A HIGH RESOLUTION PARALLEL ICE SHEET MODEL INCLUDING FAST, SLIDING FLOW: ADVANCED DEVELOPMENT AND APPLICATION

ED BUELER[1], REGINE HOCK[2], DAVID A. MAXWELL[1], AND MARTIN TRUFFER[2]

The following pages are a one page draft table of contents followed by the "Scientific, Technical, and Managment Section" of the proposal. There is a 15 page maximum for this section, but the references are not included in that page total, and nor are Biographical Sketches, Letters of Support, and so on. Those other sections are being prepared separately. Text in ALL CAPS *still needs drafting.*

[1]Dept. of Mathematics and Statistics, University of Alaska, Fairbanks 99775-6660; e-mail ffelb@uaf.edu. [2]Geophysical Institute, UAF 99775-7320.
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Scientific, Technical, and Management Section

1. Introduction

Ice sheets flow viscously and "slide" in subregions on an easily deformable composite basal layer of ice, liquid water, and till. Modeling this flow is a computational fluid dynamics problem of substantial current importance. It is important because, in particular, the Greenland ice sheet is responding more quickly than expected to global warming (Alley and Anandakrishnan, 2007; Das and others, 2008; Joughin and others, 2008; Luthcke and others, 2006). A comparable response is occurring in the West Antarctic ice sheet (Velicogna and Wahr, 2006; Rignot and others, 2008). The IPCC AR4 report identifies ice sheet dynamical behavior as the most significant uncertainty for predicting sea level rise (Pachauri and others, 2008). The current rate of sea level rise is at the upper limit of, or outside of, the range of predictions from existing global circulation and climate models for the period 1990–2006 (Rahmstorf and others, 2007), but these models lack ice dynamics components.

Though some ice sheet models have been available for coupling to climate models (Ritz, 1997), the response of these ice sheet models to climate changes is limited by their lack of effective approximations to the type of fast flow in outlet glaciers and ice streams which is observed. An increase in fast outlet glacier flow of the Greenland ice sheet is believed to account for a large portion of its contribution to sea level rise (Rignot and Kanagaratnam, 2006; Truffer and Fahnestock, 2007). Likewise, such fast flow features, both ice streams and outlet glaciers (Truffer and Echelmeyer, 2003), account for 90% of the discharge rate of the Antarctic ice sheet (Bamber and others, 2000).

We propose to address the need for ice sheet models with fast flow features, and the need for effective ice sheet model components within global climate models, by advancing the development of the existing Parallel Ice Sheet Model ("PISM", `www.pism-docs.org`). PISM is an open source code developed by PI Bueler, Jed Brown, Craig Lingle, and others at the University of Alaska, Fairbanks (UAF). This existing model *already includes the parallel solution of a fast sliding flow model for ice streams and ice shelves.* PISM makes high resolution simulation possible through the use of the massive parallelism of existing supercomputers. PISM currently includes a vertically integrated membrane ("longitudinal") stress balance model within a well-documented, comprehensive ice sheet model, as explained in the prior work section below. This existing model is well-suited to the additional development which we propose here.

Our proposed work includes fundamental model development, especially addressing the initialization of ice sheet models and the careful removal of some simplifications. We will apply the model to the Greenland and Antarctic ice sheets, both through model runs at UAF and through collaboration with teams and climate centers already using PISM. Such collaboration is addressed in part in the attached letters of support.

2. Objectives

At the end of the grant period, at the end of 2012,

- PISM will incorporate the results of principled inverse modeling to initialize prognostic runs. This will use satellite and airborne remote sensing, especially of surface velocity, to compute the most likely basal conditions at the current time. Complementary paleoglacial runs will integrate ice sheet history into the temperature and velocity fields.
- Some simplifications present in the 2008 implementation of PISM, especially relating to shallowness, will have been carefully removed while maintaining the ability to model ice sheets everywhere at high resolution. Uniformly high resolution, specifically the use of 5 to 2 km grids for Greenland and 10 to 5 km grids for Antarctica, will be achieved by parallel implementations of all included models. This high resolution will yield effective dynamical models of ice streams and better models of outlet glaciers.
- PISM will couple to regional and global circulation/climate models through a standardized interface using a realistic model for mass and energy balance at the surface of the ice.
- PISM will model the Greenland and Antarctic ice sheets in coupled simulations with regional and global climate models. "Offline" simulations will occur at UAF, especially as parameter sensitivity experiments, while collaborators at climate centers (see letters of support) will perform coupled runs. Coupled runs will be prepared for the IPCC with these collaborators.
- PISM will be documented, stable, and open source. Users at climate centers will routinely get good advice on usage though email with the developers at UAF. Users will routinely verify and validate significant PISM submodels, using tools included in the PISM download, to gain confidence in the model and to understand its limitations.

3. Prior work

Existing capability in PISM is a strong basis for development and application of an IPCC-class ice sheet model as described in the Research Announcement.

