

```

(Debug) In[106]:= SetDirectory[
    "/Users/marco/marco.live/TEX/2014/BUTTER&JAM/svn/code/Mathematica"]
(Debug) Out[106]= /Users/marco/marco.live/TEX/2014/BUTTER&JAM/svn/code/Mathematica

(Debug) In[107]:= << "wInv.ma"

(Debug) In[108]:= ForwardVisit = Function[tw, FVLastStep@Fold[FVStep, FVBase, tw]];
BackwardVisit = Function[tw, BVLastStep@Fold[BVStep, BVBase, tw]];

wRevInitBase = {};
wRevInitStep[tail_, element_] :=
  Module[{u, v, g1, g2, m, stop, sn, rs, fwdv, fwdg2, fwdm},
    (
      u = element[[1]];
      v = element[[2]];
      g1 = element[[3]];
      g2 = element[[4]];
      m = element[[5]];
      stop = element[[6]];
      sn = element[[7]];
      rs = element[[8]];
      fwdv = element[[9]];
      fwdg2 = element[[10]];
      fwdm = element[[11]];

      Prepend[tail, {u, v, g1, g2, m, stop, B, B, 0, 0, 0}]
    )
  ]

wRevInit = Function[tw, Fold[wRevInitStep, wRevInitBase, tw]];

```

```
(Debug) In[113]:= (*second example*)
```

```
u1 = {0, 1, 0, 0}
p1 = {1, 0, 1, 1}
```

```
(* paper example *)
U = {0, 1, 0, 0}
p = {1, 0, 1, 1}
```

```
baseinput =
{U,
 p,
 {0, 0, 0, 1},
 {0, 0, 0, 0},
 p,
 {0, 0, 0, 0},
 {B, B, B, B},
 {B, B, B, B},
 {0, 0, 0, 0},
 {0, 0, 0, 0},
 {0, 0, 0, 0}}
}
```

```
base = MapThread[List, baseinput];
TableForm@base
```

```
(Debug) Out[113]= {0, 1, 0, 0}
```

```
(Debug) Out[114]= {1, 0, 1, 1}
```

```
(Debug) Out[115]= {0, 1, 0, 0}
```

```
(Debug) Out[116]= {1, 0, 1, 1}
```

```
(Debug) Out[117]= {{0, 1, 0, 0}, {1, 0, 1, 1}, {0, 0, 0, 1}, {0, 0, 0, 0}, {1, 0, 1, 1}, {0, 0, 0, 0},
 {B, B, B, B}, {B, B, B, B}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

```
(Debug) Out[118]//TableForm=
```

0	1	0	0	1	0	B	B	0	0	0
1	0	0	0	0	0	B	B	0	0	0
0	1	0	0	1	0	B	B	0	0	0
0	1	1	0	1	0	B	B	0	0	0

```
(Debug) In[169]:= F0 = wRevInit[BackwardVisit@ForwardVisit@Reverse@base];
F1 = wRevInit[BackwardVisit@ForwardVisit@Reverse@F0];
F2 = wRevInit[BackwardVisit@ForwardVisit@Reverse@F1];
F3 = wRevInit[BackwardVisit@ForwardVisit@Reverse@F2];
TableForm@F0
TableForm@F1
TableForm@F2
TableForm@F3
```

```
in1 : {0, 1, 1, 0, 1, 0, B, B, 0, 0, 0}
```

```
in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B}, {}}
```

```
stopt :B
u      :0
snt    :B
sn      :B
rst     :B
rs      :B
glb     :1
```

```

case : stopt=B;snt=B;u=0;gl=1
{{0, B, 0, B, B, B, 0, 0, 1, 0, 1}, {B, B, B, B, B, B, B, B, B, B, B}}
in1 : {0, 1, 0, 0, 1, 0, B, B, 0, 0, 0}
in2 : {Prepend, {0, B, 0, B, B, B, 0, 0, 1, 0, 1}, {}}

stopt :B
u      :0
snt    :0
sn     :B
rst    :0
rs     :B
glb    :0

case : stopt=B;snt=0;rst=0
{{0, 1, 1, B, 1, B, 0, 0, 1, 0, 1}, {0, B, 0, 0, B, B, 0, 0, 1, 0, 1}}
in1 : {1, 0, 0, 0, 0, 0, B, B, 0, 0, 0}
in2 : {Prepend, {0, 1, 1, B, 1, B, 0, 0, 1, 0, 1}, {{0, B, 0, 0, B, B, 0, 0, 1, 0, 1}}}

