

The Operational Amplifier Abstraction

Lecture 17

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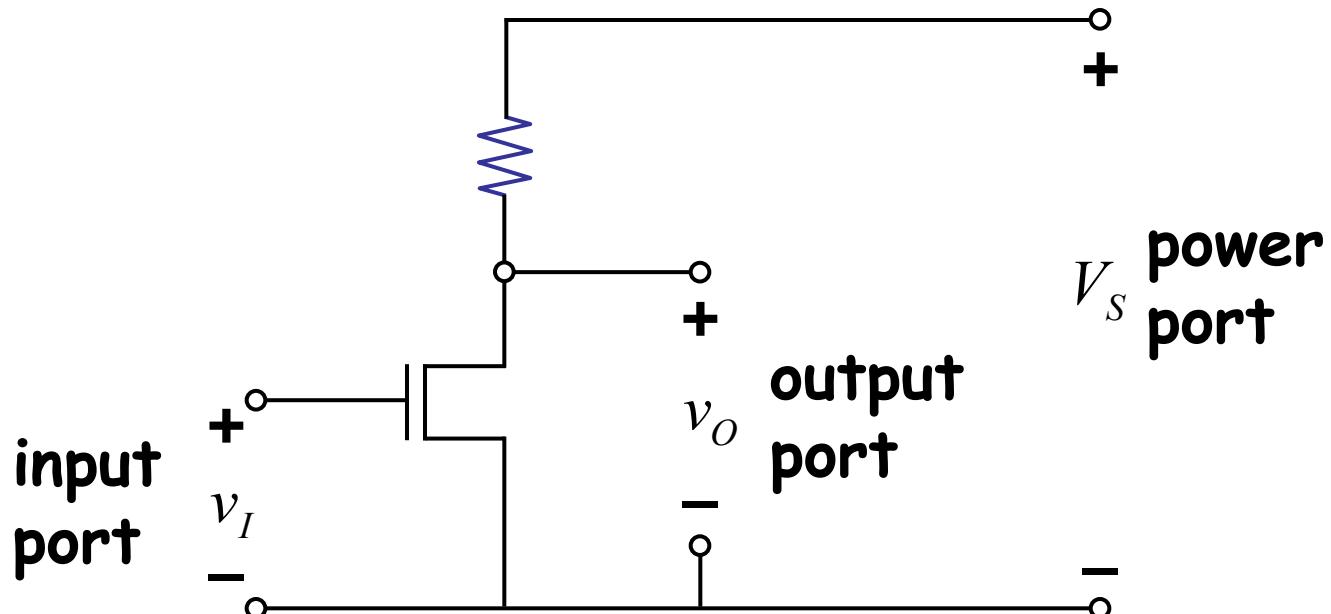
Computer Science and Engineering
Seoul National University

Slide credits: Prof. Anant Agarwal at MIT

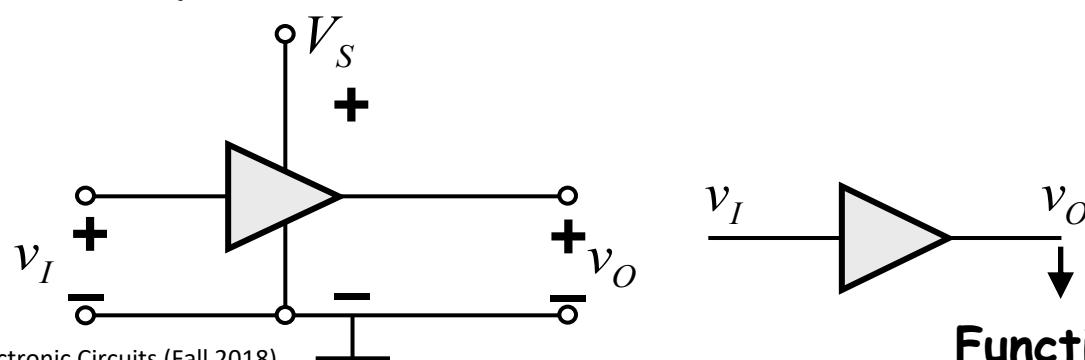
V-code: ???

