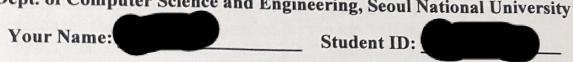
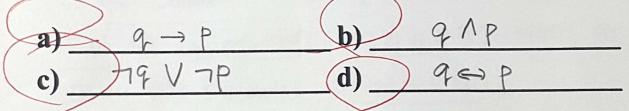
## [4190.101] Discrete Mathematics Midterm Exam 1 (2016 Spring)

Dept. of Computer Science and Engineering, Seoul National University

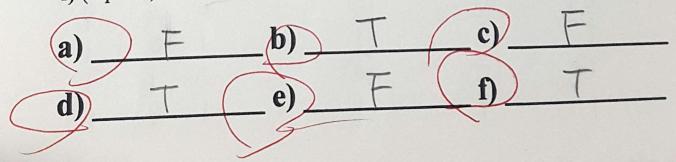




- 1. [4 points] Let p be the proposition "I will do every exercise in this book" and q be the proposition "I will get an "A" in this course." Express each of these as a combination of p and q.
  - a) (1 point) I will get an "A" in this course only if I do every exercise in this book.
  - b) (1 point) I will get an "A" in this course and I will do every exercise in this book.
  - c) (1 point) Either I will not get ah "A" in this course or I will not do every exercise in this book.
  - d) (1 point) For me to get an "A" in this course it is necessary and sufficient that I do every exercise in this book.



- 2. [6 points] Let P(m, n) be the statement "m divides n" where the domain for both variables consists of all positive integers. (By "m divides n" we mean that n = k\*m for some integer k.) Determine the truth values of each of these statements.
  - a) (1 point) P(4, 5)
  - **b)** (1 point) P(2, 4)
  - c) (1 point)  $\forall m \forall n P(m, n)$
  - (1) point) P(2, 4)
  - $\exists n \forall m P(m, n)$ **e**) (1 point)
  - $\forall$ n P(1, n) f) (1 point)



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3. [10 points] With given symbols in below, show that following statements are consistent or not: "If Miranda does not take a course in discrete mathematics, then she will not graduate." "If Miranda does not graduate, then she is not qualified for the job." "If Miranda reads this book, then she is qualified for the job." "Miranda does not take a course in discrete mathematics but she reads this book."

## [Symbols]

- t: Miranda takes a course in discrete mathematics.
- g: Miranda graduates (from her university).
- q: Miranda is qualified for the job.
- r: Miranda reads this book.

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4. [10 points] Show that  $(p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r)$  is a tautology.

|           | P) | 9.50 | 11 th | et truth | n table? | 2216t.<br>(PUq) 1/GpVr) | qVr | (p/q)nGp/r)<br>-> q/r |
|-----------|----|------|-------|----------|----------|-------------------------|-----|-----------------------|
| 0= false  | 0  | 0    | 0     | (0)      | 1        | 0)                      | 0   | 1                     |
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|           | 1  | 0    | 0     | 1        | 6)       | 0                       | (0) | 1                     |
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|           | 1  | 1    | 1     | - 1      | 1        |                         | 1   | 1                     |

PTTOIRE ZIZDAN CHEY Page 2/6 SHISTED HOSTER, OF Tantalogy of Ch.

5. [10 points] Prove that if x is irrational and  $x \ge 0$ , then  $\sqrt{x}$  is irrational. 升音哲学/好艺叶, 又外 Trotina 部2 x20일 CCH, The Pratinal of the of the 2th 2000 of the rational number 2011 THEN 22 Portinal = DCF. (rational number of M BASE ET THE) (THEO IT) THEM IX X JZ = X & total of tet. Halatole Ench essea. Tetaky 27+ Irrafinality 2200/240 DE E Trroffinal State. good!  $\frac{21}{21}$   $\frac{119}{129}$   $\frac{$ 6. [10 points] Prove that 3<sup>n</sup> < n! if n is an integer greater than 6. 华生的 刊出的是 什些时任. 1. 到伤时; 对安架内의证是 794. 39 = 2189 < 7! = 5040 0/EL. 2. 1/5KOIM, 3 < n! 0/240, 3nt/ < (n+1)! 062 \$21th. 3 < n! ole 3x3 < 3xn! ole, < (nH) Xn! ofth (nenolez) TOTALM 3 ntl < (ntl) ! 32 9049 % a. 1,271 thu, 6842 84 non Chank, 30< n! of systa. Page 3/6

- 7. [10 points] We define the Ulam numbers by setting  $U_1 = 1$  and  $U_2 = 2$ .

