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Dear Editors of the UNLV Gaming Research & Review Journal:

Thank you for the careful review of our submitted manuscript and the decision of “Revisions Requested for Acceptance.” Please see our included resubmission. We hope that we have sufficiently addressed all concerns raised by the reviewers to their and your satisfaction. We detail our edits below in our responses to each reviewer.

**Reviewer 1:**

Literature, Aldous and Diaconis (1986): Check Bayer and Diaconis ("...Dovetail

Shuffle...") to see if it's a more accurate reference.

**We thank the reviewer for this comment and have double-checked this. We believe that the Aldous and Diaconis paper we cited is the original source for the seven shuffles claim.**

Literature, Shuffling Processes A through C: I had to reread the procedures a

few times to understand how "collisions" worked in these shuffling procedures.

It looks like you took the language word-for-word from the original source, and

that source didn't do a good job of explaining things either. It might help if

you are more clearer on the data structure at the beginning of a process's

explanation. What you're really doing is starting with n separate numbered bins,

with Card k starting by itself in Bin k. For Shuffling Process A, at each stage,

you randomly choose a card, pull it out of the bin it's in (let's call it Bin

j), and then either put it in Bin (j-1), put it back in Bin j, or put it in Bin

(j+1). I would express the probabilities as 0.25, 0.50, and 0.25 (It looks

cleaner in text as decimals.). If you drop the card into an empty bin, fine.

It's there by itself. If there are already other cards in the bin, then you've

got rules for how it collides or gets mixed in with the cards already there. At

the end you scoop the cards in Bin 1, followed by the cards in Bin 2, and so on.

**We agree that our descriptions could benefit from additional clarity and thank the reviewer for their suggestions. We have re-worked this portion of the manuscript accordingly.**

Methodology, Initial left-hand and right-hand movements: Figure 1 is a little

deceiving in the sense that the four piles of cards aren't always the same

length. It would help if you defined an initial general data structure for the

deck. After reading through the wash, roll, and slice sections a few times, my

sense is that the data structure in general consists of four vectors v1 to v4 of

potentially different lengths, and four numbers n1 to n4 with the lengths of the

four vectors (to make the indexing easier). Each of the numbers 1 to 52 occurs

exactly once somewhere in the four vectors, and n1 + n2 + n3 + n4 = 52. For the

initial split of a sorted deck into four equal piles, n1 = n2 = n3 = n4 = 13, v1

= 1:13, v2 = 14:26, v3 = 27:39, and v4 = 40:52 (using R's syntax).

**We thank the reviewer for this detailed suggestion and have updated the “initial setup” section of the manuscript.**

Methodology, Rolling: Show a numerical example. Start with an initial data

structure (An imbalanced one where n1, n2, n3, and n4 are all different might be

useful.). Point out where you are getting vec\_len. Pick a number and say that

you drew it from the truncated Poisson distribution. Show what the new post-roll

data structure looks like.

**We thank the reviewer for this comment and have updated the roll section with more detail and an example as suggested.**

Methodology, Slice: Rewrite this section also, adding an illustrative example.

The vice/versa wording is confusing. A left-handed inner slice moves cards from

v2 to v3. A left-handed outer slice moves cards from v1 to v2. Also, be specific

about which cards move to the front or back of which vectors. This paper is

already highly technical (like all of the Diaconis-related papers on shuffling),

so you might as well lean into it and use vectors and indices.

**We thank the reviewer for this comment and have added additional explanation and an example.**

Results, the locations of the previous community cards: While it is interesting

that the last hand's community cards don't end up in Locations 43 to 47 after a

wash, it's not that important to the next hand. Big picture, it doesn't matter

if that card is in Locations 21, 31, 41, 45, or 52. It's not in the hand. The

more practical question is whether or not a card that a player sees (as one of

their hole cards, at showdown, on the board) shows up in the next hand.

**We completely agree with the reviewer regarding the relevance of where cards go if they are not in the hand; however, these figures were showing the previous locations of the cards that end up in the positions that will result in them being community cards in the current hand. We thank the reviewer for pointing out this confusion and have added clarity to the manuscript.**

Results, card neighbors: What is the practical significance of card neighbors?

Let's say that a player's cards get mucked as neighbors, and that the board gets

scooped into being neighbors. In order for two neighboring cards to show up

together again, they either have to be spaced the right number of cards apart to

be a person's hole cards, they have to be together on the flop, or they have to

be exactly two locations apart to have a burn card in between.

**We thank the reviewer for this comment. The practical significance of this was explained better in Merz & Chi (2022) and we agree that it is a bit sparse in the present manuscript. We have added detail on this point.**

**Reviewer 2:**

My most significant concern is that the authors cite three alternative algorithms for modelling

the wash shuffle from an arXiv paper (White, 2019), but do not compare their results with

predictions from this paper. This should be done, especially considering the paucity of previous

research on the wash shuffle. Although the algorithm is plausible, plausibility does not equate

to correctness. Proving correctness is probably not possible without performing an

unrealistically large experimental study; however, the authors could try to compare with these

previously discussed algorithms. The White paper is quite abstract and mathematical, which

might make direct comparison with this work difficult. However, White’s algorithms are

relatively simple compared to this work, and it would be very easy to simulate them in a section

of this paper for direct comparison. This addition would strengthen the paper very significantly

because it would ground the new simulations with previous results, thus strengthening

confidence in the validity of this work. Such a comparison might also tell us something

interesting about the sensitivity of the new results to the simulation mechanics.

**We agree with the reviewer’s concerns and have added results demonstrating Shuffling Processes A, B and C from White, 2019 to Figure 4 in the manuscript.**

I also note that the White paper is referenced as an “unpublished manuscript”. I don’t think this is the correct way to reference this paper, which is published on arXiv, but not refereed.

**We thank the reviewer for pointing this out and have changed “an unpublished manuscript” to “an unrefereed arXiv manuscript.”**

More explanation of the choice of parameters would help in a few places. For example, why

choose g=3 in the truncated gamma distribution? Why should an “inner slice” occur with a

probability of 70% and an “outer slice” with a probability of 30%, etc? The parameters chosen

are reasonable, but the choice should be better motivated in the text of the paper. Even more

importantly, the sensitivity of the results on all adjustable parameters should be determined

and discussed. i.e. Are the results robust with respect to parameter choices, within reasonable

ranges?

**We thank the reviewer for this comment and have added both additional explanation of the choice of parameter values, and commentary on the sensitivity of our results to those choices.**

Minor concerns:

In the discussion of Shuffling Process C, the mention of “vertices in a d-dimensional grid” comes

across as cryptic without including some context. A bit of explanation here would help.

The notation in Figure 1 and associated mechanics should be better described, especially the

1A, 1B, 2A, 2B, etc. notation, which is barely described in the text. This is the only part of the

paper where I felt that the writing was unclear, but it is unfortunately a critically important

section for understanding the algorithm. The description here should be expanded, with enough

detail that another research could in principle reproduce the simulations.

**We thank the reviewer for this comment and have revised this section to add additional clarity.**

Figure 1’s caption is very short for such an important figure. More description here would help

the reader to understand the shuffling mechanism.

**We completely agree with this comment and have added more detail to this figure caption.**

Finally, there is a typo in the first paragraph of the Discussion section: “demostrate”.

**We thank the reviewer for pointing this out and have fixed this typo.**

Thank you very much for your consideration of our resubmission and we look forward to hearing from you soon.

Sincerely,

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