

Binary Tree Traversals

Data structures

Spring 2017

Binary Tree Traversal

- Many binary tree operations are done by performing a **traversal** of the binary tree.
- In a traversal, each element of the binary tree is **visited** exactly once.
- During the **visit** of an element, all action (make a clone, display, evaluate the operator, etc.) with respect to this element is taken.

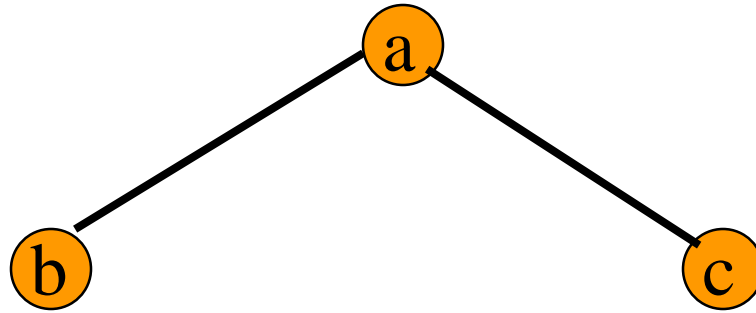
Binary Tree Traversal Methods

- Preorder
- Inorder
- Postorder
- Level order

Preorder Traversal

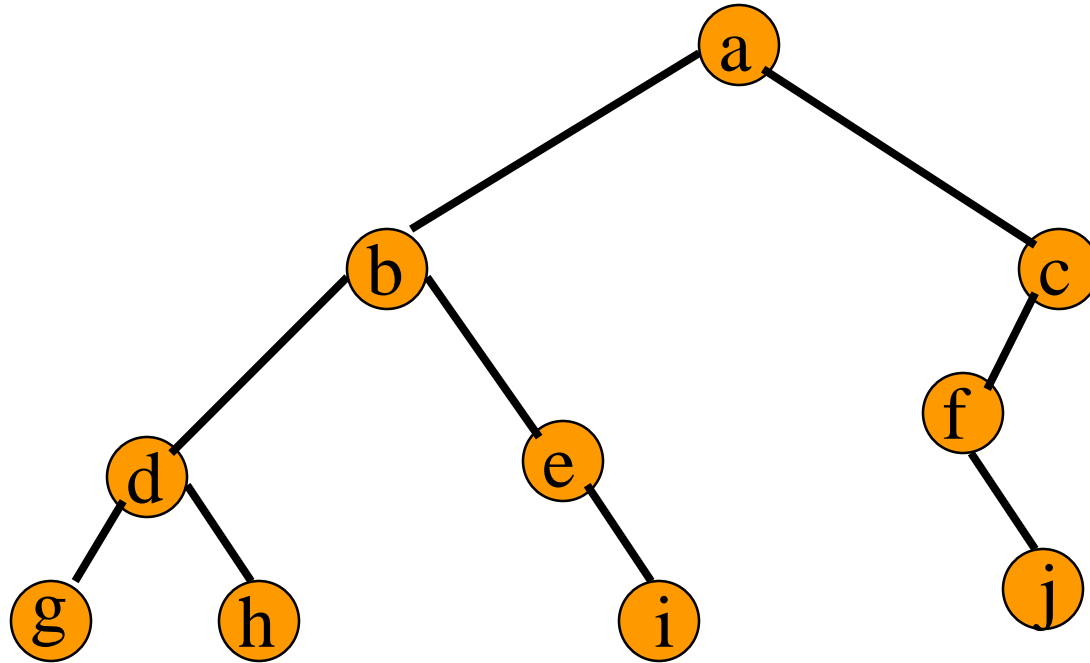
```
public static void preOrder(BinaryTreeNode t)
{
    if (t != null)
    {
        visit(t);
        preOrder(t.leftChild);
        preOrder(t.rightChild);
    }
}
```

Preorder Example (visit = print)



