Community detection in networks with unobserved edges

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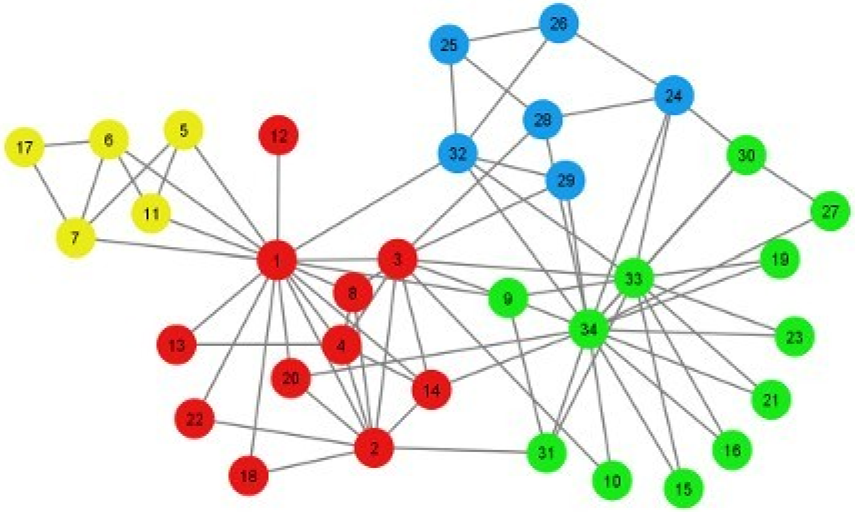
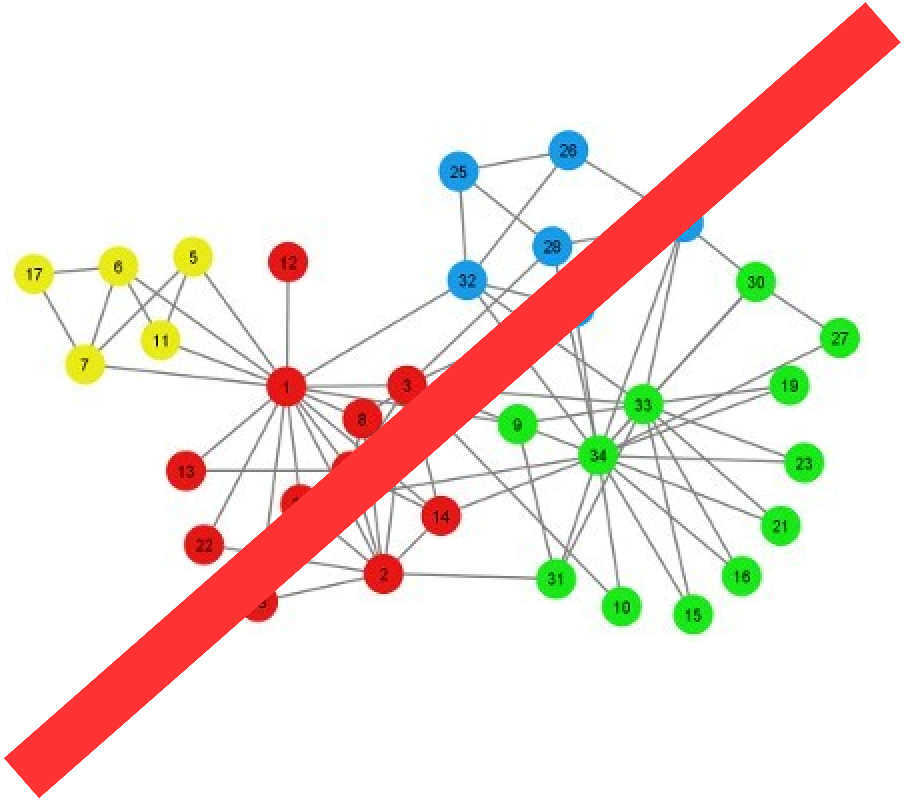
Community detection



Aim: partition the network according similarity of link structure

Community detection



Aim: partition the network according similarity of link structure

But we observe signals on nodes and no links!

