Profile Ranking Adaptive Choice-Based Conjoint Analysis: A Simple Extension to Utility-Based Analysis for Small Populations

Abstract

To analyze Adaptive Choice-Based Conjoint (ACBC) survey samples from small populations, a new methodology called profile ranking based ACBC (PR-ACBC) is proposed as a simple extension to utility based ACBC. PR-ACBC offers a form of validation especially useful for small survey data with high variances in partworth utilities. Without requiring knowledge of partworth utilities, PR-ACBC begins with a simple computation using choice task data to obtain both individual and sample mean profile attribute level (PAL) rankings. The Maximum likelihood estimation of profile rankings for both a known population size N (via multivariate hypergeometric distribution) and unknown N (via Lagrange multiplier optimization) is then used to obtain point estimates of population profile and PAL rankings. The sample profile and PAL rankings are also used for multidimensional scaling (MDS) which offers a two-dimensional visual representation of similarities/dissimilarities in respondents and PALs. PR-ranking methodology is then compared with partworth utilities with respect to accuracy of choice task predictions and ranking of attribute importances. Finally, PR-ACBC methodology is applied to a recent survey administered to a small population of disaster relief organizations.

1 Introduction

Adaptive Choice-Based Conjoint (ACBC) analysis surveys are a widely-utilized, well-developed, and highly effective type of conjoint analysis (Orme and Chrzan, 2017). Profile-Ranking (PR-) ACBC uses data from the choice-task stage based on the Method of Paired Comparisons (MPC) (Thurstone 1927), and then applies ranking theory (Alvo and Yu 2014) to obtain profile attribute level rankings akin to partworth utilities. Unlike Maximum Difference (MaxDiff) scaling which is complementary to but not directly integrable into ACBC (Sawtooth 2013), PR is easy to integrate into ACBC for use with small but geographically separated populations (eg. $N \leq 50$) where large variances in partworth utilities may hamper population inferences. Letting n be the number of survey respondents, t the number of choice tasks, a the number of alternatives per set, and c the maximum number of levels in any attribute, a rule of thumb for utility-based statistical inference is $n > \frac{s}{at}10^3$ (Orme 2014). PR-methodology serves as a check of conclusions based on partworth-utilities for the case $n < N < \frac{s}{at}10^3$. Following the first application of ACBC to disaster relief (Gralla et al. 2014), we used Sawtooth's Lighthouse platform to structure our choice-task stage as a single-elimination tournament beginning with 16 profiles close to the respondent's #1 profile revealed in the "Build Your Own" (BYO) stage. This platform uses a hierarchical-Bayesian Markov-Chain Monte-Carlo (HB MCMC) simulation (Rossi et. al. 2005) to estimate partworth utilities and their variances. Profile attribute level (PALs) rankings which can easily be computed without utilities serve to validity profile choice predictions and ranking of attribute importances based on part-worth utilities. PALs are also used in multi-dimensional scaling (MDS) (Alvo and Yu 2014) to give a 2-dimensional visual representation showing similarities in respondents and their ranking of attribute levels.

In Section 2, we introduce basic PR-ACBC methodology by means of a very simple generic ("toy") survey with only 4 profiles constructed from 2 attributes each with 2 levels. We begin with a fundamental observation that the exact sample profile rankings directly obtainable from choice tournament data can not be obtained by multiple linear regression (part-worth utilities). PR-ACBC then proceeds to analyze survey tournament data without requiring any knowledge of partworth utilities. Maximum likelihood estimate (MLE) population rankings for known population sizes are obtainable by a discrete multivariate hypergeometric distribution (Oberhofer and Kaufman 1987), and for unknown population sizes by multivariable calculus optimization using Lagrange multipliers (Stewart 2016). Population ranking intervals which must contain the unknown population profile rankings are easily computed from sample profile rankings.Similarly, the PAR-intervals which must contain the actual population attribute level rankings are easily obtained from sample data. Sample PARs are also useful for multi-dimensional scaling (MDS) which show similarities in respondents and PAL rankings. This is useful for comparing and contrasting sample subgroups. In Section 3 we illustrate PR methodology using a recent ACBC survey deployed to both faith-based and non-faith based disaster relief organizations. The context motivating this methodological study is a sequel to a novel application of ACBC in disaster-response research (Gralla et. al. 2014). Finally, in Section 4, we suggest a couple of major directions for further research in PR-ACBC.

2 PR-ACBC Methodology

In this section we introduce PR-ACBC methodology using a simple“toy” survey.

2.1 Simple Example

Consider a generic ACBC survey with just 2 attributes each having 2 levels. We designate the 4 possible profiles $A = 11, B = 10, C = 01, D = 00$, where $X = x_1x_2$ designates that profile X has level x_1 for the first attribute and level x_2 for the second attribute. Suppose we have obtained by anonymous survey choice tournament results for $n = 4$ respondents from a population of size $N > 4$. A sample tournament outcome is shown in Figure 1

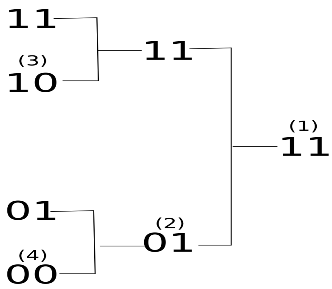


Figure 1: A respondent whose tournament data ranks profile A=11 first, C=01 second, B=10 is ranked third and D=00 fourth.

This respondent profile ranking is denoted ACBD, meaning profile A=11 is ranked 1 (tournament winner), profile C=01 is ranked 2 (runner-up), profile B=10 third, and D=00 ranked 4th. The rationale for the latter 2 rankings is that assuming transitivity in match outcomes, the highest B could be ranked if all profiles were paired is 2, while D could only be ranked as high as 3.

In our toy survey with just 4 possible profiles, there are already 4!=24 possible profile rankings. In the next section we consider an actual survey of disaster relief organizations (DROs) with 4 attributes containing 3 levels each, administered to a population of roughly $N = 50$ faith based disaster relief organizations. This survey has 81 profiles and 81!=5 797 126 020 747 367 985 879 734 231 578 109 105 412 357 244 731 625 958 745 865 049 716 390 179 693 892 056 256 184 534 249 745 940 480 000 000 000 000 000 000 profile rankings, For a sample size of $n = 13$, and a choice task stage which provides each respondent's ranking of 16 profiles, a meaningful ranking of all the profiles is evidently impossible. As a result, our approach to ranking profiles, one more akin to using partworth utilities to determine profile choices, is to rank individual attribute levels rather than profiles as a whole.