PISM has been developed since 2001 with support from NASA Cryospheric Sciences grant NAG5-11371. It was started from the isothermal shallow ice approximation numerical model of the Antarctic ice sheet used in (Lingle and Troshina, 1998). In the 2003–2006 period Master's student Jed Brown (a collaborator on the current proposal) and PI Bueler rewrote all code to create PISM as a versatile open source C++ ice sheet modeling code based upon PETSc, the Portable Extensible Toolkit for Scientific computing (Balay and others, 2007). PISM is now thermomechanically coupled, it uses membrane stress balance models in addition to the shallow ice approximation, and it incorporates a number of significant recent modeling advances.

Conservation of mass and energy. PISM includes numerical approximations of the mass continuity equation, the conservation of energy equation, and the two well-understood shallow approximations of the stress balance (conservation of momentum) equations for nonlinearly-viscous ice flow.

The implementation of the mass continuity equation in PISM demonstrates that time-stepping for this mass continuity computation is stable and easily parallelizable for all stress balance models in PISM (below), and proposed for PISM (next section). Our conditionally stable scheme for time-stepping can be summarized as an explicit diffusion scheme for the part of the flux which is parallel to the surface gradient along with an upwind scheme for the part of the flux which is orthogonal to the surface gradient (c.f. Morton and Mayers, 2005).

The conservation of energy model also couples to the velocity field through advection and strain heating. It applies for all stress balance models. PISM runs already show that this energy conservation model is easily parallelizable, and PISM's implementation is verified by exact solutions Bueler and others (2007a) included in the PISM download. Additionally the age of the ice is also modeled by PISM, and a conduction-only conservation of energy model applies in the bedrock. The basal boundary condition for the conservation of energy model is a geothermal flux condition typically applied at some hundreds of meters down into the bedrock below the ice sheet. Geothermal flux which varies in the map-plane can be used by PISM. Such a map of geothermal flux distribution is potentially derivable using magnetic susceptability determined by satellite observations (Fox Maule and others, 2005).

Constitutive relation. Ice is modeled in PISM as an isotropic nonlinearly-viscous fluid. The detailed form of the flow law ("constitutive relation") is flexible, however, and a number of forms are already implemented. Besides the standard temperature-dependent forms of Glen's power law, PISM also includes the constitutive relation of Goldsby and Kohlstedt (2001). Two versions of each flow law are necessarily implemented in PISM because different stress balance models (below) demand each form. In particular, a flow law must allow computation of deviatoric stresses from strain rates (viscosity form) and computation the other direction.

Stress balance 1: shallow ice approximation (SIA). This stress balance model, the basic "lubrication" approximation (Fowler, 1997) of ice sheet flow sticking (frozen) to the bed, is appropriate for large fractions (by area) of an ice sheet. It will remain an option for all PISM users, in part because we will compare internally the predictions of more elaborate stress balance models with those of the nonsliding SIA as an element of the identification of ice streams and outlet glaciers.

The nonsliding SIA determines the ice flow velocity by vertical integration based on the temperature in the ice column, using the surface slope at that location (Fowler, 1997). Integration of the incompressibility equation gives the vertical velocity. PISM's implementation of this velocity computation is easily parallelizable and is verifiable by exact solutions (Bueler and others, 2005, 2007a).

The literature contains many ice sheet modeling applications of the SIA model with additional basal sliding which is a function of the local (in the map plane) driving stress (Greve, 2000; Huybrechts and de Wolde, 1999, among many others). This basal sliding is "switched on" either when the temperature of the basal ice reaches the pressure-melting point or at locations where the bed is at low elevation. Such a model generates a jump discontinuity in the horizontal velocity and thus a formally-infinite vertical velocity because ice is an incompressible fluid. This phenomenon is one practical explanation of why a membrane (Hindmarsh, 2006) or "longitudinal" stress model is necessary. Next we describe the membrane stress balance appropriate for ice shelves and shallow ice streams.

Stress balance 2: shallow *shelf* approximation (SSA). In the case where the applied stress at the base of the ice is zero as in ice shelves (MacAyeal and others, 1996; Weis and others, 1999), or when the stress applied to the base of the ice from sliding is small (MacAyeal, 1989; Schoof and Hindmarsh, 2008+), the stress balance can be well-approximated by a different vertically-integrated stress balance, the SSA. We emphasize that this model is effective when used for diagnostically modeling ice shelves (MacAyeal and others, 1996); see also Figure 2 below.