Motivating examples…

Identify assets whose prices vary coherently to better manage risk

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Identify regions of the brain to predict the onset of psychosis and learn about the ageing of the brain

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Identify assets whose prices vary coherently to better manage risk

Identify climate zones to better understand factors affecting our climate

Is there really a network?

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We don’t have to directly observe something to believe it is true

Common practise

- Calculate pairwise correlations between signals (e.g. Pearson’s).
- Threshold (and Binarize) the matrix of correlations.
- Perform community detection on this (notional) network

Problems

- This procedure commonly invokes point-estimates at each step
  - Does not capture the uncertainty of individual links

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  - Does not capture the uncertainty of individual links

- Unclear how to include missing data.

- No intrinsic/clear notion of the right number of communities.

The signals we observe from many nodes are driven by a few latent factors

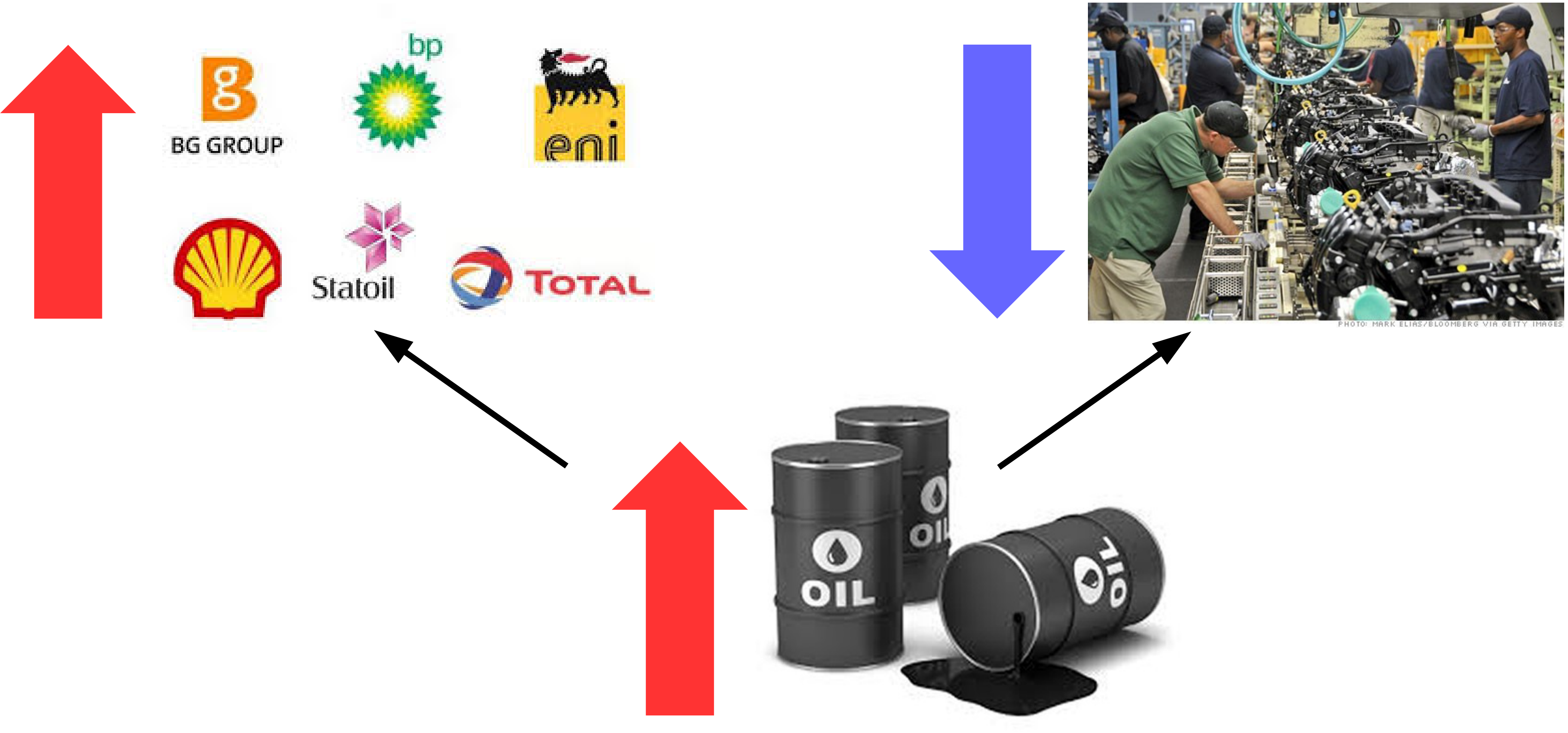
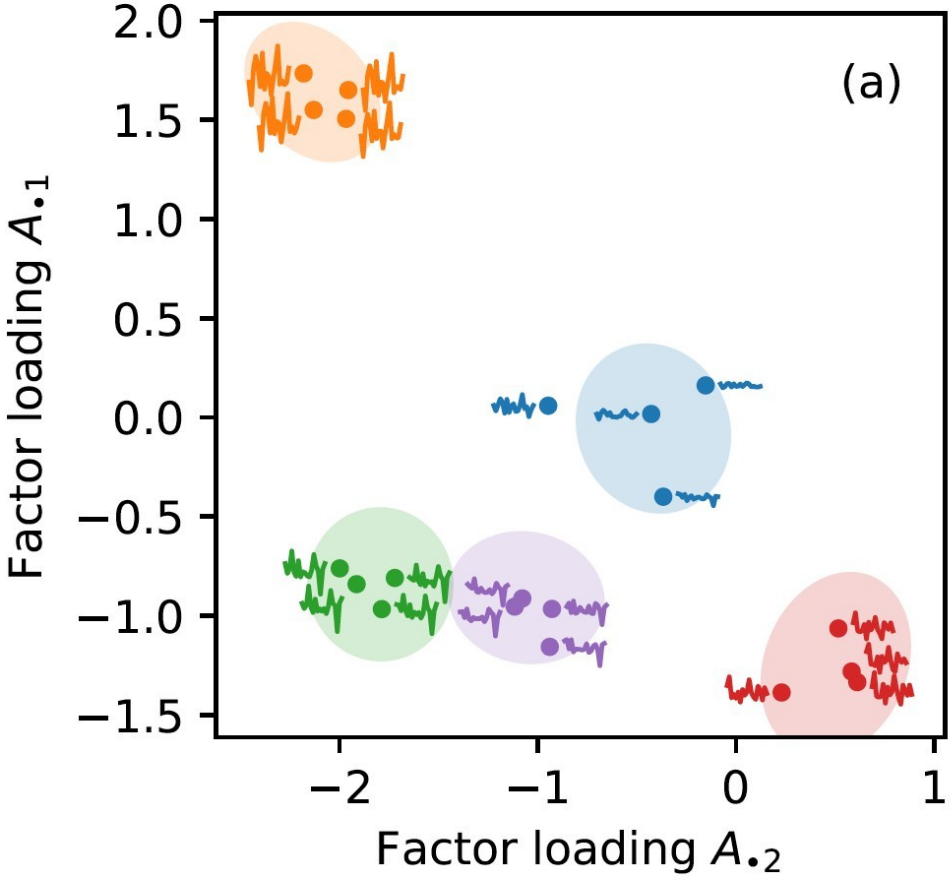


PHOTO: MARK ELIAS/BLOOMBERG VIA GETTY IMAGES

The signals we observe from many nodes are driven by a few latent factors



Notion of a community is: a group of nodes that influenced similarly by the latent factors

$$y_{ti} | A, x, \tau \sim \mathrm{Normal}\left(\sum_{q=1}^{p} x_{tq} A_{iq}, \tau_i^{-1}\right)$$

