University of Moratuwa, Sri Lanka

Department of Electronic & Telecommunication Engineering

Hodgkin Huxley Analysis

BM2102 Modelling and Analysis of Physiological Systems

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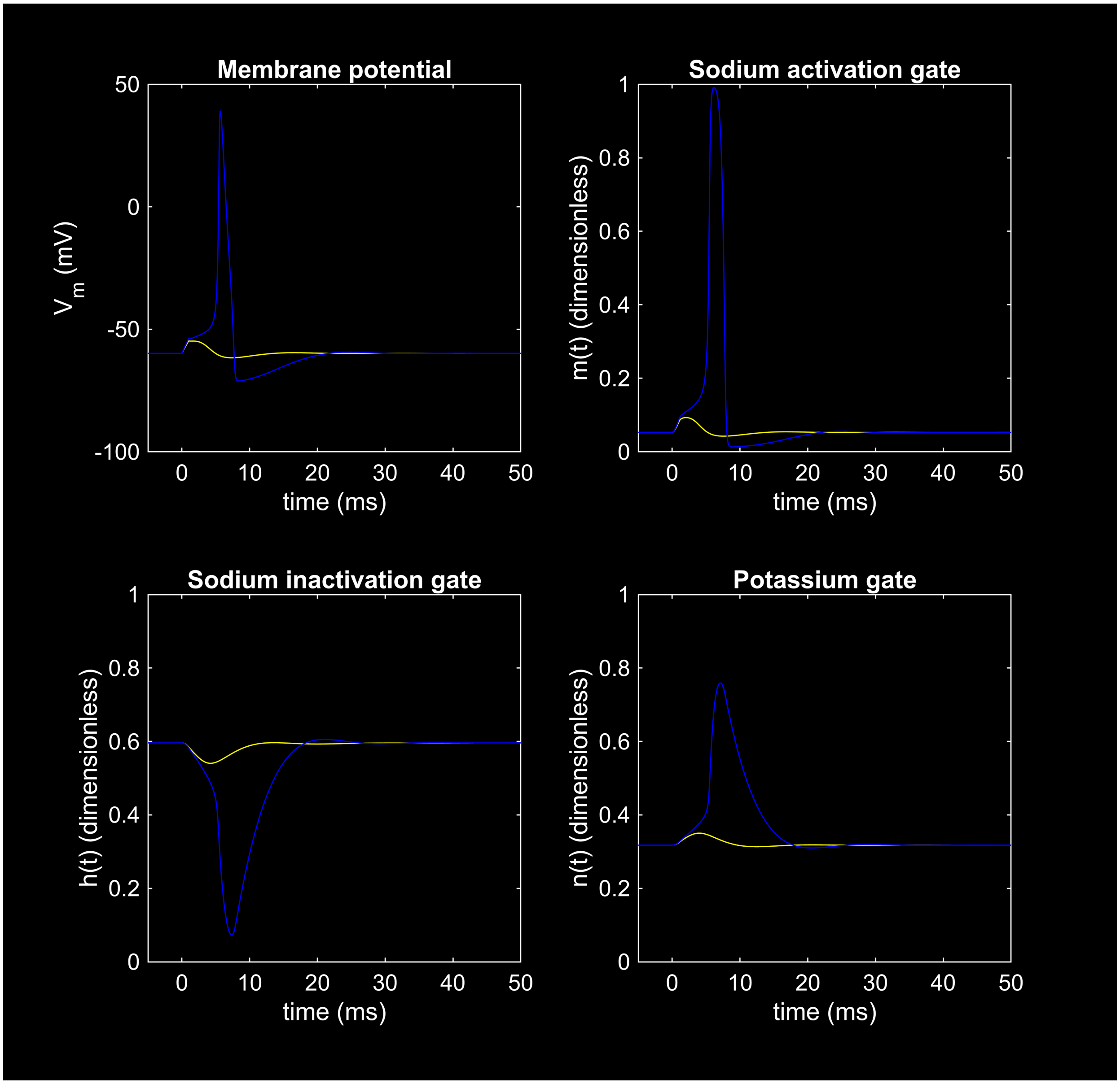
Threshold

```
hhconst
amp1 = 6;
width1 = 1;
hhmplot(0,50,0);
amp1 = 7;
hhmplot(0,50,1);
```

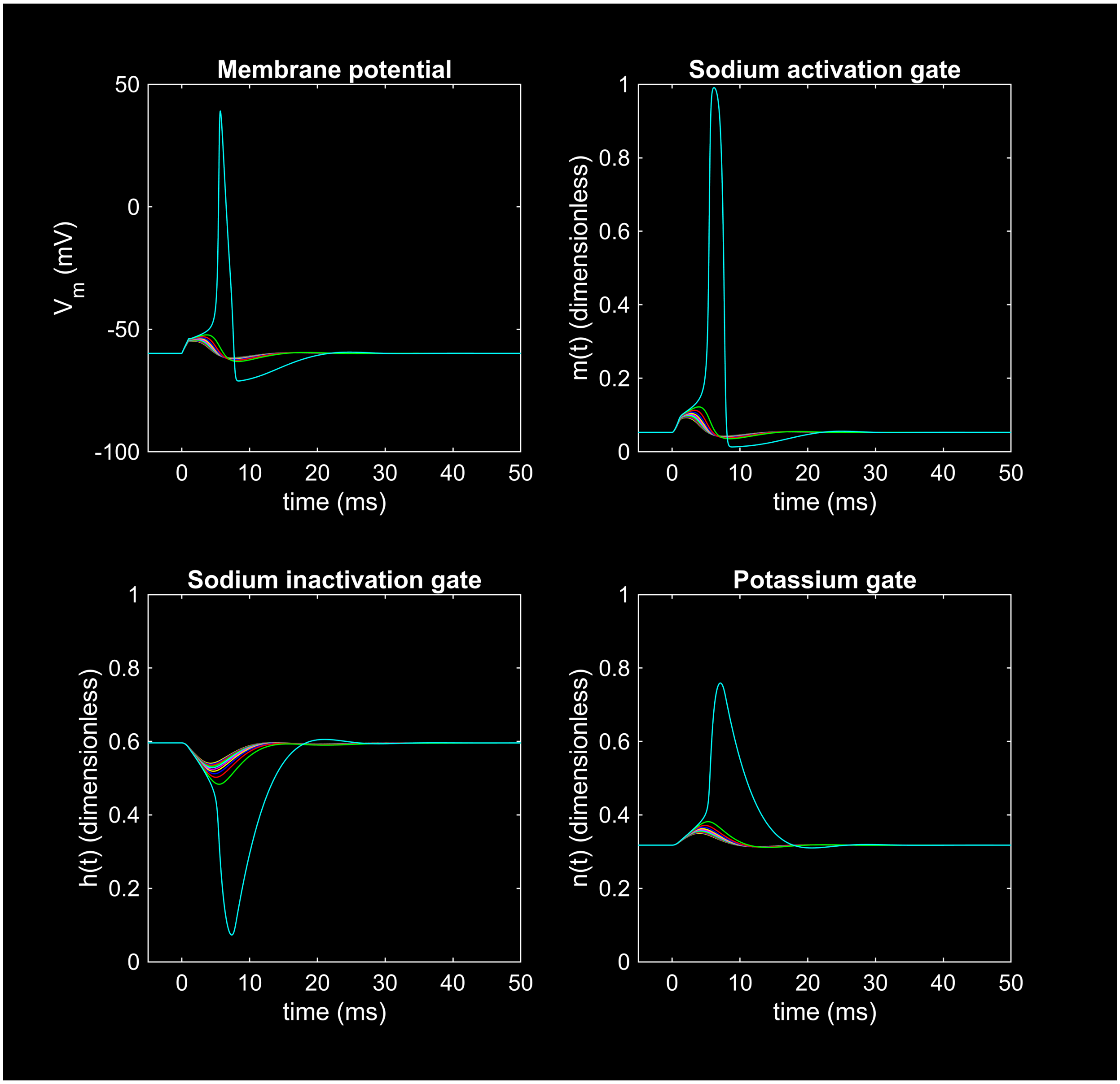


As there are no action potentials (AP) when the amplitude is A_1, but there are APs when the amplitude is A_2, the threshold should lie between A_1 and A_2.

Question 01

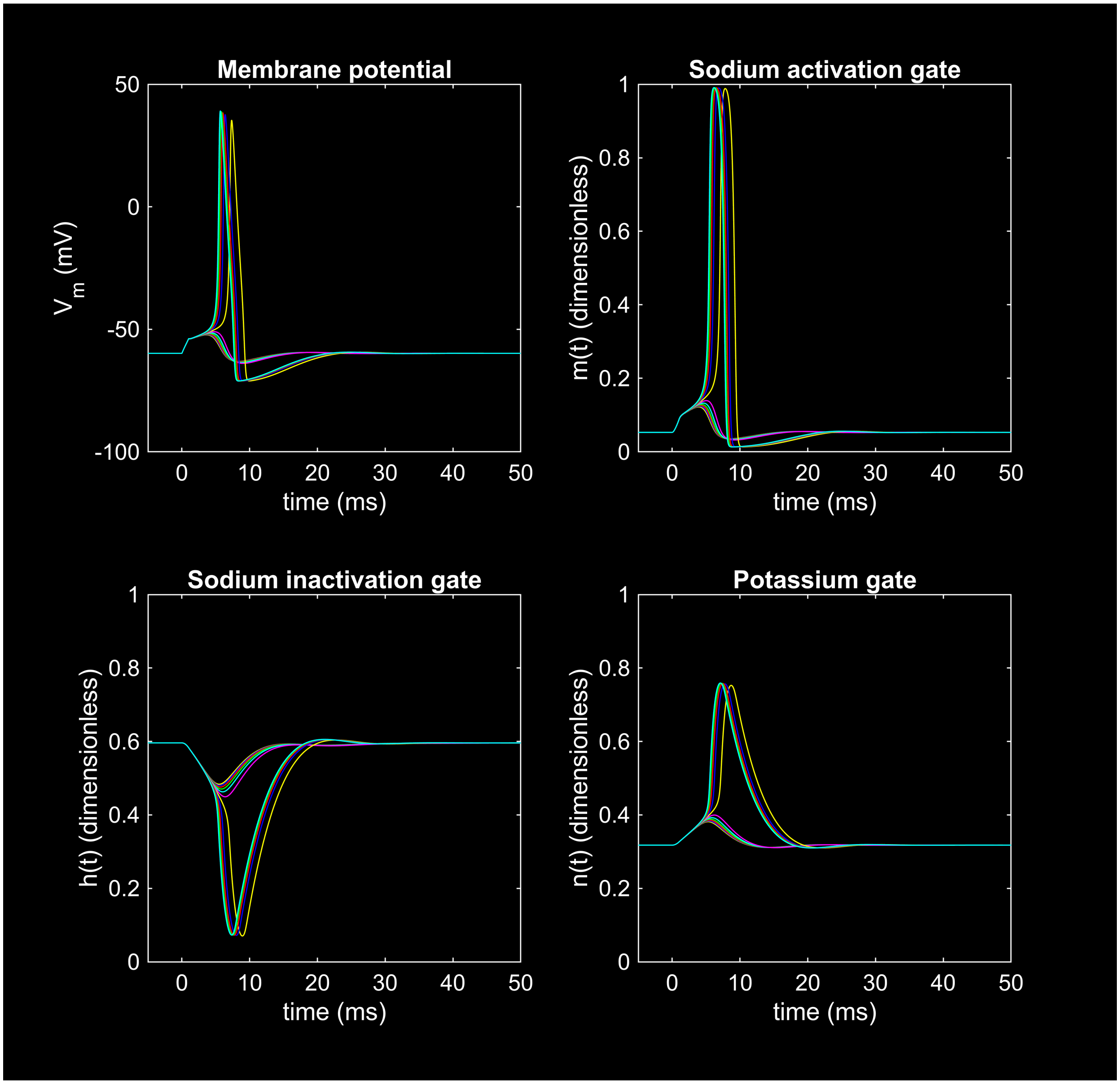
```
pause on
hhconst;
```

```
amp1 = 6;
width1 = 1;
hhmplot(0,50,0);
for i = 1:10
 amp1 = amp1+0.1;
 hhmplot(0,50,i);
 pause(1);
end
```



As there are no action potentials (AP) when the amplitude is 6.9,
but there are APs when the amplitude is 7.0, the threshold should lie between 6.9 and 7.0.
Since the amplitude was increased by 0.1 and only one action potential was observed at the 10th step,
the last subthreshold amplitude was $6 + 0.9 = 6.9$.
Therefore, the threshold lies in the interval $[6.9\ \mu\text{A/cm}^2, 7.0\ \mu\text{A/cm}^2]$.

```
pause on
hhconst;
amp1 = 6.90;
width1 = 1;
hhmplot(0,50,0);
for i = 1:10
 amp1 = amp1+0.01;
 hhmplot(0,50,i);
 pause(1);
end
```