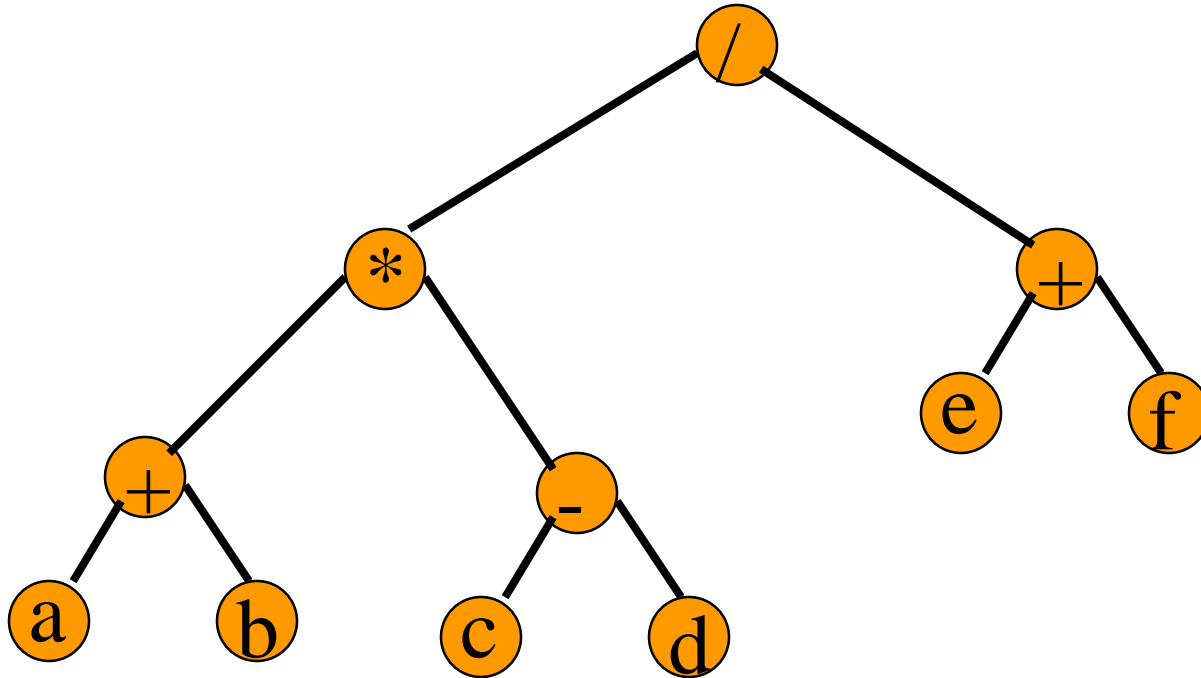
a b c

Preorder Example (visit = print)



a b d g h e i c f j

Preorder Of Expression Tree



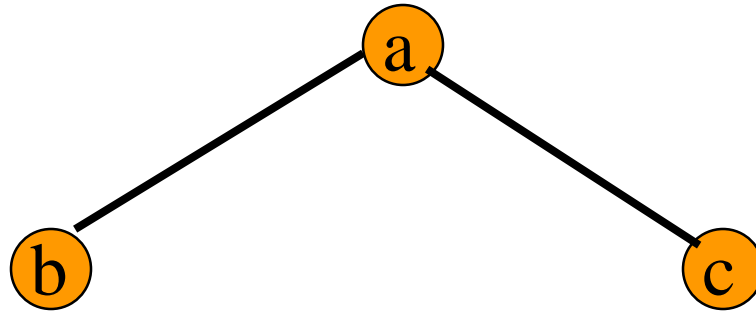
$/ * + a b - c d + e f$

Gives prefix form of expression!

Inorder Traversal

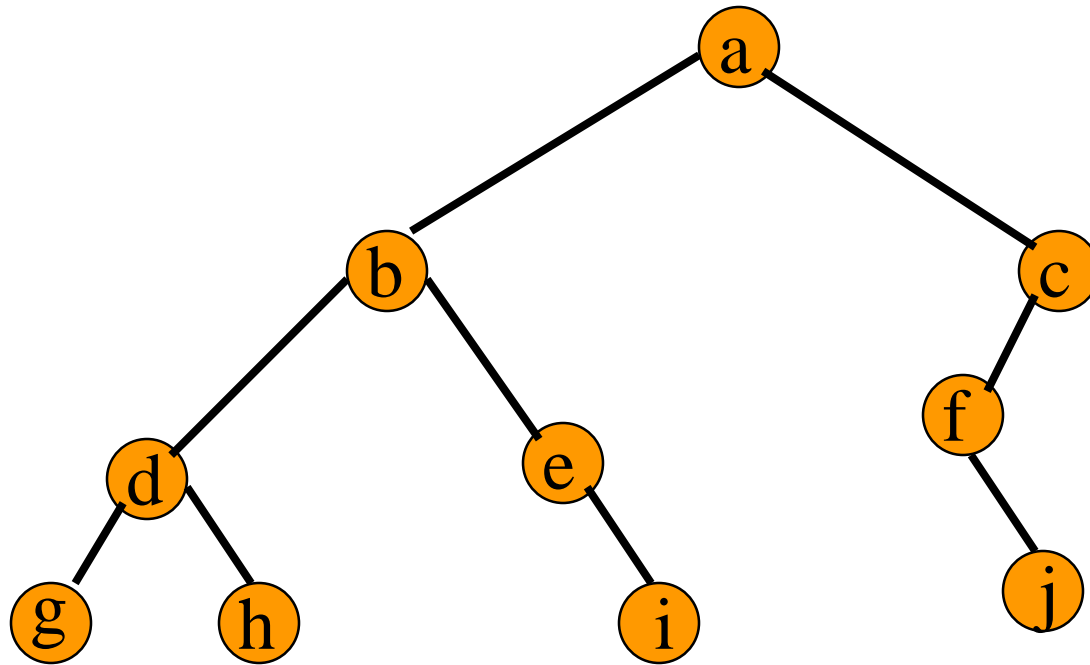
```
public static void inOrder(BinaryTreeNode t)
{
    if (t != null)
    {
        inOrder(t.leftChild);
        visit(t);
        inOrder(t.rightChild);
    }
}
```


