University of Moratuwa, Sri Lanka

Department of Electronic & Telecommunication Engineering

Branched Cylinders: Dendritic Tree Approximations

BM2102 Modelling and Analysis of Physiological Systems

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Question 01

Branch Condition

At $X = 0$:

$$\frac{dV_1}{dX}\bigg| X = 0 = (-r_i\lambda_c)_1 I\text{app}$$

$$V_1(X) = A_1e^{-X} + B_1e^{X}$$

For $0 \leq X \leq L_1$:

$$-A_1e^{-X} + B_1e^{X} \quad \text{at} \quad X = 0 = (-r_i\lambda_c)1I\text{app}$$

$$\begin{aligned} -A_1 + B_1 &= (-r_i\lambda_c)1I\text{app} \\ A_1 - B_1 &= (r_i\lambda_c)1I\text{app} \qquad (1) \end{aligned}$$

Boundary conditions:

$$V(L_{21}) = 0, \quad V(L_{22}) = 0 \quad \text{(End voltage zero)}$$

For $L_1 \leq X \leq L_{21}$:

$$A_{21}e^{-X} + B_{21}e^{X} = V_{21}(X)$$

For $L_1 \leq X \leq L_{22}$:

$$A_{22}e^{-X} + B_{22}e^{X} = V_{22}(X)$$

At $X = L_{21}$:

$$\begin{aligned} A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} &= V(L_{21}) \\ A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} &= 0 \qquad (2) \end{aligned}$$

At $X = L_{22}$:

$$\begin{aligned} A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} &= V(L_{22}) \\ A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} &= 0 \qquad (3) \end{aligned}$$

Nodal Condition

Continuity:

$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1)$$

Sample voltage at the common point:

$$A_1e^{-L_1} + B_1e^{L_1} = A_{21}e^{-L_1} + B_{21}e^{L_1} = A_{22}e^{-L_1} + B_{22}e^{L_1} \qquad (*)$$

Expanding:

$$\begin{aligned} A_1e^{-L_1} + B_1e^{L_1} &= A_{21}e^{-L_1} + B_{21}e^{L_1} \\ A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} &= 0 \qquad (4) \end{aligned}$$

$$\begin{aligned} A_{21}e^{-L_1} + B_{21}e^{L_1} &= A_{22}e^{-L_1} + B_{22}e^{L_1} \\ A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} + B_{22}e^{L_1} &= 0 \qquad (5) \end{aligned}$$

Continuation

Differentiation:

$$\frac{dV_{21}}{dX}\bigg| X = L_1 = -A21e^{-L_1} + B_{21}e^{L_1}$$

$$\frac{dV_{22}}{dX}\bigg| X = L_1 = -A22e^{-L_1} + B_{22}e^{L_1}$$

Applying KCL at $X = L_1$:

$$-\frac{1}{(r_i\lambda_c)1}\ \frac{dV_1}{dX}\bigg| X = L_1 = -\frac{1}{(r_i\lambda_c)21}\ \frac{dV21}{dX}\bigg| X = L_1 + -\frac{1}{(r_1 i\lambda_c)22}\ \frac{dV_{22}}{dX}\bigg|_{X=L_1}$$

Substituting the derivatives:

$$-\frac{1}{(r_i\lambda_c)1}\left(-A_1e^{-L_1} + B_1e^{L_1}\right) = -\frac{1}{(r_i\lambda_c)21}\left(-A_{21}e^{-L_1} + B_{21}e^{L_1}\right) + -\frac{1}{(r_i\lambda_c)22}\left(-A22e^{-L_1} + B_{22}e^{L_1}\right)$$

Expanding:

$$\frac{-A_1e^{-L_1}}{(r_i\lambda_c)1} + \frac{B_1e^{L_1}}{(r_i\lambda_c)_1} + \frac{A21e^{-L_1}}{(r_i\lambda_c)21} - \frac{B21e^{L_1}}{(r_1\lambda_c)21} + \frac{A22e^{-L_1}}{(r_i\lambda_c)22} - \frac{B22e^{L_1}}{(r_i\lambda_c)_{22}} = 0 \quad (6)$$