stopt :B
u      :1
snt    :0
sn     :B
rst    :0
rs     :B
glb    :0

case : stopt=B;snt=0;rst=0
{{1, 1, 0, B, 1, B, 0, 0, 0, 0, 0}, {0, 1, 1, 0, 1, B, 0, 0, 1, 0, 1}}
in1 : {0, 1, 0, 0, 1, 0, B, B, 0, 0, 0}
in2 : {Prepend, {1, 1, 0, B, 1, B, 0, 0, 0, 0, 0},
      {{0, 1, 1, 0, 1, B, 0, 0, 1, 0, 1}, {0, B, 0, 0, B, B, 0, 0, 1, 0, 1}}}

stopt :B
u      :0
snt    :0
sn     :B
rst    :0
rs     :B
glb    :0

case : stopt=B;snt=0;rst=0
{{0, 0, 1, B, 0, B, 0, 0, 1, 0, 1}, {1, 1, 0, 0, 1, B, 0, 0, 0, 0, 0}}

last step input :
{Prepend, {0, 0, 1, B, 0, B, 0, 0, 1, 0, 1}, {{1, 1, 0, 0, 1, B, 0, 0, 0, 0, 0},
      {0, 1, 1, 0, 1, B, 0, 0, 1, 0, 1}, {0, B, 0, 0, B, B, 0, 0, 1, 0, 1}}}

branch = stopt=B;rst=0;

out      = {{0, 1, 0, 0, 1, B, B, 0, B, B, B},
            {0, 0, 1, 0, 0, B, 0, 0, 1, 0, 1}, {1, 1, 0, 0, 1, B, 0, 0, 0, 0, 0},
            {0, 1, 1, 0, 1, B, 0, 0, 1, 0, 1}, {0, B, 0, 0, B, B, 0, 0, 1, 0, 1}}
in1 : {0, 1, 0, 0, 1, B, B, 0, B, B, B}
in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B}, {}}

stopt :B
rst    :B
rs     :0
stop   :B
u      :0
v      :1

case : stopt=B;rs=0;

```

```

{{0, 1, 0, 0, 1, B, B, 0, B, B, B}, {B, B, B, B, B, B, 0, B, B, B}}

in1 : {0, 0, 1, 0, 0, B, 0, 0, 1, 0, 1}

in2 : {Prepend, {0, 1, 0, 0, 1, B, B, 0, B, B, B}, {}}

stopt :B
rst    :0
rs     :0
stop   :B
u      :0
v      :0

case : stopt=B;rs=0;

{{0, 0, 1, 0, 0, B, 0, 0, B, B, B}, {0, 1, 0, 0, 1, B, B, 0, B, B, B}}

in1 : {1, 1, 0, 0, 1, B, 0, 0, 0, 0, 0}

in2 : {Prepend, {0, 0, 1, 0, 0, B, 0, 0, B, B, B}, {{0, 1, 0, 0, 1, B, B, 0, B, B, B}}}

stopt :B
rst    :0
rs     :0
stop   :B
u      :1
v      :1

case : stopt=B;rs=0;

{{1, 1, 0, 0, 1, B, 0, 0, B, B, B}, {0, 0, 1, 0, 0, B, 0, 0, B, B, B}}

in1 : {0, 1, 1, 0, 1, B, 0, 0, 1, 0, 1}

in2 : {Prepend, {1, 1, 0, 0, 1, B, 0, 0, B, B, B},
       {{0, 0, 1, 0, 0, B, 0, 0, B, B, B}, {0, 1, 0, 0, 1, B, B, 0, B, B, B}}}

stopt :B
rst    :0
rs     :0
stop   :B
u      :0
v      :1

case : stopt=B;rs=0;

{{0, 1, 1, 0, 1, B, 0, 0, B, B, B}, {1, 1, 0, 0, 1, B, 0, 0, B, B, B}}

in1 : {0, B, 0, 0, B, B, 0, 0, 1, 0, 1}

in2 : {Prepend, {0, 1, 1, 0, 1, B, 0, 0, B, B, B}, {{1, 1, 0, 0, 1, B, 0, 0, B, B, B},
       {0, 0, 1, 0, 0, B, 0, 0, B, B, B}, {0, 1, 0, 0, 1, B, B, 0, B, B, B}}}

stopt :B
rst    :0
rs     :0
stop   :B
u      :0
v      :B

case : stopt=B;rs=0;