Review

■ MOSFET amplifier – 3 ports

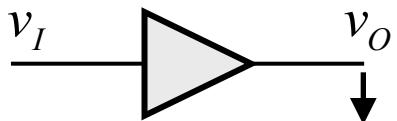


■ Amplifier abstraction

Function of v_I

Review

V-code: ???



Function of v_I

- Can use as an abstract building block for more complex circuits (of course, need to be careful about input and output).
- Today
Introduce a more powerful amplifier abstraction and use it to build more complex circuits.

V-code: ???

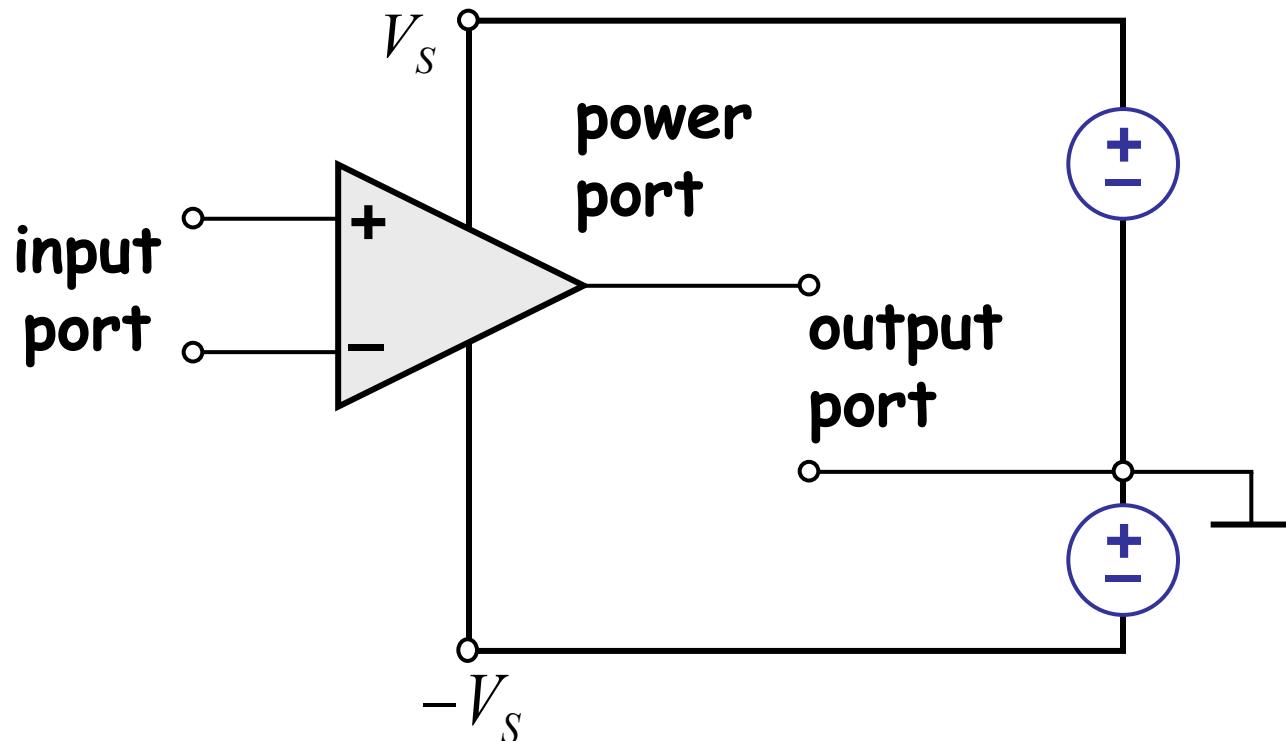
Outline

Textbook: 15.1, 15.2, 15.3, 15.5, 15.6, 15.8

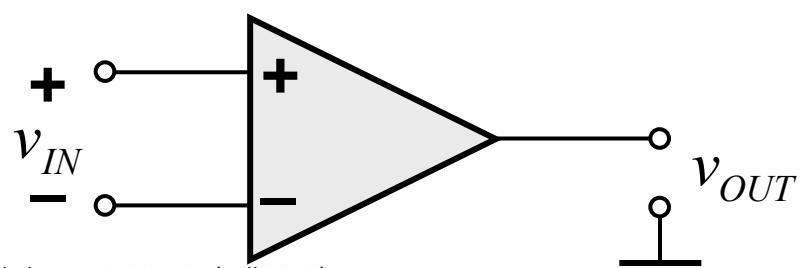
- **Device Properties of the Operational Amplifier**
- Simple Op Amp Circuits
- Op Amp RC Circuits
- Positive Feedback

V-code: ???

