A MODEL FOR NON-LINEAR IMBEDDING OF GRAPHICAL DATA

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In this problem, we start with an adjacency matrix X. The i, j entry of X is 1 if node i shares a directed edge with node j and 0 if it does not. An undirected graph being equal to it's transpose.

We view the adjacency matrix X as a random realization of a low rank non-linear model. In other words, for each node there is a position vector $Z_i = (z_{i1}, ..., z_{ik})$, where z_{ij} represents a position and an intensity score η_i that represents a propensity to talk with neighbors.

In the case of a undirected graph, we will model the probability of an edge between nodes i and j as

$$P(X_{ij} = 1|Z_i, Z_j) = exp(-(Z_i, Z_j))^{-\eta_i - \eta_j}$$

where d is some divergence (or distance) measure we decide[1].

For now, we will consider optimizing this problem using stochastic gradient descent. This will be done by sampling pairs of indices in which one pair contains an edge and the other does not.

[1]It might seem natural to just consider distance metrics, not divergence. However, I believe there maybe a reasonable argument for looking at metrics that do not follow the triangle inequality, so I'm leaving this idea open for now