Question 02

$$AX = b$$

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{21}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{-L_1} & e^{-L_1} & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{-L_1} \\ \frac{-e^{-L_1}}{(r_i\lambda_c)1} & \frac{e^{L_1}}{(r_i\lambda_c)_1} & \frac{e^{-L_1}}{(r_i\lambda_c)21} & \frac{-e^{-L_1}}{(r_i\lambda_c)21} & \frac{e^{-L_1}}{(r_i\lambda_c)22} & \frac{-e^{L_1}}{(r_i\lambda_c)_{22}} \end{pmatrix}$$

$$X = \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix}$$

$$b = \begin{pmatrix} (r_i x_c)1I\text{opp} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

First, write the 6 equations from $AX = b$:

$$\text{1st row:} \quad A_1 - B_1 = (r_i x_c)_1 I_{\text{opp}} \tag{1}$$

$$\text{2nd row:} \quad e^{-L_{21}} A_{21} + e^{L_{21}} B_{21} = 0 \tag{2}$$

$$\text{3rd row:} \quad e^{-L_{22}} A_{22} + e^{L_{21}} B_{22} = 0 \tag{3}$$

$$\text{4th row:} \quad e^{-L_1} A_1 + e^{L_1} B_1 - e^{-L_1} A_{21} - e^{-L_1} B_{21} + e^{-L_1} A_{22} = 0 \tag{1}$$

$$\text{(Factor:)} \quad e^{-L_1}(A_1 - A_{21} - B_{21} + A_{22}) + e^{L_1} B_1 = 0 \tag{4}$$

$$\text{5th row:} \quad e^{-L_1} A_{21} + e^{L_1} B_{21} - e^{-L_1} A_{22} - e^{-L_1} B_{22} = 0 \tag{2}$$

$$\text{(Factor:)} \quad e^{-L_1}(A_{21} - A_{22} - B_{22}) + e^{L_1} B_{21} = 0 \tag{5}$$

$$\text{6th row:} \quad -\frac{e^{-L_1}}{(r_i\lambda_c)_1} A_1 + \frac{e^{L_1}}{(r_i\lambda_c)_1} B_1 + \frac{e^{-L_1}}{(r_i\lambda_c)_{21}} A_{21} - \frac{e^{-L_1}}{(r_i\lambda_c)_{21}} B_{21} + \frac{e^{-L_1}}{(r_i\lambda_c)_{22}} A_{22} - \frac{e^{L_1}}{(r_i\lambda_c)_{22}} B_{22} = 0 \tag{6}$$

```matlab


% Dimensions of compartments
d1 = 75e-4;             % cm
d21 = 30e-4;            % cm
d22 = 15e-4;            % cm
l1 = 1.5;               % dimensionless
l21 = 3.0;              % dimensionless
l22 = 3.0;              % dimensionless

% Electrical properties of compartments
Rm = 6e3;               % Ohms cm^2
Rc = 90;                % Ohms cm
Rs = 1e6;               % Ohms

% Calculated coefficients
c1 = 2*(Rc*Rm)^(1/2)/pi;
rl1 = c1*d1^(-3/2);
rl21 = c1*d21^(-3/2);
rl22 = c1*d22^(-3/2);

% Applied current
iapp = 1e-9;            % Amps

% Coefficient matrices
A = [1 -1 0 0 0 0;
     0 0 exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21
        rl1*exp(-l1)/rl22 -rl1*exp(-l1)/rl22];

b = [rl1*iapp 0 0 0 0 0]';
```

Question 03

MATLAB, solving $x = A\backslash b$. **X in matrix:**

$$\begin{pmatrix} 0.0007 \\ 0.0000 \\ 0.0011 \\ -0.0000 \\ 0.0011 \\ -0.0000 \end{pmatrix}$$