{{0, B, 0, 0, B, B, 0, 0, B, B, B}, {0, 1, 1, 0, 1, B, 0, 0, B, B, B}}

last step input : {Prepend, {0, B, 0, 0, B, B, 0, 0, B, B, B},
                    {{0, 1, 1, 0, 1, B, 0, 0, B, B, B}, {1, 1, 0, 0, 1, B, 0, 0, B, B, B},
                     {0, 0, 1, 0, 0, B, 0, 0, B, B, B}, {0, 1, 0, 0, 1, B, B, 0, B, B, B}}}

in1 : {0, 1, 1, 0, 1, B, B, B, 0, 0, 0}

in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B}, {}}

```

```

stopt :B
u      :0
snt    :B
sn      :B
rst     :B
rs      :B
glb     :1

case : stopt=B;snt=B;u=0;gl=1

{{0, B, 0, B, B, B, 0, 0, 1, 0, 1}, {B, B, B, B, B, B, B, B, B, B}}

in1 : {1, 1, 0, 0, 1, B, B, B, 0, 0, 0}

in2 : {Prepend, {0, B, 0, B, B, B, 0, 0, 1, 0, 1}, {}}

stopt :B
u      :1
snt    :0
sn      :B
rst     :0
rs      :B
glb     :0

case : stopt=B;snt=0;rst=0

{{1, 1, 1, B, 1, B, 0, 0, 1, 0, 1}, {0, B, 0, 0, B, B, 0, 0, 1, 0, 1}}

in1 : {0, 0, 1, 0, 0, B, B, B, 0, 0, 0}

in2 : {Prepend, {1, 1, 1, B, 1, B, 0, 0, 1, 0, 1}, {{0, B, 0, 0, B, B, 0, 0, 1, 0, 1}}}}

stopt :B
u      :0
snt    :0
sn      :B
rst     :0
rs      :B
glb     :1

case : stopt=B;snt=0;rst=0

{{0, 1, 1, B, 1, B, 0, 0, 0, 0, 0}, {1, 1, 1, 0, 1, B, 0, 0, 1, 0, 1}}

in1 : {0, 1, 0, 0, 1, B, B, B, 0, 0, 0}

in2 : {Prepend, {0, 1, 1, B, 1, B, 0, 0, 0, 0, 0},
  {{1, 1, 1, 0, 1, B, 0, 0, 1, 0, 1}, {0, B, 0, 0, B, B, 0, 0, 1, 0, 1}}}}

stopt :B
u      :0
snt    :0
sn      :B
rst     :0
rs      :B
glb     :0

case : stopt=B;snt=0;rst=0

{{0, 0, 1, B, 0, B, 0, 0, 1, 0, 1}, {0, 1, 1, 0, 1, B, 0, 0, 0, 0, 0}}

last step input :
  {Prepend, {0, 0, 1, B, 0, B, 0, 0, 1, 0, 1}, {{0, 1, 1, 0, 1, B, 0, 0, 0, 0, 0},
    {1, 1, 1, 0, 1, B, 0, 0, 1, 0, 1}, {0, B, 0, 0, B, B, 0, 0, 1, 0, 1}}}}

branch = stopt=B;rst=0;

out      = {{0, 1, 0, 0, 1, B, B, 0, B, B, B},
  {0, 0, 1, 0, 0, B, 0, 0, 1, 0, 1}, {0, 1, 1, 0, 1, B, 0, 0, 0, 0, 0},
  {1, 1, 1, 0, 1, B, 0, 0, 1, 0, 1}, {0, B, 0, 0, B, B, 0, 0, 1, 0, 1}}

in1 : {0, 1, 0, 0, 1, B, B, 0, B, B, B}

in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B}, {}}

```

```

stopt :B
rst    :B
rs     :0
stop   :B
u      :0
v      :1
case : stopt=B;rs=0;

{{0, 1, 0, 0, 1, B, B, 0, B, B, B}}, {B, B, B, B, B, B, B, 0, B, B, B}}

in1 : {0, 0, 1, 0, 0, B, 0, 0, 1, 0, 1}

in2 : {Prepend, {0, 1, 0, 0, 1, B, B, 0, B, B, B}}, {}}

stopt :B
rst    :0
rs     :0
stop   :B
u      :0
v      :0
case : stopt=B;rs=0;

{{0, 0, 1, 0, 0, B, 0, 0, B, B, B}}, {0, 1, 0, 0, 1, B, B, 0, B, B, B}}

in1 : {0, 1, 1, 0, 1, B, 0, 0, 0, 0, 0}

in2 : {Prepend, {0, 0, 1, 0, 0, B, 0, 0, B, B, B}}, {{0, 1, 0, 0, 1, B, B, 0, B, B, B}}}

stopt :B
rst    :0
rs     :0
stop   :B
u      :0
v      :1
case : stopt=B;rs=0;

{{0, 1, 1, 0, 1, B, 0, 0, B, B, B}}, {0, 0, 1, 0, 0, B, 0, 0, B, B, B}}

in1 : {1, 1, 1, 0, 1, B, 0, 0, 1, 0, 1}

in2 : {Prepend, {0, 1, 1, 0, 1, B, 0, 0, B, B, B}},
      {{0, 0, 1, 0, 0, B, 0, 0, B, B, B}}, {0, 1, 0, 0, 1, B, B, 0, B, B, B}}}

stopt :B
rst    :0
rs     :0
stop   :B
u      :1
v      :1
case : stopt=B;rs=0;

