Analysis of Algorithms

Final Exam : 13 June 2002

1. Consider the 0-1 knapsack problem where $v_i = w_i$ (i.e., Given n items with integer weights $w_1, \ldots, w_n$ and an integer W, find the maximum load that is at most W). Give a dynamic-programming solution to the 0-1 knapsack problem that runs in $O(nW)$.

2. Give a sequence of m Make-Set, Union, and Find operations, n of which are Make-Set operations, that takes $\Omega(m \log n)$ time when we use union-by-rank only.

3. Show the discovery time and finishing time for each vertex and the depth-first forest of depth-first search of the following graph when vertices are considered in their increasing order.

4. Suppose that we run Johnson's algorithm on a directed graph G with weight function w. Show that if G contains a 0-weight cycle c, then $\widehat{w}(u, v) = 0$ for every edge (u, v) in c.

5. Construct the residual network for the following graph and flow, and find an augmenting path in the residual network.

6. Describe how to find a maximum matching in a bipartite graph by using the Ford-Fulkerson method. What is the time complexity of this algorithm?