Because of the comparison to the Blatter (1995) model which we propose to implement in parallel (see Approach section), we present the SSA stress balance equation here. Let H be the ice thickness and h the surface elevation. Let $\bar{\nu}$ be the vertical average of the nonlinear and temperature-dependent viscosity. Let $\tau_{b,1}, \tau_{b,2}$ be the shear stress applied by sliding at the base of the ice by the bedrock or till. Then the vertically averaged horizontal velocity (u_1, u_2) satisfies

$$\frac{\partial}{\partial x_1}\left[2\bar{\nu}H\left(2\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}\right)\right]+\frac{\partial}{\partial x_2}\left[\bar{\nu}H\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\right]+\tau_{b,1}=\rho g H\frac{\partial h}{\partial x_1}, \tag{1}$$

and a second very similar equation involving $\tau_{b,2}$ (MacAyeal, 1989). Note that in this case, unlike the SIA and equation (3) below, the horizontal velocity does not vary in the ice column. If the ice is floating then $\tau_b = 0$. For grounded ice the basal shear stress τ_b may be linearly (MacAyeal, 1989) or nonlinearly (Schoof, 2006, uses a plastic till) related to the sliding velocity.

PISM uses this model in the case of floating ice. It has been used for ice shelf dynamics in whole Antarctic ice sheet models where the grounded flow is modeled by the SIA (Huybrechts and de Wolde, 1999). PISM's parallel implementation of the (nonlinear)

SSA requires the iterated solution of nontrivially-coupled linear systems, but this is precisely the kind of problem for which PETSc is designed (Balay and others, 2007). The SSA for ice shelves is verifiable using an exact solution (in preparation but already implemented in PISM). The plastic till free boundary problem for the SSA is also verifiable in PISM based on an exact solution (Schoof, 2006).

MacAyeal (1989) initiated the use of equation (1) for the stress balance in diagnostic, and inverse (Joughin and others, 2001, 2004), calculations for ice streams with minimal basal topography (e.g. for the NE Greenland ice stream). Known difficulties with applying this model to ice streams for prognostic calculations using a linear till include the required *a priori* identification of sliding regions and the lack of previously-identified stable time-stepping methods for ice surface evolution (Hulbe and MacAyeal, 1999). PISM has solved these difficulties by an explicit mass continuity time-stepping scheme (above) and the use of the Schoof (2006) version of the SSA.

Stress balance 3: A version of SSA used as a "sliding law" for the SIA. The theoretical step, made by Schoof (2006), was to identify the regions of sliding ice as solving an associated free boundary problem in the case where the till is modeled as plastic. Plastic till satisfies the $\epsilon = 0$ case of the left-hand equation (Paterson, 1994):

$$\tau_b = \tau_c \frac{u}{|u|^{1-\epsilon}\,(C_{\text{crit}})^{\epsilon}}, \qquad \tau_c = (\tan\phi)\,(\rho g H - p_w) \tag{2}$$

Here u is the sliding velocity, τ_c is called the yield stress (in the $\epsilon \to 0$ limit), and C_{crit} is a velocity which can be regarded as the critical sliding velocity at which the basal shear stress no longer increases significantly with increasing velocity (if $0 < \epsilon \ll 1$); $C_{\text{crit}} = 100$ m/a is reasonable. The parameter ϕ, called the till friction angle, describes the strength of the till (Paterson, 1994). Note $\rho g H$ is the "overburden" pressure and p_w is the pore water pressure, so the second equation states that the yield stress is proportional to the effective pressure on the till. In PISM the pore water pressure is a function of the amount of locally-stored water in the till, computed by integrating the basal melt or refreeze rate.

Schoof (2008+) has shown that power law approximations of plasticity, with $0 < \epsilon \ll 1$, yield similar ice dynamics. In the plastic or nearly-plastic power law cases the application of the SSA becomes practical on whole ice sheet scale at fine resolution because only the ice streams with sufficiently weak base and sufficiently strong applied (membrane-like) stresses actually slide. PISM shows that this kind of problem can be solved in parallel on a fine grid at whole ice sheet scale, with iterative linear solutions handled by PETSc.

In this third stress balance, the vertically-constant horizontal velocity field produced by the plastic till (or power law) version of the SSA is combined, as usual for a sliding law, with the SIA velocity field, which has "vertical shear" in planes parallel to the geoid, to yield the final horizontal velocity field at a given time step. This final horizontal

