

Analysis of Algorithms

Final Exam : 9 June 2004

1. Prove that any B-tree with n keys, height h , and minimum degree t satisfies

$$h \leq \log_t \frac{n+1}{2}.$$

2. Give an example of a directed graph with negative-weight edges for which Dijkstra's algorithm produces incorrect answers.
3. Let $l_{ij}^{(m)}$ be the minimum weight of any path from vertex i to vertex j that contains at most m edges. We can compute matrix $L^{(m)}$ by multiplying the input matrix W . Compute $L^{(2)}$ and $L^{(3)}$ for the following input.
4. The Floyd-Warshall algorithm requires $\Theta(n^3)$ space, since we compute $d_{ij}^{(k)}$ for $1 \leq i, j, k \leq n$. Show that the following procedure, which simply drops all the superscripts, is correct, and thus only $\Theta(n^2)$ space is required.

```
D ← W;  
for k ← 1 to n do  
    for i ← 1 to n do  
        for j ← 1 to n do  
            dij ← min(dij, dik + dkj);  
return D;
```

5. The *edge connectivity* between vertices u and v in an undirected graph G is the minimum number of edges that must be removed to disconnect u and v . Show how the edge connectivity between two vertices can be solved by running a maximum flow algorithm.
6. Write the definitions of the hamiltonian cycle problem and the traveling salesman problem. Prove that the traveling salesman problem is NP-complete.