Observed time series

Latent factor time series

Factor loadings

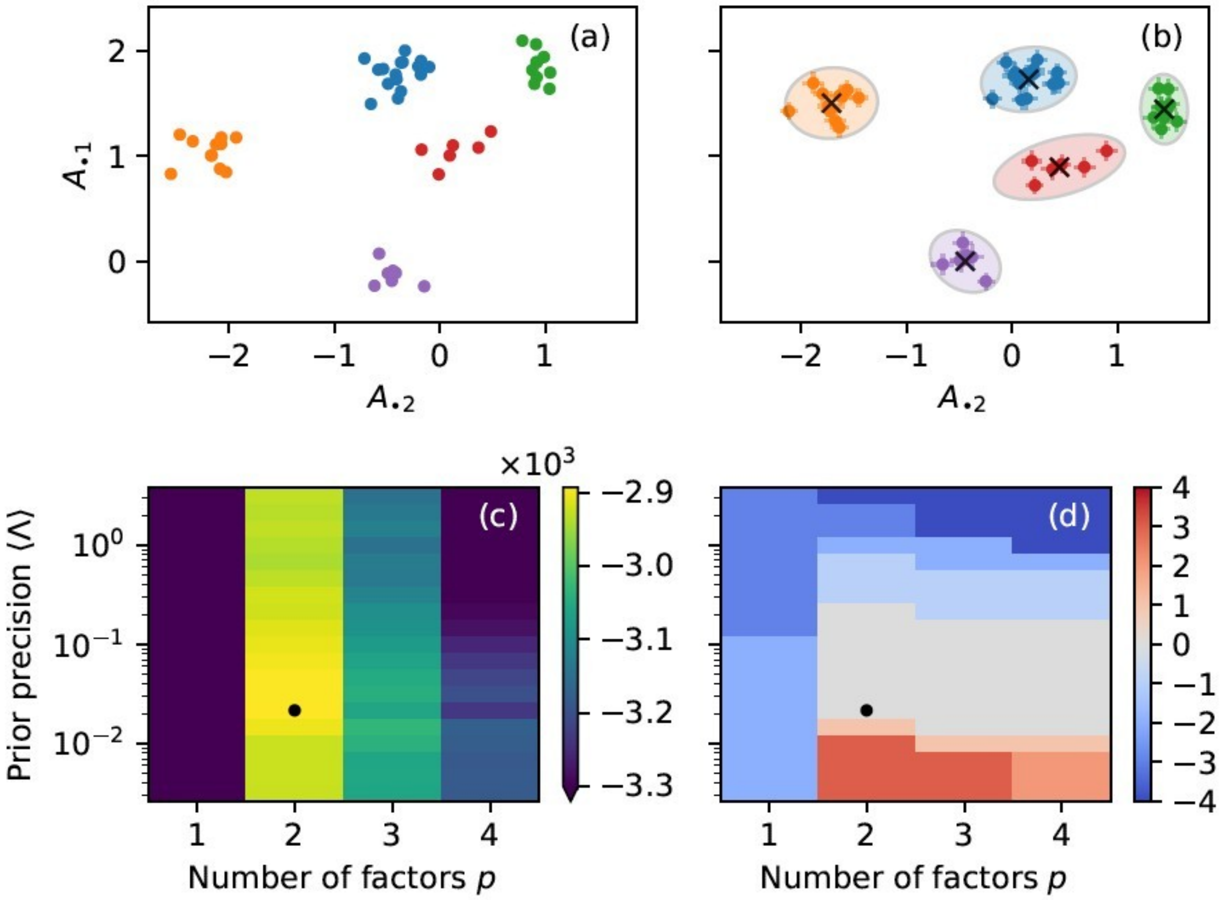
$$y_{ti}|A, x, \tau \sim \text{Normal}\left(\sum_{q=1}^{p} x_{tq}A_{iq}, \tau_i^{-1}\right)$$

$$A_i \sim \sum_{k=1}^{K} z_{ik}\text{Normal}\left(\overset{\text{Community mean}}{\mu_k}, \overset{\text{Community precision}}{\Lambda_k^{-1}}\right),$$

$$\text{where } z_{ik} = \begin{cases} 1 & \text{if } g_i = k \\ 0 & \text{otherwise} \end{cases}.$$