Our toy survey has just four profile levels which we denote (x_1, x_2) where x_1 is the attribute number (1 or 2) and x_2 the level number (0 or 1). Note that each level appears in exactly two profiles. For each attribute level, we compute a respondent's profile attribute level (PAL) ranking as the average of the two profile rankings in which the level appears. For example since $(1,0)$ appears in profiles C and D, if a respondent's profile ranking is ACBD, then the $(1,0)$ PAL ranking is (2+4)/2=3. (See Table 1)

Profile Ranking	PAL Ranking (1,1)	(1,0)	(2,1),	(2,0)
ABCD	1.5	3.5	2	3
ACBD	2	3	1.5	3.5
BADC	1.5	3.5	3	2
CBAD	2.5	2.5	2	3
m	1.875	3.375	2.125	2.875
s	.4787	.25	.6292	.6292

Table 1: Toy survey sample PAL rankings ($n = 4$, m=mean, s=standard deviation, profiles A=11,B=10,C=01, and D=00.)

In our DRO survey, there are a total of 12 PALs (x_1, x_2) with $x_1 \in \{1, 2, 3, 4\}$ and $x_2 \in \{1, 2, 3\}$. Each PAL occurs in 27 out of the 81 profiles and has roughly a 99.85% chance of appearing in a in a single elimination round of 16 tournament (and hence have a ranking between 1 and 16). PALs not appearing in a tournament are assigned a ranking of 24.5 (the average of rankings 17 through 32 which would be assigned to the first round losers had one earlier round been included in the tournament.) In short, PAL rankings are far more meaningful than profile rankings.

The main questions we will develop in the sequel are:

- POPULATION INFERENCES: *what can we infer from sample PAL rankings about the population PAL rankings?*; and
- APPLICATION TO REAL SURVEYS: *How well do PAL rankings predict choice-tasks and attribute importances in the DRO survey?*

2.2 A Fundamental Observation

In this section we show that least squares multiple regression (LSRM) will not in general give exact sample profile rankings and their corresponding PAL rankings. In other words, part-worth utilities can only approximate sample PAL rankings.

Least squares multiple linear regression (LSRM) can be used to predict sample PAL rankings as we will now explain using our toy survey. Let U denote the level of attribute 1, V the level of attribute 2, and Y the ranking of a profilw with levels U and V. Table 2 gives the dataset $\{(U_i, V_i, Y_i)\}$ $(i = 1, ..., 16)$ where

$$\begin{aligned} U_i &= \text{1 if attribute 1 has level 1, and 0 if it has level 2} \\ V_i &= \text{1 if attribute 2 has level 1, and 0 if it has level 2} \\ Y_i &= \text{Respondent's ranking of a profile with } U = U_i,\ V = V_i. \end{aligned}$$

U	V	Respondent 1	Respondent 2	Respondent 3	Respondent 4
1	1	Y_1	Y_2	Y_3	Y_4
1	0	Y_5	Y_6	Y_7	Y_8
0	1	Y_9	Y_{10}	Y_{11}	Y_{12}
0	0	Y_{13}	Y_{14}	Y_{15}	Y_{16}

Table 2: Sample profile rankings by respondent ($n = 4$).

This dataset has certain properties:

- Each column consists of a respondent's profile rankings and so contains the numbers 1,2,3 and 4.
- Table 2 can also be represented in the form of Table 3, by which we see that

$$\sum U_i = \sum V_i = \sum U_i^2 = \sum V_i^2 = 8,$$

and

$$\sum U_i V_i = 4,$$

where the symbol $\sum$ represents $\sum\limits_{n=1}^{16}$.

U	V	Rank
1	1	Y_1
1	1	Y_2
1	1	Y_3
1	1	Y_4
1	0	Y_5
1	0	Y_6
1	0	Y_7
1	0	Y_8
0	1	Y_9
0	1	Y_{10}
0	1	Y_{11}
0	1	Y_{12}
0	0	Y_{13}
0	0	Y_{14}
0	0	Y_{15}
0	0	Y_{16}

Table 3: Dataset's ranking structure with respondents combined.

Using least squares multiple linear regression (LSMR) on the dataset in Table 3, we estimate each sample profile ranking Y_i as $\hat{Y}_i$:

$$\hat{Y}_i = c_0 + c_1 U_i + c_2 V_i,$$

where the regression coefficients c_0, c_1, c_2 are determined by minimizing the sum of squared residuals (SSR):

$$SSR = \sum_{n=1}^{16}(Y_i - \hat{Y}_i)^2 = \sum_{n=1}^{16}(Y_i - (c_0 + c_1 U_i + c_2 V_i))^2.$$

To minimize the SSR, we set the partial derivatives with respect to c_0, c_1 and c_2, equal to zero:

$$\frac{\partial SSR}{\partial c_0} = \frac{\partial SSR}{\partial c_1} = \frac{\partial SSR}{\partial c_2} = 0.$$

This yields the linear system:

$$\begin{cases} nc_0 + c_1 \sum U_i + c_2 \sum V_i = \sum Y_i \\ c_0 \sum U_i + c_1 \sum U_i^2 + c_2 \sum U_i V_i = \sum U_i Y_i \\ c_0 \sum V_i + c_1 \sum U_i V_i + c_2 \sum V_i^2 = \sum V_i Y_i \end{cases},$$

which is equivalent to the matrix equation:

$$\begin{bmatrix} n & \sum U_i & \sum V_i \\ \sum U_i & \sum U_i^2 & \sum U_i V_i \\ \sum V_i & \sum U_i V_i & \sum V_i^2 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \sum Y_i \\ \sum U_i Y_i \\ \sum V_i Y_i \end{bmatrix}.$$

Simplifying the sums and using Cramer's Rule we obtain the regression coefficients c_0, c_1 and c_2:

$$c_0 = \frac{\begin{bmatrix} \sum Y_i & 8 & 8 \\ \sum U_i Y_i & 8 & 4 \\ \sum V_i Y_i & 4 & 8 \end{bmatrix}}{256} = \frac{\begin{bmatrix} \sum Y_i & 2 & 2 \\ \sum U_i Y_i & 2 & 1 \\ \sum V_i Y_i & 1 & 2 \end{bmatrix}}{16},$$

$$c_1 = \frac{\begin{bmatrix} 16 & \sum Y_i & 8 \\ 8 & \sum U_i Y_i & 4 \\ 8 & \sum V_i Y_i & 8 \end{bmatrix}}{256} = \frac{\begin{bmatrix} 4 & \sum Y_i & 2 \\ 2 & \sum U_i Y_i & 1 \\ 2 & \sum V_i Y_i & 2 \end{bmatrix}}{16}, \text{ and}$$

$$c_2 = \frac{\begin{bmatrix} 16 & 8 & \sum Y_i \\ 8 & 8 & \sum U_i Y_i \\ 8 & 4 & \sum V_i Y_i \end{bmatrix}}{256} = \frac{\begin{bmatrix} 4 & 2 & \sum Y_i \\ 2 & 2 & \sum U_i Y_i \\ 2 & 1 & \sum V_i Y_i \end{bmatrix}}{16}.$$

