V-code: ???

First-Order Transients in Linear Electrical Networks (1)

Lecture 12
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Jae W. Lee (jaewlee@snu.ac.kr)
Computer Science and Engineering
Seoul National University

Slide credits: *Prof. Anant Agarwal at MIT*

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Outline

Textbook: 10.1, 10.3

- **Analysis of RC Circuits**
- Intuitive Analysis

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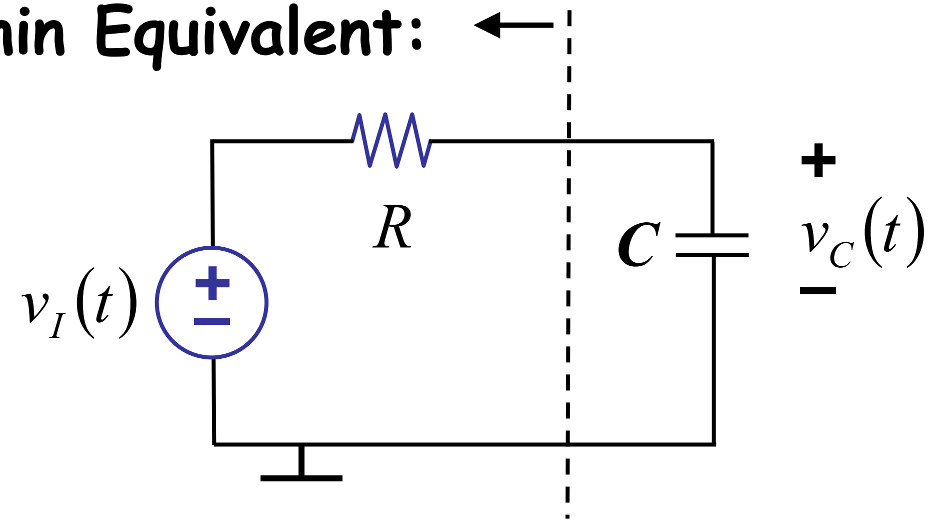
Analysis of RC Circuits

- What is $v_C(t)$?

Thévenin Equivalent:



Apply node method:

$$\frac{v_C - v_I}{R} + C\frac{dv_C}{dt} = 0$$

$$RC\frac{dv_C}{dt} + v_C = v_I \quad \begin{cases} t \geq t_0 \\ v_C(t_0) \text{ given} \end{cases}$$

units of time

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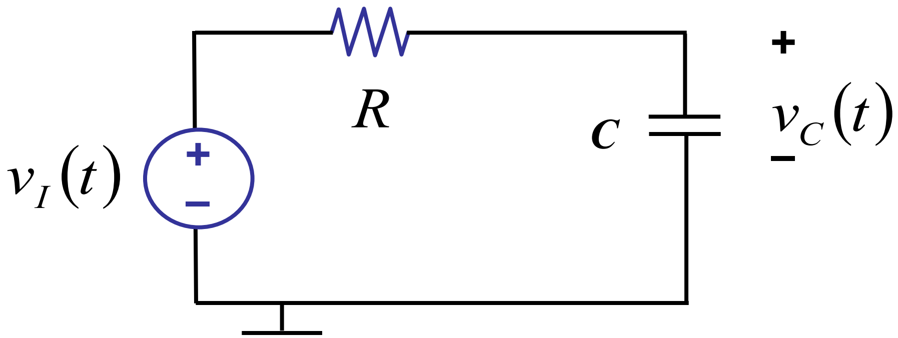
Analysis of RC Circuits

Let's do an example:



$$v_I(t) = V_I$$

$$v_C(0) = V_0 \quad \textbf{given}$$

$$RC\,\frac{dv_C}{dt} + v_C = V_I \qquad \text{———} \quad \otimes$$

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Analysis of RC Circuits

Let's do an example:

$$v_I(t) = V_I$$

$$v_C(0) = V_0 \quad \textbf{given}$$

$$RC\,\frac{dv_C}{dt} + v_C = V_I \quad \text{———} \quad \otimes$$

$$v_C(t) = v_{CH}(t) + v_{CP}(t)$$

total **homogeneous** **particular**

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Analysis of RC Circuits

Example…

$$v_C(t) = v_{CH}(t) + v_{CP}(t)$$

total homogeneous particular

Method of homogeneous and particular solutions:

1. Find the particular solution.
2. Find the homogeneous solution.
3. The total solution is the sum of the particular and homogeneous solutions.