{{1, 1, 1, 0, 1, B, 0, 0, B, B, B}}, {0, 1, 1, 0, 1, B, 0, 0, B, B, B}}

in1 : {0, B, 0, 0, B, B, 0, 0, 1, 0, 1}

in2 : {Prepend, {1, 1, 1, 0, 1, B, 0, 0, B, B, B}}, {{0, 1, 1, 0, 1, B, 0, 0, B, B, B},
      {0, 0, 1, 0, 0, B, 0, 0, B, B, B}}, {0, 1, 0, 0, 1, B, B, 0, B, B, B}}}

stopt :B
rst    :0
rs     :0
stop   :B
u      :0
v      :B
case : stopt=B;rs=0;

{{0, B, 0, 0, B, B, 0, 0, B, B, B}}, {1, 1, 1, 0, 1, B, 0, 0, B, B, B}}

last step input : {Prepend, {0, B, 0, 0, B, B, 0, 0, B, B, B},
      {{1, 1, 1, 0, 1, B, 0, 0, B, B, B}}, {0, 1, 1, 0, 1, B, 0, 0, B, B, B}},
      {0, 0, 1, 0, 0, B, 0, 0, B, B, B}}, {0, 1, 0, 0, 1, B, B, 0, B, B, B}}}

```

```

in1 : {1, 1, 1, 0, 1, B, B, B, 0, 0, 0}
in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B, B}, {}}

stopt :B
u      :1
snt    :B
sn      :B
rst     :B
rs      :B
glb     :1
case : stopt=B;snt=B;u=1

{{1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}, {B, B, B, B, B, B, B, B, B, B, B, B}}

in1 : {0, 1, 1, 0, 1, B, B, B, 0, 0, 0}
in2 : {Prepend, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}, {}}

stopt :0
u      :0
snt    :0
sn      :B
rst     :B
rs      :B
glb     :1
case : stopt=0;u=0;snt=0

{{0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}

in1 : {0, 0, 1, 0, 0, B, B, B, 0, 0, 0}
in2 : {Prepend, {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {{1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}}

stopt :0
u      :0
snt    :0
sn      :B
rst     :B
rs      :B
glb     :1
case : stopt=0;u=0;snt=0

{{0, 0, 1, 0, 0, 0, 0, B, B, B, B}, {0, 1, 1, 0, 1, 0, 0, B, B, B, B}}

in1 : {0, 1, 0, 0, 1, B, B, B, 0, 0, 0}
in2 : {Prepend, {0, 0, 1, 0, 0, 0, 0, B, B, B, B},
  {{0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}}

stopt :0
u      :0
snt    :0
sn      :B
rst     :B
rs      :B
glb     :0
case : stopt=0;u=0;snt=0

{{0, 1, 0, 0, 1, 0, 0, B, B, B, B}, {0, 0, 1, 0, 0, 0, 0, B, B, B, B}}

last step input :
  {Prepend, {0, 1, 0, 0, 1, 0, 0, B, B, B, B}, {{0, 0, 1, 0, 0, 0, 0, B, B, B, B},
    {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}}

branch = stopt=0;snt=0;

out      = {{0, 1, 0, 0, 1, 1, B, B, B, B, B}, {0, 0, 1, 0, 0, 0, 0, B, B, B, B},
  {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}

in1 : {0, 1, 0, 0, 1, 1, B, B, B, B, B}

```

```

in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B, B}, {}}

stopt :B
rst    :B
rs     :B
stop   :1
u      :0
v      :1

case : stopt=B;rs=B;stop=1;

{{0, 1, 0, 0, 1, 1, B, B, B, B, B, B}, {B, B, B, B, B, B, B, B, B, B, B, B}}

in1 : {0, 0, 1, 0, 0, 0, 0, B, B, B, B}

in2 : {Prepend, {0, 1, 0, 0, 1, 1, B, B, B, B, B, B}, {}}

stopt :1
rst    :B
rs     :B
stop   :0
u      :0
v      :0

case : stopt=1

{{0, 0, 1, 0, 0, 1, 0, B, B, B, B, B}, {0, 1, 0, 0, 1, B, B, B, B, B, B}}

in1 : {0, 1, 1, 0, 1, 0, 0, B, B, B, B}

in2 : {Prepend, {0, 0, 1, 0, 0, 1, 0, B, B, B, B, B}, {{0, 1, 0, 0, 1, B, B, B, B, B, B}}}

stopt :1
rst    :B
rs     :B
stop   :0
u      :0
v      :1

case : stopt=1

{{0, 1, 1, 0, 1, 1, 0, B, B, B, B, B}, {0, 0, 1, 0, 0, B, B, B, B, B, B}}

in1 : {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}

in2 : {Prepend, {0, 1, 1, 0, 1, 1, 0, B, B, B, B, B},
  {{0, 0, 1, 0, 0, B, B, B, B, B, B}, {0, 1, 0, 0, 1, B, B, B, B, B, B}}}