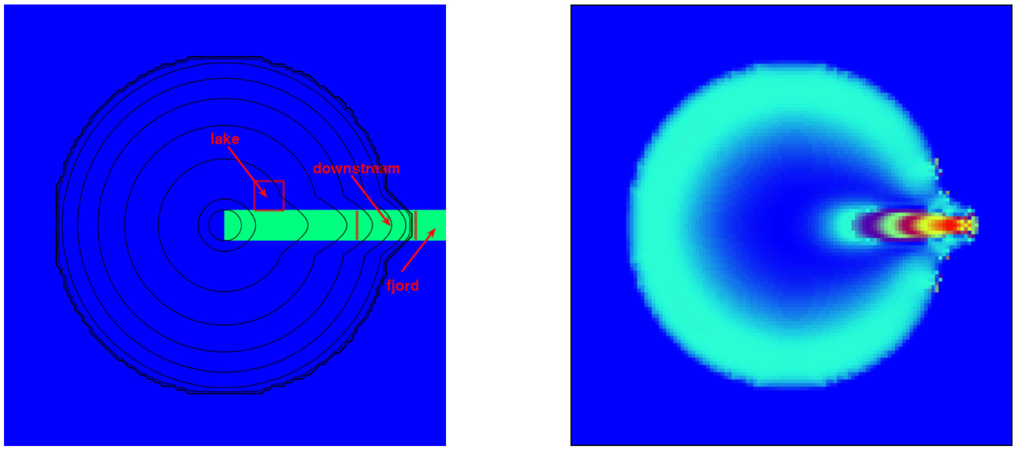

Figure 1. Left: Color shows till friction angle ϕ (green is $5°$ while blue is $20°$), and contours show the initial surface elevation of the model ice cap with radius about 550 km. The weak $5°$ till strip allows a broad 100km wide ice stream to form where the base is melted and the membrane stresses exceed the yield stress. The red notations describe additional experiments (not shown). Right: Color shows resulting PISM model flow speed at the end of a 5k year prognostic run on a 12.5km grid; fastest red flow is 270 meters per year.

velocity is used to compute vertical velocity, and in the conservation of mass and energy equations, and the age equation.

Such a model was presented by the PI at AGU 2007 and is already implemented within PISM (Bueler and Brown, 2008+; Bueler and others, 2008). It is promising as a first unified, thermocoupled, time-dependent view of shallow ice flow with both vertical shear and membrane-stress-modulated sliding. This model is based on a combination of the velocity fields computed at each time step from the nonsliding SIA and the Schoof (2006) version of the SSA, using plastic or power law till, and using the current temperature field. We believe this model is the three dimensional thermocoupled extension of the Pollard and DeConto (2007) flowline model.

This model uses as its input a map of till friction angle ϕ. An example input friction angle map and the resulting stable streaming flow speed is shown in Figure 1. Thus this model may be initialized from the output of inverse modeling efforts like those that have been applied to the NE Greenland ice stream (Joughin and others, 2001) and the Siple coast ice streams (Joughin and others, 2004). Such incorporation of inverse results is an important part of our proposed work (next section).

Positive degree day (PDD) model. A PDD model is already included in PISM (Calov and Greve, 2005), to compute the net surface mass balance from the rate of snow accumulation and the yearly temperature cycle. This model is less physical than our proposed surface energy- and mass-balance model (below). The surface mass-/energy-balance components of PISM are coupling components when PISM interacts with an atmospheric model in a regional or global climate model. Development of tools for coupling, along with sustaining collaborations with climate modelers (see letters of support) is part of the proposed work.

Earth deformation model. A newly-developed Earth deformation model (Bueler and others, 2007b) is used in PISM. The model includes more effects, including wavelength-dependent mode decay, than the standard ELRA model (Greve, 2001) which it generalizes. The spectral numerical implementation of this model is extremely fast, so earth deformation represents a small fraction of 1% of PISM computation time. This model is also significant for prognostic modeling because it allows the inclusion into ice sheet models of an observed current uplift map instead of requiring an artificial assumption of unloaded bed at some time in the past (Greve, 2001). Such an “observed current uplift map” could be computed by other earth deformation models of arbitrary sophistication (c.f. Ivins and James, 2005), while in the case of the Greenland ice sheet enough bedrock is exposed near the margin to suggest that the distributed rate of current uplift will be well-constrained by GPS-derived data. The use of PISM as a sophisticated high resolution ice sheet model coupled to an effective earth deformation model, having the ability to respect current uplift measurements, will help address the interpretation of GRACE result (Velicogna and Wahr, 2006).

Numerics. Most of the numerical methods in PISM are of finite difference type. PISM adaptively and automatically determines stable time steps (Bueler and others, 2007a; Bueler, 2008). All computations are on a regular and rectangular three-dimensional grid. The grid can be chosen at the command line. Regridding can be done at any time, for example taking the result of a rough grid computation and interpolating it onto a finer grid or vice versa.

The grid is automatically broken into one-processor portions at run-time; the user merely specifies the number of available processors. These portions communicate at their boundaries, and such communication is handled by PISM and PETSc invisibly to the user. PETSc is specifically designed to support high resolution fluid simulation on massively parallel supercomputers. PETSc is implemented using the most common method of parallel programming, namely MPI (Gropp and others, 1999). By using PETSc, PISM is already capable of using fine horizontal resolution everywhere on an ice sheet.

As noted in the Research Announcement, fine resolution is necessary for capturing fast flow features of significance to the ice sheet mass balance, which are usually confined to narrow (10–50 km wide) features within ice sheets. PISM has existing ability to model ice sheets at a resolution necessary to resolve their flow structure, however.