   Use the initial conditions to solve for the remaining constants.

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Analysis of RC Circuits

① Particular solution

$$RC\frac{dv_{CP}}{dt}+v_{CP}=V_I$$

$$\boxed{v_{CP}=V_I}$$ **works**

$$RC\underbrace{\cancel{\frac{dV_I}{dt}}}_{0}+V_I=V_I$$

In general, use trial and error.

v_{CP} : any solution that satisfies the original equation ⓧ

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Analysis of RC Circuits

② Homogeneous solution

$$RC\frac{dv_{CH}}{dt}+v_{CH}=0 \qquad \text{(y)}$$

v_{CH} : solution to the homogeneous equation (y)
(set drive to zero)

$$v_{CH}=Ae^{st}$$ assume solution of this form. A, s ?

$$RC\frac{dAe^{st}}{dt}+Ae^{st}=0$$

$$RCAs\cancel{e^{st}}+A\cancel{e^{st}}=0$$

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Analysis of RC Circuits

② Homogeneous solution (cont…)

Discard trivial $A = 0$ solution,

$$RCs + 1 = 0$$

Characteristic equation

$$\Rightarrow \quad s = -\frac{1}{RC}$$

or $$\boxed{v_{CH} = Ae^{\frac{-t}{RC}}}$$

RC **called time constant τ**

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Analysis of RC Circuits

$$v_C = v_{CP} + v_{CH}$$

$$v_C = V_I + A\,e^{\frac{-t}{RC}}$$

3 Total solution

Find remaining unknown from initial conditions:

Given, $v_C = V_0$ at $t = 0$

so, $V_0 = V_I + A$

or $A = V_0 - V_I$

thus $$v_C = V_I + \left(V_0 - V_I\right) e^{\frac{-t}{RC}}$$

also $$i_C = C\frac{dv_C}{dt} = -\frac{\left(V_0 - V_I\right)}{R} e^{\frac{-t}{RC}}$$

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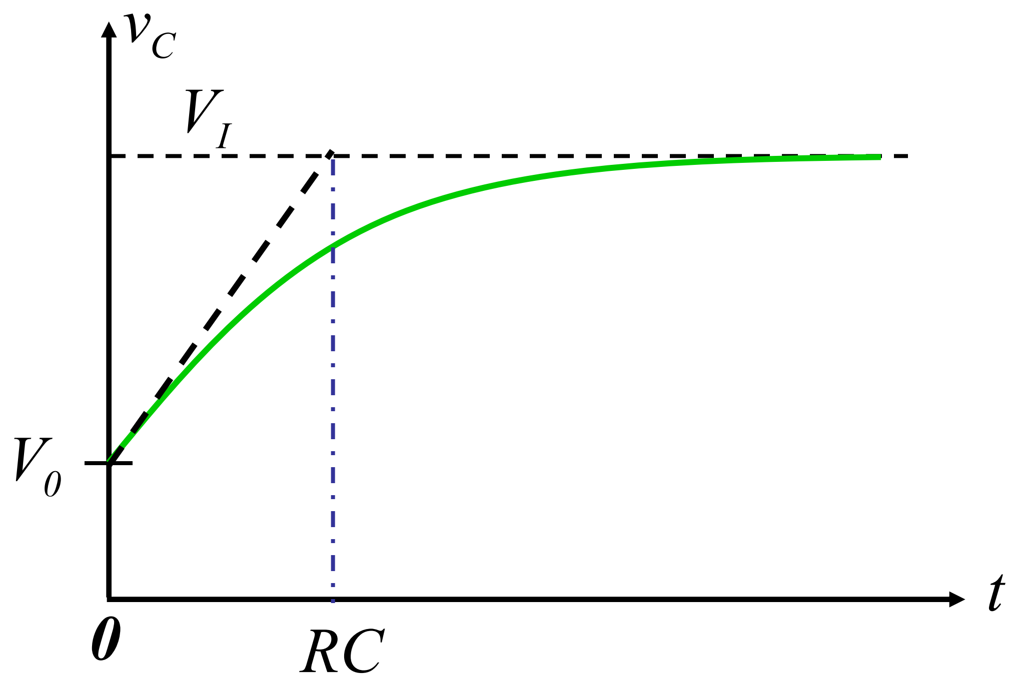
Analysis of RC Circuits

