# Physics Intersection Project
# Traffic Light Equation Derivations

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December 3, 2017

## 1 Green Light

The time that the green light stays lit is dependent on the time it takes for all of the cars waiting at the intersection to pass. To account for a specified or average amount of cars piled up at the intersection, only the last car in line is considered.

### Finding the distance a car travels when accelerating to the speed limit

*All lengths and distances in this lab are measured in meters.*
Let $a$ = car rate of acceleration

$$v = \cancel{v_o} + at$$

$$t = \frac{v}{a} = \frac{v_{speedLimit}}{a}$$

$$\bar{v} = \frac{v_{speedLimit}}{2} \quad because\ v_o\ is\ zero$$

$$\boxed{\Delta x_{distAccel} = \bar{v}t = \frac{v_{speedLimit}^2}{2a}}$$

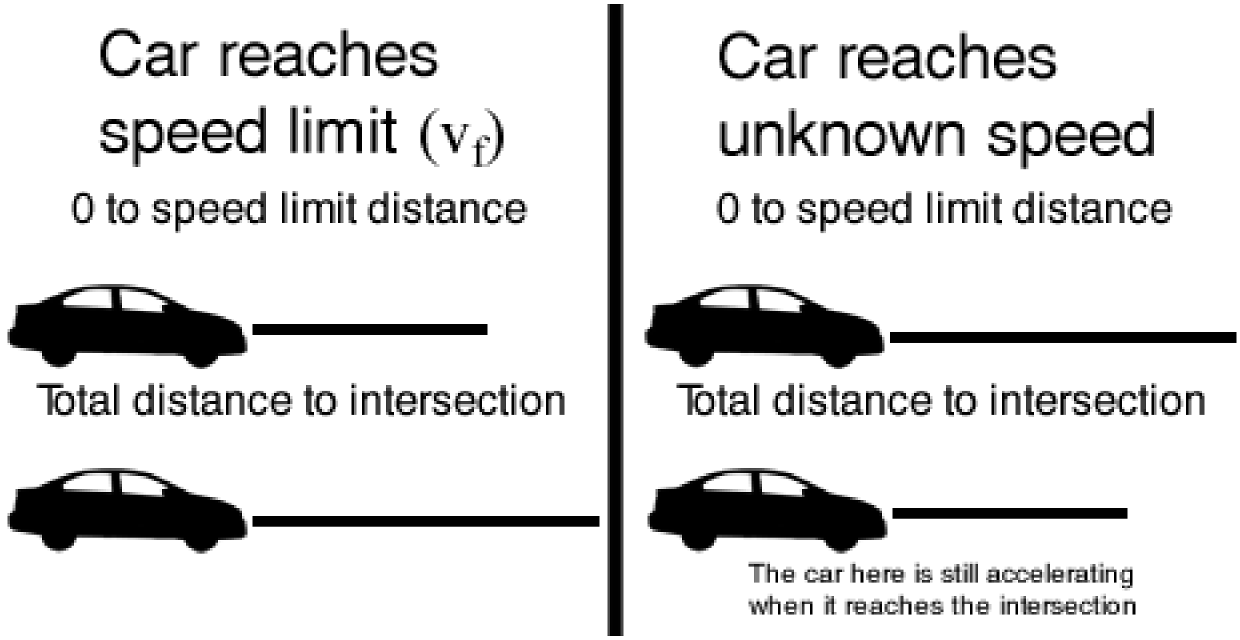
### Finding the distance that the last car in line needs to travel

Each car has a car length and a gap behind the car in front of it (except for the first car).

Let $n$ = position/order of the last car

$$\boxed{\Delta x_{distToIntersection} = n\ (gapLength + carLength)}$$

## Green light time equation (2 scenarios)




As shown above, there are two scenarios that the green light must account for when provided with a different number of cars waiting in a lane.

### Scenario 1

The last car reaches the speed limit and passes the intersection after it is at full speed, meaning that the acceleration distance is less than the distance to the intersection.

$$\boxed{\Delta t_{green1} = \frac{v_{speedLimit}}{a} + \frac{\Delta x_{distToIntersection} - \Delta x_{distAccel}}{v_{speedLimit}}}$$

The first part of the above equation is the time it takes for the car to accelerate to the speed limit. Added to the first part is the remaining time the car cruises at the speed limit to reach the intersection.

### Scenario 2

The last car doesn't reach the speed limit. Even when passing the intersection, the car is still accelerating, so the final velocity is unknown.