Assignment 2 - Branched Cylinders: Dendritic Tree Approximations

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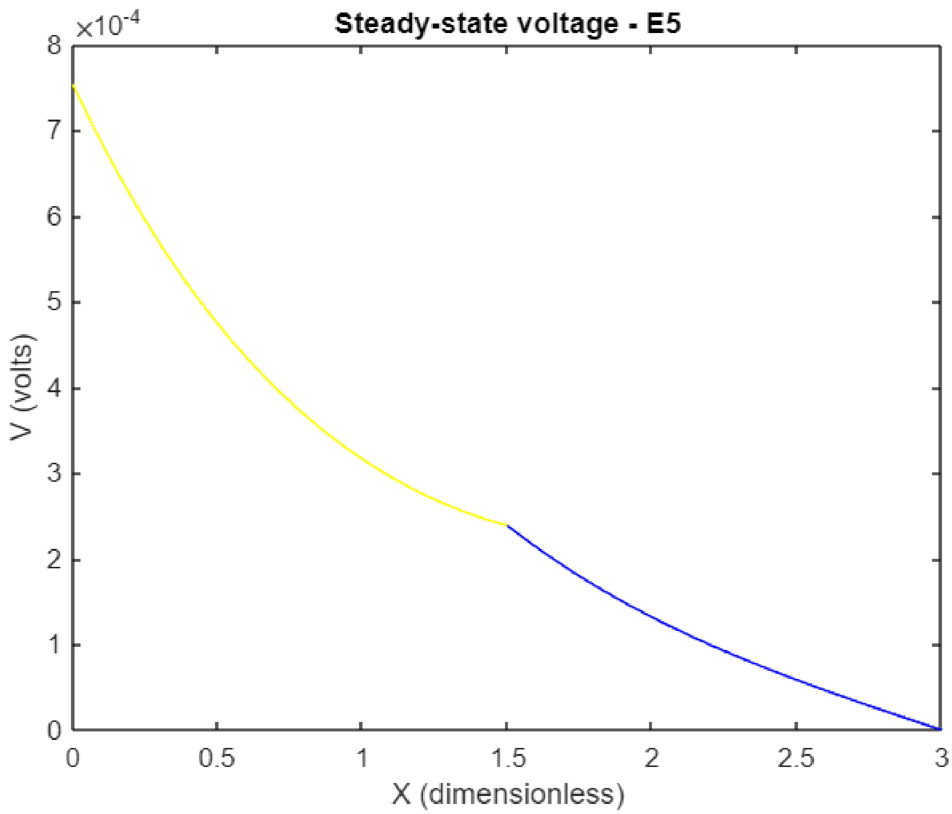
Question 4

```
y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



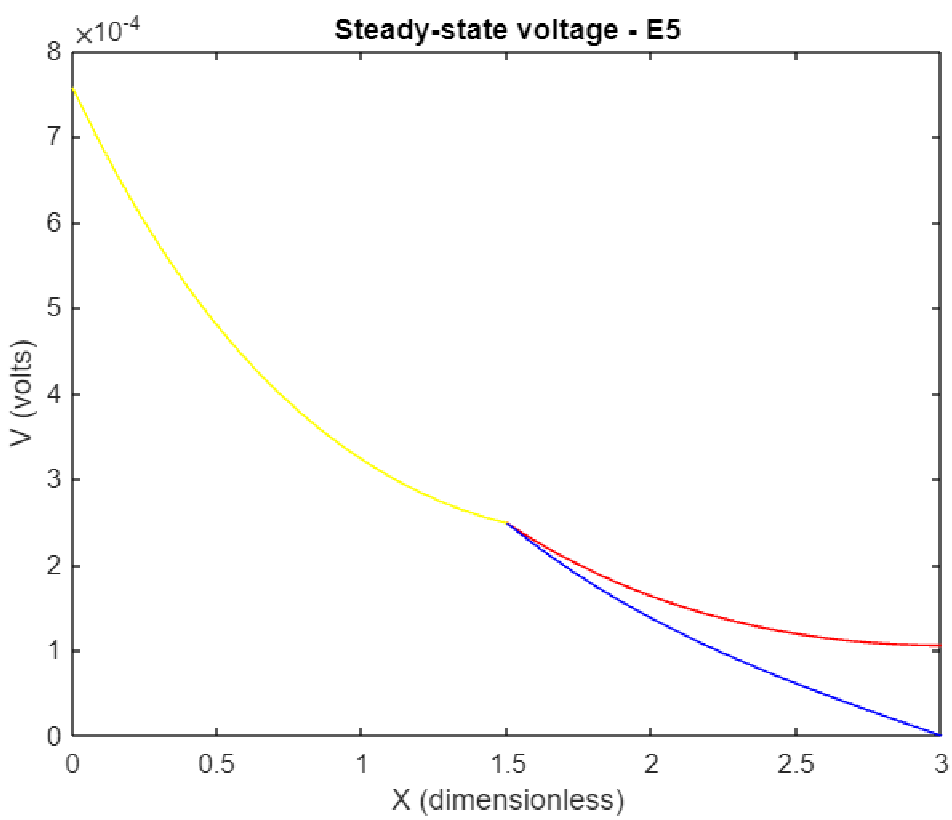
What do you note about the steady state voltage profile in the two daughter branches?

The daughter branches are shown using red and blue lines. However, only the blue line