The LSMR predicted profile rankings are given by:

$$\hat{Y}_A = c_0 + c_1 + c_2 = \frac{2\sum U_i Y_i + 2\sum V_i Y_i - \sum Y_i}{16},$$

$$\hat{Y}_B = c_0 + c_1 = \frac{-6\sum V_i Y_i + \sum U_i Y_i + \sum Y_i}{16},$$

$$\hat{Y}_C = c_0 + c_2 = \frac{-2\sum U_i Y_i + 2\sum V_i Y_i + \sum Y_i}{16}, \text{ and}$$

$$\hat{Y}_D = c_0 = \frac{-2\sum U_i Y_i - 2\sum V_i Y_i + 3\sum Y_i}{16}.$$

The corresponding actual sample profile rankings obtained by averaging the respondent rankings are:

$$\bar{Y}_A = \frac{Y_1 + Y_2 + Y_3 + Y_4}{4},$$

$$\bar{Y}_B = \frac{Y_5 + Y_6 + Y_7 + Y_8}{4},$$

$$\bar{Y}_C = \frac{Y_9 + Y_{10} + Y_{11} + Y_{12}}{4}, \text{ and}$$

$$\bar{Y}_D = \frac{Y_{13} + Y_{14} + Y_{15} + Y_{16}}{4}.$$

Let $\bar{f}(X)$ denote the average sample ranking of PAL X. For example $f(U = 1) = \frac{\bar{Y}_A + \bar{Y}_B}{2} = \frac{\sum_{i=1}^{8} Y_i}{8}$. On the other hand, the LSMR prediction for this PAL's sample ranking is $\hat{f}(X) = (\hat{Y}_A + \hat{Y}_B)/2 = c_0 + c_1 + \frac{c_2}{2}$. The relationship between the sample PAL rankings and LSMR predicted PAL rankings is shown in Table 4).

PAL	$\bar{f}(X)$	$\hat{f}(X)$
U=1	$\frac{\bar{Y}_A+\bar{Y}_B}{2}$	$c_0 + c_1 + \frac{c_2}{2}$
U=0	$\frac{\bar{Y}_C+\bar{Y}_D}{2}$	$c_0 + \frac{c_2}{2}$
V=1	$\frac{\bar{Y}_A+\bar{Y}_C}{2}$	$c_0 + c_2 + \frac{c_1}{2}$
V=0	$\frac{\bar{Y}_B+\bar{Y}_D}{2}$	$c_0 + \frac{c_1}{2}$

Table 4: Predicted PAL rankings using LSMR coefficients.

Table 5 shows that the average sample profile rankings and LSRM predicted profile rankings are different for each of the four profiles. In this case, the regression coefficients are $c_0 = 3.75$, $c_1 = -1.25$, $c_2 = -1.25$.

		Respondents						
U	V	R 1	R 2	R 3	R 4	Sample Rank	Predicted Sample Rank	|Residual Error|
1	1	1	1	3	1	1.5	3.75-1.25(1)-1.25(1)=1.25	.25
1	0	2	3	1	3	2.25	3.75-1.25(1)-1.25(0)=2.5	.25
0	1	3	2	2	2	2.25	3.75-1.25(0)-1.25(1)=2.5	.25
0	0	4	4	4	4	4	3.75-1.25(0)-1.25(0)=3.75	.25

Table 5: Predicted rankings for the sample outcomes in Table 1.

Geometrically, if the four points representing the sample profile rankings are co-planar, there is no error; otherwise, the LSMR predicted profile rankings will have a residual error (Figure 2). Such a geometric interpretation is not possible for surveys involving more than 2 attributes, in which case standard LSMR residual analysis indicates the error in sample profile rankings using regression coefficients.

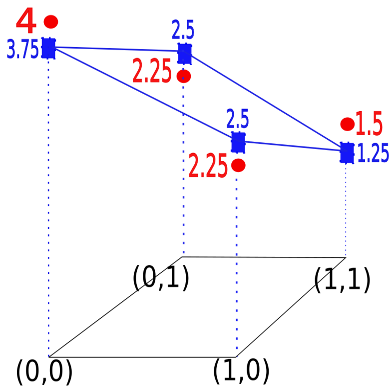


Figure 2: The LSRM predicted sample profile rankings ($\hat{Y} = 3.75 - 1.25U - 1.25V$ (square vertices) have residual errors as the actual sample profile rankings (dots) do not belong to the plane of regression.

LSMR will not in general predict exact PAL rankings, as the latter are computed directly from the sample profile rankings (Table 6).

	Respondent PAL Ranking					$c_0 = 3.75, c_1 = -1.25, c_2 = -1.25$	
PAL	R 1	R 2	R 3	R 4	Sample PAL Rank	Predicted Sample Rank	$\lvert error \rvert$
$U = 1$	1.5	2	2.5	2	1.875	$c_0 + c_1 + \frac{c_2}{2} = 1.875$	0
$U = 0$	3.5	3	2.5	3	3	$c_0 + \frac{c_2}{2}$=3.125	.125
$V = 1$	2	1.5	1.5	1.5	1.625	$c_0 + c_2 + \frac{c_1}{2} = 1.875$	.2
$V = 0$	3	3.5	3.5	3.375	2.875	$c_0 + \frac{c_1}{2} = 3.125$	.25

Table 6: Actual vrs. LSMR predicted PAL rankings for the sample outcomes in Table 1.

2.3 Maximum Likelihood Estimation of PAL-ranking Intervals

We will proceed to develop PR-ACBC methodology without assuming any knowledge of partworth utilitities. Since sample PAL rankings are easily computed from sample profile rankings, we discuss how to obtain the population profile rankings most likely to have given the observed sample profile rankings.

2.3.1 Known Population Size

Let us assume our population has size $N = 7$, and that our sample was a random selection of $n = 4$ out of these 7. The number $N = 7$ is for simplicity of illustrating the relevant cnmputations only, and could be any number $N > n = 4$. We will now use a multivariate hypergeometric distribution to obtain the maximum likelihood estimate (MLE) population profile rankings, meaning the population which was must likely to have yielded the observed sample profile rankings.

Suppose in our sample with $n = 4$, three different profile rankings O_1, O_2, O_3 are observed, with ranking O_2 ocurring twice in the sample. We create what we shall call a *factor table*, whose k largest factors $f_{ij} = 1 + (n_i/j)$ are used to determine the MLE population (see Table 7)

Choose the $k=3$ largest factors f_{ij} for a population size $N=n+k=n+3$.				
+3	$f_{13}=4/3$	$f_{23}=5/3$	$f_{33}=4/3$	
+2	$f_{12}=3/2$	$f_{22}=2$	$f_{32}=3/2$	
+1	$f_{11}=2$	$f_{21}=3$	$f_{31}=2$	
Number observed in sample:	$n_1=1$	$n_2=2$	$n_3=1$	Sample size: $n=4$
Ranking:	1=O_1	2=O_2	3=O_3	

Table 7: Probability factor table. The $k=3$ largest factors $f_{ij}=1+(n_i/j)$ are used to determine the MLE population with size $N=n+k=4+3=7$.