stopt :1
rst    :B
rs     :B
stop   :0
u      :1
v      :1

case : stopt=1

{{1, 1, 1, 0, 1, 1, 0, B, 1, 0, 1}, {0, 1, 1, 0, 1, B, B, B, B, B, B}}

last step input :
{Prepend, {1, 1, 1, 0, 1, 1, 0, B, 1, 0, 1}, {{0, 1, 1, 0, 1, B, B, B, B, B, B},
  {0, 0, 1, 0, 0, B, B, B, B, B, B}, {0, 1, 0, 0, 1, B, B, B, B, B, B}}}

in1 : {1, 1, 1, 0, 1, B, B, B, 0, 0, 0}

in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B, B}, {}}

stopt :B
u      :1
snt    :B
sn     :B
rst    :B
rs     :B
glb    :1

case : stopt=B;snt=B;u=1

```



```

{{1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}, {B, B, B, B, B, B, B, B, B, B, B}}
in1 : {0, 1, 1, 0, 1, B, B, B, 0, 0, 0}
in2 : {Prepend, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}, {}}

stopt :0
u      :0
snt    :0
sn     :B
rst    :B
rs     :B
glb    :1
case : stopt=0;u=0;snt=0

{{0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}
in1 : {0, 0, 1, 0, 0, B, B, B, 0, 0, 0}
in2 : {Prepend, {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {{1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}}

stopt :0
u      :0
snt    :0
sn     :B
rst    :B
rs     :B
glb    :1
case : stopt=0;u=0;snt=0

{{0, 0, 1, 0, 0, 0, 0, B, B, B, B}, {0, 1, 1, 0, 1, 0, 0, B, B, B, B}}
in1 : {0, 1, 0, 0, 1, B, B, B, 0, 0, 0}
in2 : {Prepend, {0, 0, 1, 0, 0, 0, 0, B, B, B, B},
        {{0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}}

stopt :0
u      :0
snt    :0
sn     :B
rst    :B
rs     :B
glb    :0
case : stopt=0;u=0;snt=0

{{0, 1, 0, 0, 1, 0, 0, B, B, B, B}, {0, 0, 1, 0, 0, 0, 0, B, B, B, B}}
last step input :
{Prepend, {0, 1, 0, 0, 1, 0, 0, B, B, B, B}, {{0, 0, 1, 0, 0, 0, 0, B, B, B, B},
        {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}}
branch = stopt=0;snt=0;

out      = {{0, 1, 0, 0, 1, 1, B, B, B, B, B}, {0, 0, 1, 0, 0, 0, 0, B, B, B, B},
        {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}
in1 : {0, 1, 0, 0, 1, 1, B, B, B, B, B}
in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B}, {}}

stopt :B
rst    :B
rs     :B
stop   :1
u      :0
v      :1
case : stopt=B;rs=B;stop=1;

{{0, 1, 0, 0, 1, 1, B, B, B, B, B}, {B, B, B, B, B, B, B, B, B, B, B}}
in1 : {0, 0, 1, 0, 0, 0, 0, B, B, B, B}

```

```

in2 : {Prepend, {0, 1, 0, 0, 1, 1, B, B, B, B, B}, {}}

stopt :1
rst   :B
rs    :B
stop  :0
u     :0
v     :0

case : stopt=1

{{0, 0, 1, 0, 0, 1, 0, B, B, B, B}, {0, 1, 0, 0, 1, B, B, B, B, B}}

in1 : {0, 1, 1, 0, 1, 0, 0, B, B, B, B}

in2 : {Prepend, {0, 0, 1, 0, 0, 1, 0, B, B, B, B}, {{0, 1, 0, 0, 1, B, B, B, B, B}}}

stopt :1
rst   :B
rs    :B
stop  :0
u     :0
v     :1

case : stopt=1

{{0, 1, 1, 0, 1, 1, 0, B, B, B, B}, {0, 0, 1, 0, 0, B, B, B, B, B}}

in1 : {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}

in2 : {Prepend, {0, 1, 1, 0, 1, 1, 0, B, B, B, B},
        {{0, 0, 1, 0, 0, B, B, B, B, B}, {0, 1, 0, 0, 1, B, B, B, B, B}}}

stopt :1
rst   :B
rs    :B
stop  :0
u     :1
v     :1

case : stopt=1

{{1, 1, 1, 0, 1, 1, 0, B, 1, 0, 1}, {0, 1, 1, 0, 1, B, B, B, B, B}}

last step input :
{Prepend, {1, 1, 1, 0, 1, 1, 0, B, 1, 0, 1}, {{0, 1, 1, 0, 1, B, B, B, B, B},
        {0, 0, 1, 0, 0, B, B, B, B, B}, {0, 1, 0, 0, 1, B, B, B, B, B}}}