can be seen, indicating that it overlaps the red line. This suggests that the steady-state voltage profiles of both daughter branches are identical. This conclusion is also supported by the results in Question 3, where the given values confirm this observation., $A_{21} = A_{22}$ and $B_{21} = B_{22}$.

Question 5

Part (a)

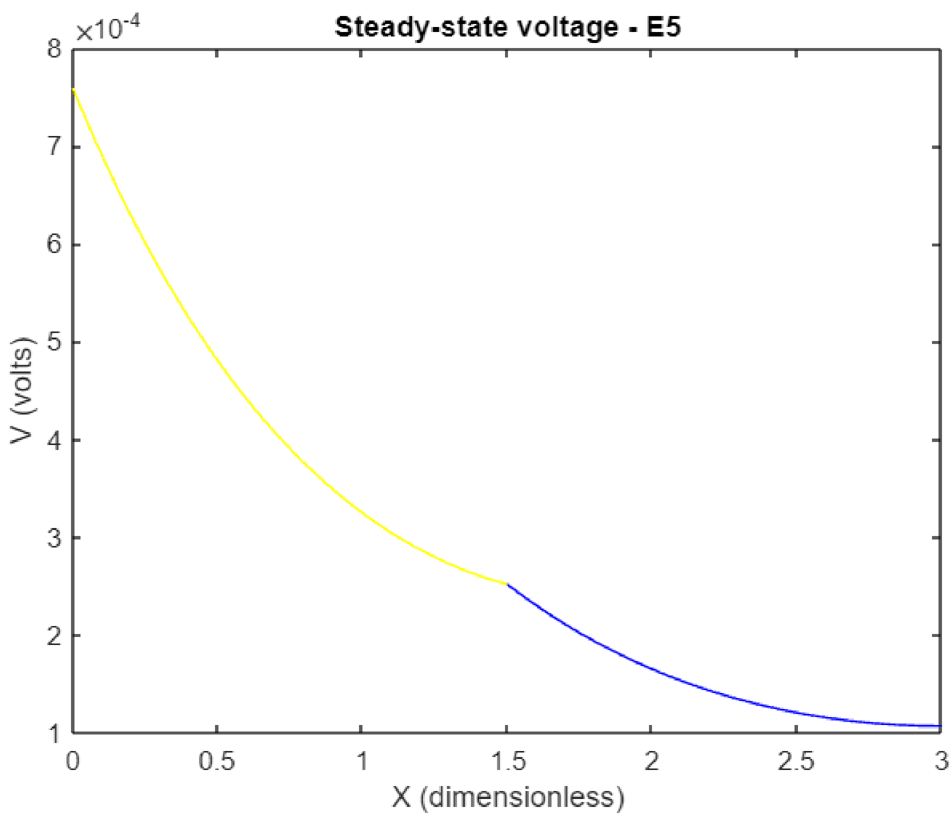
```
% Make a Copy of the original A matrix so that it won't be changed
A_a = A;