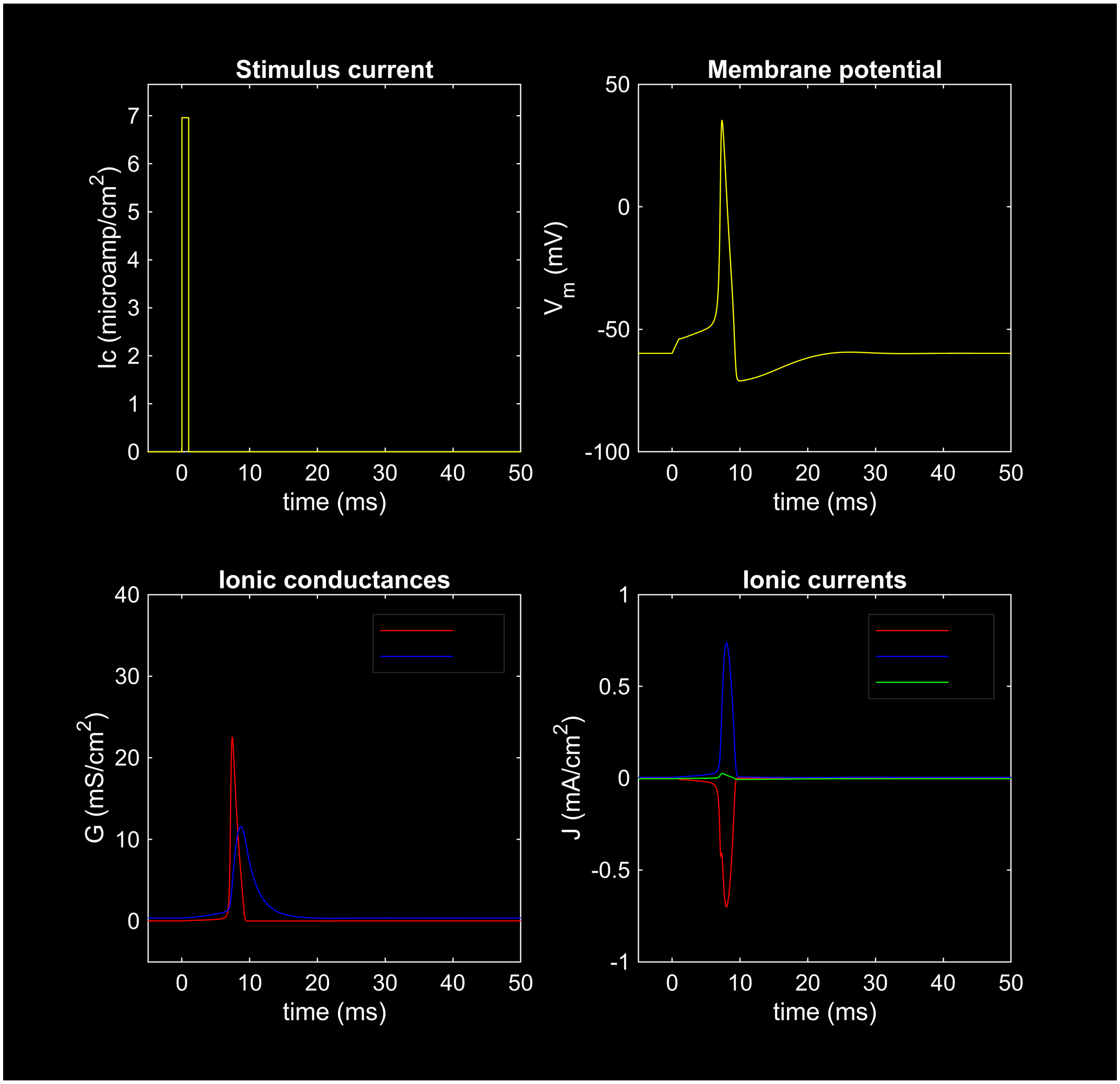


In the action potential plot, 5 color bands are indicating possible action potentials at amplitudes : 7, 6.99, 6.98, 6.97, and 6.96.

The minimum of these values is 6.96 μA/cm^2, which is considered the threshold amplitude.

Question 02

```
clf;
amp1 = 6.96;
[qna,qk,ql]=hhsplot(0,50);
```




```
qna+qk+ql
```

```
ans = 6.9620
```

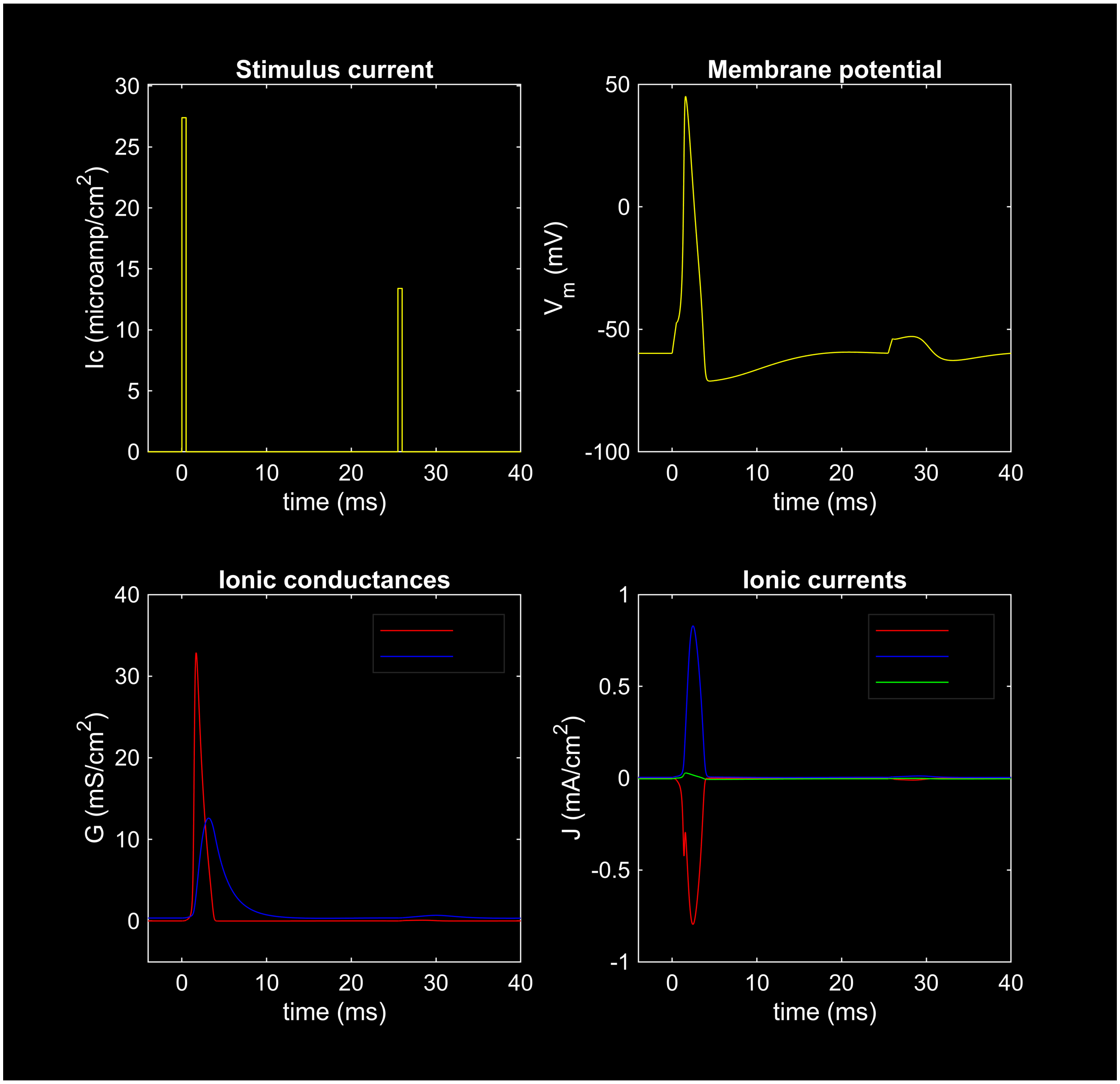
$$\int_{t_0}^{t_f} \sum J_k(t)\,dt = q_{Na} + q_K + q_L \quad \text{(expected total charge from individual ion currents)}$$

$$\int_{t_0}^{t_f} \sum_k J_k(t)\,dt = \int_{t_0}^{t_f} J_{\mathrm{ei}}(t)\,dt \quad \text{(total input current equals sum of individual currents)}$$

The computed results are $6.96\mu\mathrm{A/cm}^2$ and $6.9620\mu\mathrm{A/cm}^2$.
Due to minor numerical errors, these values are not exactly equal.
However, they are theoretically equal as expected.

Refractoriness

```
amp1 = 27.4;
width1 = 0.5;
delay2 = 25;
amp2 = 13.4;
width2 = 0.5;
hhsplot(0,40);
```

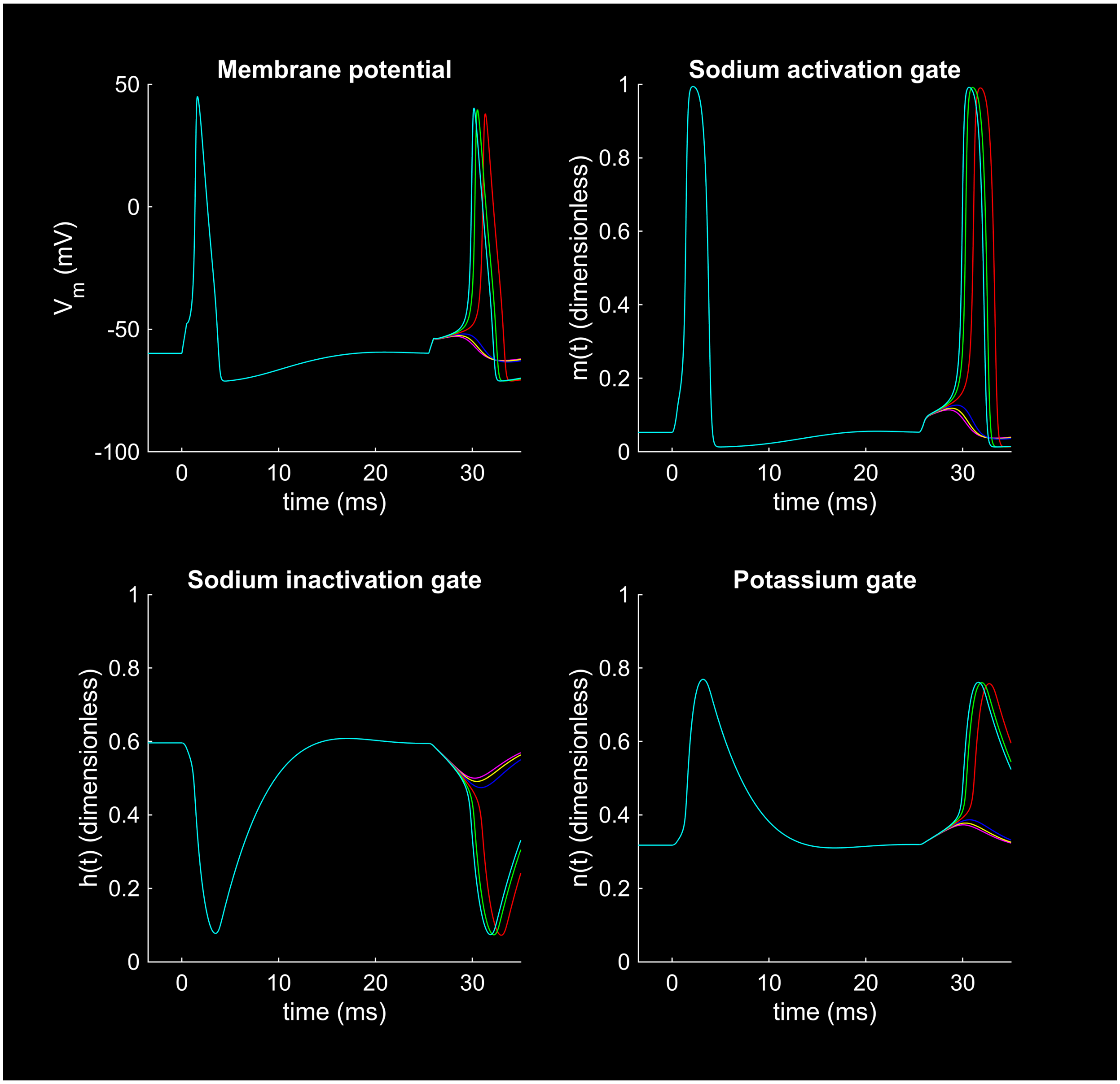


Question 03

Delay 25

```
close;
amp2=13.4;
amp1 = 26.8;
```

```
width1 = 0.5;
delay2 = 25;
width2 = 0.5;
for i = 1:6
 hhmplot(0,35,i);
 amp2 = amp2+0.1;
end
```