(Debug) Out[173]/TableForm=
  0  1  0  0  1  B  B  B  0  0  0
  0  0  1  0  0  B  B  B  0  0  0
  1  1  0  0  1  B  B  B  0  0  0
  0  1  1  0  1  B  B  B  0  0  0

(Debug) Out[174]/TableForm=
  0  1  0  0  1  B  B  B  0  0  0
  0  0  1  0  0  B  B  B  0  0  0
  0  1  1  0  1  B  B  B  0  0  0
  1  1  1  0  1  B  B  B  0  0  0

(Debug) Out[175]/TableForm=
  0  1  0  0  1  B  B  B  0  0  0
  0  0  1  0  0  B  B  B  0  0  0
  0  1  1  0  1  B  B  B  0  0  0
  1  1  1  0  1  B  B  B  0  0  0

(Debug) Out[176]/TableForm=
  0  1  0  0  1  B  B  B  0  0  0
  0  0  1  0  0  B  B  B  0  0  0
  0  1  1  0  1  B  B  B  0  0  0
  1  1  1  0  1  B  B  B  0  0  0

(Debug) In[135]:= << "wInv.ma"
TableForm@F1
X6 = ForwardVisit@Reverse@F1;
TableForm@X6

```

(Debug) Out[136]//TableForm=

0	1	0	0	1	B	B	B	0	0	0
0	0	1	0	0	B	B	B	0	0	0
0	1	1	0	1	B	B	B	0	0	0
1	1	1	0	1	B	B	B	0	0	0

```

in1 : {1, 1, 1, 0, 1, B, B, B, 0, 0, 0}
in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B, B}, {}}

stopt :B
u      :1
snt    :B
sn      :B
rst     :B
rs      :B
glb     :1

case : stopt=B;snt=B;u=1

{{1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}, {B, B, B, B, B, B, B, B, B, B, B, B}}

in1 : {0, 1, 1, 0, 1, B, B, B, 0, 0, 0}
in2 : {Prepend, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}, {}}

stopt :0
u      :0
snt    :0
sn      :B
rst     :B
rs      :B
glb     :1

case : stopt=0;u=0;snt=0

{{0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}

in1 : {0, 0, 1, 0, 0, B, B, B, 0, 0, 0}
in2 : {Prepend, {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {{1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}}

stopt :0
u      :0
snt    :0
sn      :B
rst     :B
rs      :B
glb     :1

case : stopt=0;u=0;snt=0

{{0, 0, 1, 0, 0, 0, 0, B, B, B, B}, {0, 1, 1, 0, 1, 0, 0, B, B, B, B}}

in1 : {0, 1, 0, 0, 1, B, B, B, 0, 0, 0}
in2 : {Prepend, {0, 0, 1, 0, 0, 0, 0, B, B, B, B},
  {{0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}}

stopt :0
u      :0
snt    :0
sn      :B
rst     :B
rs      :B
glb     :0

case : stopt=0;u=0;snt=0

{{0, 1, 0, 0, 1, 0, 0, B, B, B, B}, {0, 0, 1, 0, 0, 0, 0, B, B, B, B}}

last step input :
  {Prepend, {0, 1, 0, 0, 1, 0, 0, B, B, B, B}, {{0, 0, 1, 0, 0, 0, 0, B, B, B, B},
    {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}}

branch = stopt=0;snt=0;

out      = {{0, 1, 0, 0, 1, 1, B, B, B, B, B}, {0, 0, 1, 0, 0, 0, 0, B, B, B, B},
  {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}}

```

```
(Debug) Out[138]//TableForm=
```

0	1	0	0	1	1	B	B	B	B	B
0	0	1	0	0	0	0	B	B	B	B
0	1	1	0	1	0	0	B	B	B	B
1	1	1	0	1	0	0	B	1	0	1

```
(Debug) In[158]:= << "wInv.ma";
X7 = BackwardVisit@X6;
TableForm@X7

F2 = wRevInit[X7];
TableForm@F2
```

```

in1 : {0, 1, 0, 0, 1, 1, B, B, B, B, B}
in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B}, {}}

stopt :B
rst    :B
rs     :B
stop   :1
u      :0
v      :1

case : stopt=B;rs=B;stop=1;