Inorder Example (visit = print)



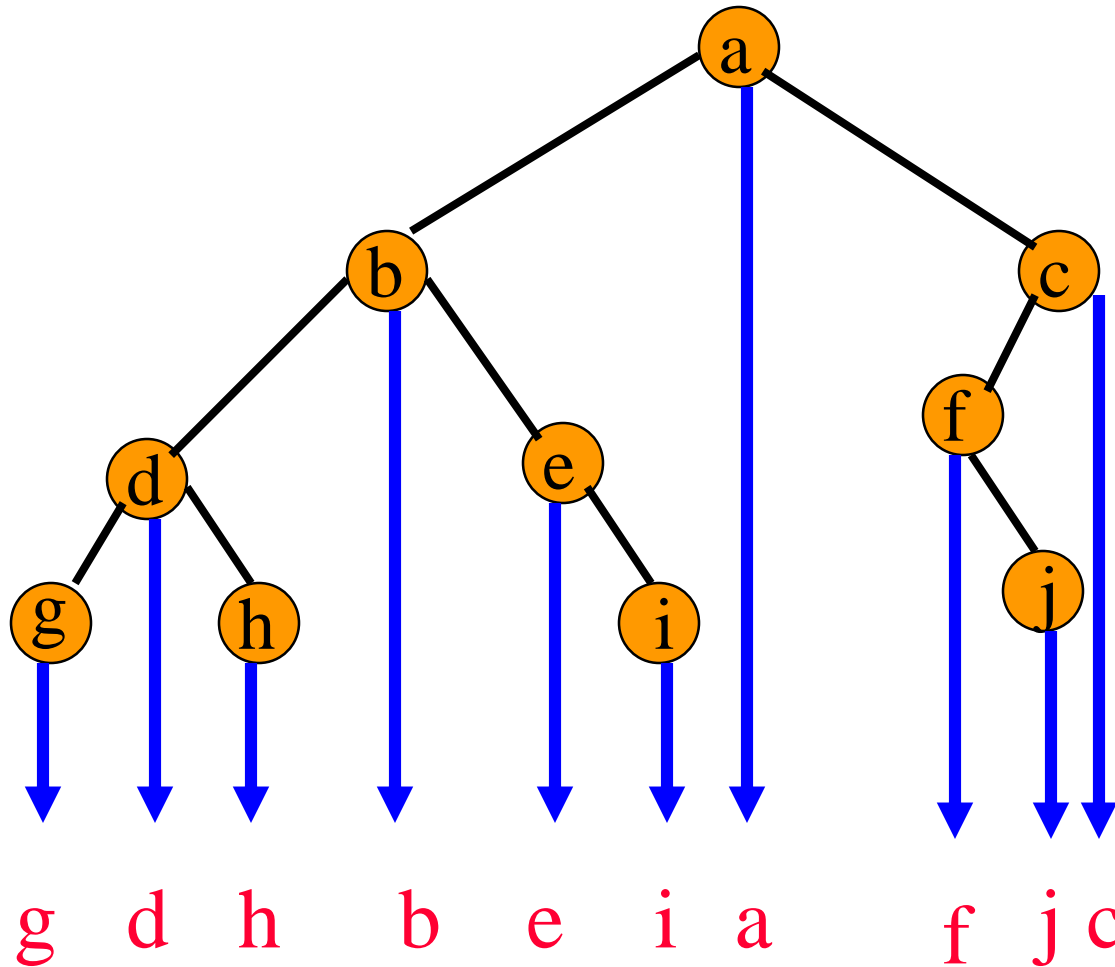
b a c

Inorder Example (visit = print)