For instance, more than a year ago PISM simulated the entire Antarctic ice sheet using the thermomechanically coupled SIA on 480 processors at the Arctic Regional Supercomputing Center (Fairbanks, Alaska), for 100 model years, on an unprecedented 5 km horizontal grid resolution (and 200 million degrees of freedom). This capability will be extended to more complete stress balance during the grant period.

Open source, well-documented, and compatible. PISM is open source: `www.pism-docs.org`. Readers of this proposal can download the *User's Manual* or even the source code and run PISM themselves. The seventy page *User's Manual* includes realistic tutorial examples as well as complete documentation on command line options and on installation. Comments in the source code comprise an eighty page *Reference Manual* for programmers (Bueler, 2008).

PISM has been designed with coupling to global circulation models (GCMs) in mind, though this capability is not yet proven. The input and output format for PISM is CF 1.0-compliant NetCDF (`www.unidata.ucar.edu/software/netcdf`). Because PISM is already an MPI program, coupling to GCMs can potentially be either offline *or* in parallel. The state of the model can be viewed graphically at runtime or by examining output files in NetCDF or Matlab format.

Applications to ice sheets and ice shelves. The EISMINT-Greenland (Ritz, 1997) and EISMINT-Ross (MacAyeal and others, 1996) intercomparison experiments are already included in PISM distribution. A user can already, within hours of the free download of a copy of PISM, model *in parallel and at a resolution choosable at run-time* the Greenland ice sheet, based on publicly-available (though low resolution) data. Similarly for the Ross ice shelf. These intercomparison experiments are tutorial examples (Bueler and others, 2008). Figure 2 shows the velocity fields which result from such PISM runs.

4. Approach and proposed work

We believe the current state of PISM is a compelling basis on which to build the improved model described in the Objectives. The current state of PISM is based on fundamental theoretical advances of the last five years, and puts us at the threshhold of greatly improved models for the Greenland and Antarctic ice sheets. These models are being created at UAF and in collaboration with modeling groups at the Danish Climate Center and the Potsdam Institute for Climate Impact Research among others (see letters of support). Our approach is to work as a team on the following five threads.

4.1. **Incorporate inverse modeling results.** Ice sheet modeling differs from some other problems in geophysical fluid dynamics because it is difficult to observe the current state of dynamically-important fields within and at the base of an ice sheet. It is necessary to infer the state of these fields from remote sensing observations at the

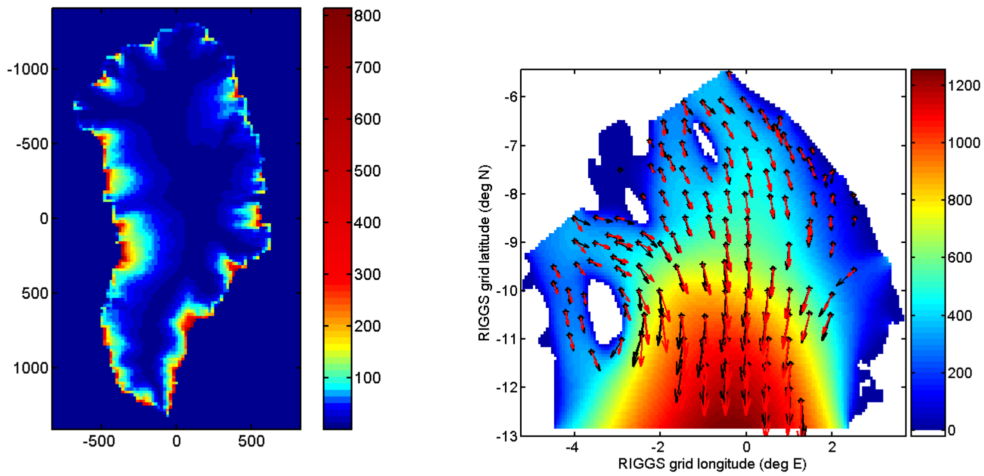

FIGURE 2. Color shows PISM-modeled flow speed. *Left*: The Greenland ice sheet, based on EISMINT-Greenland data. *Right*: The Ross ice shelf, based on EISMINT-Ross (MacAyeal and others, 1996) data, and compared to RIGGS observations (Bentley, 1984); red arrows = modeled, black = observed.

surface, by inverse modeling. This proposal emphasizes inverse modeling as a tool to initialize predictive models, not its direct use for understanding glaciers and ice sheets.

The temperature within an ice sheet is an integrated form of the paleoclimate record and of previous ice sheet geometries, according to the conservation of energy model. Ice sheet models attempt to reconstruct this temperature field by "spinning up" through long paleoglacial runs. We do this when applying PISM to the EISMINT-Greenland experiment, for example (Bueler and others, 2008). But this "spin up" process is itself a form of inverse modeling, necessary because the current temperature field within the ice is not generally observable, so indirect paleo records (ice and seabed cores) and boundary conditions must be used instead.

