Analysis of Algorithms

Final Exam : 12 June 2000

1. Design a divide-and-conquer algorithm to find the minimum and maximum elements in a set of n numbers. Prove that your algorithm uses at most $3n/2$ comparisons (for $n = 2^k$).

2. The input is a set S of n real numbers, and a real number x. Design an algorithm to determine whether there are two elements of S whose sum is exactly x. Your algorithm should run in $O(n \log n)$ time.

3. Describe the Floyd-Warshall algorithm for the all-pairs shortest-paths problem. Describe how the output of the Floyd-Warshall algorithm can be used to detect the presence of a negative-weight cycle.

4. Given a flow network $G = (V, E)$, let f_1 and f_2 be functions from $V \times V$ to R. The flow sum $f_1 + f_2$ is the function from $V \times V$ to R defined by

$$(f_1 + f_2)(u, v) = f_1(u, v) + f_2(u, v)$$

   for all $u, v \in V$. If f_1 and f_2 are flows in G, which of the three flow properties must the flow sum $f_1 + f_2$ satisfy, and which might it violate?

5. Show heights, excesses, and flow after each discharge in the execution of Lift-To-Front for the following network. Assume that initially $L = (v_1, v_2)$, and the neighbor lists are $N[v_1] = (s, v_2, t)$ and $N[v_2] = (s, v_1, t)$.