Functions

The *coverage* problem is a problem relates to *graph theory*. We can define the *submodular and monotone* using the notation from graph theory. Given a graph $G = (V, E)$. Our search space is the vertex space V. The submodular function can be defined as:

$$f : 2^V \to \mathbb{R}$$

$A \subseteq B \subseteq 2^V$ are sets of vertices, it is monotonic when:

$$A \subseteq B \implies f(A) \leq f(B).$$

it is *submodular*, given that $v \in V \setminus B$ when:

$$f(B \cup \{v\}) - f(B) \leq f(A \cup \{v\}) - f(A).$$

Objective Functions

I will defined $\boldsymbol{v} \subseteq V$ be a set of vertices, the relevant notation will be written in bold with a lowercase letter. The bold, lowercase letter indicate a set of values.

Uniform k-constraint cost

The *uniform cost* function $c : 2^V \to \mathbb{R}_{\geq 0}$ is a linear function with weight $\boldsymbol{w}$ and defined as:

$$c(\boldsymbol{v}) = \sum_{i}^{n} w_i \cdot c(v_i) \leq k \in \mathbb{R},$$
$$\text{where } w_i = 1,\ c(v_i) = 1$$

Notice that $c(v_i)$ is the cost of the vertex being chosen in a set not the cost of its out-going edge. The value for $\boldsymbol{w} = \mathbf{1}$ and $c(v_i) = 1$ is given.

Fitness Function

For the *Maximum Coverage*

$$\arg\max_{\boldsymbol{v} \subseteq V} f(\boldsymbol{v}) = |N(\boldsymbol{v})|, \qquad \text{where } N(\boldsymbol{v}) \text{ is the out-going neighbours of } \boldsymbol{v}$$

For the *Max Influence*, this is a probabilistic problem, therefore we need to find the *seed set* $\boldsymbol{v} \in V$ that has the highest expected influence value.

$$\arg\max_{\boldsymbol{v} \subseteq V} f(\boldsymbol{v}) = \mathbb{E}[IC(\boldsymbol{v})]$$

> $IC(.)$ is a *Independence Cascade process*, in short, this is a propagation process that how a vertex v can influence vertex u with probability $p_{v,u}$ from time $t \to t + 1$.

Overall, the fitness of a search point is a 2D vector given by. Therefore this is a bi-objective optimisation problem, defined as

$$
\begin{aligned}
\arg\max_{\boldsymbol{v}\subseteq V} \mathsf{F}(\boldsymbol{v}) &= (f'(\boldsymbol{v}), -c(\boldsymbol{v})) \\
&\text{subject to} \\
&c(\boldsymbol{v}) \leq 10 \\
&f'(\boldsymbol{v}) = \begin{cases} f(\boldsymbol{v}), & c(\boldsymbol{v}) \leq k = 10 \\ (k = 10) - f(\boldsymbol{v}) \quad , & c(\boldsymbol{v}) > k = 10 \end{cases}
\end{aligned}
$$

Dominance Formulation

For any two given solution sets $\boldsymbol{v} \succ \boldsymbol{u}$. We have that

$$
\begin{aligned}
&\mathsf{F}(\boldsymbol{v}) \succ \mathsf{F}(\boldsymbol{u}) \\
\iff &\begin{cases} f'(\boldsymbol{v}) \geq f'(\boldsymbol{u}) \\ c(\boldsymbol{v}) \leq c(\boldsymbol{u}) \\ \mathsf{F}(\boldsymbol{v}) \neq \mathsf{F}(\boldsymbol{u}) \end{cases}
\end{aligned}
$$

Thus, we can say that

$$
\mathsf{F}(\boldsymbol{v}) \succ \mathsf{F}(\boldsymbol{u}) \iff \begin{cases} f'(\boldsymbol{v}) \geq f'(\boldsymbol{u}) \\ c(\boldsymbol{v}) < c(\boldsymbol{u}) \end{cases}
$$

And also,

$$
\mathsf{F}(\boldsymbol{v}) \succeq \mathsf{F}(\boldsymbol{u}) \iff \begin{cases} f'(\boldsymbol{v}) \geq f'(\boldsymbol{u}) \\ c(\boldsymbol{v}) \leq c(\boldsymbol{u}) \end{cases}
$$