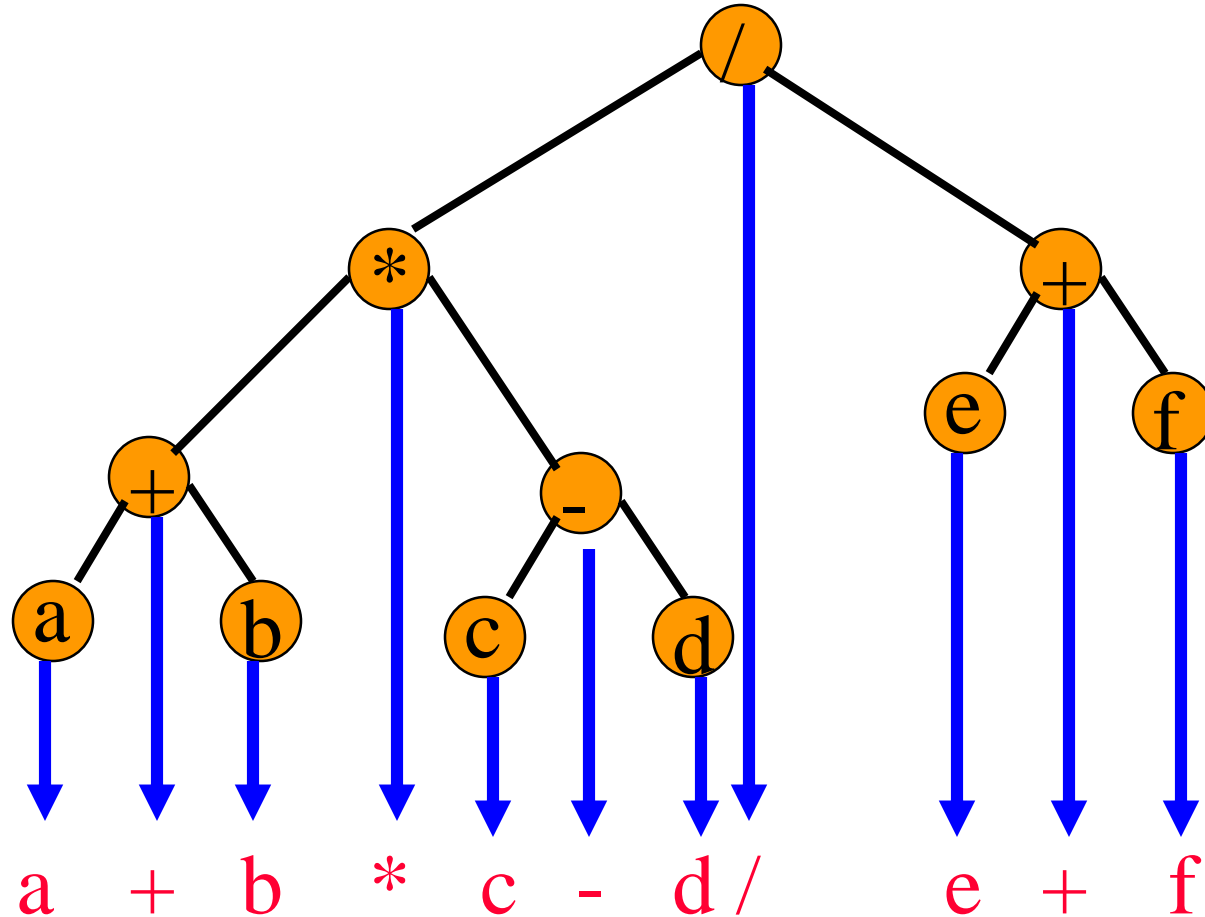


g d h b e i a f j c

Inorder By Projection (Squishing)



Inorder Of Expression Tree

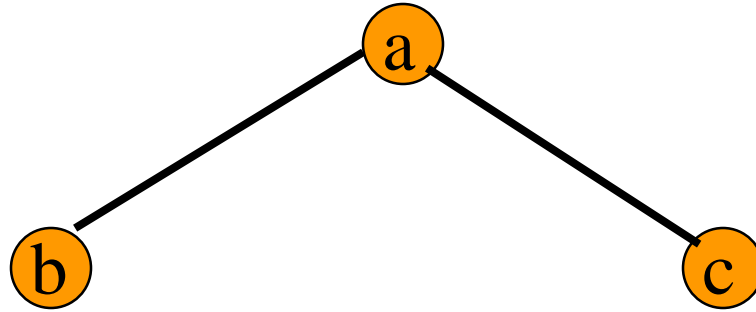


Gives infix form of expression (sans parentheses)!