A complementary approach for basal boundary conditions is provided by the theory of inverse problems. Control theory has been used (MacAyeal, 1993; Joughin and others, 2004) to solve for basal shear stresses in shallow models. Thorsteinsson and others (2003) used bed-to-surface transfer functions to determine bed characteristics (profile and slipperiness) for a linear, non-shallow inverse problem. Steps have been taken to generalize this method to nonlinear, non-shallow problems (Raymond and Gudmundsson, 2005). We propose to use the results of such improved inverse methods to generate superior initial data for PISM.

Co-investigators Maxwell and Truffer, who are also supported by NSF grant ARC 0724860 for the 2007–2010 period to do inverse modeling in glaciology, have addressed the ill-posed problem of inferring basal fields from overly specified data at the surface using a full Stokes model as the equation of stress balance (Maxwell and others, 2008+). This technique assumes a known interior temperature field. It used surface velocities and shear stresses to infer velocities and stresses at the base. Application of the method to compute longitudinal flow through a two-dimensional cross-section of Athabasca glacier was validated with measurements of velocity and internal deformation at depth (Raymond, 1971). Our near term objective is to extend this method to three–dimensional domains with the Blatter (1995) dynamical model, equation (3) below.

The method used by Maxwell and others (2008+) does not assume a particular boundary condition relating basal velocities and stresses. This approach has the advantage of making a minimal number of hypotheses. However, forward modeling already requires a basal sliding law like equation (2), and inclusion of such a law into an inverse technique can lead to sharper results. We will investigate how to do this in a principled manner. For instance, for a power law till described by equation (2), can practical inverse schemes be found to identify the current value of the till friction angle parameter? The inverse method will be evaluated in part by how the resulting prognostic model performs when incorporating the inverse-modeling-derived parameters

One type of available observation of the inside of an ice sheet flow are the isochrones from ice penetrating radar (analyzed in Hindmarsh and others, 2006, for example). Isochrones are derived from ice penetrating radar like the Polar Radar for Ice Sheet Measurements (`http://ku-prism.org/`). Techniques for assimilating these will be examined and implemented if they provide additional information.

Our particular objective is to identify the best techniques for coupling inverse modeling computation with glaciologically credible sliding relations. We will thereby reduce modeling uncertainty related to difficult-to-observe fields at the bases of ice sheets by fully exploiting satellite and airborne observations. In particular, the ice surface velocities are derived from NASA satellite observations, for example from the Modified Antarctic Mapping Mission (Jezek and others, 2003).

4.2. **Improve ice flow dynamical models.** We will implement the Blatter stress balance model (Blatter, 1995; Pattyn, 2003) in PISM, while maintaining optional use of the vertically-integrated equations described in the prior work. This stress balance model is the equation

$$\frac{\partial}{\partial x_1}\left[2\nu\left(2\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}\right)\right]+\frac{\partial}{\partial x_2}\left[\nu\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\right]+\frac{\partial}{\partial z}\left(\nu\frac{\partial u_1}{\partial z}\right)=\rho g\frac{\partial h}{\partial x_1}, \tag{3}$$

and a second very similar equation involving the strain rate $\partial u_2/\partial z$ (Blatter, 1995). Here the notation is the same as in the SSA model (1) in the prior work section. The experience of implementing (1) in parallel is directly applicable to the parallel implementation of this less-shallow model. That is because equations (3) essentially replace the basal shear stress term τ_b in equations (1) with a vertical shear gradient term (involving derivatives with respect to z).

Very recent theoretical results suggest that the Blatter model both generalizes the shallower stress balances already in PISM (Schoof and Hindmarsh, 2008+), and gives a well-posed model with a variety of sliding laws including plastic and power law till models (Schoof, 2008+). Furthermore, the isochrones and streamlines predicted by the Blatter model are known to be excellent approximations of the full Stokes result when that is known (Hindmarsh and others, 2006). In other words, we have both practical experience with implementing equations like (3), and a reasonable expectation that solving these equations will yield effective models with sliding.

We will use exact solutions for verification of our approximation of (3). In fact such an exact solution already appears, for the flowline case, in (Glowinski and Rappaz, 2003), though we will explore other possibilities for effective verification.

It is essential to note the important differences between relatively shallow ice streams like the NE Greenland ice stream or the Siple coast ice streams, and outlet glaciers with constrained to narrow channels like Jakobshavns Isbrae and the Pine Island Glacier (Truffer and Echelmeyer, 2003). (Here "shallow" simply refers to a low thickness to width ratio for cross sections of the fast flowing part, not to models.) We will carefully investigate the effectiveness of the Blatter model (3) for such outlet glaciers. Note that the addition of a complete stress balance is not, by itself, sufficient to model outlet glaciers. In addition the basal parameters must be correctly identified, presumably by inverse modeling using rich surface observations, and the modeling must be done at high resolution as noted in the Research Announcement. These later issues can be expected to be easier to resolve using a stress balance of intermediate complexity like equations (3).