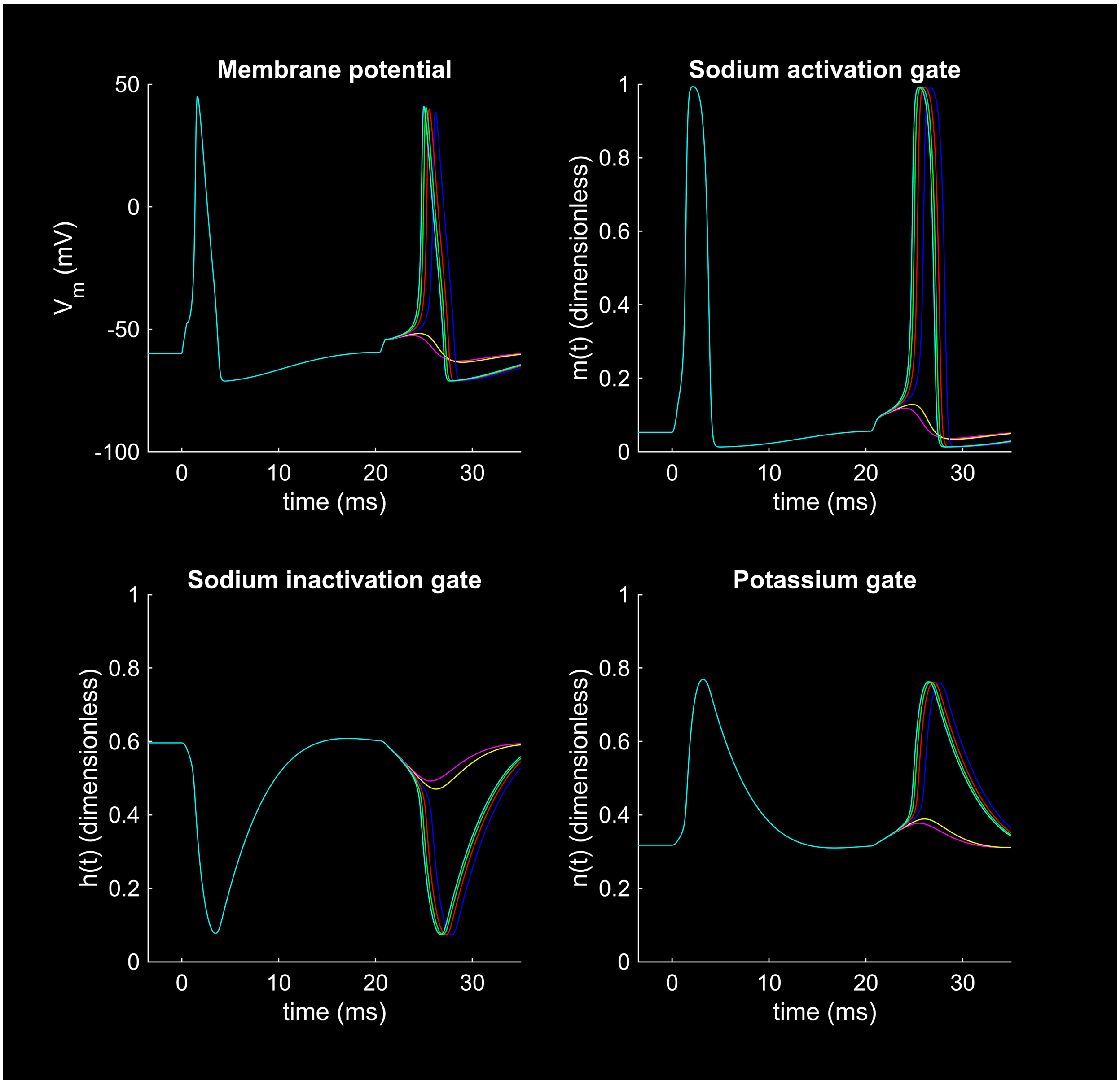


As the plot for the delay 25 ms, 13.7μA/cm^2, of the second impulse is needed to elicit an AP.

Delay 20

```
close;
amp1 = 26.8;
width1 = 0.5;
```

```
delay2 = 20;
amp2 = 11.4;
width2 = 0.5;
for i = 1:6
 hhmplot(0,35,i);
 amp2 = amp2+0.1;
end
```

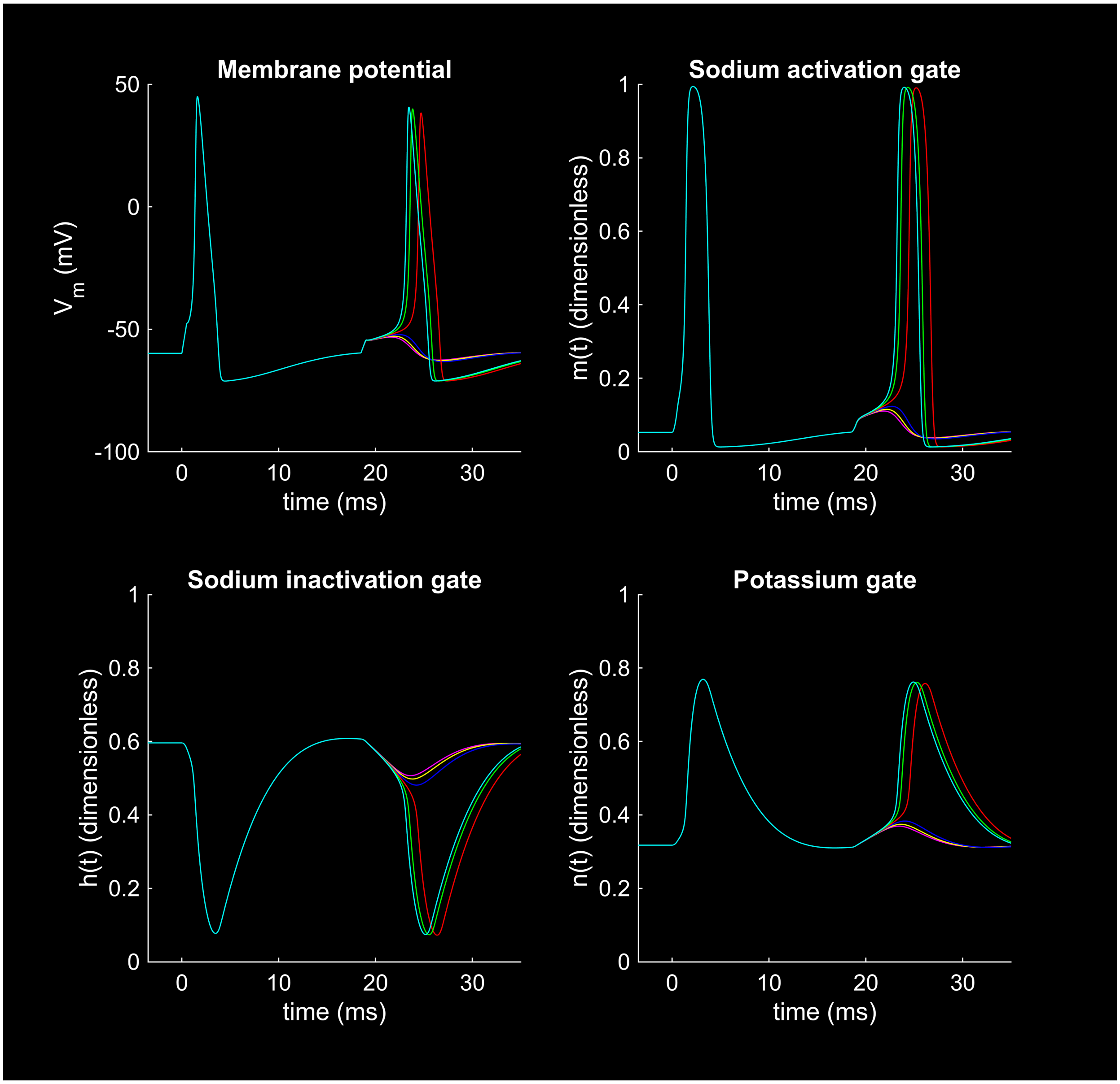


As the plot for the delay 20 ms, 11.6μA/cm^2, of the second impulse is needed to elicit an AP.

Delay 18

```
close;
amp1 = 26.8;
width1 = 0.5;
delay2 = 18;
amp2 = 11;
```

```
width2 = 0.5;
for i = 1:6
 hhmplot(0,35,i);
 amp2 = amp2+0.1;
end
```

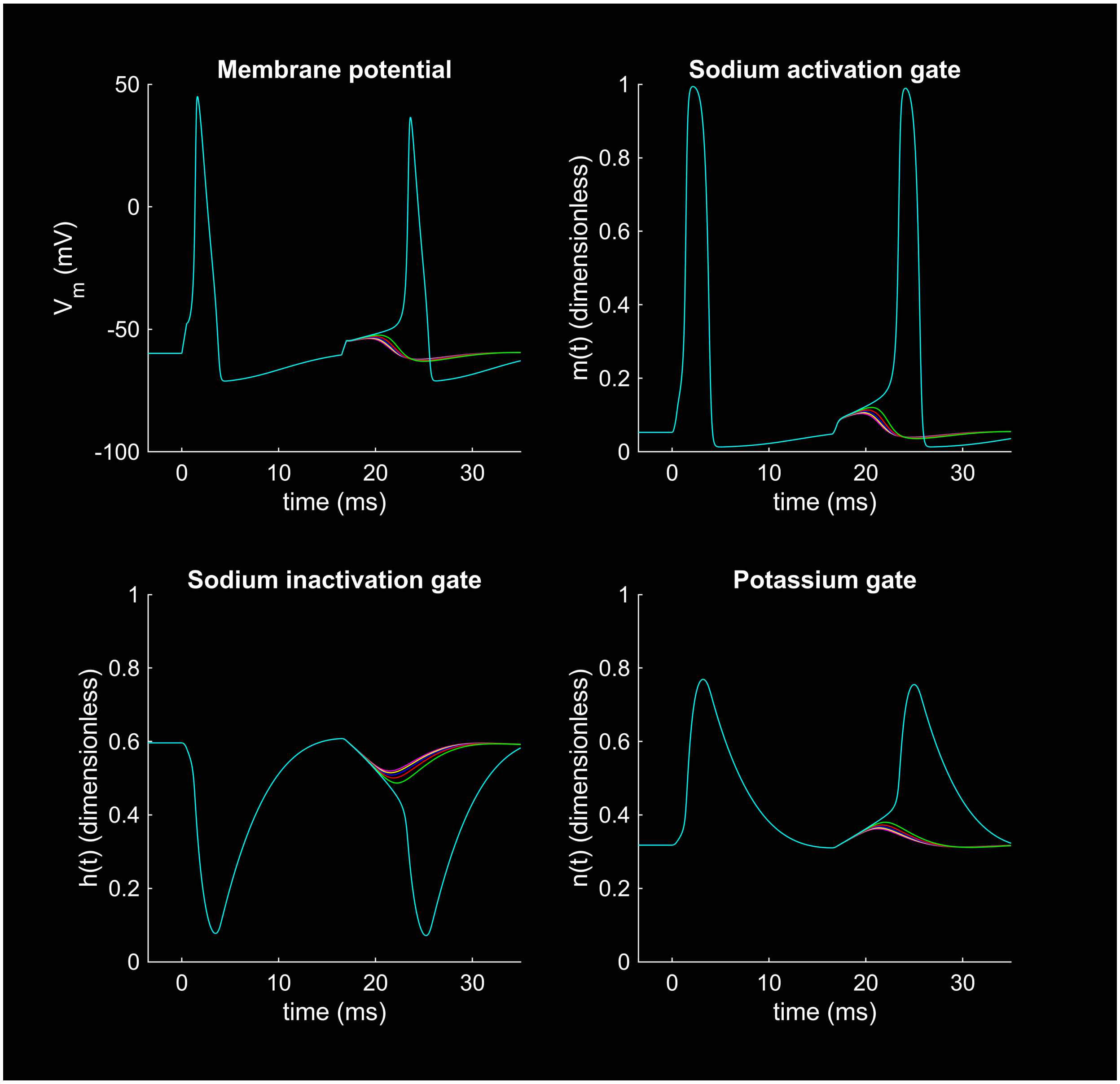


As the plot for the delay 18 ms, 11.3μA/cm^2, of the second impulse is needed to elicit an AP.

