

GATE IN 2009 – Question 8 Complete Analysis

Question 8

Q.8 The minimal sum-of-products expression for the logic function f represented by the given Karnaugh map is:

		PQ			
RS		00	01	11	10
00		0	1	0	0
01		0	1	1	1
11		1	1	1	0
10		0	0	1	0

- (A) $QS + PR'S + PQR + P'RS + P'QR'$ (C) $P'RS' + P'Q'R + PR'S + PQ'R$
 (B) $Q'S + P'RS + P'Q'R + PR'S + PQ'R$ (D) $PR'S + PQR + P'RS + P'QR'$

Answer: (A) $QS + PR'S + PQR + P'RS + P'QR'$

(GATE IN 2009)

Question Analysis

- The given problem involves a four-variable logic function $f(P, Q, R, S)$ represented using a Karnaugh map.
- The variables P and Q form the column indices, while R and S form the row indices, arranged in Gray code order 00, 01, 11, 10.
- Each cell containing a value 1 represents a minterm where the function output is equal to 1.
- The objective is to obtain the minimal Sum-of-Products (SOP) expression by grouping adjacent 1's.

- Valid groupings must be formed in powers of two such as 1, 2, or 4 cells.
- While forming groups, variables that change within the group are eliminated, and only constant variables remain in the product term.
- After systematic grouping of all possible 1's, the minimized expression becomes:

$$f = QS + PR'S + PQR + P'RS + P'QR'.$$
- This expression represents the minimal realization of the given logic function.

Required Components and Connections

S.No	Component
1	Arduino Uno Board
2	Breadboard
3	Seven Segment Display
4	220Ω Resistor
5	Jumper Wires
6	USB Cable

Segment	Arduino Pin
a	2
b	3
c	4
d	5
e	6
f	7
g	8
VCC	5V
GND	GND

Logic Description

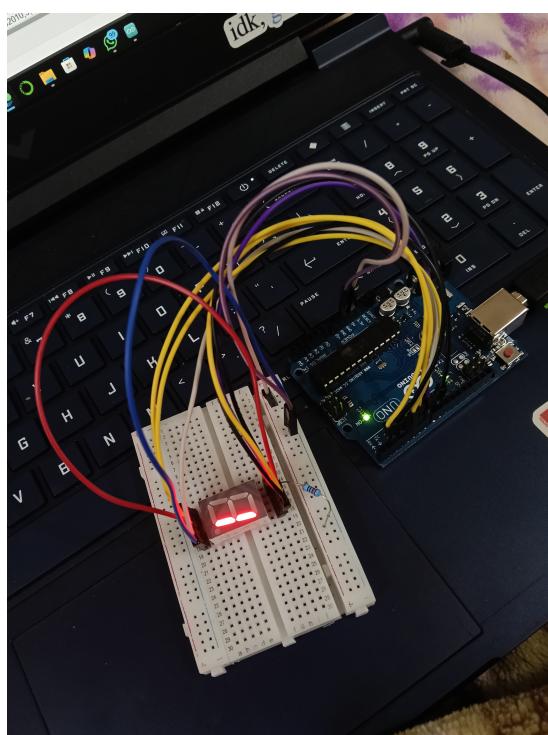
- The logic function is implemented using AND-OR realization based on the simplified SOP expression.
- Each product term such as QS , $PR'S$, and PQR is implemented using AND operations.
- Complemented variables P' and R' are generated using NOT operations.
- All product terms are finally combined using OR operations to produce the output f .
- The final Boolean expression ensures minimal hardware complexity.

Code Uploading Steps

1. Create a new folder for BLINK
2. Write The code in main.asm
3. Run the main.asm with command ”avr-gcc main.asm”. It will compile the code and creates .hex file
4. Copy the .hex file to ArduinoDroid folder
5. connect the Arduino UNO to mobile with OTG cable
6. Upload the hex file using ”upload precompiled” option
7. Observe the ouput and verify the expression

Experimental Truth Table

P	Q	R	S	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1



Conclusion

- The Karnaugh map was analyzed systematically to derive the minimal SOP expression.
- Proper grouping of adjacent 1's eliminated redundant variables and reduced complexity.
- The simplified expression obtained is $f = QS + PR'S + PQR + P'RS + P'QR'$.
- Hardware implementation verified the theoretical simplification.
- Therefore, the minimal SOP expression corresponds to option (A).