Charge distribution on a conducting plane via method of moments

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Abstract

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Contents

1 Introduction

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2 Theory

Consider a Hilbert space $\mathbb{H}$ over $L^2(\mathbb{R}^n)$ equipped with the inner product defined as

$$\langle f|g\rangle = \int_{\mathbb{R}} f(x)g(x)\,\mathrm{d}x\,. \tag{1}$$

We are interested in the inhomogeneous operator equation

$$\mathcal{L}(f) = g, \tag{2}$$

where $\mathcal{L} : D \subseteq \mathbb{H} \to \mathbb{H}$ denotes a linear operator, g represents a known function, and f is the unknown function to be determined. To solve this equation, we employ a countable basis $\mathfrak{B} = (f_1, f_2, \dots)$ of $\mathbb{H}$ and express f as an infinite series expansio

$$f = \sum_{l=1}^{\infty} c_l f_l, \tag{3}$$

where $(f_l)_{l=1}^{\infty}$ is often called set of expansion functions. Substituting the expansion (3) into equation (2) yields

$$\sum_{l=1}^{\infty} c_l \mathcal{L}(f_l) = g. \tag{4}$$

Let us introduce another basis $\mathfrak{C} = (w_1, w_2, \dots)$ of $\mathbb{H}$, commonly referred to as weighting functions or test functions. Taking the inner product of equation (4) with each w_k leads to the system

$$\sum_{k=1}^{\infty} c_l \langle w_k|\mathcal{L}(f_l)\rangle = \langle w_k|g\rangle \quad k = 1, 2, \dots \tag{5}$$

The system of equations (5) can be expressed in matrix form as

$$\mathbf{L}\mathbf{F} = \mathbf{G}, \tag{6}$$

where $\mathbf{L}_{kl} = \langle w_k | \mathcal{L}(f_l) \rangle$, $\mathbf{F}_l = c_l$, and $\mathbf{G}_k = \langle w_k | g \rangle$. For a regular matrix $\mathbf{L}$, the solution is given by

$$\mathbf{F} = \mathbf{L}^{-1}\mathbf{G}. \tag{7}$$

Consequently, substituting (7) into (4), we obtain the solution

$$f = \sum_{l=1}^{\infty} \mathbf{F}_l f_l. \tag{8}$$

The nature of the solution—whether approximate or exact—depends on the properties of the operator $\mathcal{L}$ and the choice of expansion functions f_l and test functions w_k. While the matrix $\mathbf{L}$ may be of infinite order with a theoretically obtainable inverse, practical implementations typically use finite-dimensional bases, allowing for numerical matrix inversion techniques. In the special case where the test functions are chosen identical to the expansion functions ($w_k = f_k$), the method is known as the Galerkin's method [1].

3 Implementation

where $\mathcal{L}$ is a linear operator.

The method of moments generally transforms operators $\mathcal{L} : L^2 \to L^2$ to matrix operators $M : l^2 \to l^2$.

We begin with the integral formulation for electric potential ϕ_e given in [2]

$$\phi_e(\boldsymbol{r}) = \int_\Sigma \frac{q_e(\boldsymbol{r}')}{\|\boldsymbol{r} - \boldsymbol{r}'\|} \, \mathrm{d}\boldsymbol{r}' \,, \tag{9}$$

where Σ is the source region, q_e is the electric charge density. Now we introduce the following basis function

4 Discussion

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References

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[2] A. Zangwill, *Modern Electrodynamics*, ser. Modern Electrodynamics. Cambridge University Press, 2013.