Delay 16

```
close;
amp1 = 26.8;
width1 = 0.5;
delay2 = 16;
amp2 = 12.2;
```

```
width2 = 0.5;
for i = 1:6
 hhmplot(0,35,i);
 amp2 = amp2+0.1;
end
```

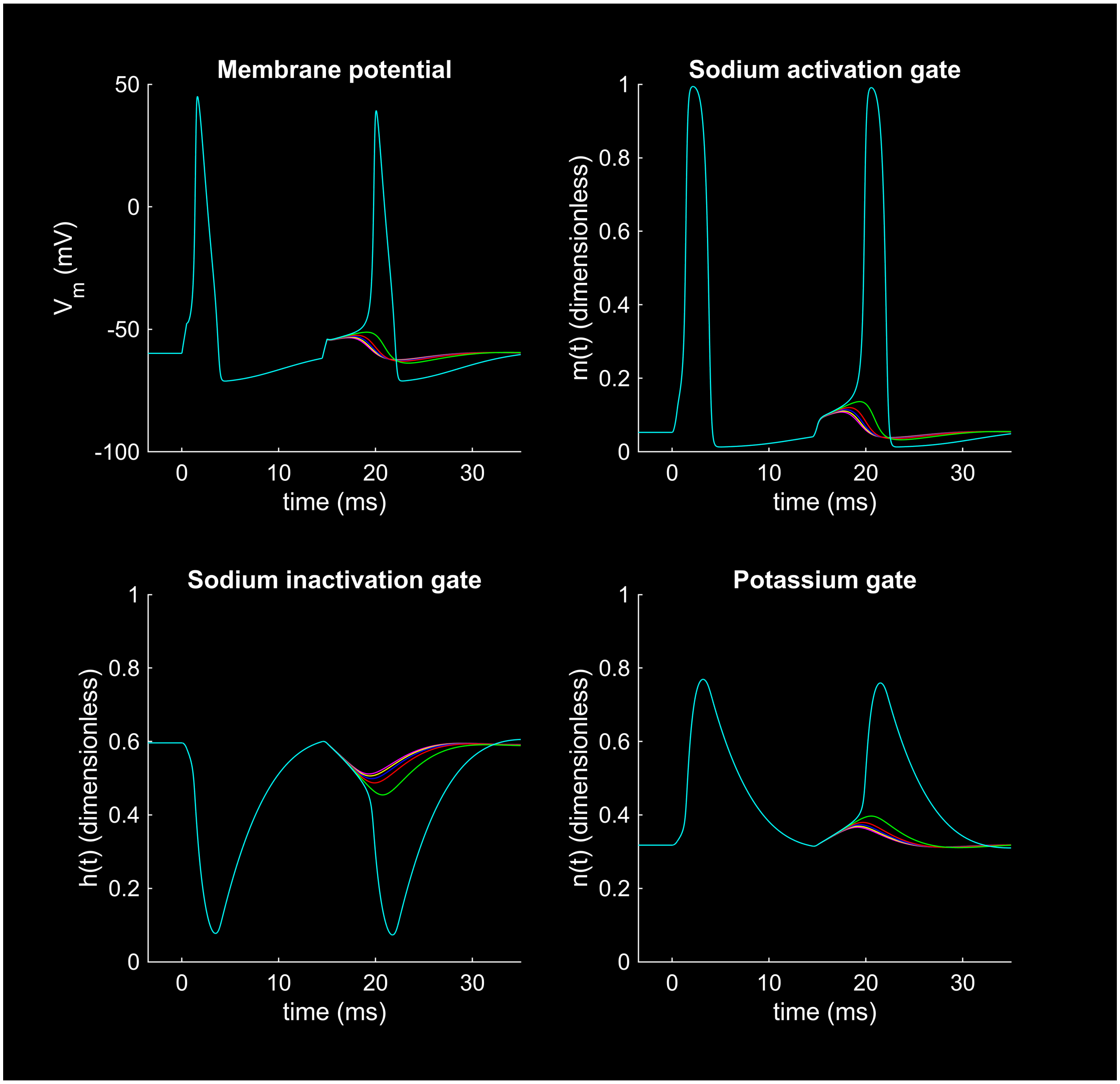


As the plot for the delay 16 ms, 12.7μA/cm^2, of the second impulse is needed to elicit an AP.

Delay 14

```
close;
amp1 = 26.8;
width1 = 0.5;
delay2 = 14;
amp2 = 16.5;
width2 = 0.5;
for i = 1:6
```

```
 hhmplot(0,35,i);
 amp2 = amp2+0.1;
end
```

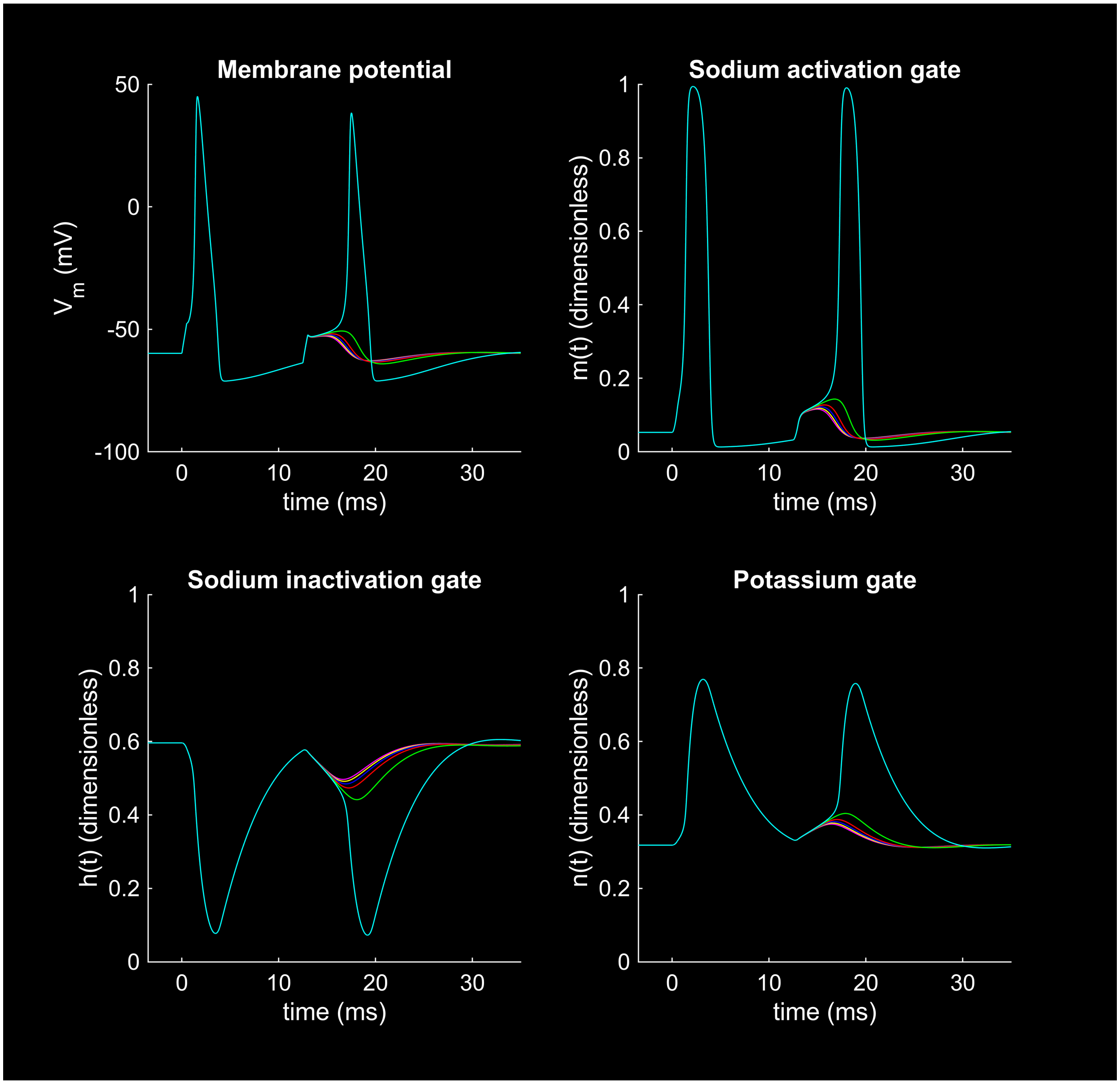


As the plot for the delay 14 ms, 17 μA/cm^2, of the second impulse is needed to elicit an AP.

Delay 12

```
close;
amp1 = 26.8;
width1 = 0.5;
delay2 = 12;
amp2 = 25;
width2 = 0.5;
for i = 1:6
```

```
 hhmplot(0,35,i);
 amp2 = amp2+0.1;
end
```

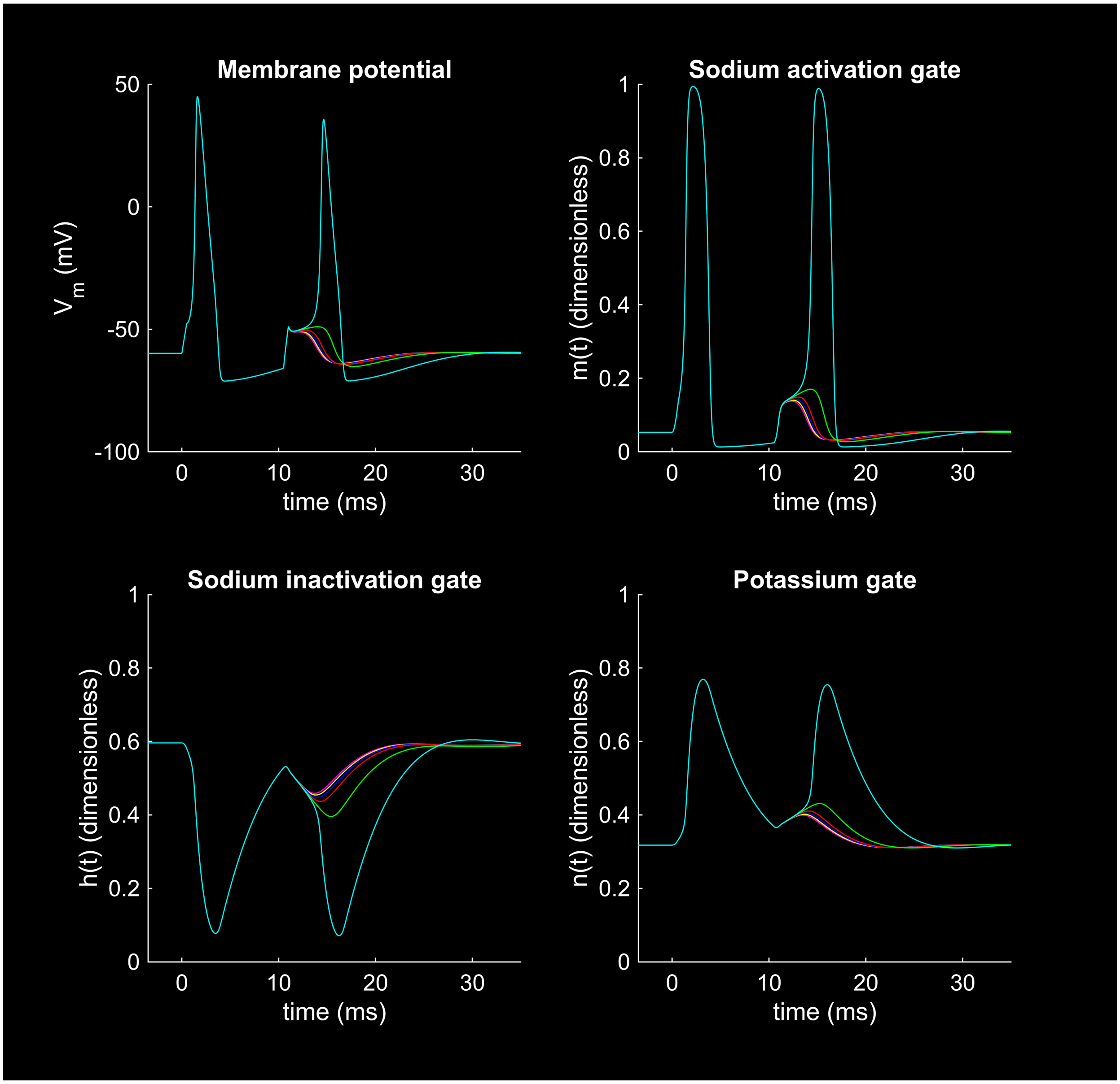


As the plot for the delay 12 ms, $25.5\mu\text{A/cm}^2$, of the second impulse is needed to elicit an AP.