Let N_j denote the number of rankings O_j in the population. The number of factors a_j chosen from column j in the factor table indicates that a population with $N_j=n_j+a_j$ is a MLE. In our case, the latter is not unique. One choice of 3 largest factors is $f_{11}=2, f_{21}=3, f_{22}=2$ so that $a_1=1$, $a_2=2$ and $a_3=0$. A population Y in which $N_1=n_1+a_1=2$, $N_2=n_2+a_2=4$, and $N_3=n_3+a_3=1$ is a MLE. Another choice of 3 largest factors is $f_{11}=2, f_{21}=3, f_{31}=1$ so that $a_1=1$, $a_2=1$ and $a_3=0$. Population Z in which $N_1=n_1+a_1=2$, $N_2=n_2+a_2=3$, and $N_3=n_3+a_3=2$ is also a MLE. We can verify this is so by computing the respective probabilities p_Y and p_Z that our observed sample arises from respective populations Y and Z :

$$p_Y=\frac{C(2,1)C(4,2)C(1,1)}{C(7,4)}=\frac{f_{11}\cdot f_{21}f_{22}}{C(7,4)} \tag{1}$$

and

$$p_Z=\frac{C(2,1)C(3,2)C(2,1)}{C(7,4)}=\frac{f_{11}\cdot f_{21}\cdot f_{31}}{C(7,4).} \tag{2}$$

This procedure generalizes to any number of rankings m which appear in a sample of size n (Oberhofer and Kaufman (1987)). Let $n=\sum_{j=1}^{m} n_j$ where n_j is the number of ranking j appearing in the sample. Form the probability factor table with $f_{ij}=1+\frac{n_i}{j}$ ($i=1,...,r$, $j=1,...,m$ and $N=n+r$.) Choose the r largest factors in the latter table and let a_j be the number of factors chosen in column j. Then a population with $N_j=n_j+a_j$ ($j=1,...,m$) is a MLE.

2.3.2 Unknown Population Size

Let us assume now that rather than having a specified size, the population size is an unknown value N. In this case, we wish to determine the probability p_i that a member of the population has ranking O_i. As before, we assume that our sample is drawn randomly from the population. The probability p that the observed sample consisting of one O_1, two O_3's and one O_9 is

$$p=f(p_1,p_3,p_9)=\frac{4!}{1!2!1!}[p_1p_3^2p_9], \tag{3}$$

where $g(p_1,p_3,p_9)=p_1+p_3+p_9=1$. The values of p_1^*, p_3^*, and p_9^* which maximize $H(p_1,p_3,p_9)=\ln(f(p_1,p_3,p_9))$ (and hence also maximizes $p=f(p_1,p_3,p_9)$) are obtained using Lagrange multipliers:

$$\nabla H(p_1^*,p_3^*,p_9^*) \;\;=\;\; \lambda\nabla g(p_1^*,p_3^*,p_9^*),$$

and therefore

$$\begin{aligned}\frac{1}{p_1^*} &= \lambda\\ \frac{2}{p_2 3^*} &= \lambda\\ \frac{1}{p_9^*} &= \lambda.\end{aligned}$$

(The scalar quantity λ is called a Lagrange multiplier.) Using $p_1^* + p_3^* + p_9^* = 1$ gives $\frac{1}{\lambda} + \frac{2}{\lambda} + \frac{1}{\lambda} = 1$ and so $\lambda = 4$. Hence, the values $p_1^* = \frac{1}{4}, p32^* = \frac{1}{2}$, $p_9^* = \frac{1}{4}$ maximize the probability of the observed sample outcomes.

In general, let n_k be the number of sample outcomes O_k $(k = 1, 2, ..., K)$ and let p_k be the probability that a respondent in the population has outcome O_k $(k = 1, 2, ..., K)$. The likelihood function $f(p_1, p_2, ..., p_K)$ giving the probability of observing the sample values $n_1, ..., n_K$ is given by

$$f(p_1, ..., p_K) = \frac{n!}{n_1!n_2! \cdots n_K!} \prod_{k=1}^{K} p_k^{n_k}, \tag{4}$$

with $\sum_{k=1}^{K} n_k = n$ and $\sum_{k=1}^{K} p_k = 1$. We seek to find the values $p_1^*, ..., p_K^*$ which maximize the likelihood function f, or equivalently, the log-likelihood function

$$H(p_1, ..., p_K) = \ln f = \ln(n!) - \sum_{k=1}^{K} n_k! + \sum_{k=1}^{K} n_k \ln(p_k), \tag{5}$$

subject to the constraint $g(p_1, ..., p_K) = p_1 + p_2 + ... + p_K = 1$. Properties of gradients imply that the optimal values p_i^* must satisfy

$$\nabla H(p_1^*, ..., p_K^*) = \lambda \nabla g(p_1^*, ..., p_K^*). \tag{6}$$

It follows that for $k = 1, ..., K$,

$$\frac{n_k}{p_k^*} = \lambda. \tag{7}$$

Hence, $n = \sum_{k=1}^{K} n_k = \lambda \sum_{k=1}^{K} p_k^* = \lambda$, and so the probabilities $p_k^* = \frac{n_k}{n}$ give the maximum likelihood of the observed sample outcomes n_k $(k = 1, 2, ..., K)$. For any sample of size n and number n_k of observed outcomes O_k $(k = 1, 2, ...K)$, the maximum likelihood probabilities $p_k^* = \frac{n_k}{n}$ indicate that for a population of size N, the expected number N_k of outcomes O_k is given by $E(N_k) = p_k N$. A maximum-likelihood population could be simulated by augmenting the observed n sample outcomes, where the probability of outcome O_k at each draw is given by p_k. For a large number of such randomly constructed populations of size N, for each k the average number of population outcomes O_k is approximately $p_k N$.

2.4 Profile Ranking Intervals

Maximum likelihood provides a point estimates into the population profile rankings and their corresponding PAL rankings. It is easy to construct intervals guaranteed to include the actual population profile and attribute level rankings.

First, suppose a profile X in a sample of size n has mean profile ranking $\rho_n(X)$. It is easy to construct an interval which contains the mean ranking $\rho_N(X)$ for any population size $N > n$:

$$1 + \frac{n}{N}(\rho_n(X) - 1) \leq \rho_N(X) \leq 4 - \frac{n}{N}(4 - \rho_n(X)) \tag{8}$$

This interval containing $\rho_N(X)$ is obtained by either (i) assigning the rank 1 to X for all $N - n$ members of the population not in the sample (lower bound for $\rho_N(X)$); or (ii) assigning the rank 4 to X for all $N - n$ non-sample population members (upper bound for $\rho_k(X)$). Let $\lambda = \frac{N-n}{N}$ be the fraction of the population not included in the sample. It is easy to show that

$$\rho_n(X) - \lambda(\rho_n(X) - 1) \leq \rho_N(X) \leq \rho_n(X) + \lambda(4 - \rho_n(X)). \tag{9}$$

We call (9) a *profile ranking interval* for profile X. The length of this interval is 3λ, where the 3 arises algebraically as the difference between the extreme rankings 1 and 4.

