Computer1 Homework01

Junho Lee, Yonsei University

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We need to solve

$$\frac{d^2\zeta}{dt^2} = \frac{1}{\zeta^3} - \frac{1}{\zeta^2} \tag{1}$$

with intial condition

$$\begin{aligned} \zeta(t_0) &= 0.9 \\ \zeta'(t_0) &= 0 \end{aligned} \tag{2}$$

Let domain D of $\zeta(t)$ be $[t_0, t_1]$.
To solve equation (1), I divide the domain D into n pieces.

$$x_i = (t_1 - t_0)\frac{i}{n} + t_0 \tag{3}$$

for $0 \leq i \leq n$.
Now define

$$\zeta_i \coloneqq \zeta(x_i) \tag{4}$$

Then we can approximate ζ_i'' as following

$$\zeta_i'' \approx \frac{\zeta_{i+1} - 2\zeta_i + \zeta_{i-1}}{h^2} \tag{5}$$

, where $h = (t_1 - t_0)/n$.
Substitude (5) to (1), then

$$\frac{\zeta_{i+1} - 2\zeta_i + \zeta_{i-1}}{h^2} = \frac{1}{\zeta_i^3} - \frac{1}{\zeta_i^2} \tag{6}$$

for $1 \leq i \leq n-1$.
To evaluate ζ_i, I modify (6) as

$$\zeta_i = \frac{h^2}{\zeta_{i-1}^3} - \frac{h^2}{\zeta_{i-1}^2} + 2\zeta_{i-1} - \zeta_{i-2} \tag{7}$$

, for $2 \leq i \leq n$.

ζ_0 and ζ_1 are needed to solve recurrence equation (7). ζ_0 is explicity given as 0.9, but ζ_1 is not.

To find ζ_1, I use following approximation.

$$\zeta_0' \approx \frac{\zeta_1 - \zeta_0}{h} \tag{8}$$

By (8), ζ_1 is

$$\zeta_1 = h\zeta_0' + \zeta_0 \tag{9}$$

Now we can solve recurrence equation (7) to numerically approximate the solution of (1).