Analysis of Algorithms
## Midterm Exam: 24 October 2006

1. Prove the following.

   a. $(n-1)^2 = O(n^2)$

   b. $f(n) = O(g(n))$ and $g(n) = O(h(n))$ imply $f(n) = O(h(n))$

2. Illustrate the decision tree for insertion sort on four elements.

3. Given $A[1..n]$ and k where $k \ll n$, the partial sort problem is to return the k smallest numbers in order from the given n numbers. Find an efficient algorithm for the partial sort problem. Explain the time complexity of your algorithm.

4. Consider the following red-black tree.
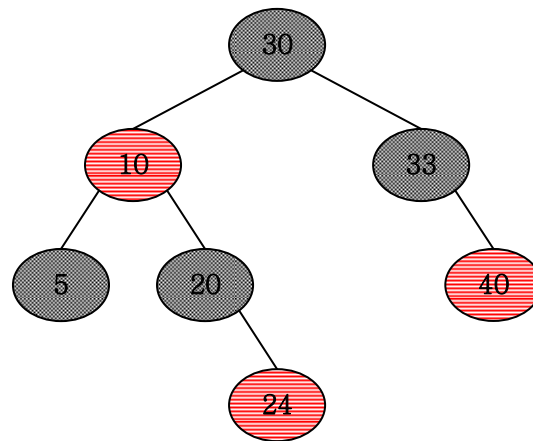
   a. Show the red-black tree after inserting 15.

   b. Show the red-black tree after inserting 12.

5. Explain the four steps of developing a dynamic programming algorithm by using the matrix chain multiplication as an example.

6. Let $G = (V, E)$ be a weighted, directed graph with $V = \{1, ..., n\}$ and edge weight $w(i, j)$. Assume that $w(i, j) = 0$. Given $u, v \in V$, we want to find the shortest path from u to v. Find a dynamic programming algorithm for this problem. (Write down your dynamic programming recurrence and the time complexity of your algorithm.)