In other words we will carefully remove the flaws of the shallow ice approximation (SIA) while maintaining performance (in the sense of high resolution, in particular). Note that a concrete step in this direction has been described as "stress balance 3" in the prior work (c.f. Bueler and Brown, 2008+).

Ongoing intercomparison participation (c.f. Bueler, 2008+), and comparison of exact and numerical solutions of full Stokes to various shallow approximations in collaboration with J. Brown (note letter of collaboration), will determine precise needs for more or less shallowness in the models. These techniques will also be used to address the flow modeling near the grounding line, and at calving fronts, of marine ice sheets.

An additional planned thermodynamical model improvement is worth noting, namely a reformulation of the conservation of energy model to model enthalpy instead of temperature as the primary variable. This will be done in consultation with the permafrost group at UAF (e.g. Nicolsky and others, 2007).

4.3. **Improve surface mass-/energy-balance model and climate model coupling.** PISM, as do most ice sheet models, employs a positive degree-day approach to model the melt component of the mass balance. Although a gross simplification of the melt physics, this approach is widely used due to low data requirements and generally good performance (Hock, 2003). It is well-suited for multi-millenia model runs.

Since melt is more adequately described by a surface energy balance model (Hock, 2005), and the necessary input data can be obtained from a global or regional climate model, we will develop surface mass balance schemes for PISM that are based on the surface energy balance (Hock, 2005). The near-surface output of relevant variables from climate models will be used to derive parameterizations of the each energy balance component appropriate for ice sheets. Ice-sheet wide albedo simulations will be validated by remote sensing derived albedo. Both approaches, degree-day and energy balance modeling, will be compared and evaluated by comparison to previous efforts to model the recent surface mass balance of the ice sheets (e.g. Bougamont and others, 2007; Box and others, 2006; Fettweis, 2007; Hanna and others, 2005).

A related issue is the details of coupling to regional and global circulation models. For example, we foresee the requirement of near machine precision mass and energy conservation as a concern in coupled runs, and this will be resolved by analysis of the existing methods and additional post-processing of the time-step as needed. Also, since the large data input required for energy balance modeling is only available for much shorter time-scales (centuries) a focus of this work will include developing approaches how to spin up the ice sheet model over thousands of years, while the GCM output only is available for hundreds of years. Standards for modeling coupling established by the Community Ice Sheet Model project will be followed when possible (see Lipscomb letter of support).

We will also couple PISM with the Regional Climate Model HIRHAM4 in collaboration with G. Adalgeirsdottir from the Danish Climate Centre. As noted in the letter of support from the Danish Climate Centre, this collaboration is underway. Note HIRHAM4 runs at 25 km grid resolution covering Greenland and surrounding seas driven by the GCM ECHAM5/MPI-OM at the boundaries and coupled to HYCOM ocean model on a 20 km grid. Fully transient simulation covering 1950–2080 are available forced by observations until 2000 and by emission scenario A1B thereafter.

A collaboration to apply PISM to the Antarctic ice sheet is underway with the Potsdam Institute for Climate Impact Research, as described in the letter from A. Levermann.

4.4. **Apply PISM to the Greenland and Antarctic ice sheets.** As described already, application of PISM in the grant period will be primarily through collaboration with climate modeling experts at major centers. The fully coupled ice sheet and climate models described above will be used to make projections of the ice sheet changes on a century time-scale.

We will also simulate the Greenland and Antarctic ice sheets, covering the coming century especially, using PISM "offine" forced by AOGCM output. This approach allows more flexibility to include a range of IPCC climate scenarios, for example. We will produce a suites of results for the same emission scenario and same essential ice-sheet model but with a range of appropriate parameter values. Note that this will require development of adequate downscaling techniques to be able to use the GCM data in a transient mode (e.g. Radic and Hock, 2006).

A planned experiment, by the core UAF investigators, is to consider the effect, especially on the interior ice sheet, of enhanced calving front fluxes in a PISM Greenland model. This can be done by decreasing the sliding resistance or increasing heat flux (conceptually from melt water)into the base of the ice. A longer term goal is a detailed model for ocean/ice sheet interaction at the calving front (c.f. letter of support from D. Holland), but this is not necessary to perform the experiment.

4.5. **Maintain PISM open source usability and performance.** We will work with and support PISM users, existing and new. We will maintain PISM usability and availability as an open source scientific program. We will maintain and improve the PISM documentation. We will work to make PISM installation as smooth as possible.

We will work with ARSC staff (see letter of support) on profiling PISM runs and improving their performance on high performance computers. For example the PI will continue to mentor a undergraduate ARSC Intern each summer, with the typical task of improving performance.

5. Management plan

This plan is organized by dividing responsibilities among the investigators. This division is not intended to be rigid.