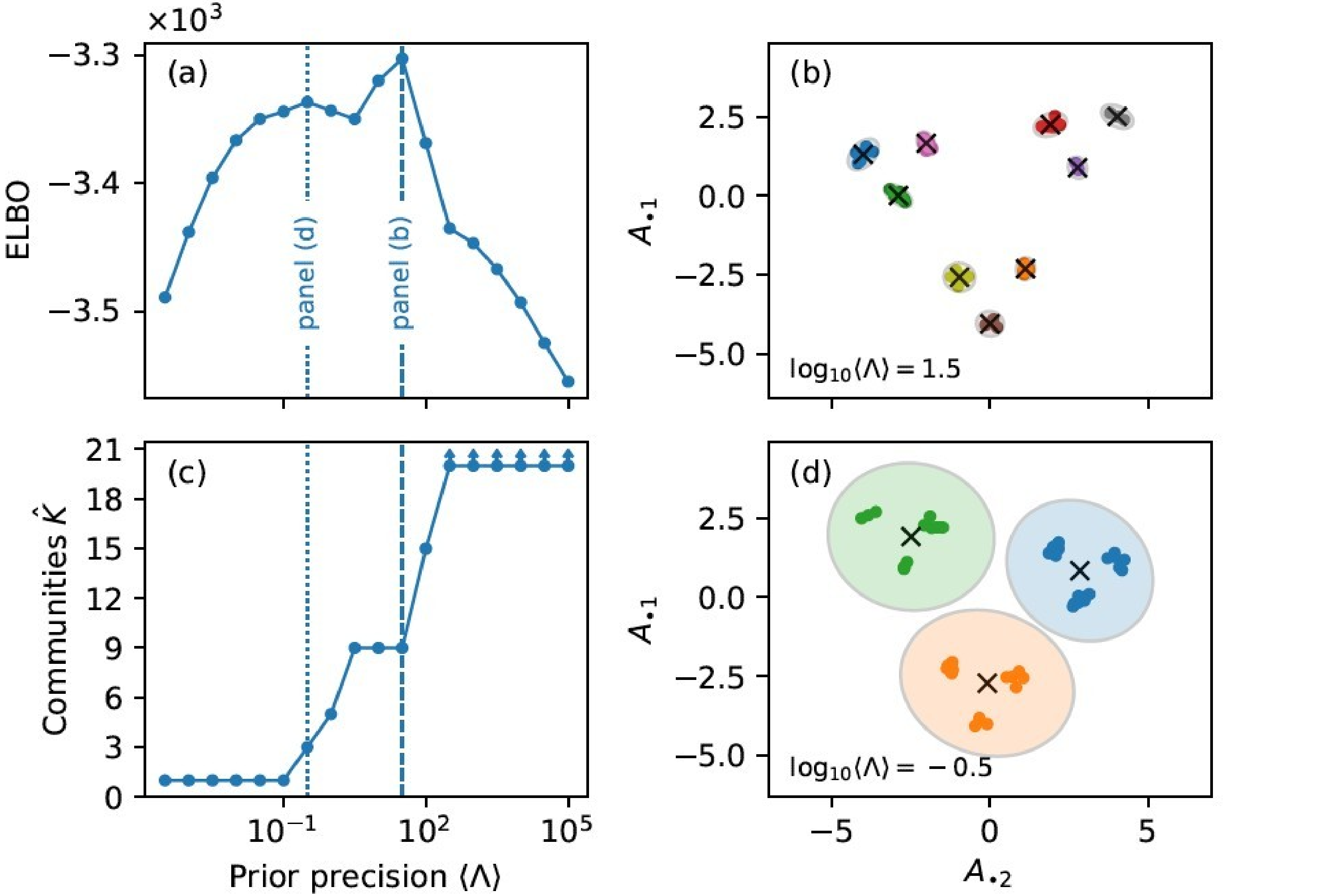
Generated

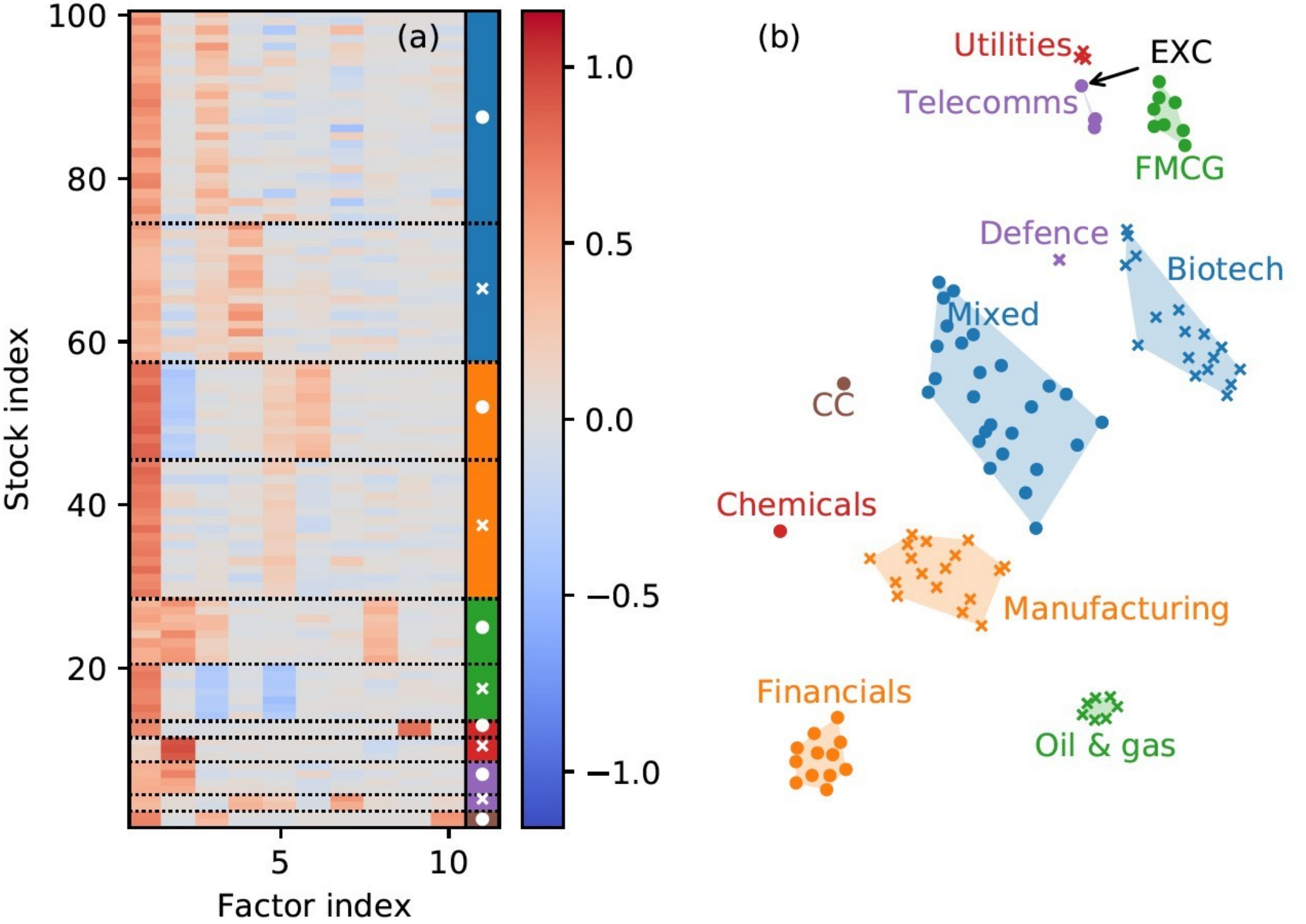
Inferred



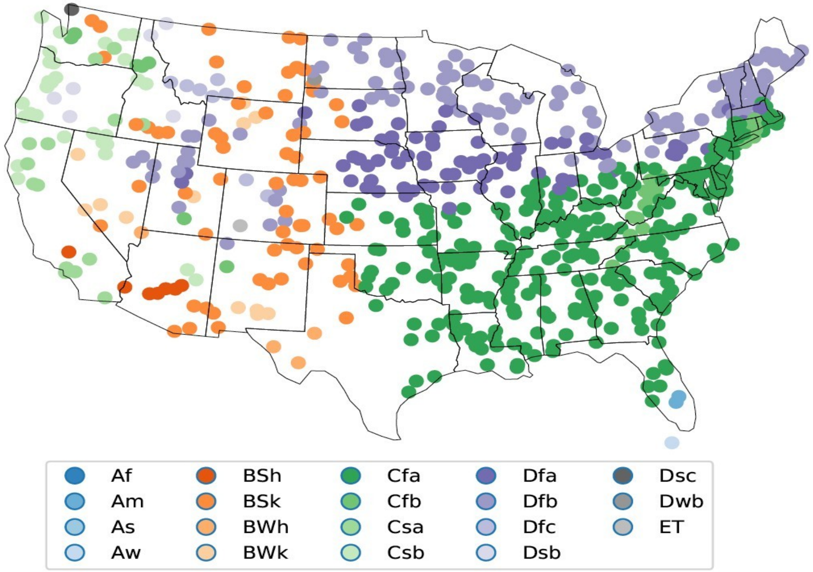
Lower bound on the marginal likelihood (ELBO)

Difference between $K_{generated}$ and $K_{inferred}$

