hoffmann

Nick Jones

Renaud Lambiotte

Preprint available: arXiv:1808.06079

Code available: https://github.com/tillahoffmann/time_series/

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