% Update the required boundary condition for part (a)
A_a(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];

x_a = A_a\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x_a(1)*exp(-y1) + x_a(2)*exp(y1);
v21 = x_a(3)*exp(-y21) + x_a(4)*exp(y21);
v22 = x_a(5)*exp(-y22) + x_a(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



Part (b)

```
% Make a Copy of the A_a matrix so that it won't be changed
```

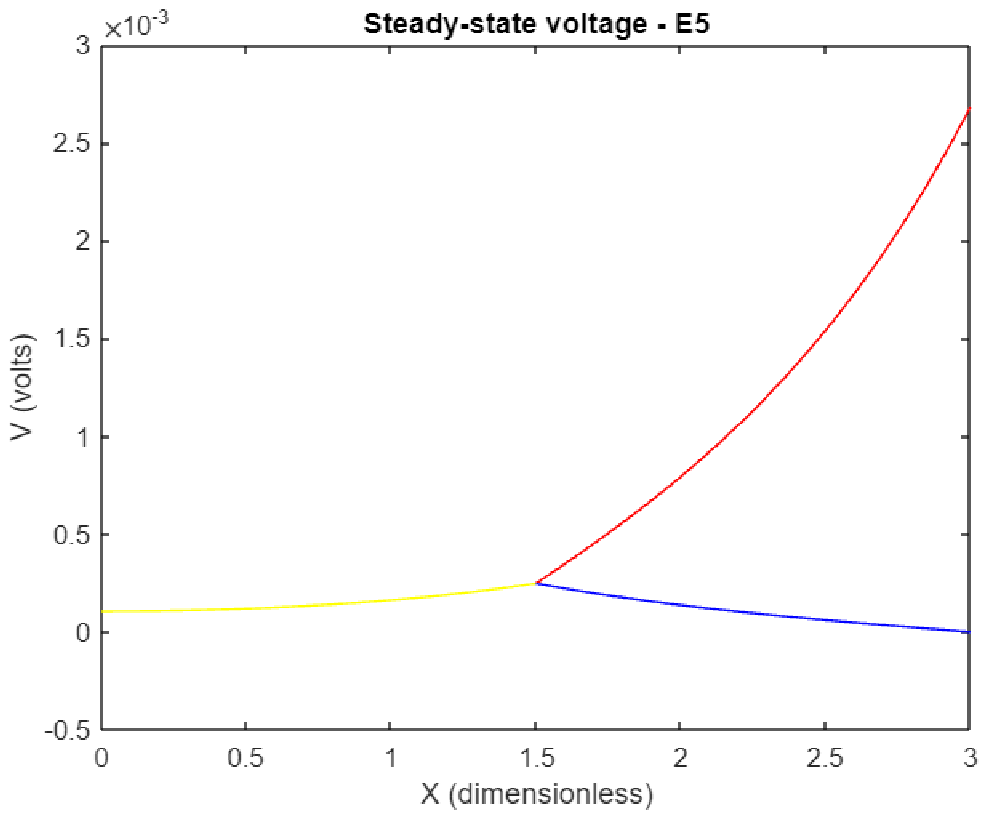
```
A_1 = A_a;

% Update the required boundary condition for part (b)
A_1(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];

x_b = A_1\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x_b(1)*exp(-y1) + x_b(2)*exp(y1);
v21 = x_b(3)*exp(-y21) + x_b(4)*exp(y21);
v22 = x_b(5)*exp(-y22) + x_b(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



Part (c)

```
% Make a Copy of the A_a, b matrix so that it won't be changed
A_2 = A_a;
b_c = b;
% Update the required boundary condition for part (c)
b_c(1) = 0; b_c(2) = rl21*iapp;
```

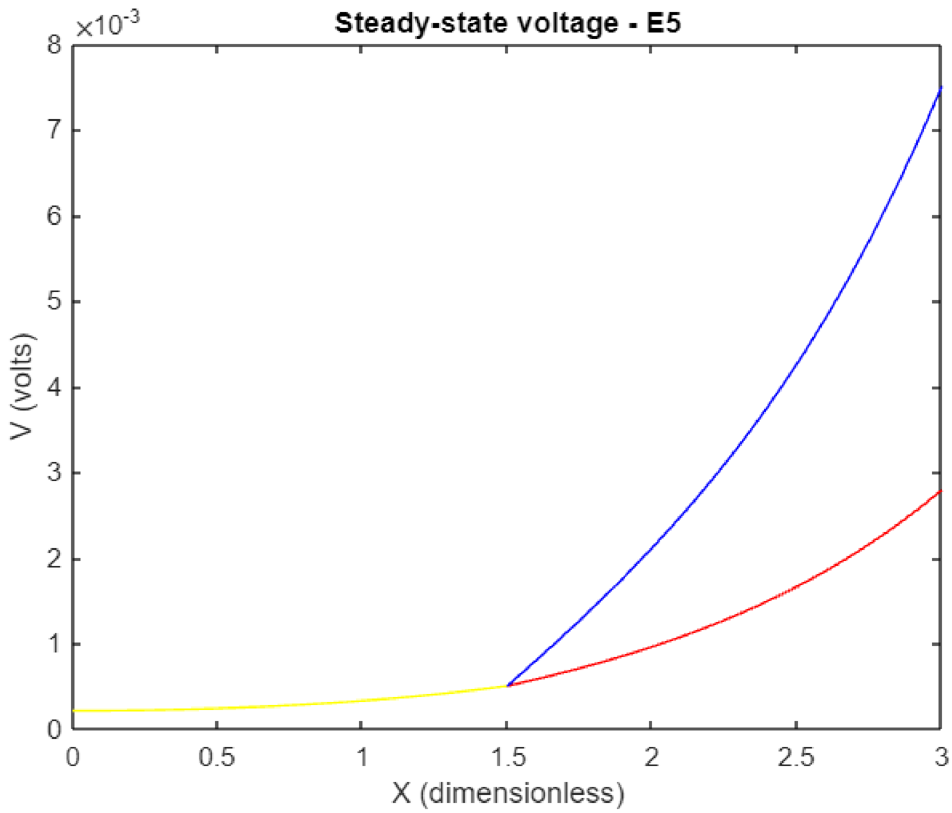
```
x_c = A_2\b_c;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x_c(1)*exp(-y1) + x_c(2)*exp(y1);
v21 = x_c(3)*exp(-y21) + x_c(4)*exp(y21);
v22 = x_c(5)*exp(-y22) + x_c(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