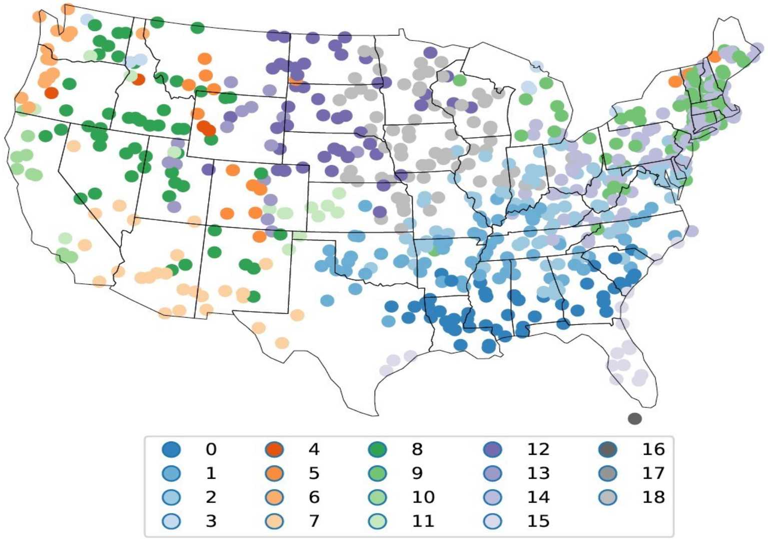

×10^3
−3.3
−3.4
−3.5
ELBO
(a)
panel (d)
panel (b)
(b)
2.5
0.0
−2.5
−5.0
A_{•1}
log_{10}⟨Λ⟩ = 1.5
(c)
21
18
15
12
9
6
3
0
Communities K̂
10^{−1}
10^2
10^5
Prior precision ⟨Λ⟩
(d)
2.5
0.0
−2.5
−5.0
A_{•1}
log_{10}⟨Λ⟩ = −0.5
−5
0
5
A_{•2}


(a)
Stock index
100
80
60
40
20
5
10
Factor index
1.0
0.5
0.0
−0.5
−1.0
(b)
Utilities
EXC
Telecomms
FMCG
Defence
Biotech
Mixed
CC
Chemicals
Manufacturing
Financials
Oil & gas