{{0, 1, 0, 0, 1, 1, B, B, B, B, B}, {B, B, B, B, B, B, B, B, B, B, B}}

in1 : {0, 0, 1, 0, 0, 0, 0, B, B, B, B}
in2 : {Prepend, {0, 1, 0, 0, 1, 1, B, B, B, B, B}, {}}

stopt :1
rst    :B
rs     :B
stop   :0
u      :0
v      :0

case : stopt=1

{{0, 0, 1, 0, 0, 1, 0, B, B, B, B}, {0, 1, 0, 0, 1, B, B, B, B, B, B}}

in1 : {0, 1, 1, 0, 1, 0, 0, B, B, B, B}
in2 : {Prepend, {0, 0, 1, 0, 0, 1, 0, B, B, B, B}, {{0, 1, 0, 0, 1, B, B, B, B, B, B}}}

stopt :1
rst    :B
rs     :B
stop   :0
u      :0
v      :1

case : stopt=1

{{0, 1, 1, 0, 1, 1, 0, B, B, B, B}, {0, 0, 1, 0, 0, B, B, B, B, B, B}}

in1 : {1, 1, 1, 0, 1, 0, 0, B, 1, 0, 1}
in2 : {Prepend, {0, 1, 1, 0, 1, 1, 0, B, B, B, B},
  {{0, 0, 1, 0, 0, B, B, B, B, B, B}, {0, 1, 0, 0, 1, B, B, B, B, B, B}}}

stopt :1
rst    :B
rs     :B
stop   :0
u      :1
v      :1

case : stopt=1

{{1, 1, 1, 0, 1, 1, 0, B, 1, 0, 1}, {0, 1, 1, 0, 1, B, B, B, B, B, B}}

last step input :
{Prepend, {1, 1, 1, 0, 1, 1, 0, B, 1, 0, 1}, {{0, 1, 1, 0, 1, B, B, B, B, B, B},
  {0, 0, 1, 0, 0, B, B, B, B, B, B}, {0, 1, 0, 0, 1, B, B, B, B, B, B}}}

```

(Debug) Out[160]//TableForm=

1	1	1	0	1	B	B	B	B	B	B
0	1	1	0	1	B	B	B	B	B	B
0	0	1	0	0	B	B	B	B	B	B
0	1	0	0	1	B	B	B	B	B	B

(Debug) Out[162]//TableForm=

0	1	0	0	1	B	B	B	0	0	0
0	0	1	0	0	B	B	B	0	0	0
0	1	1	0	1	B	B	B	0	0	0
1	1	1	0	1	B	B	B	0	0	0

```

(Debug) (Dialog) In[11]:=
  << "wInv.ma"; (X1 = ForwardVisit[Reverse[base]]);

(Debug) (Dialog) In[12]:=
  TableForm[base]
  TableForm[Reverse@X1]

(Debug) (Dialog) In[14]:=
  X2 = BackwardVisit[X1];

(Debug) (Dialog) In[15]:=
  TableForm[base]
  TableForm[Reverse@X1]
  TableForm[X2]

(Debug) In[58]:= F0 = wRevInit[BackwardVisit@ForwardVisit@Reverse@base];
  F1 = wRevInit[BackwardVisit@ForwardVisit@Reverse@F0];
  F2 = wRevInit[BackwardVisit@ForwardVisit@Reverse@F1];
  TableForm@F2

(Debug) In[88]:= TableForm@base
  TableForm@F0
  X4 = ForwardVisit@(X3 = Reverse@F0)
  TableForm@X3
  TableForm@X4

(Debug) In[47]:= << "wInv.ma";
  F1 = wRevInit@(X5 = BackwardVisit@(F0));
  TableForm@F0
  TableForm@X3
  TableForm@X4
  TableForm@X5
  TableForm@F1

(Debug) In[77]:= X7 = ForwardVisit@(X6 = Reverse@F1)
  TableForm[F1]
  TableForm@X7

```

```

in1 : {1, 1, 1, 0, 1, B, B, B, 0, 0, 0}
in2 : {#1 &, {B, B, B, B, B, B, B, B, B, B, B, B}, {}}

stopt :B
u      :1
snt    :B
sn      :B
rst     :B
rs      :B
glb     :1
case : stopt=B;snt=B;u=1

{{1, B, 1, 0, B, 0, 0, B, 1, 0, 1}, {B, B, B, B, B, B, B, B, B, B, B, B}}

in1 : {0, 1, 1, 0, 1, B, B, B, 0, 0, 0}
in2 : {Prepend, {1, B, 1, 0, B, 0, 0, B, 1, 0, 1}, {}}

stopt :0
u      :0
snt    :0
sn      :B
rst     :B
rs      :B
glb     :1
case : stopt=0;u=0;snt=0

{{0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, B, 1, 0, B, 0, 0, B, 1, 0, 1}}

in1 : {0, 0, 1, 0, 0, B, B, B, 0, 0, 0}
in2 : {Prepend, {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {{1, B, 1, 0, B, 0, 0, B, 1, 0, 1}}}

stopt :0
u      :0
snt    :0
sn      :B
rst     :B
rs      :B
glb     :1
case : stopt=0;u=0;snt=0