③ Total solution

$$v_C = V_I + \left(V_0 - V_I\right) e^{\frac{-t}{RC}}$$



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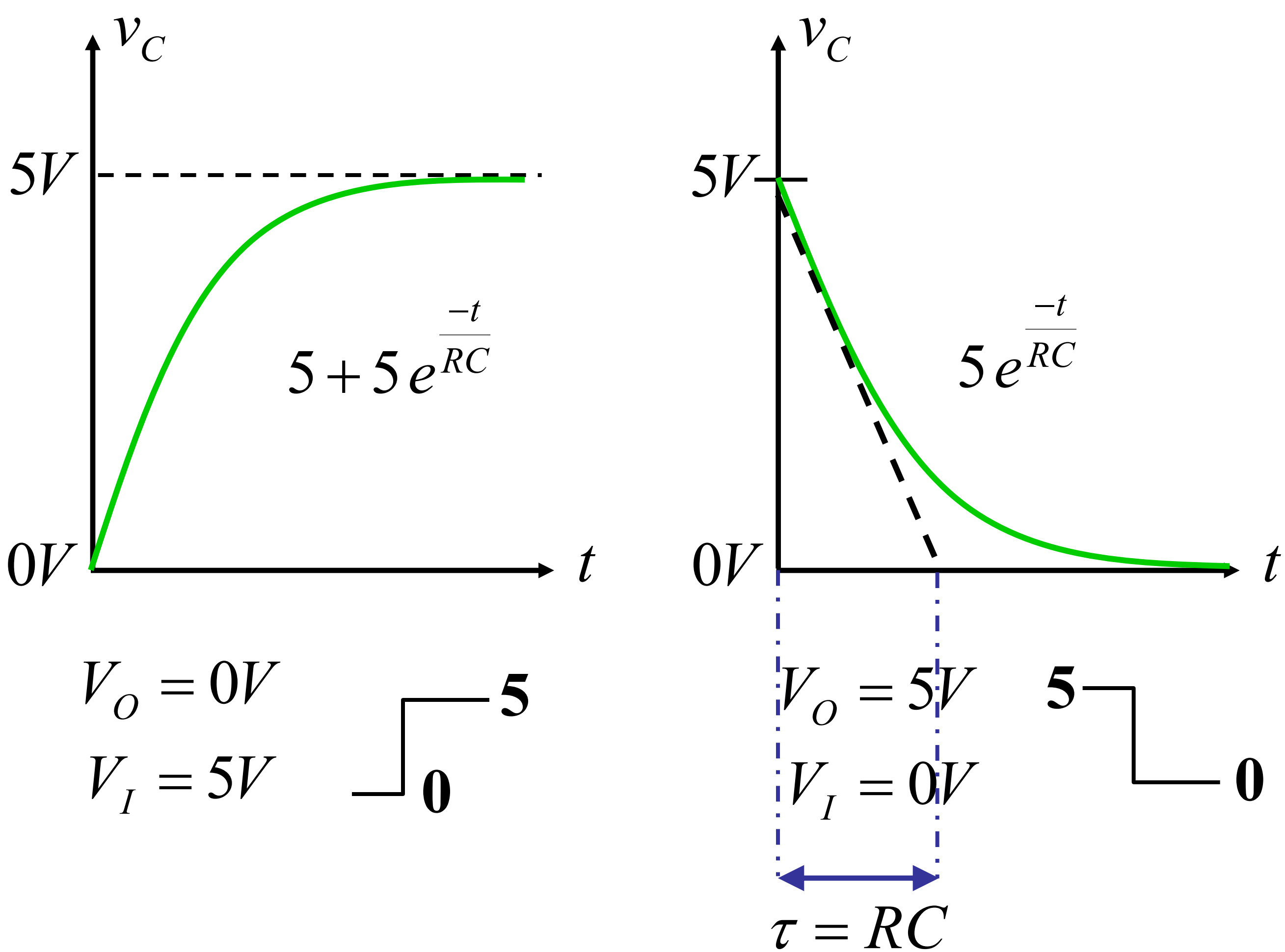
Analysis of RC Circuits