Postorder Traversal

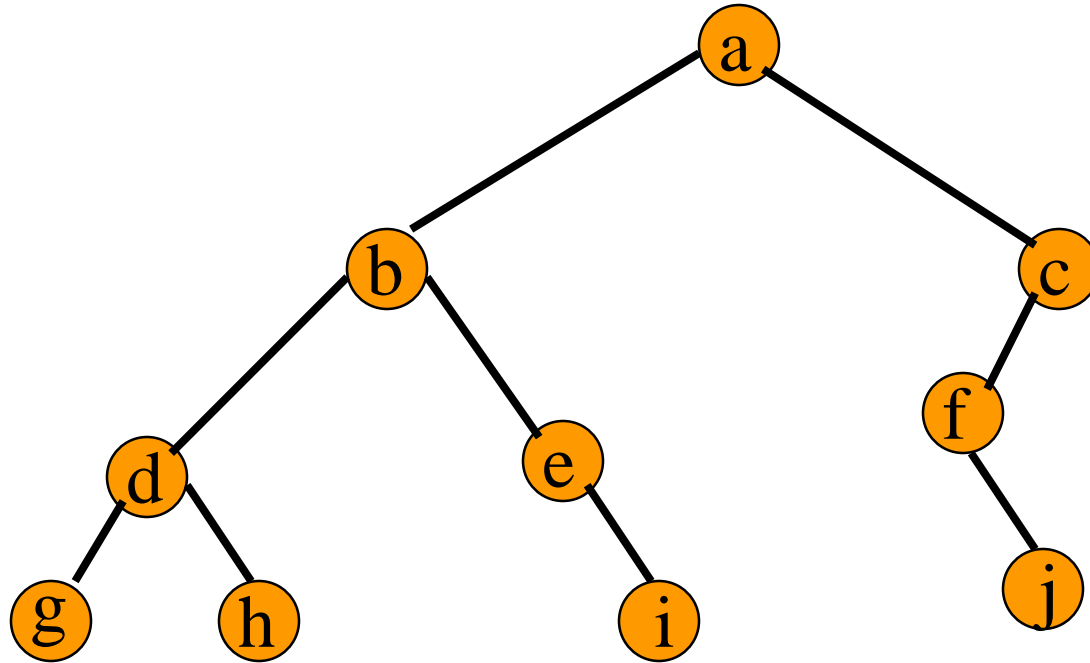
```
public static void postOrder(BinaryTreeNode t)
{
    if (t != null)
    {
        postOrder(t.leftChild);
        postOrder(t.rightChild);
        visit(t);
    }
}
```

Postorder Example (visit = print)



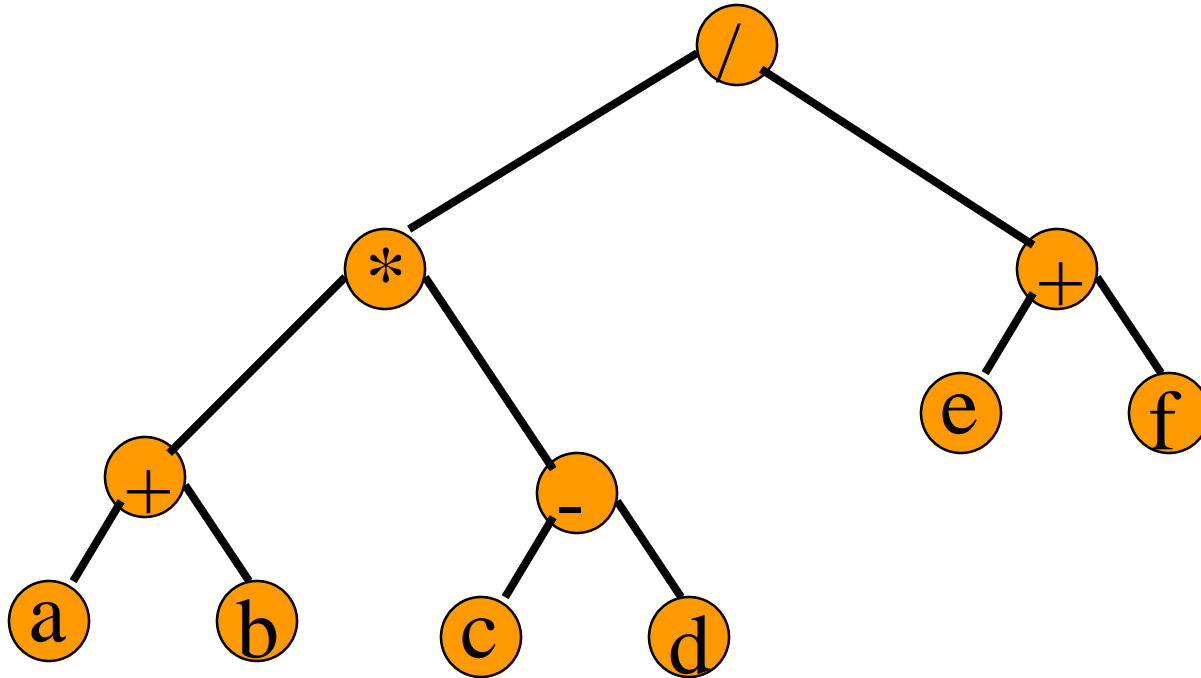
b c a

Postorder Example (visit = print)



