

## Homework Assignment #3

Instructor: Yizhou Sun

TA: Zeyu Li

## Homework Policy:

- Read the *Homework Submission Guidance* carefully before you start working on the assignment, and before you make a submission.
- This is an individual homework. Please **DO NOT** collaborate with others.
- Write your answers in a **pdf** (typeset by LaTeX) and submit it to Gradescope.

## Problem 1: pLSA Initialization

In the iterative pLSA [2] training algorithm, word distribution vectors  $\{\beta_k\}_{k=1}^K$  and topic distribution vectors  $\{\theta_d\}_{d=1}^M$ , where  $K$  denotes the number of topics and  $M$  denotes the number of documents, need to be initialized.

1. Is it a good initialization to set  $\theta_d$ 's and  $\beta_k$ 's as uniform distribution, i.e.,  $\theta_{dk} = \frac{1}{K}$  for every  $d$  and  $k$ , and  $\beta_{kw} = \frac{1}{N}$  for every  $k$  and  $w$ , where  $N$  is the total number of words in the dictionary? Why?
2. Can you give another example of bad initialization?

## Problem 2: Multinomial Naïve Bayes with Dirichlet Prior

In LDA [1], Dirichlet priors can be added to topic distributions and word distributions. Similarly, Dirichlet priors can be added to word distribution vectors  $\beta_k$ 's in multinomial naïve Bayes model, i.e.,  $\beta_k|\alpha \sim \text{Dir}(\alpha)$ , where  $\alpha$  is the parameter vector associated with the Dirichlet distribution. We use  $\mathbf{x}_d$  to denote the bag-of-words vector in document  $d$  and  $y_d$  to denote the latent topic for document  $d$ .

1. Please write down the joint distribution  $p(\mathbf{x}_d, y_d, \beta|\alpha)$ ;
2. Please write down the inference procedure, i.e., find  $y^* = \arg \max p(\mathbf{x}_d, y)$ ;
3. Please write down the posterior distribution for  $\beta$ , i.e.,  $p(\beta_k|D, \alpha)$ , where  $D = \{(\mathbf{x}_d, y_d)\}_{d=1}^M$  is the labeled document dataset with  $M$  documents. Compute the posterior mean for  $\beta$ , i.e.,  $\mathbb{E}(\beta_k|D, \alpha)$ . If  $\alpha = (1, 1, \dots, 1)$ , i.e., an all one vector with dimensionality  $N$ , where  $N$  denotes the number of words in the vocabulary, what is the posterior mean and what is the connection between it and add-1 smoothing?

(Hints: the integral of density function  $p(\beta_k|\alpha)$  over  $\beta_k$  equals to 1.)

## Problem 3: Word Embedding

Word2Vec [3] is trained based on local context window. Suppose we aggregate all the co-occurrence information between words based on local context window as done in GloVe [4], where  $X_{ij}$  is denoted as the counts that  $w_j$  has appeared in  $w_i$ 's context.

1. Is  $X_{ij}$  symmetric, i.e.,  $\forall i, j, X_{ij} = X_{ji}$ ? Why?
2. What would be the form of original objective function for skip-gram using  $X_{ij}$ ?
3. Suppose we fix negative samples before training for negative sampling, and obtain  $X_{ij}^+$  and  $X_{ij}^-$ , which denote the number of times  $w_j$  appears in  $w_i$ 's local contexts and the number of times  $w_j$  appears in  $w_i$ 's negative samples. What would be the form of negative sampling based objective function for skip-gram using  $X_{ij}^+$  and  $X_{ij}^-$ ?

## References

- [1] D. M. Blei, A. Y. Ng, and M. I. Jordan. Latent dirichlet allocation. *Journal of machine Learning research*, 3(Jan):993–1022, 2003.
- [2] T. Hofmann. Probabilistic latent semantic analysis. In *Proceedings of the Fifteenth conference on Uncertainty in artificial intelligence*, pages 289–296. Morgan Kaufmann Publishers Inc., 1999.
- [3] T. Mikolov, K. Chen, G. Corrado, and J. Dean. Efficient estimation of word representations in vector space. *arXiv preprint arXiv:1301.3781*, 2013.
- [4] J. Pennington, R. Socher, and C. Manning. Glove: Global vectors for word representation. In *Proceedings of the 2014 conference on empirical methods in natural language processing (EMNLP)*, pages 1532–1543, 2014.