PI Bueler will lead and coordinate the work among himself, the other investigators at UAF, and outside collaborators. His main responsibility is to supervise the development of PISM in close collaboration with the research scholar and post-doc. He will also participate in many details of that development. He will assist climate centers using PISM in making modeling and coupling choices, and in preparing PISM for IPCC runs (when PISM is coupled with climate models). He will make sure that a stable open source copy of PISM together with extensive documentation is maintained. Together with the co-investigators he will decide on the direction of model development. First steps will include the extension to a Blatter-type model and the inclusion of an enthalpy formulation for polythermal ice.

Co-I Maxwell is a mathematician who will work closely with Bueler on PISM numerics and in an effort to develop exact solutions for model verification. He will work with

Truffer to develop boundary inverse methods, in particular to derive basal velocity fields to be used for model initiation. The first steps will be to extend existing iterative methods to a Blatter model and eventually to full 3D Stokes flow solutions (on small spatial scales).

Co-I Truffer will lead the inverse problem part of the proposal in close collaboration with Maxwell and Bueler. He will help with compiling gridded data sets to be used in offline (i.e. non-coupled) model runs to be performed at UAF. He will also supervise a graduate student who will likely be in the Geophysics program. Together with the graduate student he will run full 3D Stokes finite element models on small spatial scales to be compared to various approximations used in PISM.

Co-I Hock is an expert on surface energy balances, and first steps of the proposed work include improving PISM's surface mass-/energy-balance model. She will compile data sets to be used in offline model runs at UAF. She will also be the primary contact for external collaborators who will be coupling PISM to existing climate models, for issues other than ice dynamics per se.

6. Relevance to NASA MAP program goals

Our proposal addresses NASA's Strategic Subgoal 3A of studying the Earth from space to advance scientific understanding and societal needs. At UAF we propose to use observations from IceSat, GRACE (Velicogna and Wahr, 2006), and other satellites, as well as airborne observations from the Polar Radar for Ice Sheet Measurements (`ku-prism.org`), for example, as gridded forms of these data become available to the whole ice sheet modeling communities. These data will be used for several purposes described in the Approach section: as the input to inverse models, for initializing prognostic ice sheet simulations, and for evaluating the quality of (validating) ice sheet model results. Moreover, advanced development of PISM as an open source program will give many researchers a high-resolution ice sheet model capable of exploiting high resolution space-based observations of the state of the ice sheets. An effective ice sheet model with fast flow, coupled to other climate model components, is part of the answer to each of the Science Questions in Strategic Subgoal 3A. Indeed, our proposal shares NASA's Research Objective 3A.5 of improving predictive capability for the future evolution of ice sheets within the global climate system.

We are responding to Appendix A.7 (Modeling, Analysis, and Prediction) of the NASA ROSES 2008 Research Announcement. Our proposed work is *precisely* aligned with both Research Themes "1.3.2 Ice Sheet Modeling" and "1.3.4 Advanced Modeling Dynamics and Numerics" in Appendix A.7.

Regarding Theme 1.3.2, the proposed work is the development and application of an existing, powerful, documented, open source ice sheet model. The Research Announcement describes a less mature possiblity for ice sheet modeling, which is to say the potential application of variable resolution (presumably finite element) modeling without any simplifying assumptions to three-dimensional temperature-dependent modeling. PISM takes a complementary, and more conservative and proven, approach, which is to approximate the continuum model on a regular, rectangular grid and then work by massive parallelism to divide a fine grid among many processors. PISM computations on hundreds of processors have been routine in the development of PISM so far (including for the results in Bueler and others, 2007a), but thousands of processors are available on modern supercomputers. (Note that, among other examples, the POP ocean circulation component of the Community Climate System Model (see `www.cgd.ucar.edu/csm/models/ccsm3.0/pop/doc/manual.pdf`) takes a similarly conservative and massively parallel approach. This fact is despite a more mature discipline-wide understanding of the computational fluid dynamics of ocean circulation compared to sliding and flowing ice.)

Regarding Theme 1.3.4, PISM has already been shown to be scale up to problems with more than 200 million degrees of freedom. This is possible because PISM has been built from the ground up with the advanced parallel numerical library PETSc. We will continue the exploitation of high performance computing tools in the grant period. In addition, regarding the same Theme, the PI and others have been focused for the last five years on the verification of numerical ice sheet models (Bueler and others, 2005, 2007b,a). This has already strongly paid off in the reliability and maintainability of PISM, and it will be continued in the grant period.

7. Conclusion

Our goal is to make PISM an IPCC class ice sheet model including glaciologically-supported models for fast flowing ice streams and outlet glaciers. Our model design may be regarded as conservative, applying a continuum model with some shallowness assumptions on a fine but regular grid using existing massively parallel computational resources. The proposers have the expertise to improve the ice dynamics core of PISM, and to make it an effective model in coupling to circulation models. We will apply PISM to the Greenland and Antarctic ice sheets through already started collaborations with major climate modeling centers and groups, as described in the letters of support.

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