US cities climate data



Koppen climate zones

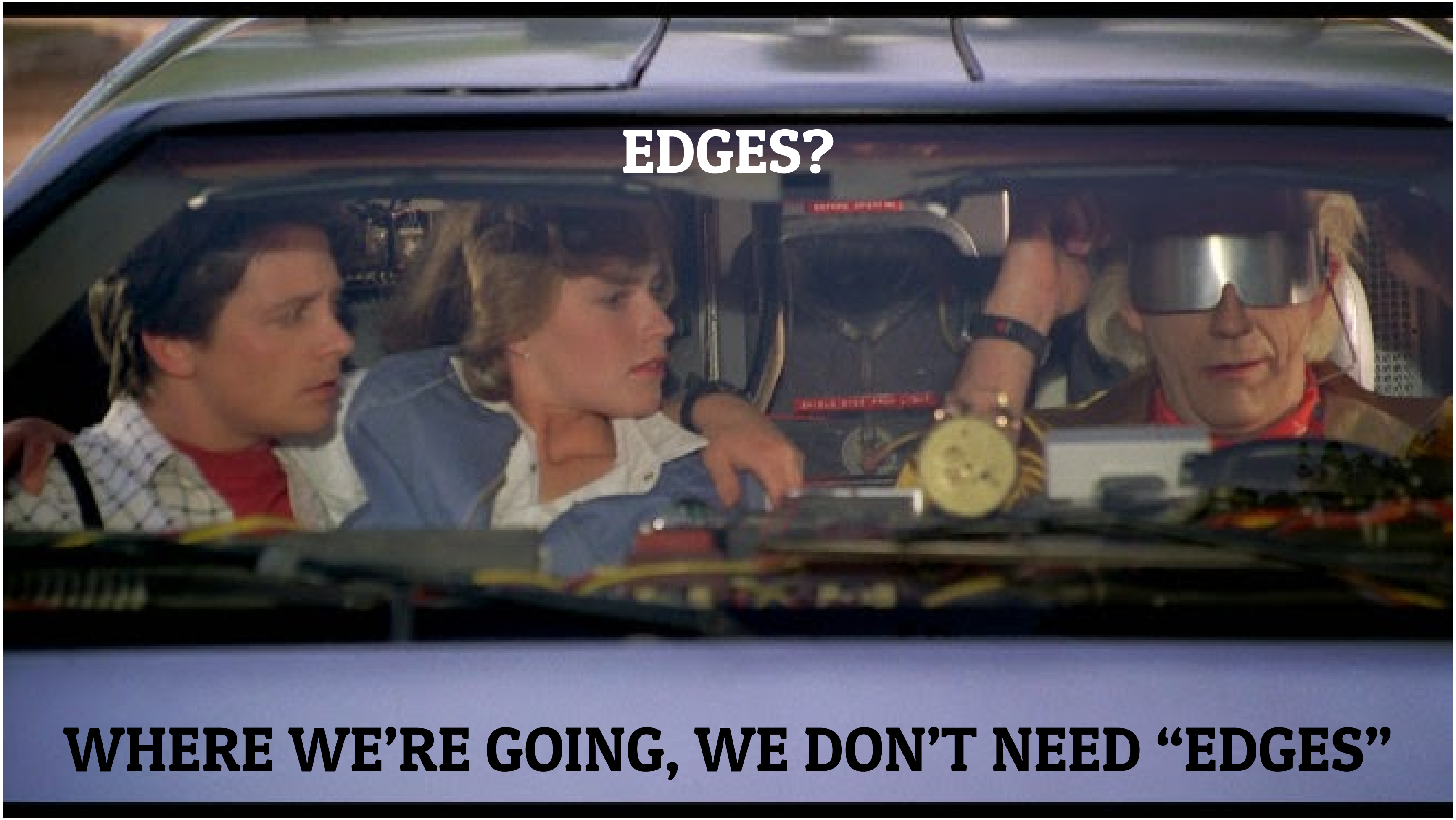


inferred climate zones

What happened to the network?

- Since we skip explicit interpretation of A our inference framework is basically a Bayesian (time-series) clustering.

- One can re-interpret AA^T as a network, or interpret distances between time-series in the latent-space as links in a network, but this is optional.


EDGES?
WHERE WE’RE GOING, WE DON’T NEED “EDGES”

In collaboration with…

Till
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Nick
Jones

Renaud
Lambiotte

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