Delay 10

```
close;
amp1 = 26.8;
width1 = 0.5;
delay2 = 10;
amp2 = 40.3;
width2 = 0.5;
for i = 1:6
```

```
 hhmplot(0,35,i);
 amp2 = amp2+0.1;
end
```

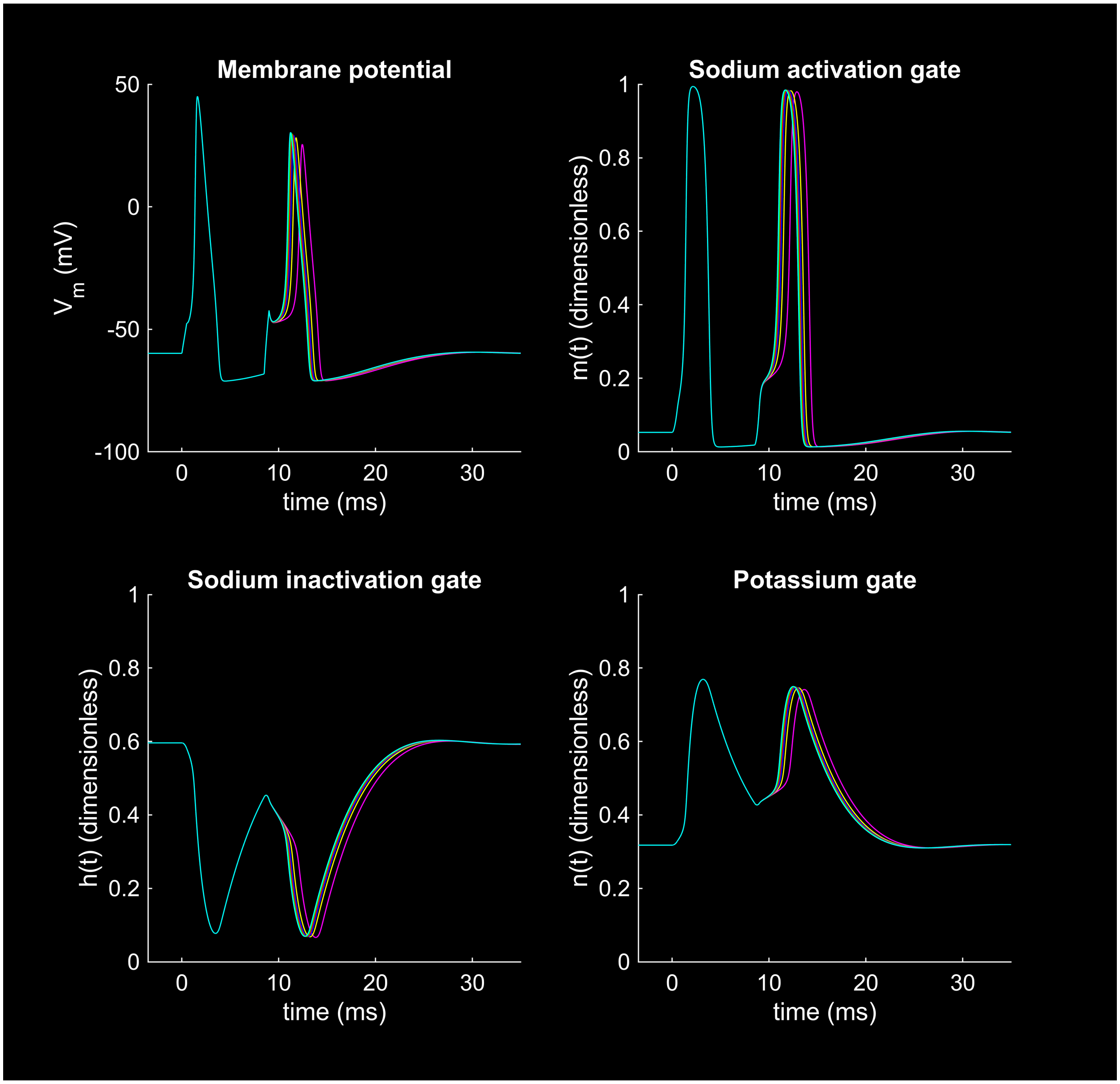


As the plot for the delay 10 ms, $40.6\mu\text{A/cm}^2$, of the second impulse is needed to elicit an AP.

Delay 08

```
close;
amp1 = 26.8;
width1 = 0.5;
delay2 = 8;
amp2 = 70.1;
width2 = 0.5;
for i = 1:6
```

```
  hhmplot(0,35,i);
  amp2 = amp2+0.1;
end
```

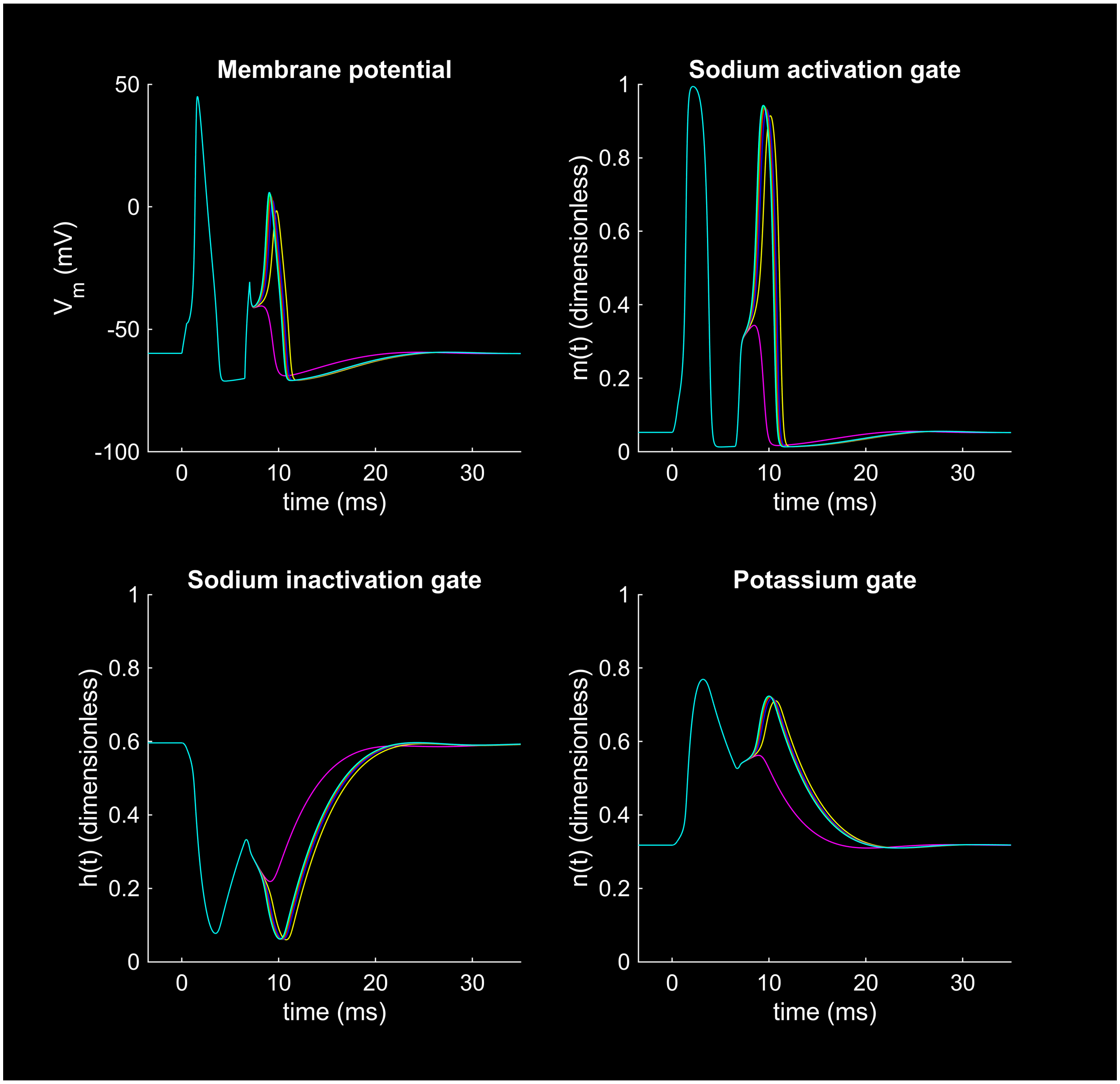


As the plot for the delay 8 ms, $70.1\mu\text{A/cm}^2$, of the second impulse is needed to elicit an AP.

Delay 06

```
close;
amp1 = 26.8;
width1 = 0.5;
delay2 = 6;
amp2 = 145.1;
width2 = 0.5;
for i = 1:6
```

```
  hhmplot(0,35,i);
  amp2 = amp2+0.1;
end
```

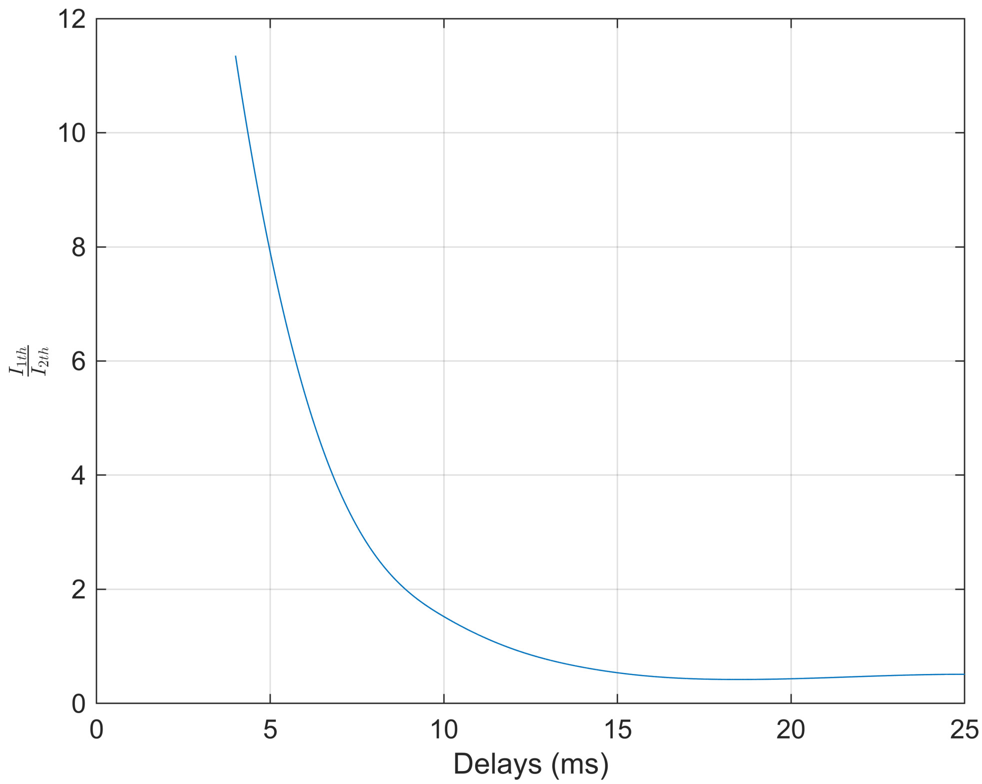


As the plot for the delay 6 ms, 145.2μA/cm^2, of the second impulse is needed to elicit an AP.

Question 04

```
close;
I2 = [145.2, 70.1, 40.8, 25.5, 17.0, 12.7, 11.3, 11.6, 13.7];
I1 = 26.8;
current_ratio= I2 / I1;
delay = [6, 8, 10, 12, 14, 16, 18, 20, 25];
x_values = linspace(4, 25, 10000); % Points to evaluate the smooth curve
```

```
fx = spline(delay, current_ratio, x_values);
plot(x_values, fx);
xlabel('Delays (ms)');
ylabel('$\frac{I_{1th}}{I_{2th}}$', 'Interpreter', 'latex');
grid on;
```



By computing the normalized threshold ratio

$$\frac{I_{2\text{th}}}{I_{1\text{th}}}$$

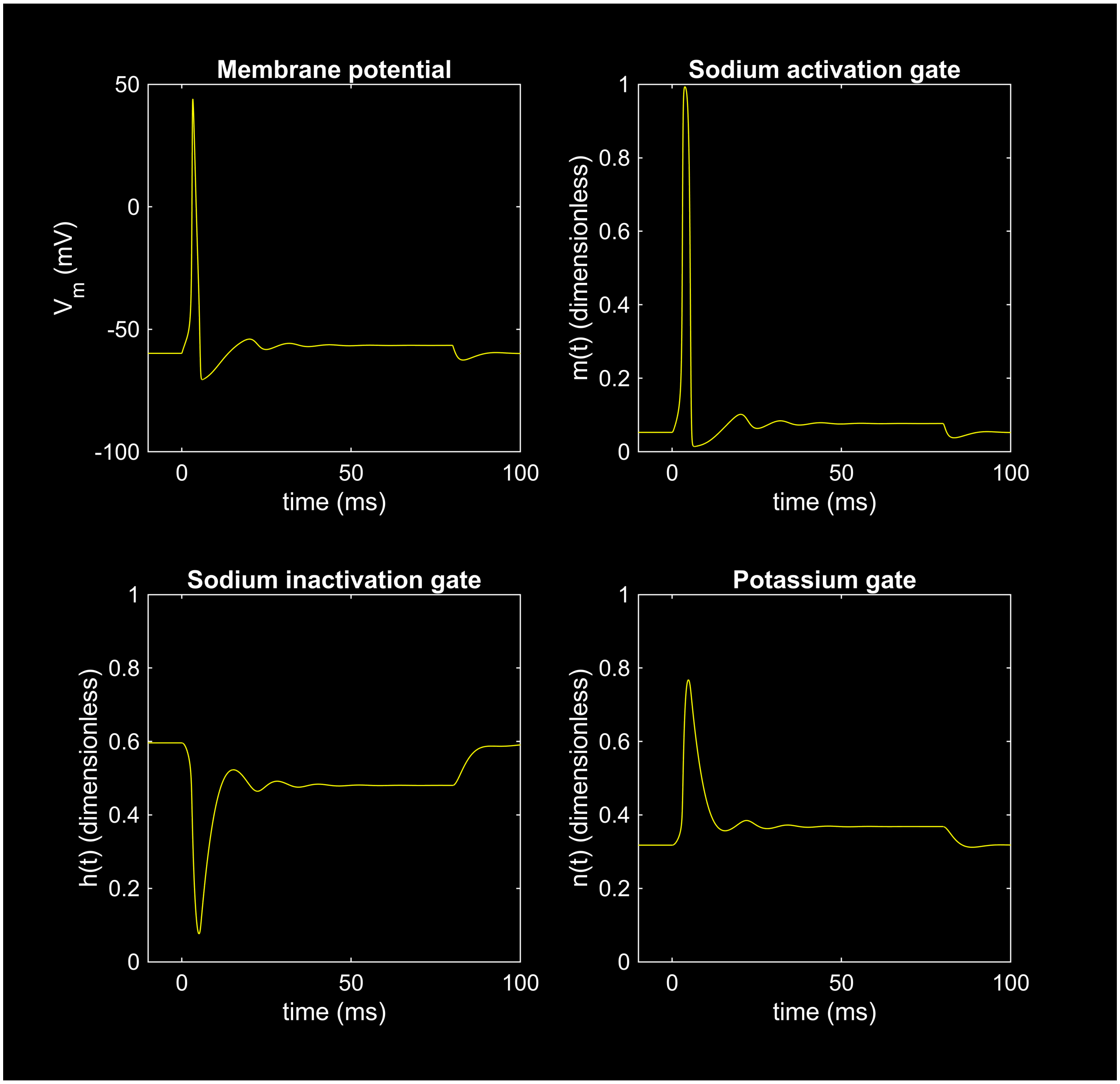
and interpolating it smoothly, we see that at a 4 ms inter-pulse interval this ratio exceeds 10.\
even a tenfold stimulus cannot evoke a second action potential—so the absolute refractory period (ARP)\
extends from 0 up to roughly 2–4 ms\
Beyond4 ms the ratio falls below infinity but remains above 1,\
meaning a second spike is possible only with a suprathreshold pulse.\
Theinstant the ratio returns to unity (found by solving the spline) occurs at about 9–11 ms, \
marking the end of the relative refractory period (RRP).\
Thus,numerically, ARP $\approx$ 2–4 ms and RRP $\approx$ 4–13 ms (full recovery by 9–11 ms).