g h d i e b j f c a

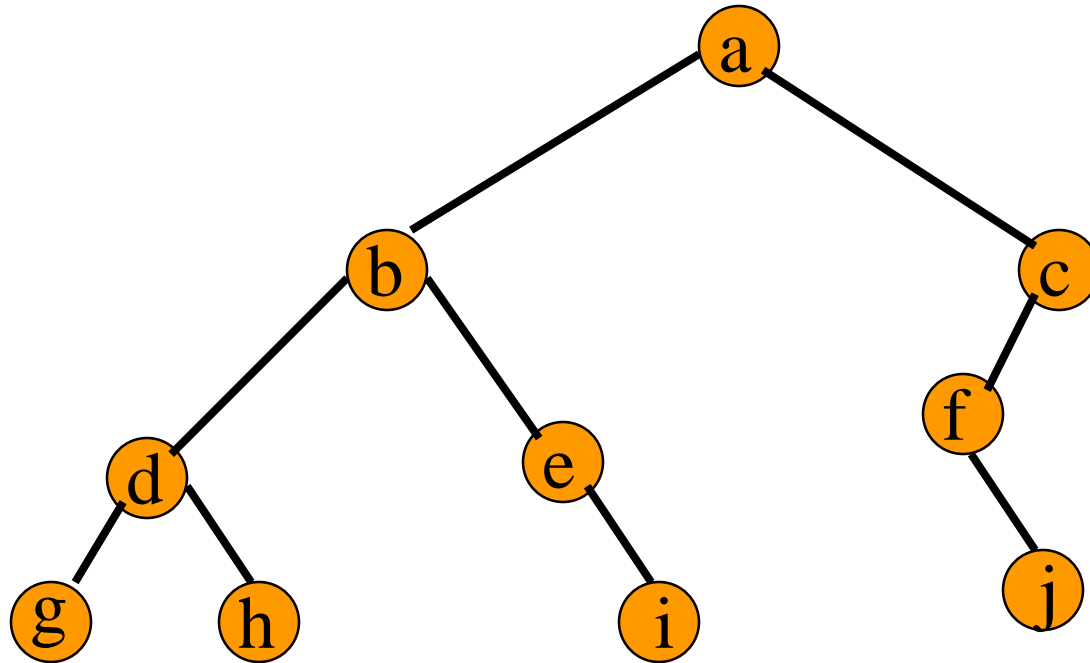
Postorder Of Expression Tree



a b + c d - * e f + /

Gives postfix form of expression!

Traversal Applications



- Make a clone.
- Determine height.
- Determine number of nodes.

Level Order

Let **t** be the tree root.

```
while (t != null)
```

```
{
```

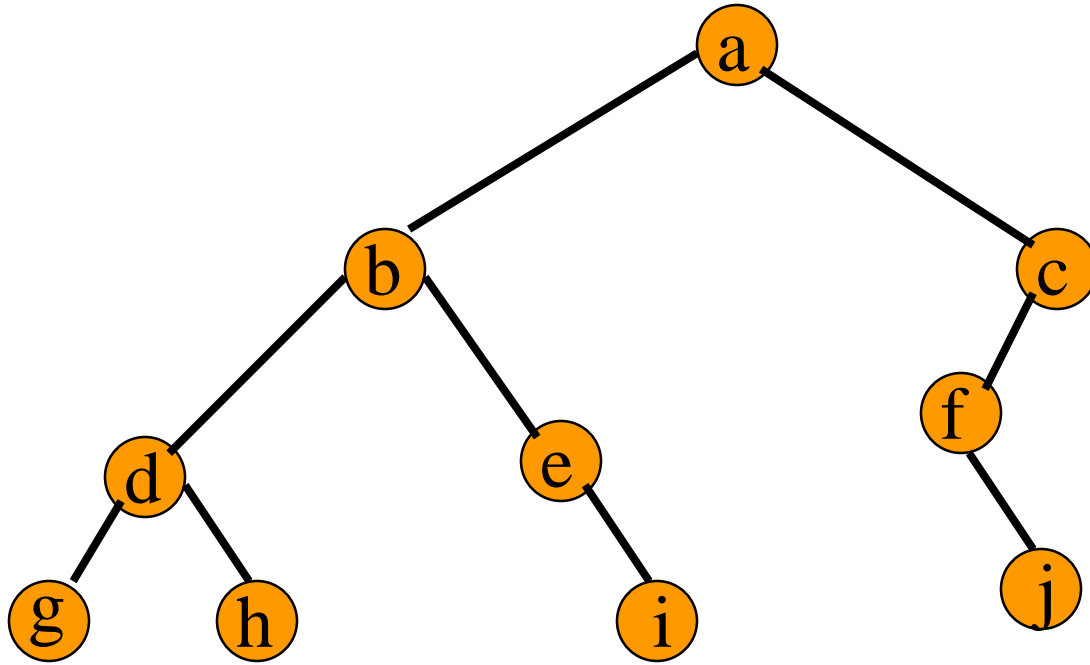
```
    visit t and put its children on a FIFO queue;
```

```
    remove a node from the FIFO queue and  
    call it t;
```

```
    // remove returns null when queue is empty
```

```
}
```