Part (d)

```
% Make a Copy of the A_c matrix so that it won't be changed
A_3 = A_2;
b_3 = b_c;

% Update the required boundary condition for part (d)
A_3(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];                  %3rd row is changed to
achieve the bottom daughter branch boundary condition in part (d) of figure 2
b_3(3) = rl22*iapp;
```

```
x_d = A_d\b_d;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x_d(1)*exp(-y1) + x_d(2)*exp(y1);
v21 = x_d(3)*exp(-y21) + x_d(4)*exp(y21);
v22 = x_d(5)*exp(-y22) + x_d(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



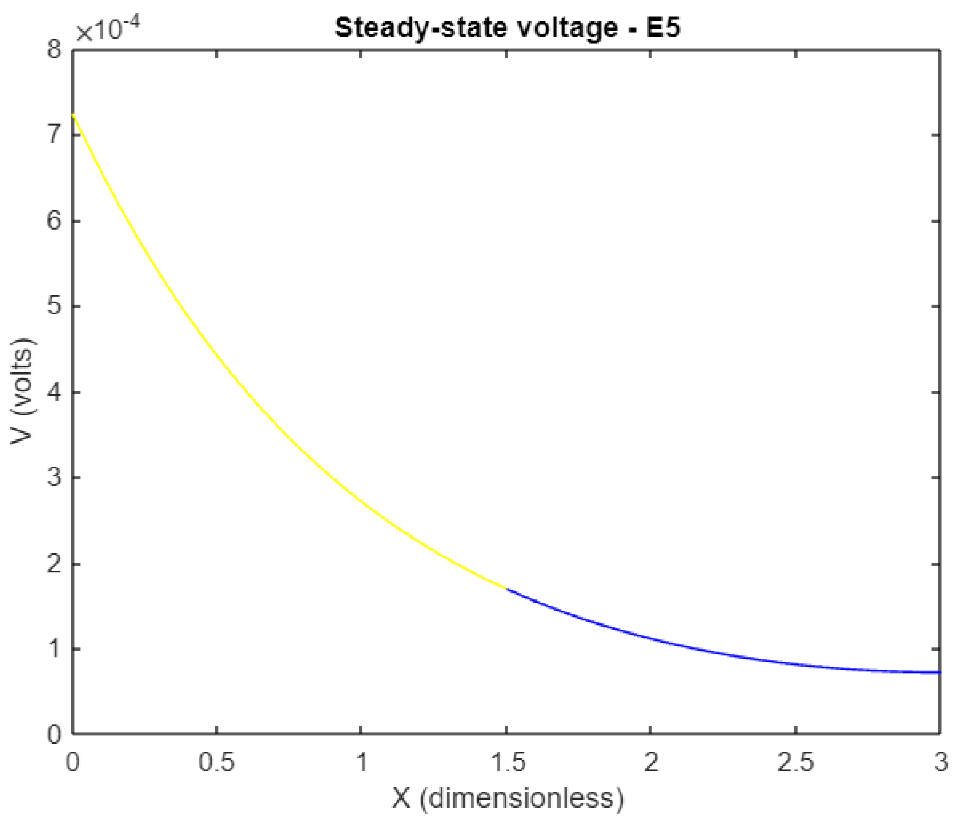
What is the meaning of the positive right hand sides of $\left.\frac{dV_{21}}{dX}\right|_{X=L_{21}}$ and $\left.\frac{dV_{22}}{dX}\right|_{X=L_{22}}$ in 2(c) and 2(d)?

By the graphs , In (c) $\left.\frac{dV_{21}}{dX}\right|_{X=L_{21}} > 0$ and in (d) both $\left.\frac{dV_{21}}{dX}\right|_{X=L_{21}} > 0$ and $\left.\frac{dV_{22}}{dX}\right|_{X=L_{22}} > 0$. This indicates that the membrane voltage of the daughter branches increases as it approaches the rightmost boundaries. The equations also confirm that an outward current flows through the branches under these conditions. Altogether, these observations show that an electrical impulse travels along the branch, increasing the membrane potential at the boundaries, and transmitting the signal from one neuron to another.

Question 6