Repetitive Activity

Question 05

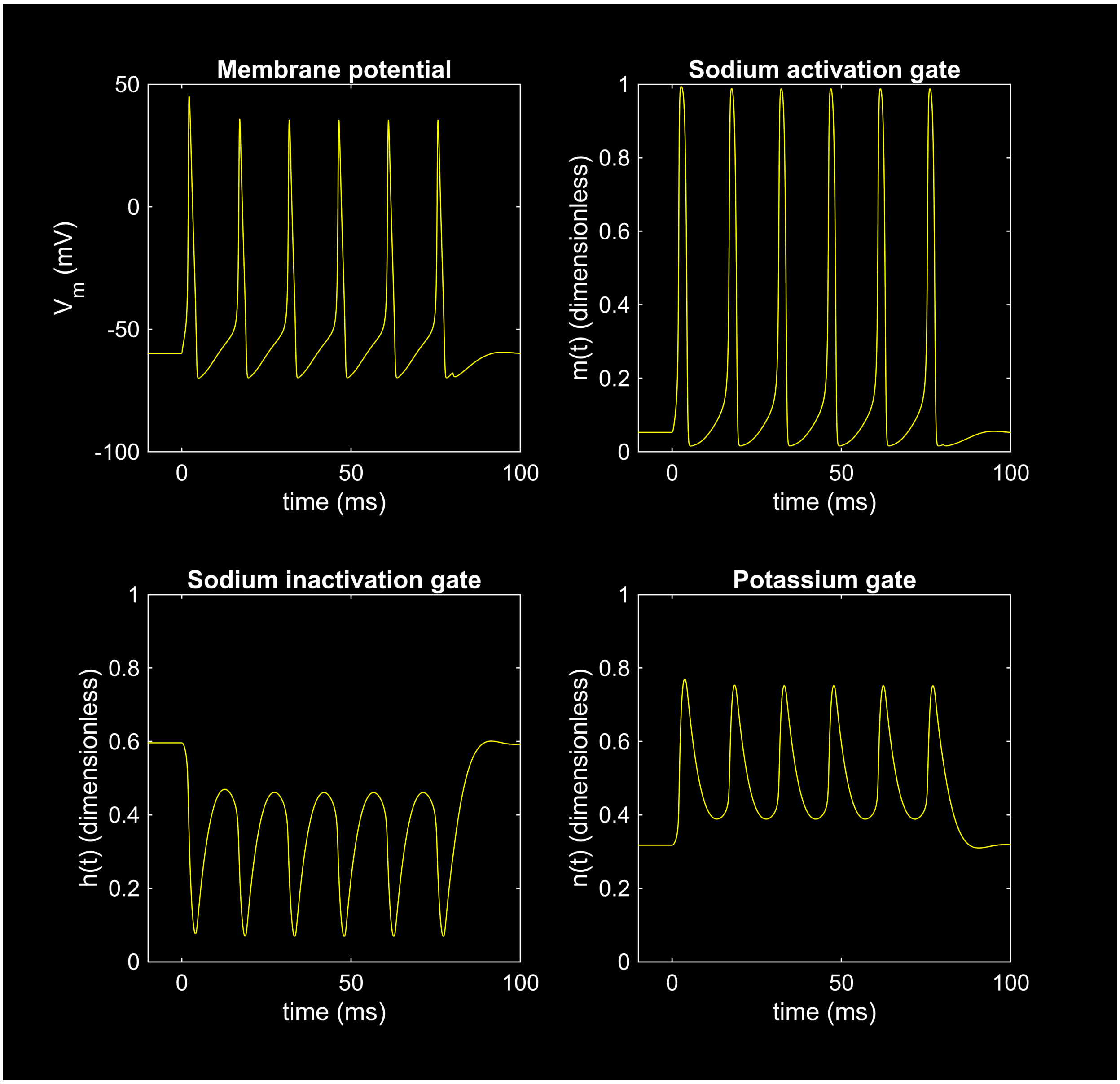
```
% Plot for amp1 = 5
```

```
close;
amp1=5;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
amp2 = 0;
hhmplot(0,100,0);
```

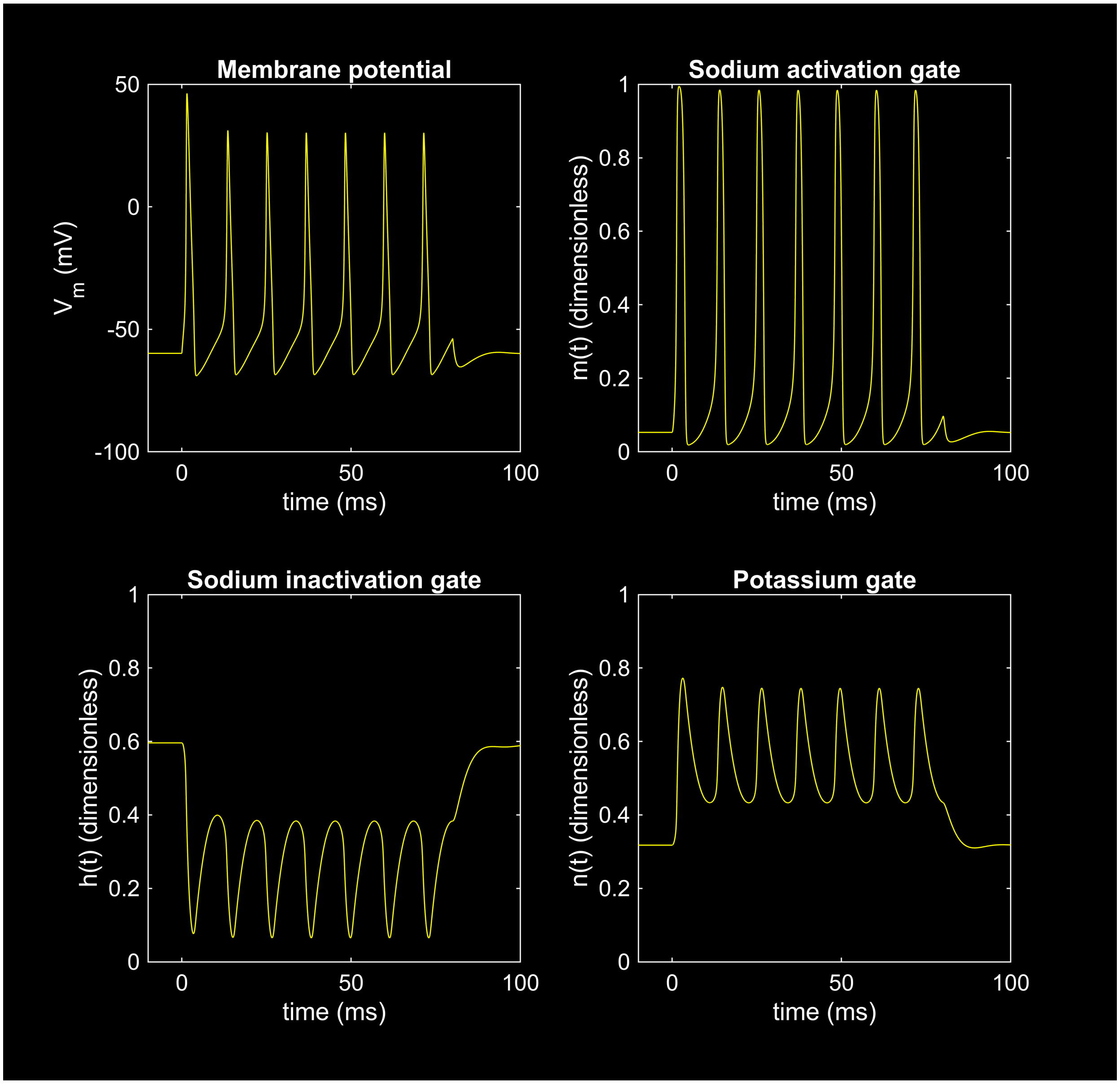



```
% Plot for amp1 = 10
close;
amp1 = 10;
width1 = 80;
delay2 = 0;
amp2 = 0;
```