Level-Order Example (visit = print)



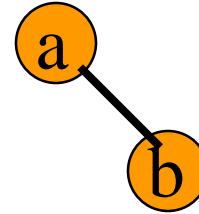
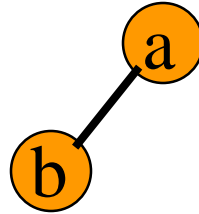
a b c d e f g h i j

Binary Tree Construction

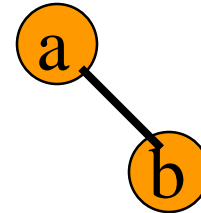
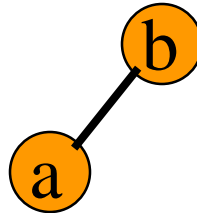
- Suppose that the elements in a binary tree are distinct.
- Can you construct the binary tree from which a given traversal sequence came?
- When a traversal sequence has more than one element, the binary tree is not uniquely defined.
- Therefore, the tree from which the sequence was obtained cannot be reconstructed uniquely.

Some Examples

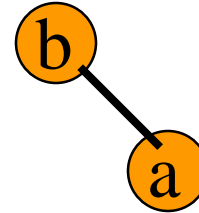
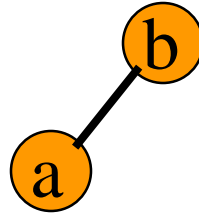
preorder
= **ab**



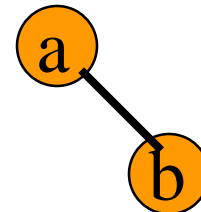
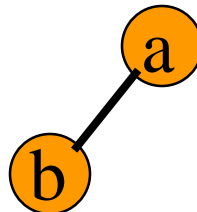
inorder
= **ab**



postorder
= **ab**



level order
= **ab**



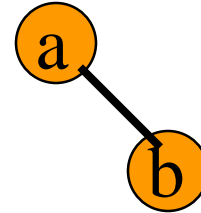
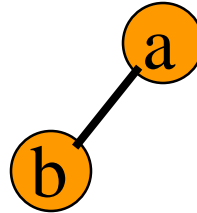
Binary Tree Construction

- Can you construct the binary tree, given two traversal sequences?
- Depends on which two sequences are given.

Preorder And Postorder

preorder = **ab**

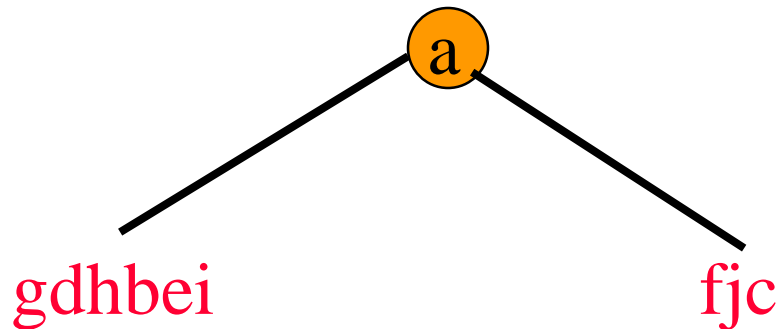
postorder = **ba**



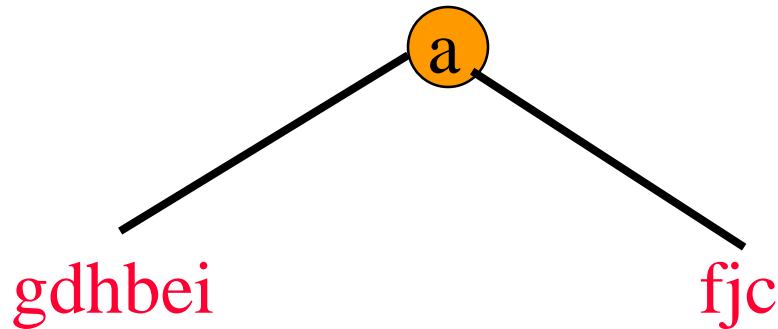
- Preorder and postorder do not uniquely define a binary tree.
- Nor do preorder and level order (same example).
- Nor do postorder and level order (same example).