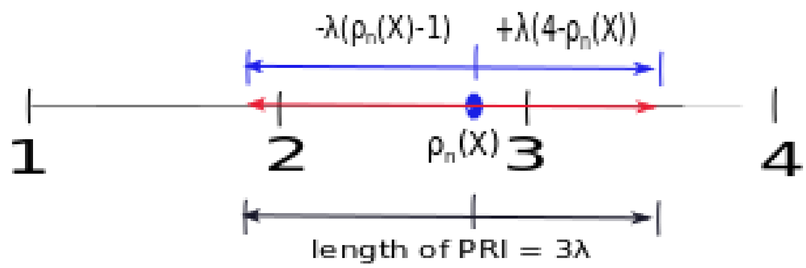


Figure 3: Given a profile X and its sample ranking $\rho_n(X)$, the length of the population profile ranking interval is determined by $\lambda = \frac{k-n}{k}$, the proportion of the population who have not taken the survey.

The profile ranking interval can be used to quantify sample bias. Assume that, out of a total population $N = 8$, two respondents have refused to take the survey, four have completed it, and the other two have not yet replied. In this case, the length of the PRI is $\frac{3(8-4)}{8} = 3/2$. If one of the non-respondents is convinced to participate, the interval length is reduced to $\frac{3(8-5)}{8} = \frac{9}{8}$, and if both non-respondents participate, then the interval is further reduced to $\frac{3(8-6)}{8} = \frac{3}{4}$. In other words, the two non-respondents cause the length of the PRI interval to be twice as large, an important consideration in seeking to elicit survey response.

In a similar way to constructing profile ranking intervals, if PAL χ has a sample ranking mean $\rho_n(\chi)$, it is easy to form a *PAL ranking interval* for any population $N > n$. For our toy survey,

$$r_n(\chi) + 2(N-n)r_n(\chi) \le r_N(\chi) \le r_n(\chi) + 8(N-n)r_n(\chi), \tag{10}$$

which is equivalent to

$$r_n(X) - \lambda(r_n(X) - 1) \le r_N(X) \le r_n(X) + \lambda(4 - r_n(X)). \tag{11}$$

with $\lambda = \frac{N-n}{N}$. Note that (9) and (11) have the same form, so the length of both intervals is 3λ.

2.5 Multimensional Scaling

One further type of analysis of PAL ranking data is a 2-dimensional geometric representation known as multidimensional scaling (MDS) (Alvo and Yu 2014). Fundamental to MDS is use of a distance measure $d(\mu, \nu)$ in which the more similar (resp. dissimilar) are a pair of rankings μ and ν, the smaller (resp. larger) is their distance. A variety of distance measures have been used for MDS, where it is conventional to number the objects being ranked $1, 2, ..., t$ and represent a ranking as a permutation, $\mu : S \to S$ where $S = \{1, 2, ..., t\}$ and $\mu(i)$ is the rank of object i. For example, Hamming distance (from coding theory) is defined as

$$d_H(\mu, \nu) = t - \sum_{i=1}^{t} I(\mu(i) = \nu(i)). \tag{12}$$

The indicator function $I(\cdot)$ equals 1 if the statement inside parenthesis is true and 0 otherwise. Hamming distance counts the number of positions where the permutations are different.

Another example is Spearman distance, which is akin to usual Euclidean distance

$$d_S(\mu, \nu) = \frac{1}{2} \sum_{i=1}^{t} (\mu(i) - \nu(i))^2. \tag{13}$$

Note that Hamming distance formula satisfies the three required metric properties:

- NON-NEGATIVITY $d_S(\mu, \nu) \ge 0$ for all μ, ν, with equality holding if and only if $\mu = \nu$;
- SYMMETRY $d_S(\mu, \nu) = d_S(\nu, \mu)$
- TRIANGLE INEQUALITY $d(\mu, \nu) + d(\nu, \sigma) \ge d(\mu, \sigma)$.

Spearman distance, however, only satisfies the first two metric properties (Alvo and Yu).

In applying MDS to PR-ACBC, we let (i, j) denote level j of attribute i, and $f(i, j)$ the average rank of level (i, j) in an individual respondent's profile ranking. For example, for the profile ranking BACD, $f(1, 1) = (1 + 2)/2 = 1.5$ since B=10 is ranked first and A=11 is ranked second; $f(20) = (2+4)/2 = 3$ since B=10 is ranked second and $D = 00$ is ranked 4th.

For our toy survey, each respondent X will have four average profile level rankings $f_X(1,1), f_X(1,0), f_X(2,1), f_X(2,0)$. Given 2 respondents X and Y, we consider the squared Euclidean distance between their PAL rankings defined as

$$d_S(X,Y) = \sum_{i,j} [f_X(i,j) - f_Y(i,j)]^2. \tag{14}$$

This distance measure can be used for an ACBC survey with any number of attributes and levels.

Once a distance measure is defined, a 2-dimensional MDS is such that rankings are represented by points in an xy Cartesian coordinate system, and the Euclidean distance between these points reflects the relative distances between rankings. Note that in this MDS, respondents R2 and R4 coincide since they have the same ranking (ACBD).

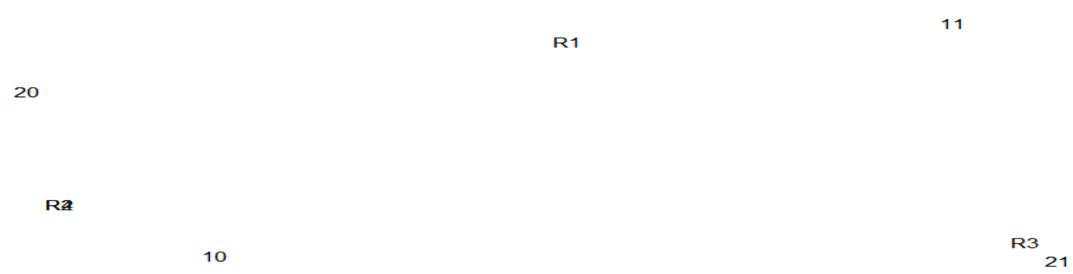


Figure 4: Example of an MDS using the distance function defined in (14) . In this case the sample profile rankings are ABCD, ACBD (twice), and BCAD

3 ACBC Survey of Humanitarian Disaster Relief Organizations

PR-based ACBC methodology is designed to analyze small populations and has many possible applications. One application involves a study of a small population ($N = 61$) of international disaster relief organizations headquartered in the U.S. or Canada as shown in Table 8.

