College of Industrial Technology

King Mongkut's University of Technology North Bangkok

เลขที่นั่งสอบ

Final Examination of Semester 1 Year: 2020

Subject: 394171 Mathematics I Section: 15-18

Date: 28 October 2020 Time: 13.00-16.00

Name _____ ID _____ Class _____

Directions: The test is designed to measure your comprehension. The test is divided into questions. There will be 10 pages (including this page) and they are worth 90 points.

- This exam paper contains no errors. If a suspected error is found, it is the student's discretion to correct it.
- Answer the questions on this test papers.
- Books, documents and lecture notes are not allowed.
- You must be in the room for one hour after the exam is started and, while taking the exam, you cannot go out except in an emergency case.
- Before leaving, make sure you do not bring this test outside.
- Do not use any electronic communication device.
- Calculators can't be used in this test.

Now begin the test.

Cheating in the exam is considered an extremely serious offence which will result in expulsion from the University.

No.	1.	2.	3.	4.	5.	6.	7.	8.	9	Total
score										

1. The following questions are unrelated.

1.1) Determine whether each relation is a function. If so, find its domain and range. If not, show a reason why it is not a function. (2 marks each)

(a) $r_1 = \{(m,m),(a,a),(t,t),(h,h)\}$.

(b) $r_2 = \{(x,y) \in R \times R | y^2 = x+5\}$.

1.2) Given $f(2x+1) = 4x^2 - 8x + 5$. Determine $f(x)$ and the value of $f(1)$. (4 marks)

2. Given the following functions $f(x)=\dfrac{1}{3x^2-2x-1}$, $g(x)=\dfrac{x}{x-1}$ and $h(x)=\dfrac{x}{x+1}$.

(a) Find $(h\circ g)(x)$ and $(h\circ g)(2)$. (3 marks)

(b) Find $\left(\dfrac{f}{g}\right)(x)$ and $D_{\frac{f}{g}}$. (4 marks)

(c) Find $\left(g^{-1}+h^{-1}\right)(x)$ and $\left(g^{-1}+h^{-1}\right)(1)$. (5 marks)

3. 3.1) Given $\cos\theta = \frac{4}{5}$ and $\sin\theta < 0$. Find $\csc\theta$ and $\tan\theta$. (4 marks)

3.2) Determine the value of the expression $\frac{\cos 316^\circ \cos 494^\circ}{\sec 314^\circ \sin 224^\circ \cos 2162^\circ}$. (6 marks)

4. 4.1) Evaluate the value of the expression $\sin\frac{5\pi}{12}\cos\frac{\pi}{4}-\cos\frac{5\pi}{12}\sin\frac{\pi}{4}$. (5 marks)

4.2) If $\cos\theta=\frac{3}{5}$ and $\frac{3\pi}{2}<\theta<2\pi$, find the value of $\sin 2\theta$. (5 marks)

5 5.1) Evaluate the value of the expression $4\sin 10^\circ \cos 40^\circ \cos 20^\circ$. (5 marks)

5.2) Simplify the expression $\sin\left(\frac{\pi}{6}+A\right)-\sin\left(\frac{\pi}{6}-A\right)+\sin\left(\frac{\pi}{3}+A\right)-\sin\left(\frac{\pi}{3}-A\right)$. (5 marks)

6 6.1) If $0 < \theta < \frac{\pi}{2}$, find the value of x from the equation

$$x^2 \cot^2\left(\frac{\pi}{2} - \theta\right) + \sec\theta \csc\left(\frac{3\pi}{2} - \theta\right) = -1. \qquad \text{(5 marks)}$$

6.2) Use trigonometric identities to show that the left hand side of the equation equals its right hand side:

$$(\sin A \cos B + \cos A \sin B)^2 + (\cos A \cos B - \sin A \sin B)^2 = 1. \qquad \text{(5 marks)}$$

7. 7.1) Find the exact value of

$$\sin\left(\arcsin\left(-\frac{4}{5}\right)\right)+\cos\left(\arcsin\left(-\frac{4}{5}\right)\right)+\tan\left(\arcsin\left(-\frac{4}{5}\right)\right).$$ (5 marks)

7.2) Evaluate the value of $\sin\left(\arccos\left(\frac{5}{13}\right)+\arcsin\left(-\frac{4}{5}\right)\right)$. (5 marks)

8. 8.1) If $\frac{a}{b} \in [-1,1]$, then determine the value of

$$\frac{a}{b}\left[\tan\left(\frac{\pi}{4}+\frac{1}{2}\arccos\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\arccos\frac{a}{b}\right)\right].$$ (5 marks)

Hint :Let $A=\frac{1}{2}\arccos\frac{a}{b}$.

8.2) If $0 \le x < 2\pi$, find all solutions of $8\sin^3 x+4\sin^2 x-6\sin x-3=0$. (5 marks)

9. To measure the height of a hill, Bob measures the angle of elevation to the top of the hill to be 24 degrees. He then moves back 198 feet and measures the angle of elevation to be 22 degrees. Find the height of the hill. ($\sin(2°) = 0.03$, $\sin(22°) = 0.37$, $\sin(24°) = 0.41$, $\cot(22°) = 2.48$, $\cot(24°) = 2.25$ and $\sin(66°) = 0.91$) (10 marks)

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