- **Examples**



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Outline

Textbook: 10.1, 10.3

- Analysis of RC Circuits
- **Intuitive Analysis**

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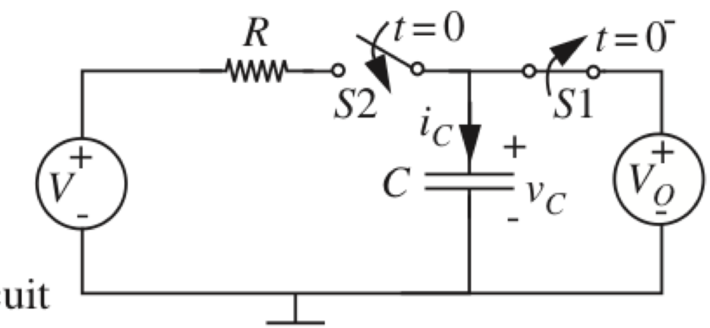
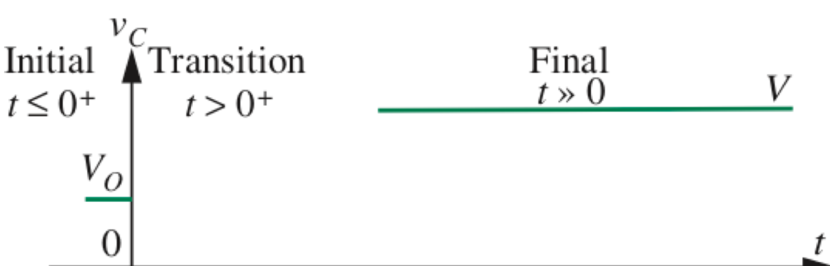
Intuitive Analysis

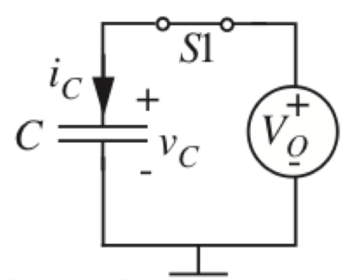
- **How to calculate $v_C(t)$ (without solving differential eq.)?**
  - For simple excitations (like step functions), the response of the first order-systems can be sketched easily using some intuition.
  - DC steady state with DC voltage or current source
    - After a long time: $t \to \infty$
      - Capacitor: **open** circuit
        (cf. Inductor: **closed** circuit)



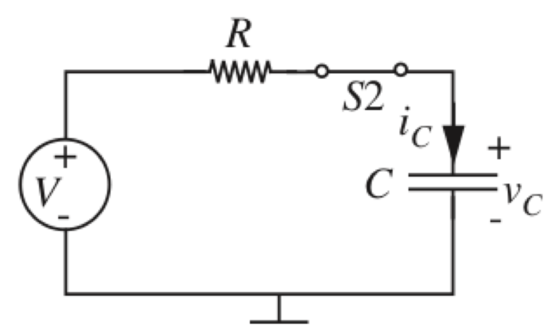
(a) Circuit



(d) Initial, transition, and final intervals



(b) $t < 0^-$



(c) $t \gg 0$, final interval

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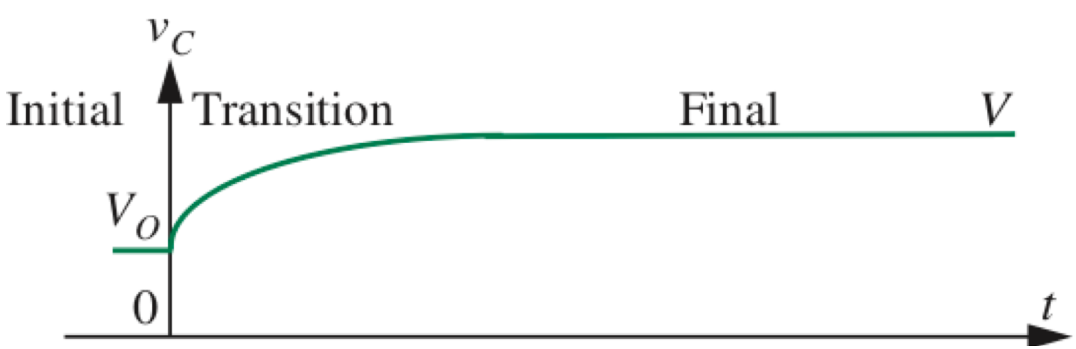
Intuitive Analysis

- **How to calculate $v_C(t)$ (without solving differential eq.)?**
  - Transition interval (t>0^+): General form

$$v_C = \text{initial value} \;\; e^{-t/\text{time constant}} \;\; + \;\; \text{final value}(1 - e^{-t/\text{time constant}}) \tag{10.62}$$

Time constant (τ) = RC

  - Complete response form



  - Can be applied to obtain capacitor current ($i_C(t)$) as well!