

Step-by-Step Solutions for Matrix Problems

Problem (b)

Given:

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix}$$

Find X such that: $(A^{-1} + B^T)X = (AB)^T$

Step 1: Compute A^{-1}

$$\det(A) = (0)(3) - (2)(1) = -2$$

$$A^{-1} = (1/-2) * \begin{bmatrix} 3 & -1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -1.5 & 0.5 \\ 1 & 0 \end{bmatrix}$$

Step 2: Compute B^T

$$B^T = \begin{bmatrix} 1 & 4 \\ 3 & 0 \end{bmatrix}$$

Step 3: Compute $A^{-1} + B^T$

$$A^{-1} + B^T = \begin{bmatrix} -1.5+1 & 0.5+4 \\ 1+3 & 0+0 \end{bmatrix} = \begin{bmatrix} -0.5 & 4.5 \\ 4 & 0 \end{bmatrix}$$

Step 4: Compute AB

$$AB = \begin{bmatrix} 0*1+1*4 & 0*3+1*0 \\ 2*1+3*4 & 2*3+3*0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 14 & 6 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 4 & 14 \\ 0 & 6 \end{bmatrix}$$

Step 5: Solve for X

$$\text{Let } M = \begin{bmatrix} -0.5 & 4.5 \\ 4 & 0 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 4 & 14 \\ 0 & 6 \end{bmatrix}$$

Compute M^{-1} :

$$\det(M) = (-0.5)(0) - (4)(4.5) = -18$$

$$M^{-1} = (1/-18) * \begin{bmatrix} 0 & -4.5 \\ -4 & -0.5 \end{bmatrix} = \begin{bmatrix} 0 & 0.25 \\ 0.2222 & 0.0278 \end{bmatrix}$$

$$X = M^{-1} * Y = \begin{bmatrix} 0 & 0.25 \\ 0.2222 & 0.0278 \end{bmatrix} * \begin{bmatrix} 4 & 14 \\ 0 & 6 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1.5 \\ 0.8889 & 3.2778 \end{bmatrix}$$

Final Answer for (b):

$$X = \begin{bmatrix} 0 & 1.5 \\ 0.8889 & 3.2778 \end{bmatrix}$$

Problem (c)

$$\text{Matrix } A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \\ -2 & 3 & -2 \end{bmatrix}$$

Compute determinant:

$$\det(A) = 1 * ((-1) * (-2) - (2) * (3)) - 2 * ((3) * (-2) - (2) * (-2)) + 0 * ((3) * (3) - (-1) * (-2))$$

$$= 1 * (2 - 6) - 2 * (-6 - (-4)) + 0 * (9 - 2)$$

$$= -4 - 2 * (-2) + 0$$

$$= -4 + 4 = 0$$

Since determinant = 0, matrix A is singular.