Analysis of Algorithms

Exam 1 : 17 April 2000

1. Give tight asymptotic bounds (i.e., Θ) for $T(n)$ in each of the following recurrences. Justify your solutions by naming the particular case of the master theorem, or by using the iteration method or the substitution method.

   (a) $T(n) = T(n/2) + T(n/3) + n$.

   (b) $T(n) = 9T(n/3) + n^3$.

2. Give an $O(n \log k)$-time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists.

3. Give a linear-time algorithm for the following problem: Given a sequence of n integers, find the majority in the sequence or determine that none exists. (A partial credit will be given for an algorithm of $\omega(n)$.)

4. Illustrate the operation of *Counting-Sort* on the array $A = (4, 7, 1, 5, 2, 2, 5, 2)$.

5. Show the red-black trees that result after successively inserting the keys 30, 20, 25, 10, 5 into an initially empty tree and then the red-black tree that results from deleting 30.