In disaster relief, effectiveness of a response may depend on the quality of collaboration between organizations with a broad diversity of religious and ideological perspectives. For effective coordination of relief, it is important that humanitarian organizations understand the unique traits and characteristics that shape their disaster response decisions. Through comparison of these factors, it is possible to design optimal partnerships and joint endeavors between organizations that may fulfill distinct, yet complimentary, humanitarian roles. Our research focuses on

Survey Population: International Disaster Relief Organizations Headuarted in the US/Canada	
Faith-based (Christian)($N_1 = 49$)	Adventist Community Services, Adventist Development and Relief Agency, AMG international, Baptist World Aid, Billy Graham Evangelistic Association (rapid response team), Blessings international, Brethren Disaster Ministries, Catholic Charities USA, Catholic Medical Mission Board, Church World Service, Christian Disaster Response, Christian Aid Ministries, Convoy of Hope, Cooperative Baptist Fellowship, Covenant World Relief, Episcopal relief and Development, Food for the Hungry, Food for the Poor, Habitat for Humanity, Hopeforce International, International Aid, Lutheran Disaster Response, Lutheran World Relief, MAP international, MedAir, Medical Teams International, Mennonite Central Committee, Mission Aviation Fellowship, Missionary expeditors, The National Baptist Convention - Office of Disaster Management, Nazarene Compassionate Ministries, Operation Blessing International, Operation Safe, Presbytarian Church in America- Mission to North America, Presbytarian Disaster Assistance, Reach Global, S. Baptist N. Amer. Mission Board, Salvation Army World Service Organization, Samaritan's Purse, Service International, SOS Children – not disaster relief, Tearfund USA, United Church of Christ Disaster Ministries, United Methodist Committee on Relief (UMCOR), Water Mission, World Concern, World Relief, World Renew, World Vision
Non faith-based ($N_2 = 12$)	All Hands and Hearts,Americares, Direct Relief, Headwaters Relief Organization, Heart to Heart, Mercy Corps, Partnership with Native Americans, Save the Children, ShelterBox USA, SBP USA, Team Rubicon.

Table 8: Small populations comprising an ACBC disaster relief organization (DRO) survey.

a few key attributes affecting our population group's decision whether or not to respond to an international humanitarian disaster.

To this end, we designed an ACBC survey that creates disaster profiles with attributes and levels for this survey are displayed in Figure 7. Different disaster scenarios are paired off in the choice task (single elimination tournament) stage, beginning with 16 profiles close to the Build-Your-Own (BYO) or ideal scenario. The tournament data is the basis for profile ranking. The survey was deployed and tournament data collected using Sawtooth's Lighthouse platform.

What is the focus of this survey?

This survey asks you to identify the type of international disaster relief scenarios your organization would respond to. Your decision may be based either on prior experience, or on the degree to which the scenario allings with your organization's core values. With input from several managers of international relief organizations, we have framed your "go/no go" decision according to the following factors:

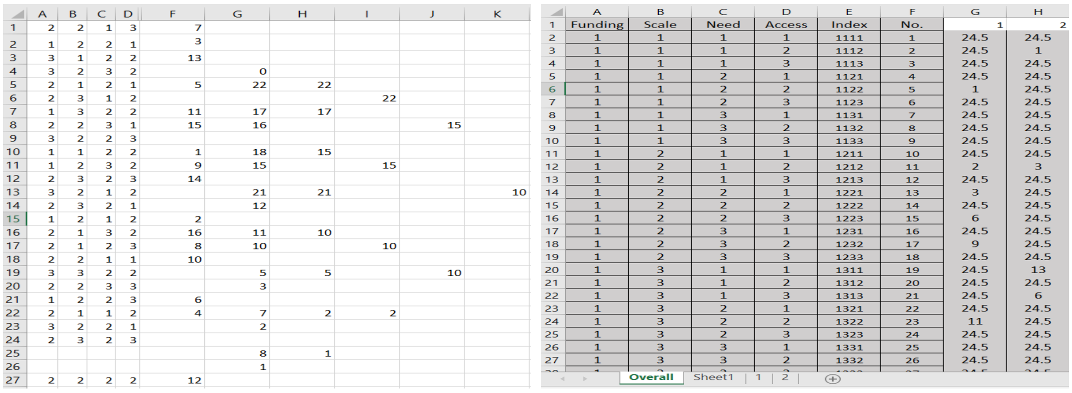
Factor	Description	Levels Considered
EXTERNAL FUNDING (Unrestricted money donated for the response.)	What amount of external funding is available for your response?	a) At least 75% of what is required b) About 50% of what is required c) Less than 25% of what is required
RESPONSE SCALE	Has the disaster been declared an IASC Level 1,2 or 3 emergency?	a) Large INGOs are responding in clusters (Level 3) b) Some INGOs are responding (Level 2) c) Few or no INGOs are responding (Level 1 or undeclared)
NEED ASSESSMENT	What is the level of need of the vulnerable for your organization's resources and/or capabilities?	a) Clear need for your organization's particular contribution b) Contribution could be of assistance but is not indispensable c) Unknown need of contribution
ACCESS TO AFFECTED COMMUNITY	What partnership provides connection to the community?	a) No pre-existing partnership is available b) A local partner has requested help c) An outside party has invited participation

Figure 5: An ACBC survey with 4 attributes consisting of 3 levels each.

3.1 Survey Data

As shown in Figure 4, our humanitarian survey consists of four attributes, each with three levels. Thus, the number of possible profiles is $3^4 = 81$. These are identified by four digit numbers $X = x_1x_2x_3x_4$ where profile X has level x_1 for the first attribute, level x_2 for the second attribute, level x_3 for the third attribute, and level x_4 for the fourth attribute. In the tournament stage of the competition, there are four rounds, in which sixteen profiles face off against each other in head to head match-ups, much like the FIFA World Cup Round of 16. The competing profiles are selected from the 81 possible profiles based on the respondent's BYO preferences. We assign a ranking of # 1 to the tournament winner, # 2 to the runner up, # 3 to the semifinalist who lost to #1, # 4 to the quarterfinalist who lost to #1, # 5 to the quarter-finalist who lost to #2 and so on. Profiles that do not appear in the tournament are ranked # 24.5 (the average of # 17 through #32).

An Excel file keeps track of each responding organization's tournament and the PAL ranking of each survey.