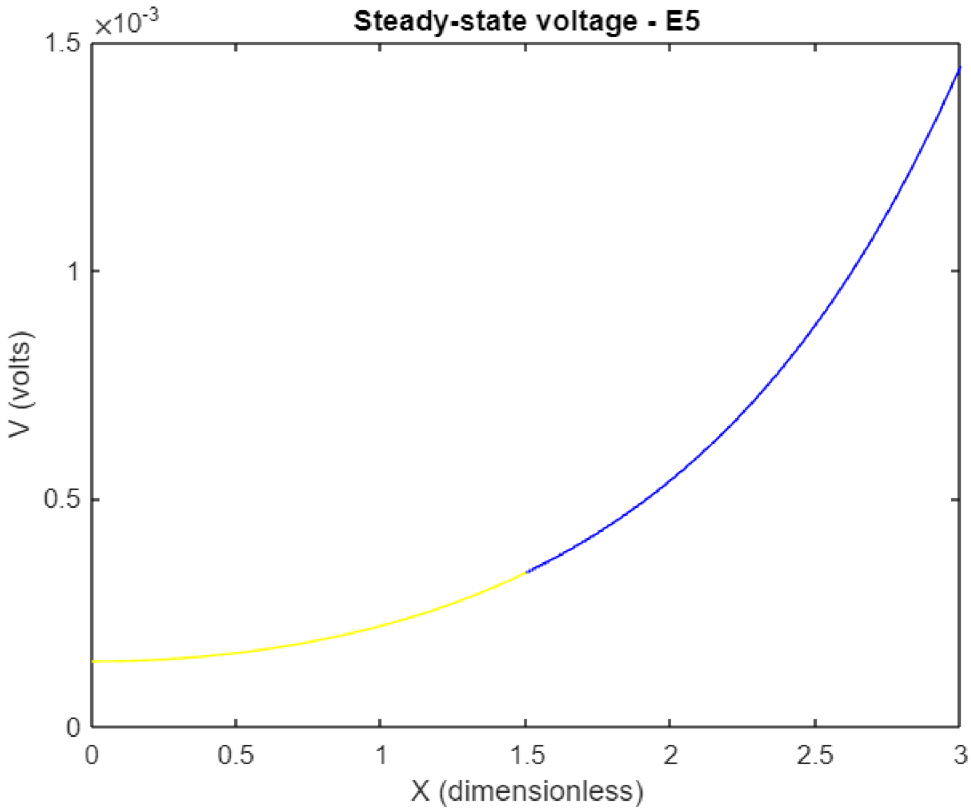
Q6 Part (b)

```
% Make a Copy of the original A matrix so that it won't be changed
A6_b = A;

% Update the required boundary conditions for part (b)
A6_b(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A6_b(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];

x6_b = A6_b\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x6_b(1)*exp(-y1) + x6_b(2)*exp(y1);
v21 = x6_b(3)*exp(-y21) + x6_b(4)*exp(y21);
v22 = x6_b(5)*exp(-y22) + x6_b(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



Q6 Part (d)

```
% Make a Copy of the original A matrix so that it won't be changed
A6_d = A;
b6_d = b;

% Update the required boundary conditions for part (d)
A6_d(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A6_d(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];
b6_d(1) = 0; b6_d(2) = rl21*iapp;
b6_d(3) = rl22*iapp;

x6_d = A6_d\b6_d;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x6_d(1)*exp(-y1) + x6_d(2)*exp(y1);
v21 = x6_d(3)*exp(-y21) + x6_d(4)*exp(y21);
v22 = x6_d(5)*exp(-y22) + x6_d(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



What do you notice?

At the branching point, the graphs transition smoothly without any abrupt changes. The voltage profiles of both daughter branches are almost identical in both cases. This is due to the fact that the daughter branches have the same diameter, allowing them to carry equal amounts of current.