  Furthermore, after determining whether the integers less than n are Ulam numbers, we set n equal to the next Ulam number if it can be written uniquely as the sum of two different Ulam numbers. Note that  $U_3 = 3$ ,  $U_4$ = 4,  $U_5 = 6$ , and  $U_6 = 8$ .

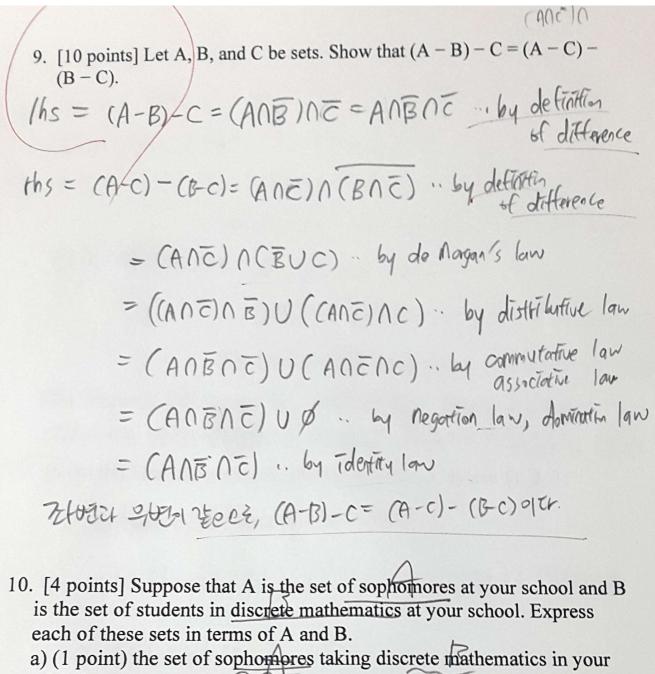
  a) (4 points) Find the 9th Ulam number,  $U_9$ b) (6 points) Prove that there are infinitely many Ulam numbers. (Hint
  - b) (6 points) Prove that there are infinitely many Ulam numbers. (Hint, use contradiction)

a)  $U_9 = 6$ 

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アミチャロロー finite ましい Unm number 21 Set 11を 至かを121 ではなられる (Un < Unique また)

8. [10 points] Can you conclude that A = B if A and B are two sets that have the same power set? Explain why.



school

b) (1 point) the set of sophomores at your school who are not taking discrete mathematics

c) (1 point) the set of students at your school who either are sophomores

or are taking discrete mathematics

d) (1 point) the set of students at your school who either are not sophomores or are not taking discrete mathematics

| (a) | ANB | (b)        | ANB |  |
|-----|-----|------------|-----|--|
| (c) | AUB | <b>d</b> ) | AUB |  |
| P/  |     |            |     |  |

11. [10 points] Prove that if n is an odd integer, then  $[n^2/4] = (n^2 + 3)/4$   $[n^2/4] = [n^2/4] = [n^2 + 3)/4$ 

T) n=4k+1 2 73t, n2=16k2+8k+1 012,

[12/4] = [4/2/2/4] = 4/4] = 4/2/2/4 | 94. 0/2 (12/2)/4 = (16/2/2/4/4)/4 = 4/2/2/4/1 2/2/6.

(i) N=4k+392 134, N=16k2+24k +9012,

Tn2/47 = [4k2+6k+2+1/4] = 4k2+6k+30164.

01/2 (n3+3)/4=(16/2°+24/4+12)/4=4/2°+6/43 21/26/21.

Odd Thegerol71 (CHEN) n= 4k, n= 4k+29 7376 2373121000+5 ELCH.

12. [6 points] Find the symmetric difference of {1, 3, 5} and {1, 2, 3}

Symmetric difference 2/232/9/ torr,

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= {5} U {2} = {2,5}