EXAMPLE 2.28 BASIC CIRCUIT ANALYSIS METHOD Solve the circuit in Figure 2.58 using the basic method.

Step 1 is to assign the branch variables. Figure 2.59 shows the circuit with the variables properly assigned.

In Step 2, we write the constituent relations:

$$v_S = -V \tag{2.153}$$

$$v_1 = i_1 R_1 \tag{2.154}$$

$$v_2 = i_2 R_2 \tag{2.155}$$

$$v_3 = i_3 R_3 \tag{2.156}$$

$$v_4 = i_4 R_4 \tag{2.157}$$

$$v_5 = i_5 R_5. \tag{2.158}$$

In Step 3, we write the KVL and KCL equations. The KVL equations with respect to the loop choice shown in Figure 2.60, are

$$v_S + v_1 + v_2 + v_4 = 0 \tag{2.159}$$

$$-v_2 + v_3 = 0 \tag{2.160}$$

$$-v_4 + v_5 = 0 \tag{2.161}$$

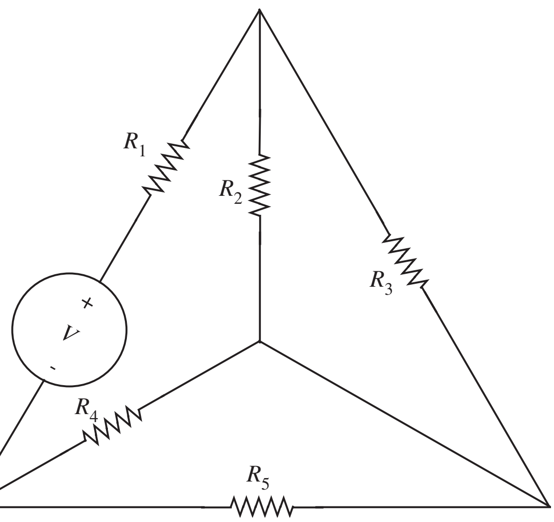


FIGURE 2.58 Circuit example.

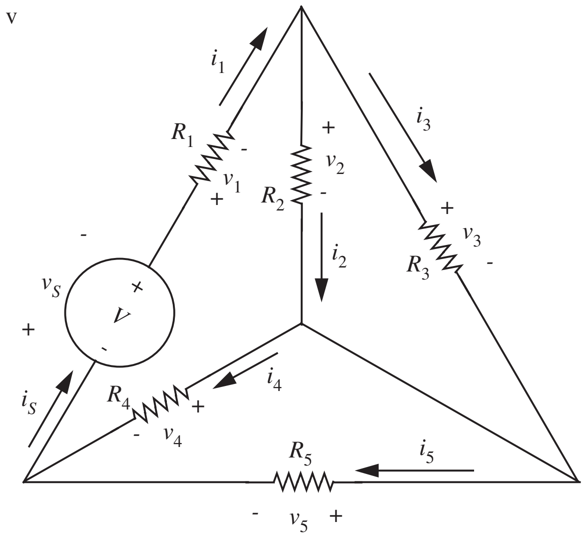


FIGURE 2.59 Circuit with properly assigned variables.

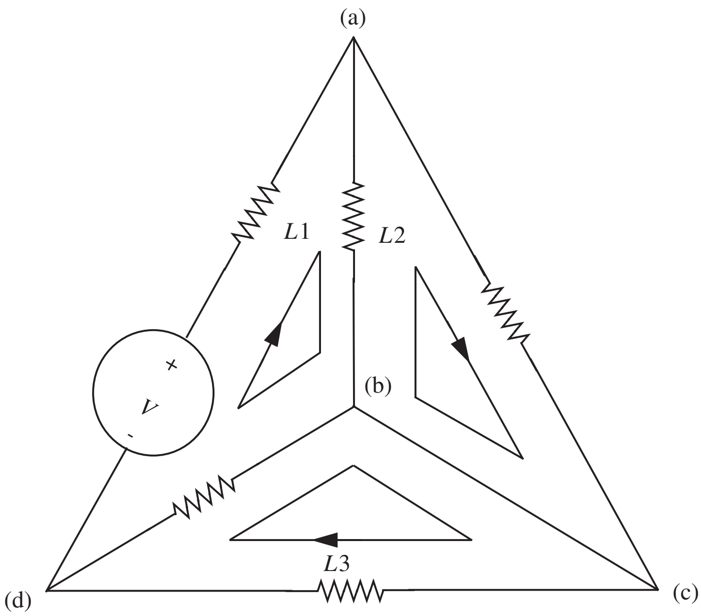


FIGURE 2.60 Loop and node choice.

At node (a), the KCL equation is

$$i_1 - i_2 - i_3 = 0. \tag{2.162}$$

Notice that nodes (b) and (c) are connected by a wire, so they yield only one KCL equation

$$i_2 + i_3 - i_4 - i_5 = 0. \tag{2.163}$$

Lastly, at node (d), we have

$$i_4 + i_5 - i_S = 0. \tag{2.164}$$

Combining the constituent relations with KVL equations, we obtain

$$-V + i_1R_1 + i_2R_2 + i_4R_4 = 0 \tag{2.165}$$

$$-i_2R_2 + i_3R_3 = 0 \tag{2.166}$$

$$-i_4R_4 + i_5R_5 = 0. \tag{2.167}$$

By adding Equations 2.162–2.164, we have

$$i_S = i_1. \tag{2.168}$$

Eliminating $i_2$ and $i_4$ and substituting back into Equations 2.166–2.167 gives us

$$i_3 = i_S\frac{R_2}{R_2 + R_3} \tag{2.169}$$

$$i_5 = i_S\frac{R_4}{R_4 + R_5} \tag{2.170}$$

$$V = i_S\left(R_1 + R_2 + R_4 - \frac{R_2^2}{R_2 + R_3} - \frac{R_4^2}{R_4 + R_5}\right) \tag{2.171}$$

$$= i_S\left(R_1 + \frac{R_2R_3}{R_2 + R_3} + \frac{R_4R_5}{R_4 + R_5}\right). \tag{2.172}$$

As a quick sanity check of the solution, one might notice that the equivalent resistance of the network around the voltage source is $R_1 + R_2\|R_3 + R_4\|R_5$, which is correctly shown by Equation 2.172.