	A	B	C	D	F	G	H	I	J	K
1	2	2	1	3	7					
2	1	2	2	1	3					
3	3	1	2	2	13					
4	3	2	3	2		0				
5	2	1	2	1	5	22	22			
6	2	3	1	2				22		
7	1	3	2	2	11	17	17			
8	2	2	3	1	15	16			15	
9	3	2	2	3						
10	1	1	2	2	1	18	15			
11	1	2	3	2	9	15		15		
12	2	3	2	3	14					
13	3	2	1	2		21	21			10
14	2	3	2	1		12				
15	1	2	1	2	2					
16	2	1	3	2	16	11	10			
17	2	1	2	3	8	10		10		
18	2	2	1	1	10					
19	3	3	2	2		5	5		10	
20	2	2	3	3		3				
21	1	2	2	3	6					
22	2	1	1	2	4	7	2	2		
23	3	2	2	1		2				
24	2	3	2	3						
25						8	1			
26						1				
27	2	2	2	2	12					

	A	B	C	D	E	F	G	H
1	Funding	Scale	Need	Access	Index	No.	1	2
2	1	1	1	1	1111	1	24.5	24.5
3	1	1	1	2	1112	2	24.5	1
4	1	1	1	3	1113	3	24.5	24.5
5	1	1	2	1	1121	4	24.5	24.5
6	1	1	2	2	1122	5	1	24.5
7	1	1	2	3	1123	6	24.5	24.5
8	1	1	3	1	1131	7	24.5	24.5
9	1	1	3	2	1132	8	24.5	24.5
10	1	1	3	3	1133	9	24.5	24.5
11	1	2	1	1	1211	10	24.5	24.5
12	1	2	1	2	1212	11	2	3
13	1	2	1	3	1213	12	24.5	24.5
14	1	2	2	1	1221	13	3	24.5
15	1	2	2	2	1222	14	24.5	24.5
16	1	2	2	3	1223	15	6	24.5
17	1	2	3	1	1231	16	24.5	24.5
18	1	2	3	2	1232	17	9	24.5
19	1	2	3	3	1233	18	24.5	24.5
20	1	3	1	1	1311	19	24.5	13
21	1	3	1	2	1312	20	24.5	24.5
22	1	3	1	3	1313	21	24.5	6
23	1	3	2	1	1321	22	24.5	24.5
24	1	3	2	2	1322	23	11	24.5
25	1	3	2	3	1323	24	24.5	24.5
26	1	3	3	1	1331	25	24.5	24.5
27	1	3	3	2	1332	26	24.5	24.5

Overall Sheet1 1 2

Figure 6: Respondent 1's tournament (left) and profile rankings for organizations 1 and 2 (right). The latter is processed by the MATLAB script in Table 9 to obtain the PAL rankings.

```
%% MATLAB script to compute PAL rankings from organized data file
clear all; close all;
n=5; %% number of respondents
%% Create respondents
for h=1:n
    org(h,1)=respondent;
end
%% Read in Organized Data
Profiles=xlsread('FBOOrganized.xlsx','Overall','A2:D82');
ProfileRankings=xlsread('FBOOrganized.xlsx','Overall','G2:K82')
%% Compute PAL Rankings by each organization
for h=1:n
for i=1:4
    for j=1:3
         org(h,1).PALRanking(i,j)=0;
        for k=1:81
        if Profiles(k,i)==j
        org(h,1).PALRanking(i,j)=  org(h,1).PALRanking(i,j) + ProfileRankings(k,h)
        end
        end
        org(h,1).PALRanking(i,j)= org(h,1).PALRanking(i,j)/27;
     end
    end
end

%% Compute sample PAL Rankings
samplePALranking=zeros(4,3);
for i=1:4
    for j=1:3
        for h=1:n
        samplePALranking(i,j)=samplePALranking(i,j)+org(h,1).PALRanking(i,j);
        end
        samplePALranking(i,j)=samplePALranking(i,j)/n;
    end
end
```

Table 9: Main MATLAB script to compute PAL rankings.

```
classdef respondent

    properties
        PALRanking
    end

   end
```

Table 10: Respondent Class used by the main script in Table 9

3.2 Survey Analysis

ORG	Profile Attribute Levels (PALs) L11	L12	L13	L21	L22	L23	L31	L32	L33	L41	L42	L43
FBO1	20.2	18.6	22.7	20.8	18.2	22.5	21.2	17.9	22.4	21.5	18.4	21.6
FBO2	21.2	21.5	18.8	21.3	21.4	18.8	17.8	21.5	22.2	22.1	17.1	22.3
FBO3	21.1	18.1	22.5	20.3	18.5	22.8	20.8	18.7	22.1	21.8	21.9	17.9
FBO4	20.6	17.7	20.3	20.6	18.1	20.1	21.2	17.8	20.7	21.6	15.6	21.6
FBO5	20.3	20.4	17.8	20.1	17.6	20.7	17.3	20.8	20.4	20.3	13.0	20.2
NGO1	20.8	21.5	19.2	21.7	18.7	21.0	17.3	21.6	22.6	22.4	18.2	21.0
NGO2	20.3	22	20.0	18.4	21.1	22.8	18.8	20.9	22.6	21.3	19.4	21.7
NGO3	18.1	21.6	21.8	19.5	20.3	21.7	17.2	22.5	21.8	20.3	19.2	21.9
NGO4	19.6	20.3	21.4	18.0	21.3	22.2	21.2	19.2	21.1	22.1	17.9	21.6
NGO5	20.6	21.5	19.4	21.2	21.4	18.8	16.6	22.3	22.7	18.4	21.6	21.4

Table 11: FBO/NGO Individual PAL rankings ($n = 5$)

ATTRIBUTE	Level 1		Level 2		Level 3	
	FBO	NGO	FBO	NGO	FBO	NGO
1 Funding	20.6	**19.93** ($\geq$ 75%)	**19.27** ($\sim$ 50%)	21.36	20.42	20.38
2 Scale	20.61	**19.78 (ISAC Level 3)**	**18.77 (ISAC Level 2)**	20.57	21.00	21.32
3 Need	19.45	**18.20 (clear need)**	**19.34 (not indispensable)**	21.31	21.58	22.14
4 Access	21.47	20.89	**18.19 Local Partner**	**19.26 Local Partner**	20.71	21.51

Table 12: FBO/NGO Sample Mean PAL rankings ($n = 5$)

