Analysis of Algorithms

Midterm Exam : 3 May 2004

1. Give asymptotically tight bounds for $T(n)$ in each of the following recurrences. Justify your solution by naming the particular case of the master theorem or by using some other method.

   (a) $T(n) = T(n/3) + T(n/4) + n$

   (b) $T(n) = 3T(n/2) + n$

2. Illustrate the operations of `Counting-Sort` on an array $A = (7, 2, 4, 8, 4, 4, 7, 5)$.

3. Show the red-black trees that result after successively inserting keys 55, 30, 47, 25, 11, 18 into an initially empty tree.

4. Explain the four steps of developing a dynamic programming algorithm by using the longest common subsequence problem as an example.

5. Determine a longest common subsequence of `abbababa` and `babaabaa`.

6. Give a dynamic programming solution to the 0-1 knapsack problem that runs in $O(nW)$ time, where n is the number of items (with weights $w_1, \ldots, w_n$ and worth $v_1, \ldots, v_n$) and W is the maximum weight that the thief can put in his knapsack. All values are integers.