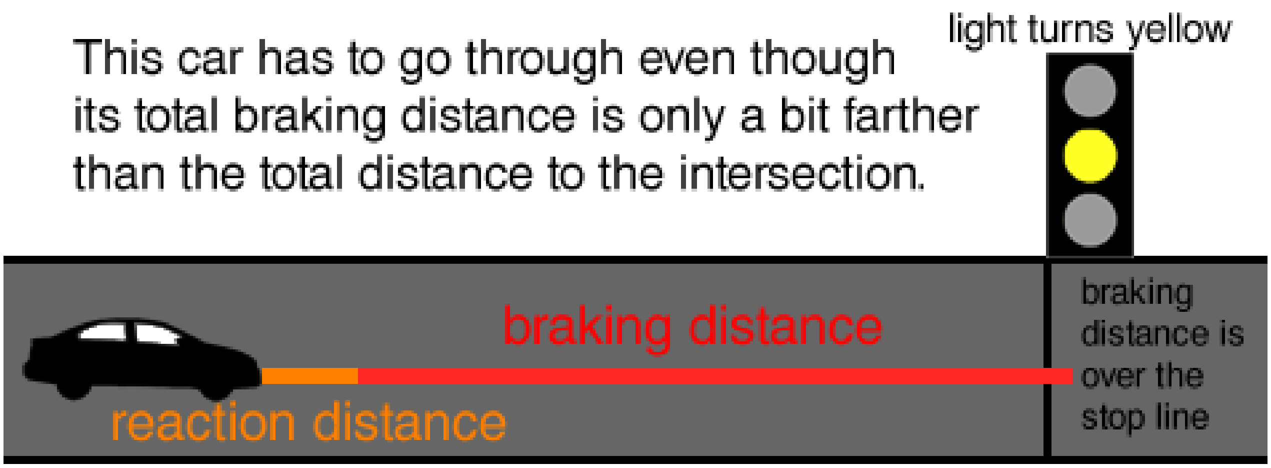
$$\Delta x = \frac{1}{2}at^2 + \cancel{v_o t + x_o}$$

$$\Delta t = \sqrt{\frac{2\Delta x}{a}}$$

$$\boxed{\Delta t_{green2} = \sqrt{\frac{2\Delta x_{distToIntersection}}{a}}}$$

## 2 Yellow Light

The time of the yellow light is dependent on the braking distance of a car approaching the intersection. If the braking distance is longer than the car's current position and the driver sees a yellow light, he or she has no choice but to go through. Therefore, the time of a yellow light is the time it takes for a driver at the braking distance to go through driving at the speed limit.




### Braking Distance

The stopping distance of a car requires another variable: braking acceleration. However, a new number must be accounted for, the reaction time. Each driver has a reaction time before braking, and this time is spent going at the speed limit.

$$v^2 = \cancel{v_0^2} - 2a\Delta x$$

$$\Delta x_{braking} = -\frac{v_{speedLimit}^2}{2a_{brake}}$$

$$\boxed{\Delta x_{braking} = -\frac{v_{speedLimit}^2}{2a_{brake}} + (t_{reactionTime})(v_{speedLimit})}$$

### Yellow light time equation

$$\boxed{\Delta t_{yellow} = \frac{\Delta x_{braking}}{v_{speedLimit}}}$$

# 3 Period between red light of one side and green of another

In a situation where a driver may have misjudged the braking distance and instead goes through only because he or she saw a yellow light (about to change to a red) right when going into the intersection, a short pause between the red light of one side of the intersection and the green light of the other is necessary.

### Intermission time equation

This equation is calculated by the total time it would take a car on the edge of the intersection to reach the other side.

$$\boxed{\Delta t_{break} = \frac{\Delta x_{intersectionWidth}}{v_{speedLimit}}}$$

# 4 Red Light

Calculating the time that the red light is lit is based on the lights of the opposing side in an intersection.

### The equation

$$\boxed{\Delta t_{red} = \Delta t_{green} + \Delta t_{yellow} + \Delta t_{break}}$$

# 5 Measurements/Researched Values

Note: measurements are from the Clarksville Rd x Princeton Hightstown Road intersection.

| Measured Data | |
|---|---|
| Variable | Measurement |
| Red light cycle (avg 3 trials) | 59.1 s |
| Green light cycle (avg 3 trials) | 60.4 s |
| Yellow light cycle (avg 3 trials) | 4.89 s |
| Width of intersection | 33.7 m |
| Speed limit | 15.6 m/s |
| # of cars waiting at red | 14.3 |
| Researched Values | |
| Variable | Value |
| Average car length | 4.50 m |
| Average gap distance | 2.00 m |
| Average braking acceleration | 3.40 $m/s^2$ |
| Average human reaction time | 0.250 s |

In the program, these measurements/values will be set as the default values in the fields.