The Ideal Gas Law at the Center of the Sun

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Chemistry students are usually taught that the ideal gas law $PV = nRT$ is not valid for real gases at high pressures, since the space taken up by the individual atoms or molecules (the "excluded" volume) is a significant fraction of the volume ("empty space") of the gas. At least one chemistry text suggests that the ideal gas law is generally valid for real gases only at pressures up to 10 atm (*1*). Thus it may come as a surprise to chemistry students to learn that solar physicists routinely apply the ideal gas law to the interiors of average stars like the sun, where the pressures are billions of atmospheres! It is educational and entertaining to examine this use of the gas law and to show how it is valid at these extreme pressures.

The sun is composed almost exclusively of hydrogen and helium gases. At the very high temperatures (millions of degrees Kelvin) deep in the solar interior, the gas atoms are ionized to form a plasma (*2*). Therefore, the space taken up by the individual gas particles, which are H and He nuclei and free electrons, is negligible compared to the gas volume. This is true even at the very great gas densities, well over 100 g/cm^3, near the center of the sun.

To illustrate this point, we will assume a gas composition at the very center of the sun of 36% H and 64% He by mass, at a density of 158 g/cm^3, in accord with the "standard model" of the sun developed by solar physicists (*3*). Assuming 100% ionization, we can calculate that in 1 cm^3 of central solar gas there are 3.4×10^{25} H nuclei, 1.5×10^{25} He nuclei, and 6.4×10^{25} free electrons, which is a total of 1.1×10^{26} gas particles. To determine the volume excluded by these gaseous particles, we assume each to be a sphere. Using nuclear radii $r_{\mathrm{H}} = 1.4 \times 10^{-13}$ cm and $r_{\mathrm{He}} = 2.2 \times 10^{-13}$ cm, based on the approximation $r_{\mathrm{nucleus}} = 1.4 \times 10^{-13}\ \mathrm{cm} \times (\text{mass number})^{1/3}$ (*4*), and an electron dimension of 10^{-16} cm (*5*), we calculate that the excluded volumes for the H and He nuclei in 1 cm^3 at the sun's center are (respectively) 3.7×10^{-13} cm^3 and 6.8×10^{-13} cm^3. The size of the free electron is negligible compared to that of the H and He nuclei, thus the total excluded volume (1.1×10^{-12} cm^3) is a mere 1.1×10^{-10} percent of the gas volume. No overcrowding here! On this point, the use of the ideal gas law is justified even at the very high density of gas at the center of the sun.

We can use the ideal gas law to calculate the temperature at the center of the sun, using a "standard model" central solar pressure of 2.5×10^{11} atm (*6*) and the aforementioned density and chemical composition. The numbers of moles of gas particles in 1 cm^3 of central solar gas are $n_{\mathrm{H\ nuclei}} = 56$, $n_{\mathrm{He\ nuclei}} = 25$, $n_{\mathrm{free\ electrons}} = 106$, giving $n_{\mathrm{total}} = 187$. Application of $PV = nRT$ yields a central solar temperature of 16×10^6 K, which is equal (considering the precision of the calculation) to the widely quoted "standard model" value of 15×10^6 K (*7*).

This result illustrates the justified use of the ideal gas law at the center of the sun, at least according to the "standard model". But this does not imply that the "standard model" gives the correct picture of conditions in the solar interior. Many other models have been proposed, with varying assumptions about chemical composition (%H and He), densities, pressures, temperatures, and percent ionization. Allen (*8*) publishes values for a composite model giving central solar conditions of 38% H, density 160 g/cm^3, pressure 3.4×10^{17} dyn/cm^2 (which is 3.4×10^{11} atm), and temperature 15×10^6 K. Application of $PV = nRT$ to these values yields $T = 21 \times 10^6$ K, considerably higher than the published value for the composite model. This suggests that factors in addition to ideal gas behavior may be at work, although the solar gas is still assumed to be ideal.

There is little question that the ideal gas law is not completely sufficient to describe conditions at the centers of stars much more massive than the sun. In such stars the "ordinary" gas pressure must be supplemented by the radiation pressure exerted by photons (*9*). And in extremely dense stars called "white dwarfs", the phenomenon known as electron degeneracy makes the stellar plasma behave more like a solid than a gas (*10*). However, Schwarzschild (*11*) has shown that radiation pressure and electron degeneracy are not important for the gas at the center of the sun.

Although direct observation of the solar interior is not possible, recent advances in solar seismography and neutrino counting (*12*) could help determine which solar models are most accurate. Regardless of the uncertainties and vagaries of these models, they all rest firmly on the ideal gas law as the fundamental equation of state for the hot, dense, high-pressure gas at the center of the sun.

The observation of sunspots destroyed the Aristotelian concept of the sun as a "perfect body". It is somehow comforting to note that our blemished sun is at least composed of a "perfect gas".

Literature Cited

1. Chang, R. *Chemistry*, 3rd ed.; Random House: New York, 1988; p 208.
2. Zeilik, M.; Smith, E. *Introductory Astronomy and Astrophysics*, 2nd ed.; Saunders: Philadelphia, 1987; p 272.
3. Nicolson, I. *The Sun*; Rand McNally: New York, 1982; p 82.
4. Mortimer, C. *Chemistry*, 6th ed.; Wadsworth: Belmont, CA, 1986; p 747.
5. Ne'eman, Y.; Kirsh, Y. *The Particle Hunters*; Cambridge: London, 1986; p 61.
6. Nicolson, I. *The Sun*; Rand McNally: New York, 1982; p 82.
7. Nicolson, I. *The Sun*; Rand McNally: New York, 1982; p 82.
8. Allen, C. W. *Astrophysical Quantities*, 3rd ed.; Oxford University: New York, 1973; p 163.
9. Zeilik, M.; Smith, E. *Introductory Astronomy and Astrophysics*, 2nd ed.; Saunders: Philadelphia, 1987; p 272.
10. Zeilik, M.; Smith, E. *Introductory Astronomy and Astrophysics*, 2nd ed.; Saunders: Philadelphia, 1987; pp 293–294.
11. Schwarzschild, M. *Structure and Evolution of Stars*; Dover: New York, 1965; pp 56 and 60.
12. Abell, G.; Morrison, D.; Wolff, S. *Exploration of the Universe*, 5th ed.; Saunders: Philadelphia, 1987; pp 509–511.