DEFIN.: Dato $(V, +, \cdot, \mathbb{R})$ vettoriale un sottoinsieme $A \subseteq V$ del tipo

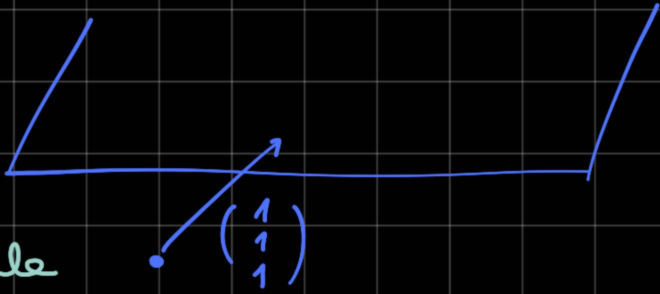
$$A = p + W = \{ v = p + w \ : \ w \in W \} \qquad p \in V$$

si chiama SOTTOSPAZIO AFFINE, con W = GIACITURA

ESEMPIO : $A = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \mathcal{L}\left(\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right)$

OSSERVA:

se $0 \in A$ oppure $p \in W$ allora il sottospazio affine e' un sottospazio vettoriale



ESEMPIO 2: In $\mathbb{R}_2[t]$

$$A = (t^2+1) + \mathcal{L}(t^2 - t, t+1) \quad \text{e' ssv.}$$

ESEMPIO 3: In $(V, +, \cdot, \mathbb{R})$ un singolo vettore

$A = v + \{0_V\}$ e' affine con giacitura il ssv. banale $W = \{0\}$

ESERCIZIO: Calcola in $(\mathbb{R}_3, +, \cdot, \mathbb{R}, <, >)$

il ssa. $A_2 \perp A_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \mathcal{L}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

passante da $p_0 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$$A_2 = \begin{pmatrix}1\\0\\-1\end{pmatrix} + W \quad \text{T.C.} \quad W \perp \mathcal{L}\begin{pmatrix}1\\0\\1\end{pmatrix}$$

Quindi $W = \mathcal{L}\begin{pmatrix}1\\0\\1\end{pmatrix}^{\perp} = \ker\begin{pmatrix}1 & 0 & 1\end{pmatrix} = \mathcal{L}\left(\begin{pmatrix}1\\0\\-1\end{pmatrix};\begin{pmatrix}0\\1\\0\end{pmatrix}\right)$

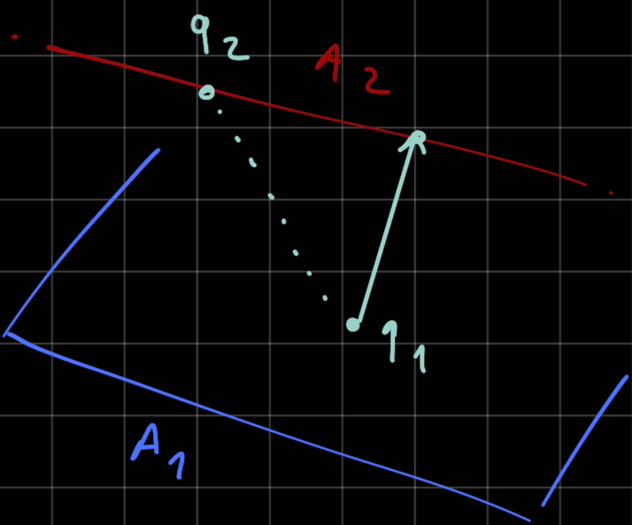
Quindi $A_2 = \begin{pmatrix}1\\0\\-1\end{pmatrix} + \mathcal{L}\left(\begin{pmatrix}1\\0\\-1\end{pmatrix};\begin{pmatrix}0\\1\\0\end{pmatrix}\right)$

Qual è la distanza tra due spazi vettoriali?
0, poichè entrambi contengono il vettore nullo

E tra due ssa?

DEF.: Dato $(V, +, \cdot, \mathbb{R}, \langle , \rangle)$ e due ssa., A_1 e A_2, la distanza fra A_1 e A_2 è

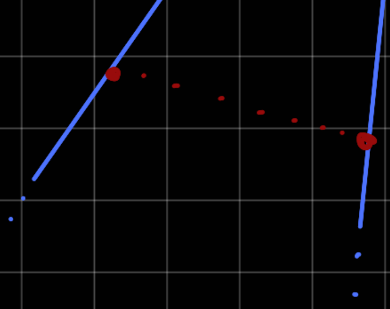
$$\mathrm{dist}(A_1, A_2) = \min\{d(q_1, q_2)\} = \ldots = \|q_1 - q_2\|$$



la formula generale è

TEOREMA: se $A_1 = p_1 + W_1$ $\quad A_2 = p_2 + W_2$:

allora $U = W_1 + W_2$



Allora

$$\text{dist}(A_1, A_2) = \| P_{U^\perp}(P_1 - P_2) \|$$

ESERCIZIO: $\text{dist}(A_1, A_2)$ in $\mathbb{R}^4$ rispetto al p.s.s.

$$A_1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \mathcal{L}\left(\begin{pmatrix}1\\0\\0\\1\end{pmatrix}; \begin{pmatrix}1\\0\\0\\-1\end{pmatrix}\right)$$

$$A_2 = \begin{pmatrix}0\\-1\\0\\0\end{pmatrix} + \mathcal{L}\begin{pmatrix}1\\1\\1\\0\end{pmatrix}$$

$$U = W_1 + W_2 = \mathcal{L}\left(\begin{pmatrix}1\\0\\0\\1\end{pmatrix}; \begin{pmatrix}1\\0\\0\\-1\end{pmatrix}; \begin{pmatrix}1\\1\\1\\0\end{pmatrix}\right)$$

$$\dim(U) = 3$$

$$U^\perp = \ker\begin{pmatrix}1 & 0 & 0 & 1\\ 1 & 0 & 0 & -1\\ 1 & 1 & 1 & 0\end{pmatrix} = \mathcal{L}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$$

↳ Mi serve una b.o.n. per fare la proiezione ortogonale

$$b_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$$

$$x = P_1 - P_2 = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}$$

$$\rightarrow P_{U^\perp}\begin{pmatrix}1\\1\\0\\0\end{pmatrix} = \left\langle \begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix} \right\rangle \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$$

$$\Rightarrow \text{dist}(A_1, A_2) = \left\| \frac{1}{2}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix} \right\| = \frac{\sqrt{2}}{2}$$