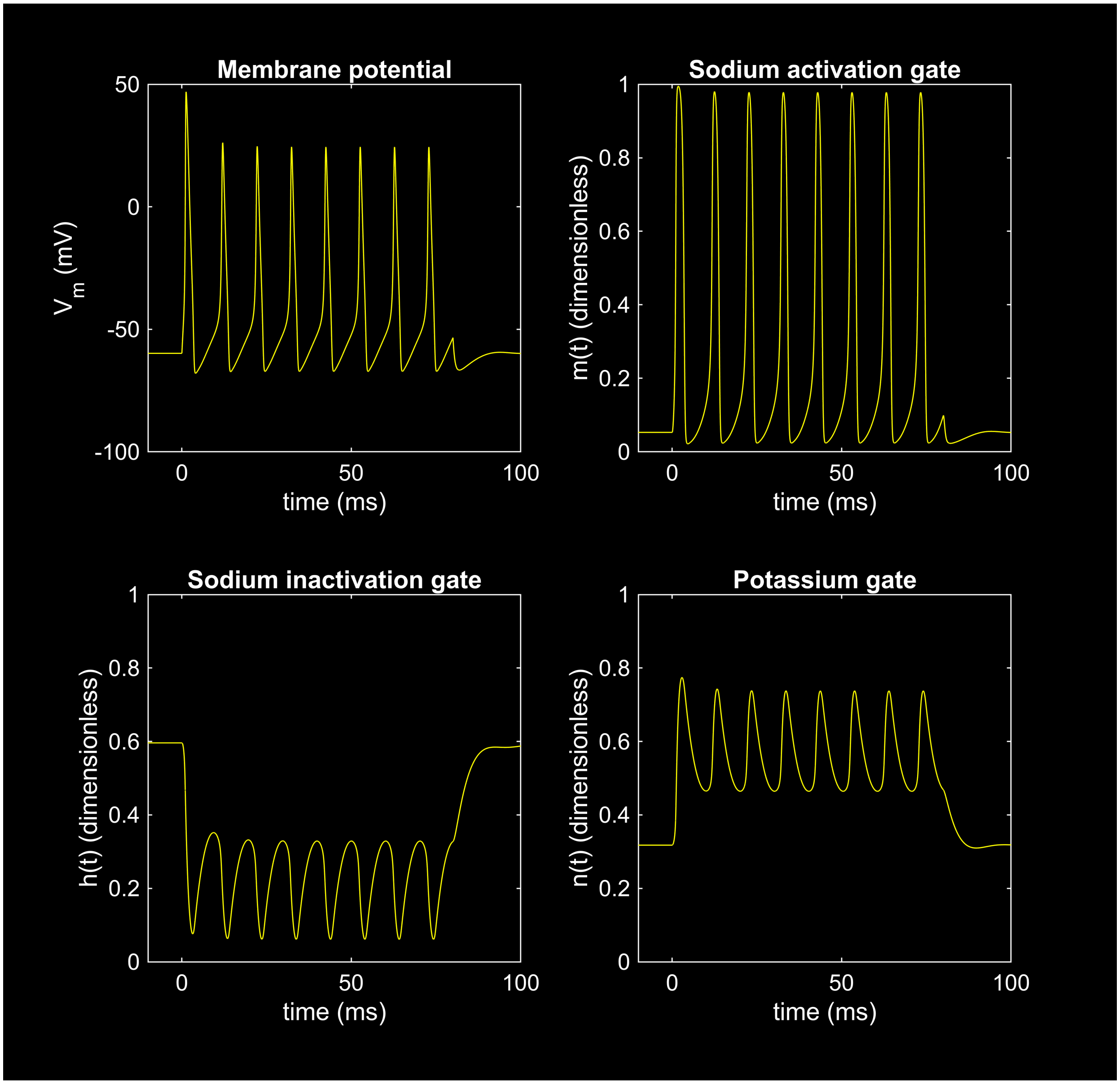
```
width2 = 0;
hhmplot(0, 100, 0);
```



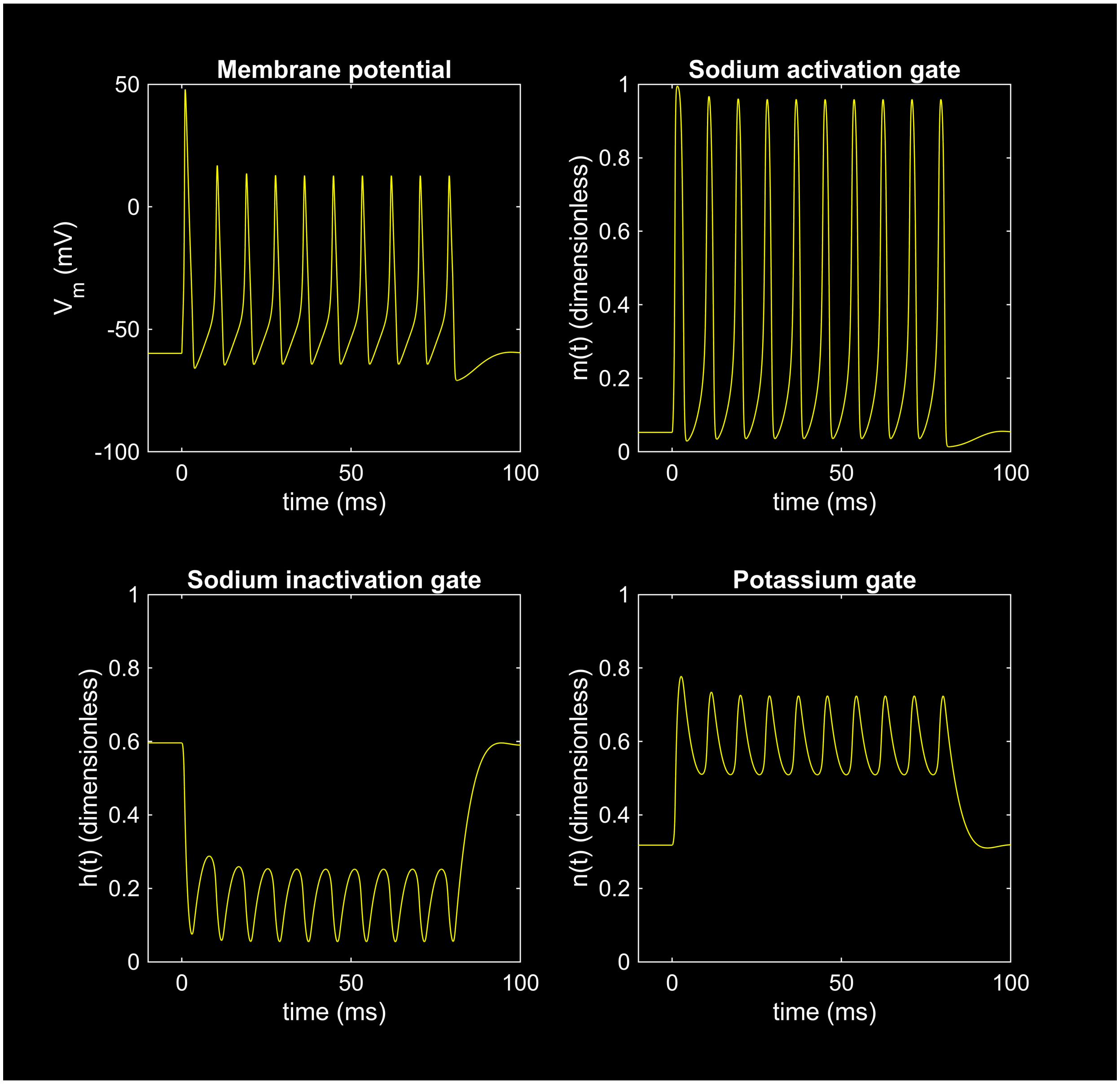

```
% Plot for amp1 = 20
close;
amp1 = 20;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0, 100, 0);
```



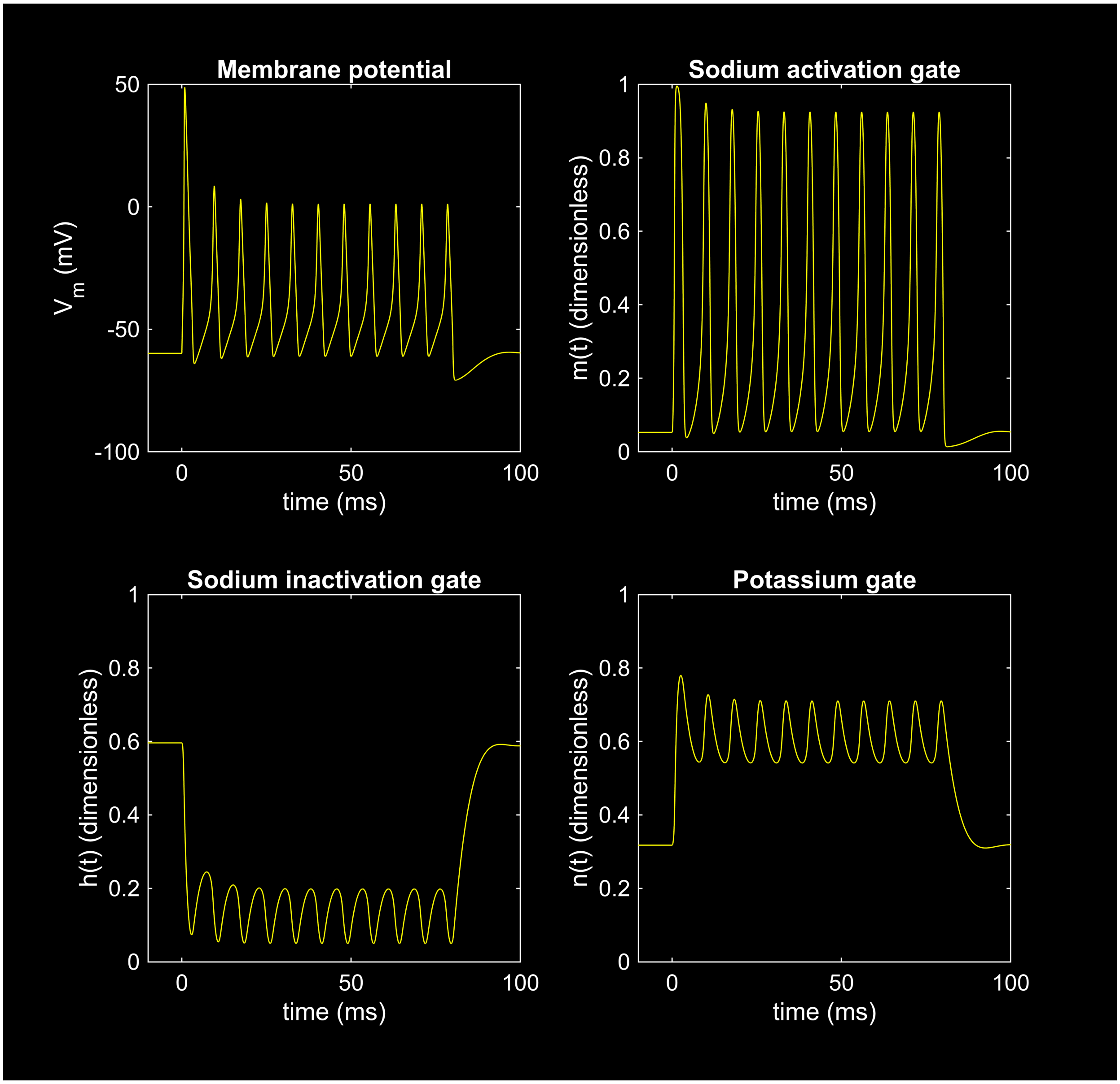

```
% Plot for amp1 = 30
close;
amp1 = 30;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0, 100, 0);
```



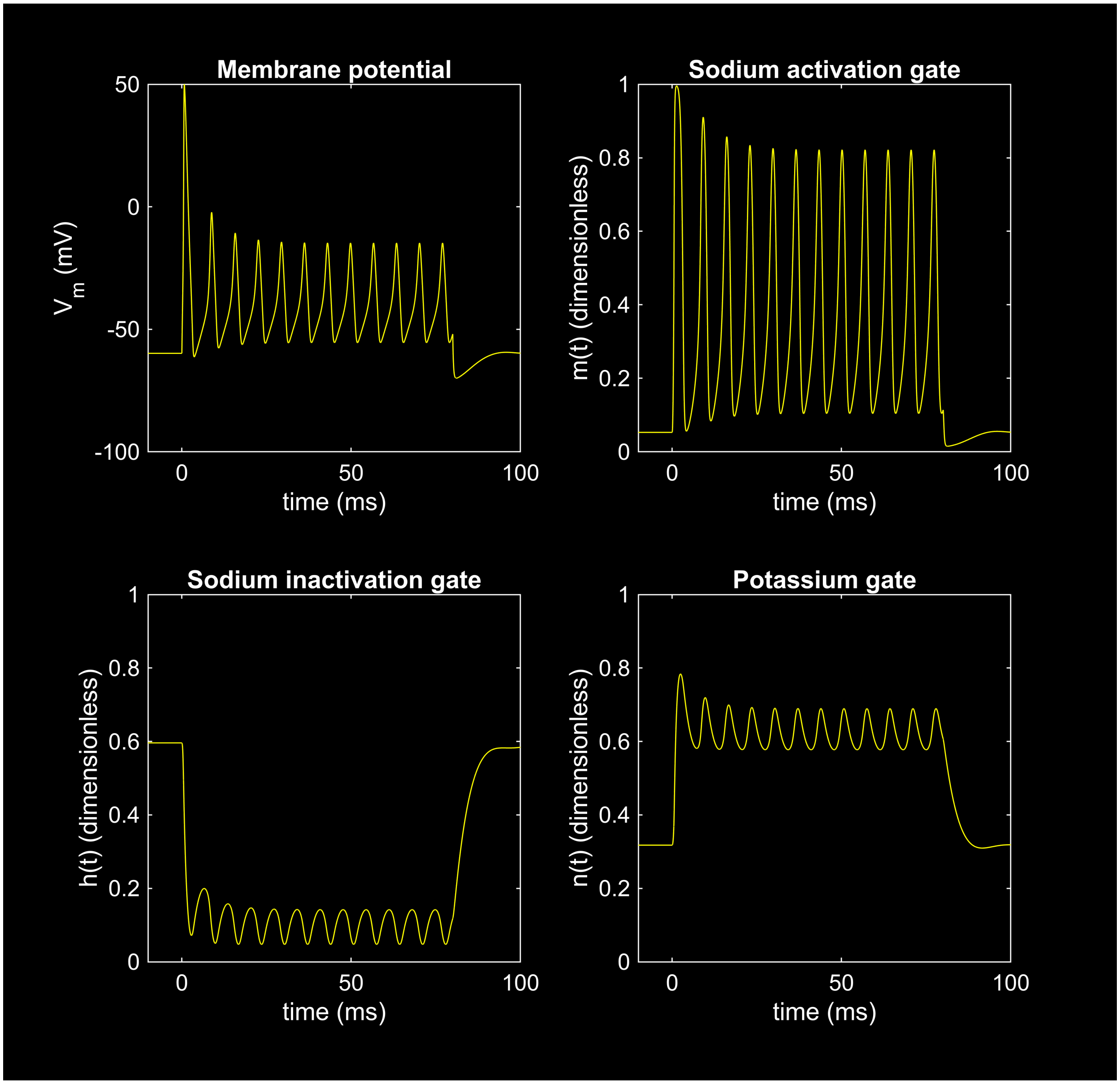

```
% Plot for amp1 = 50
close;
amp1 = 50;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0, 100, 0);
```