3.2.1 PAL ranking intervals

3.2.2 MDS

```
# Classical MDS of ACBC Disaster Relief Organization Survey 2018 (R script)

levels <- c('F11','F12','F13','F21','F22','F23','F31','F32','F33','F41','F42','F43','N11','N12','N13','N21','N22','N23','N31','N32','N33','N41','N42','N43')
respondents <- c('FBO1','FBO2','FBO3','FBO4','FBO5','NGO1','NGO2','NGO3','NGO4','NGO5')
FBO1 <- c(20.2,18.6,22.7,20.8,18.2,22.5,  21.2,17.9,22.4,21.5,18.4,21.6)
FBO2 <- c(21.2,21.5,18.8,21.3,21.4,18.8,  17.8,21.5,22.2,22.1,17.1,22.3)
FBO3 <-c(21.1,18.1,22.5,20.3,18.5,22.8,   20.8,18.7,22.1,21.8,21.9,17.9)
FBO4 <- c(20.6,17.7,20.3,20.6,18.1,20.1,  21.2,17.8,20.7,21.6,15.6,21.6)
FBO5 <- c(20.3,20.4,17.8,20.1,17.6,20.7,  17.3,20.8,20.4,20.3,13.0,20.2)
NGO1 <-c(20.8,21.5,19.2,21.7, 18.7,21.0,   17.3,21.6,22.6,22.4,18.2,21.0)
NGO2 <-c(20.3,22,20.0,18.4,   21.1,22.8,   18.8,20.9,22.6,21.3,19.4,21.7)
NGO3 <-c(18.1,21.6,21.8,19.5, 20.3,21.7,   17.2,22.5,21.8,20.3,19.2,21.9)
NGO4 <-c(19.6,20.3,21.4,18.0, 21.3,22.2,   21.2,19.2,21.1,22.1,17.9,21.6)
NGO5 <- c(20.6,21.5,19.4,21.2,21.4,18.8,   16.6,22.3,22.7,18.4,21.6,21.4)
Respondent.data <- data.frame(FBO1,FBO2,FBO3,FBO4,FBO5,NGO1,NGO2,NGO3,NGO4,NGO5)
mds1 <- cmdscale(dist(Respondent.data))
par(mai=c(.75,.75,.75,.75))
plot(mds1, type = 'n', axes = FALSE, xlab = '', ylab = '')
text(mds1[, 1], mds1a[, 2], respondents)

par(new=T)
fL11 <-c(20.2,21.2,21.1,20.6,20.3)
fL12 <-c(18.6,21.5,18.1,17.7,20.4)
fL13 <-c(22.7,18.8,22.5,20.3,17.8)
fL21 <-c(20.8,21.3,20.3,20.6,20.1)
fL22 <-c(18.2,21.4,18.5,18.1,17.6)
fL23 <-c(22.5,18.8,22.8,20.1,20.7)
fL31 <-c(21.2,17.8,20.8,21.2,17.3)
fL32 <- c(17.9,21.5,18.7,17.8,20.8)
fL33 <- c(22.4,22.2,22.1,20.7,20.4)
fL41 <- c(21.5,22.1,21.8,21.6,20.3)
fL42 <-c(18.4,17.1,21.9,15.6,13.0)
fL43 <- c(21.6,22.3,17.9,21.6,21.4)
nL11 <-c(20.8,20.3,18.1,19.6,20.6)
nL12 <-c(21.5,22,22.0,18.4,21.1)
nL13 <- c(19.2,20.0,21.8,21.4,19.4)
nL21 <-c(21.7,18.4,19.5,18.0,21.4)
nL22 <- c(18.7,21.1,20.3,21.3,21.4)
nL23 <- c(21.0,22.8,21.7,22.2,18.8)
nL31 <- c(17.3,18.8,17.2,21.2,16.6)
nL32 <- c(21.6,20.9,22.5,19.2,22.3)
nL33 <-c(22.6,22.6,21.8,21.1,22.7)
nL41 <-c(22.4,21.3,20.3,22.1,18.4)
nL42 <- c(18.2, 19.4, 19.2, 17.9, 21.6)
nL43 <- c(21.0,21.7,21.9,21.6,21.4)
Level.data <- data.frame(fL11,fL12,fL13,fL21,fL22,fL23,fL31,fL32,fL33,fL41,fL42,fL43,nL11,nL12,nL13,nL21,nL22,nL23,nL31,nL32,nL33,nL41,nL42,nL43)
mds2 <- cmdscale(dist(Level.data))
par(mai=c(.5,.5,.5,.5))
plot(mds2, type = 'n', axes = FALSE, xlab = '', ylab = '')
text(mds2[, 1], mds2[, 2], levels)
```

Figure 7: R-script used for the MDS shown in Figure 8

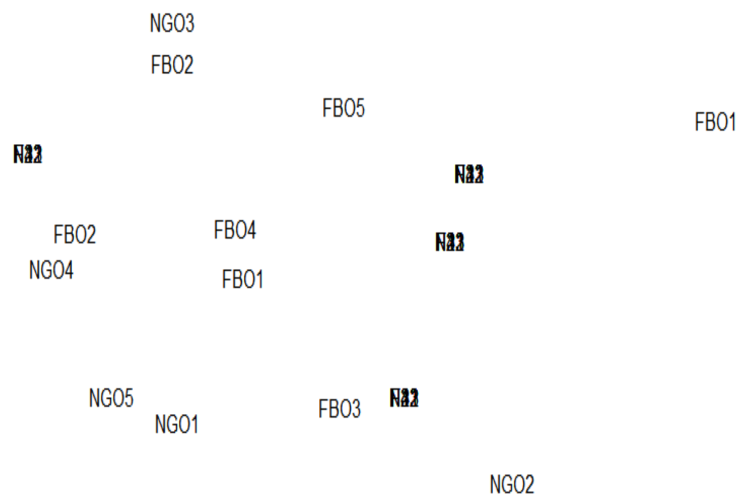


Figure 8: Multidimensional scaling showing similarity of sample respondents and attribute levels.

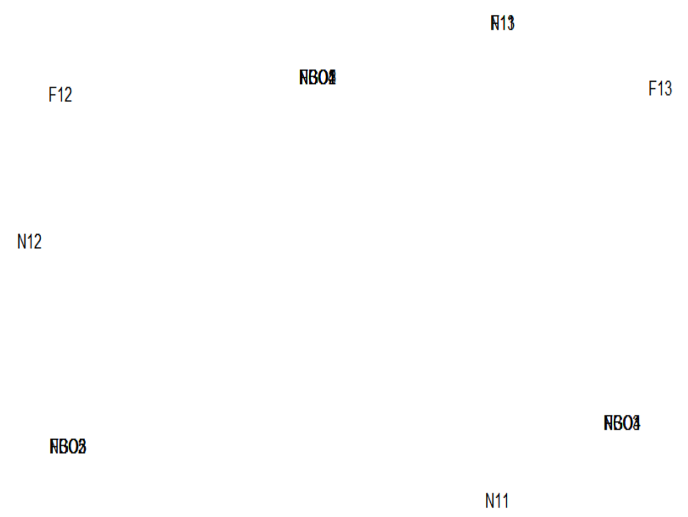


Figure 9: Attribute 1 levels only.

3.3 Comparison with Part-worth Utilities

3.3.1 Choice task prediction accuracy

3.3.2 Attribute Importances

4 Further Work

Still in its infancy, PR-ACBC methodology has several major areas open to further research including analysis of different ranking systems for various types of choice tournaments, determination of the population nd sample sizes for which PR-ACBC is most suitable, and application of PR-ACBC to other small population studies.

5 Conclusion

To complement well-established partworth-utility-based conjoint analysis methodology of survey data where the target populations are typically large, we have introduced a simple, intuitive approach to a small population's profile and level rankings based on choice-task sample data. A toy survey is used to introduce the basic concepts, and an actual survey administered to a small number of disaster relief organizations used as an example of PR-ACBC methodology.

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