{{0, 0, 1, 0, 0, 0, 0, B, B, B, B}, {0, 1, 1, 0, 1, 0, 0, B, B, B, B}}

in1 : {0, 1, 0, 0, 1, B, B, B, 0, 0, 0}
in2 : {Prepend, {0, 0, 1, 0, 0, 0, 0, B, B, B, B},
  {{0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, B, 1, 0, B, 0, 0, B, 1, 0, 1}}}

stopt :0
u      :0
snt    :0
sn      :B
rst     :B
rs      :B
glb     :0
case : stopt=0;u=0;snt=0

{{0, 1, 0, 0, 1, 0, 0, B, B, B, B}, {0, 0, 1, 0, 0, 0, 0, B, B, B, B}}

last step input :
{Prepend, {0, 1, 0, 0, 1, 0, 0, B, B, B, B}, {{0, 0, 1, 0, 0, 0, 0, B, B, B, B},
  {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, B, 1, 0, B, 0, 0, B, 1, 0, 1}}}

branch = stopt=0;snt=0;

out      = {{0, 1, 0, 0, 1, 1, B, B, B, B, B}, {0, 0, 1, 0, 0, 0, 0, B, B, B, B},
  {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, B, 1, 0, B, 0, 0, B, 1, 0, 1}}

(Debug) Out[77]= {{0, 1, 0, 0, 1, 1, B, B, B, B, B}, {0, 0, 1, 0, 0, 0, 0, B, B, B, B},
  {0, 1, 1, 0, 1, 0, 0, B, B, B, B}, {1, B, 1, 0, B, 0, 0, B, 1, 0, 1}}

```



```
(Debug) Out[78]//TableForm=
  0   1   0   0   1   B   B   B   0   0   0
  0   0   1   0   0   B   B   B   0   0   0
  0   1   1   0   1   B   B   B   0   0   0
  1   1   1   0   1   B   B   B   0   0   0
```

```
(Debug) Out[79]//TableForm=
  0   1   0   0   1   1   B   B   B   B   B
  0   0   1   0   0   0   0   B   B   B   B
  0   1   1   0   1   0   0   B   B   B   B
  1   B   1   0   B   0   0   B   1   0   1
```

```
(Debug) In[66]:= TableForm@base
TableForm@F0
```

**pp**

**p**

```

(Debug) In[21]:=
U = {0, 1, 0, 0}
pp = {1, 1, 0, 1}
p = pp.Table[x^i, {i, 0, Length[pp] - 1}]
uu = U.Table[2^i, {i, 0, Length[U] - 1}]
TestDivisibility[p_, var_] := CoefficientList[p, var][[1]] == 0
FongBase[n_, fieldpoly_] :=
  {Reverse[IntegerDigits[n, 2, 8]].Table[x^i, {i, 0, 7}], fieldpoly, 1, 0}

DEBUG = Print;
FongStep[stato_] := Module[{u, v, g1, g2},
  (
    u = stato[[1]];
    v = stato[[2]];
    g1 = stato[[3]];
    g2 = stato[[4]];
    DEBUG["u=", u, " v=", v, " g1=", g1, " g2=", g2];
    While[TestDivisibility[u, x],
      (
        u = Collect[u / x, x];
        If[TestDivisibility[g1, x],
          g1 = Collect[g1 / x, x],
          g1 = Collect[PolynomialMod[(g1 + p), 2] / x, x]
        ];
        DEBUG["IN WHILE u=", u, " v=", v, " g1=", g1, " g2=", g2];
      )
    ];
    If[u != 1,
      (
        d1 = Exponent[u, x];
        d2 = Exponent[v, x];
        DEBUG["PRIMA IF u=", u, " v=", v, " g1=", g1, " g2=", g2];
        If[d1 < d2,
          (
            aux = u;
            u = v;
            v = aux;
            aux = g1;
            g1 = g2;
            g2 = aux;
            DEBUG[" DOPO SWAP u=", u, " v=", v, " g1=", g1, " g2=", g2];
          )
        ];
        DEBUG[" SOMMO u=", u, " v=", v, " g1=", g1, " g2=", g2];
        u = PolynomialMod[u + v, 2];
        g1 = PolynomialMod[g1 + g2, 2];
        DEBUG[" FINALE u=", u, " v=", v, " g1=", g1, " g2=", g2];
      )
    ];
    {u, v, g1, g2}
  )
]

```

```

(Debug) In[57]:= p

```

```

(Debug) In[33]:= uu = 4
p = 1 + x + x^3

```

```
(Debug) In[35]:= FongBase[uu, p]  
                TableForm@NestList[FongStep, FongBase[uu, p], 8]
```