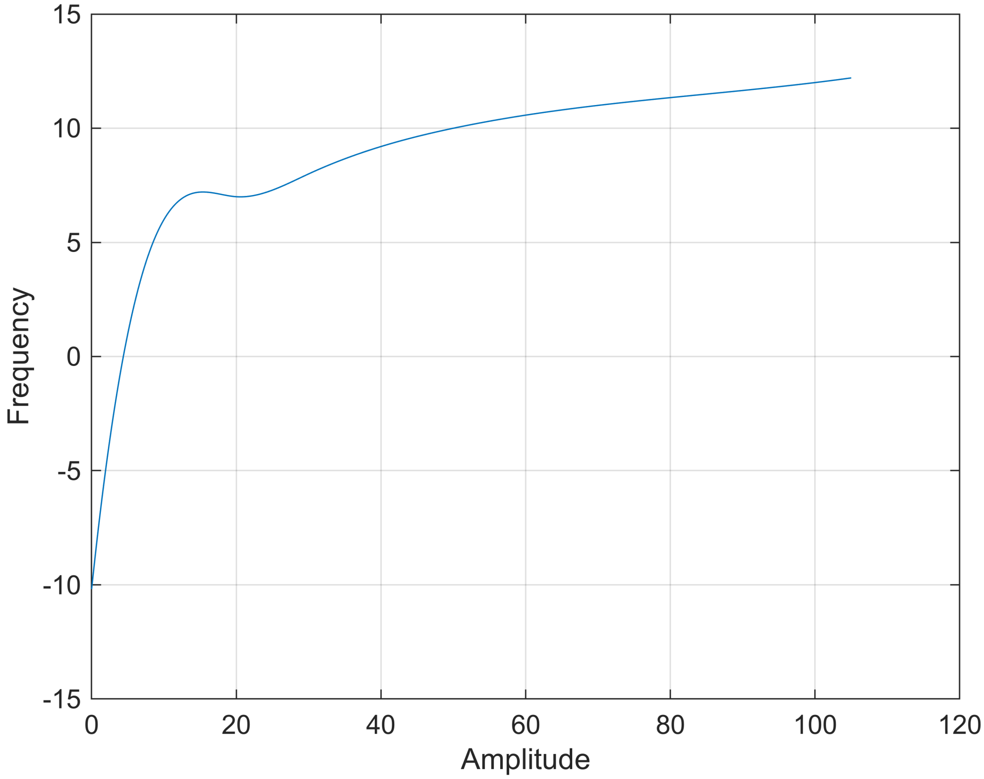

```
% Plot for amp1 = 70
close;
amp1 = 70;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0, 100, 0);
```




```
% Plot for amp1 = 100
close;
amp1 = 100;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0, 100, 0);
```



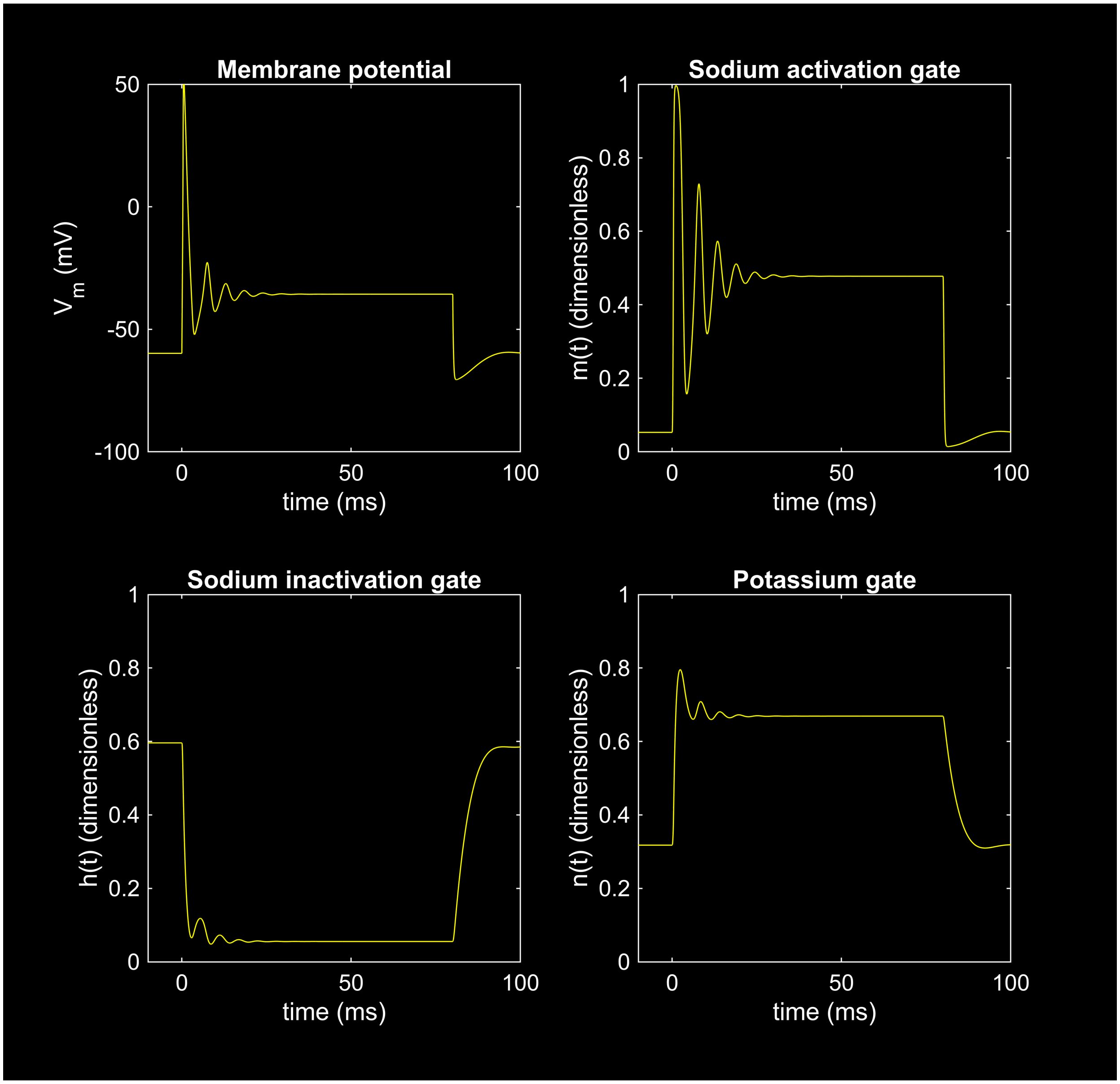

```
close;
amp = [5,10,20,30,50,70,100];
freq = [1,6,7,8,10,11,12];
x_values = linspace(0, 105, 1000); % Points to evaluate the smooth curve
fx = spline(amp, freq, x_values);
plot(x_values, fx);
xlabel('Amplitude');
ylabel('Frequency');
grid on;
```



The frequency increases with the amplitude. Initially, there is a rapid increase at small amplitudes, followed by a gradual decrease in the rate of increase. However, as the frequency rises, the amplitude of the stimulus intensity decreases.

Question 06

```
close;
amp1 = 200;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0, 100, 0);
```

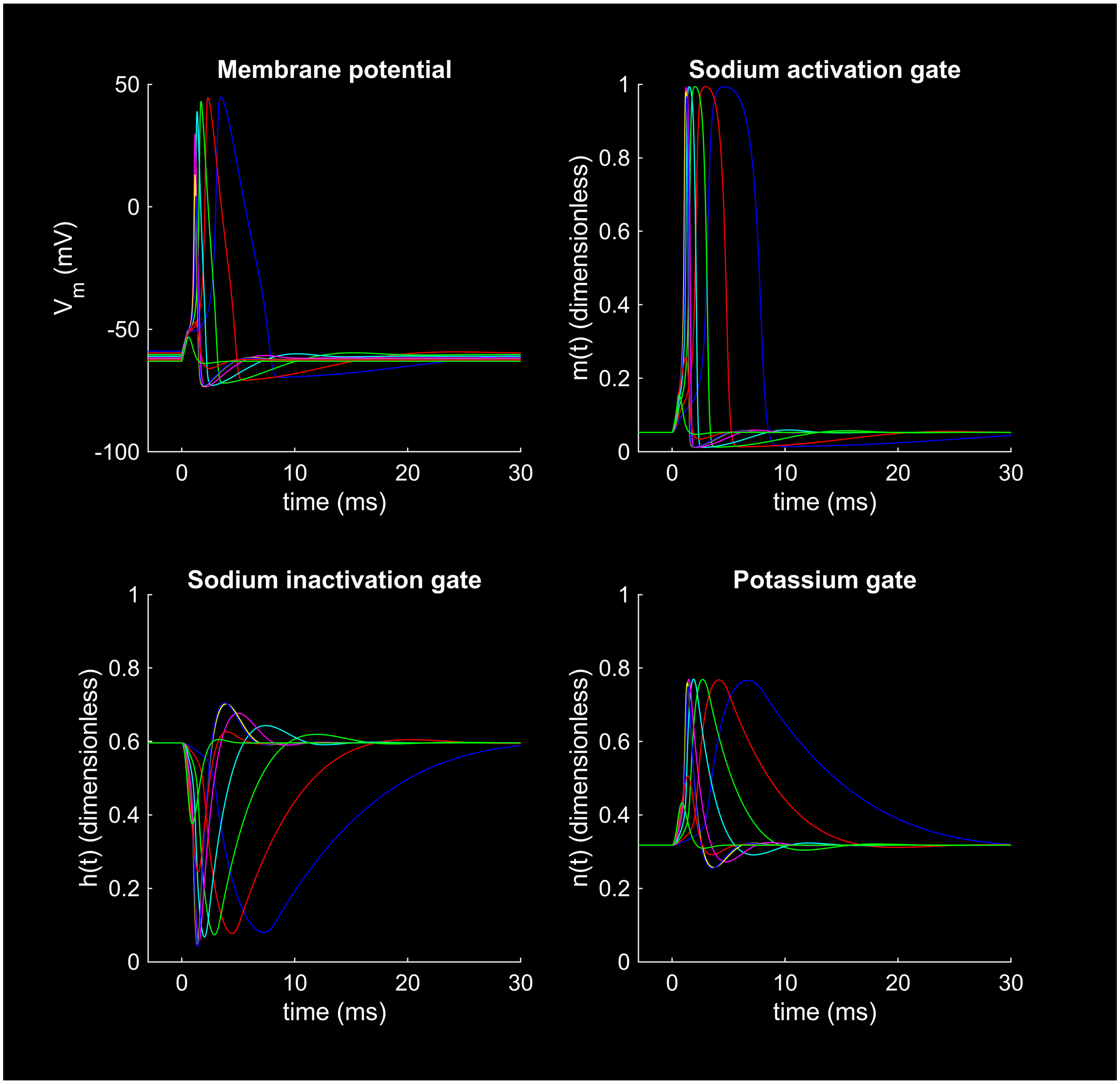

Membrane potential
Vm (mV)
time (ms)
Sodium activation gate
m(t) (dimensionless)
time (ms)
Sodium inactivation gate
h(t) (dimensionless)
time (ms)
Potassium gate
n(t) (dimensionless)
time (ms)

When the stimulating current is set to 200 μA/cm²,
the neuron fires one or two action potentials,
but then it stops firing despite continuous stimulation.
This is known as a depolarization block.
A depolarization block occurs when a sustained,
high depolarizing current keeps the membrane potential elevated,
which prevents the generation of further spikes.
Although higher current might suggest more firing,
the opposite happens due to the voltage-dependent gating variables
of the Hodgkin-Huxley model—m, h, and n.
The m gate activates Na^+ channels and responds quickly,
so it remains high during depolarization.
The h gate, which inactivates Na^+ channels, decreases over time,
and Na^+ channels become inactivated and cannot reopen.
Due to high h values, the current becomes unable to initiate new spikes.
Meanwhile, the n gate activates K^+ channels and increases slowly,
causing more K^+ channels to open and repolarization to fail.
With high m, low h, and high n values,
the membrane enters a depolarized, non-excitable state,
leading to the depolarization block.

Temperature Dependence

Question 07

```
close;
vclamp = 0;
amp1 = 20;
width1 = 0.5;
temp=[0, 5, 10, 15, 20, 24, 25, 26, 30];
for i =1:9
 tempc = temp(i);
 hhmplot(0,30,i);
end
```


Membrane potential
Vm (mV)
time (ms)
Sodium activation gate
m(t) (dimensionless)
time (ms)
Sodium inactivation gate
h(t) (dimensionless)
time (ms)
Potassium gate
n(t) (dimensionless)
time (ms)

Increasing temperature affects several key features of the action potential,
due to the temperature dependence of ion channel kinetics and membrane properties.

1. Faster ion channel kinetics:
Higher temperatures increase the rates of gating variable transitions (m, h, and n).
This speeds up both activation and inactivation of Na^+ and K^+ channels.
2. Shorter action potential duration:
As channels open and close more quickly,
the duration of the action potential becomes shorter.
3. Increased conduction velocity:
Faster depolarization and repolarization enhance signal propagation speed
along axons, especially in myelinated fibers.
4. Lower action potential threshold:
Temperature can slightly reduce the threshold for spike initiation,
making neurons more excitable.
5. Increased firing rate:
Due to faster recovery from inactivation,
neurons may fire at higher frequencies.
6. Changes in amplitude (minor effect):
Amplitude may slightly decrease at higher temperatures.
This is partly due to increased K^+ conductance,
as the n gate activates more quickly.
The enhanced K^+ current causes faster repolarization,
which can reduce the peak voltage reached during the action potential.

Overall, increased temperature enhances the speed and frequency of action potentials,
while reducing their duration and slightly lowering amplitude,
mainly due to accelerated ion channel kinetics, especially of K^+ channels.