Inorder And Preorder

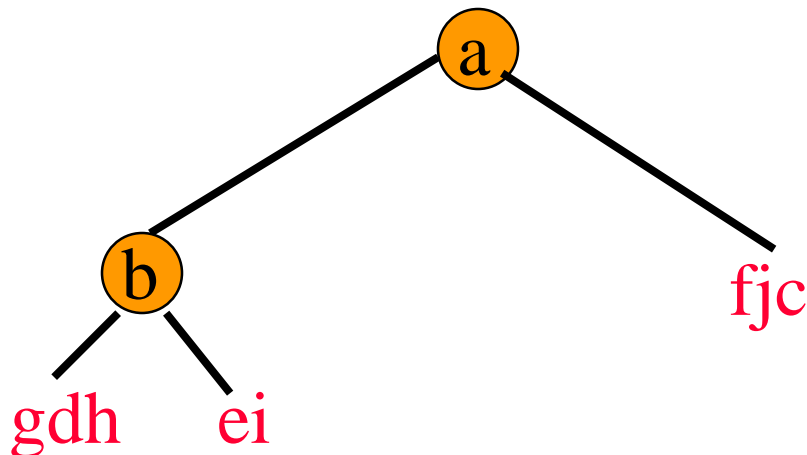
- inorder = g d h b e i a f j c
- preorder = a b d g h e i c f j
- Scan the preorder left to right using the inorder to separate left and right subtrees.
- a is the root of the tree; gdhbei are in the left subtree; fjc are in the right subtree.



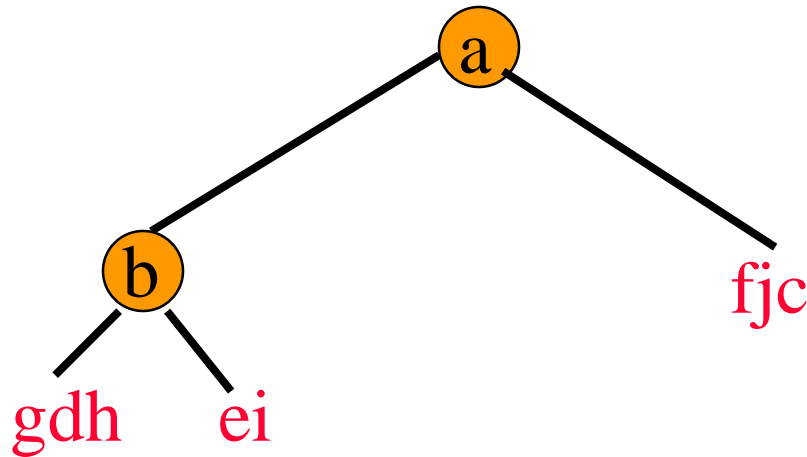
Inorder And Preorder



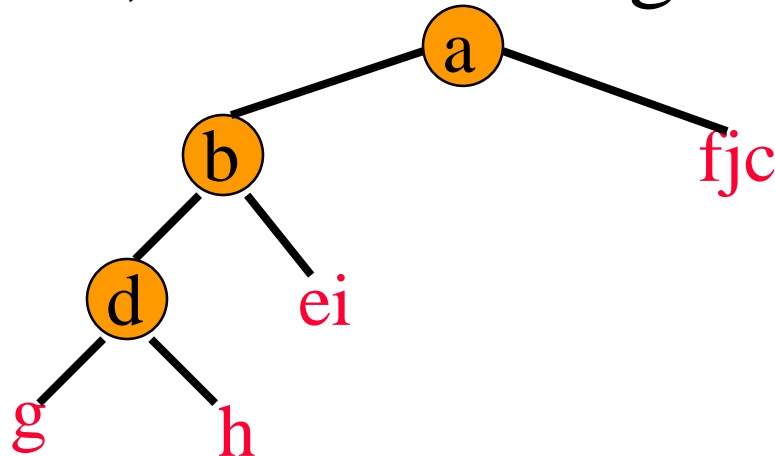
- preorder = a b d g h e i c f j
- **b** is the next root; **gdh** are in the left subtree; **ei** are in the right subtree.



Inorder And Preorder



- preorder = a b d g h e i c f j
- d is the next root; g is in the left subtree; h is in the right subtree.



Inorder And Postorder

- Scan postorder from right to left using inorder to separate left and right subtrees.
- inorder = g d h b e i a f j c
- postorder = g h d i e b j f c a
- Tree root is a; gdhbei are in left subtree; fjc are in right subtree.

Inorder And Level Order

- Scan level order from left to right using inorder to separate left and right subtrees.
- inorder = g d h b e i a f j c
- level order = a b c d e f g h i j
- Tree root is a; gdhbei are in left subtree; fjc are in right subtree.