Discrete Mathematics

Homework#5

Due date : May 7, 2018

Section 5.1)

12. Prove that

$$\sum_{j=0}^{n} \left(-\frac{1}{2}\right)^j = \frac{2^{n+1} + (-1)^n}{3 \cdot 2^n}$$

whenever n is a nonnegative integer.

Use mathematical induction to prove the inequalities in Exercises 18–30.

18. Let $P(n)$ be the statement that $n! < n^n$, where n is an integer greater than 1.

a) What is the statement $P(2)$?

b) Show that $P(2)$ is true, completing the basis step of the proof.

c) What is the inductive hypothesis?

d) What do you need to prove in the inductive step?

e) Complete the inductive step.

f) Explain why these steps show that this inequality is true whenever n is an integer greater than 1.

60. Use mathematical induction to show that $\neg(p_1 \vee p_2 \vee \cdots \vee p_n)$ is equivalent to $\neg p_1 \wedge \neg p_2 \wedge \cdots \wedge \neg p_n$ whenever $p_1, p_2, \ldots, p_n$ are propositions.

Section 5.2)

8. Suppose that a store offers gift certificates in denominations of 25 dollars and 40 dollars. Determine the possible total amounts you can form using these gift certificates. Prove your answer using strong induction.

28. Let b be a fixed integer and j a fixed positive integer. Show that if $P(b), P(b+1), \ldots, P(b+j)$ are true and $[P(b) \land P(b+1) \land \cdots \land P(k)] \rightarrow P(k+1)$ is true for every integer $k \geq b+j$, then $P(n)$ is true for all integers n with $n \geq b$.

Section 5.3)

10. Give a recursive definition of $S_m(n)$, the sum of the integer m and the nonnegative integer n.

24. Give a recursive definition of

a) the set of odd positive integers.

b) the set of positive integer powers of 3.

c) the set of polynomials with integer coefficients.

Section 5.4)

8. Give a recursive algorithm for finding the sum of the first n positive integers.

16. Prove that the recursive algorithm for finding the sum of the first n positive integers you found in Exercise 8 is correct.