Operational Amplifier (Op Amp)

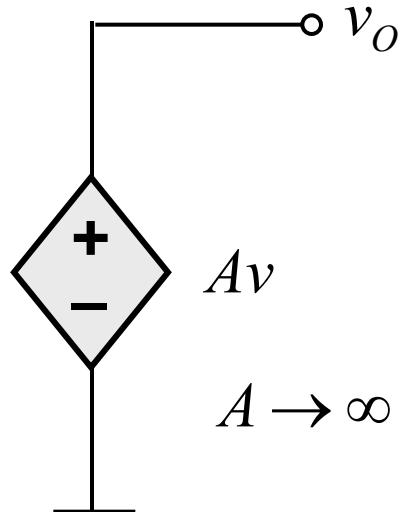
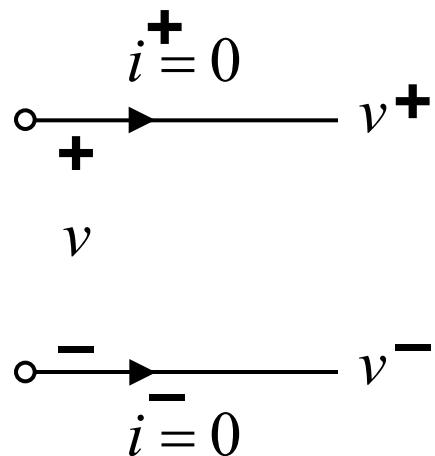


More abstract representation:



Operational Amplifier (Op Amp)

Circuit model (ideal):

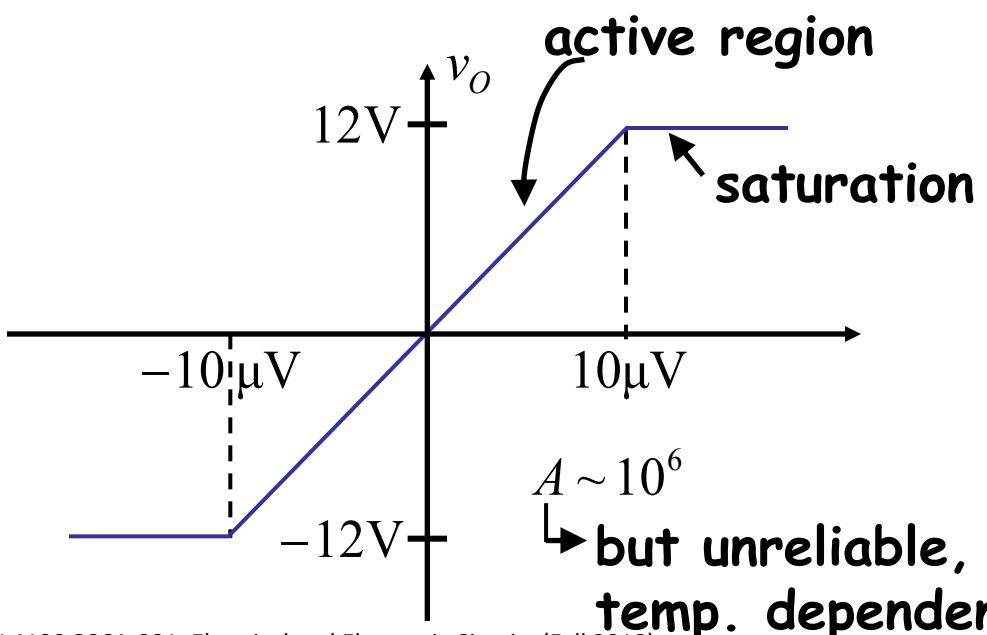
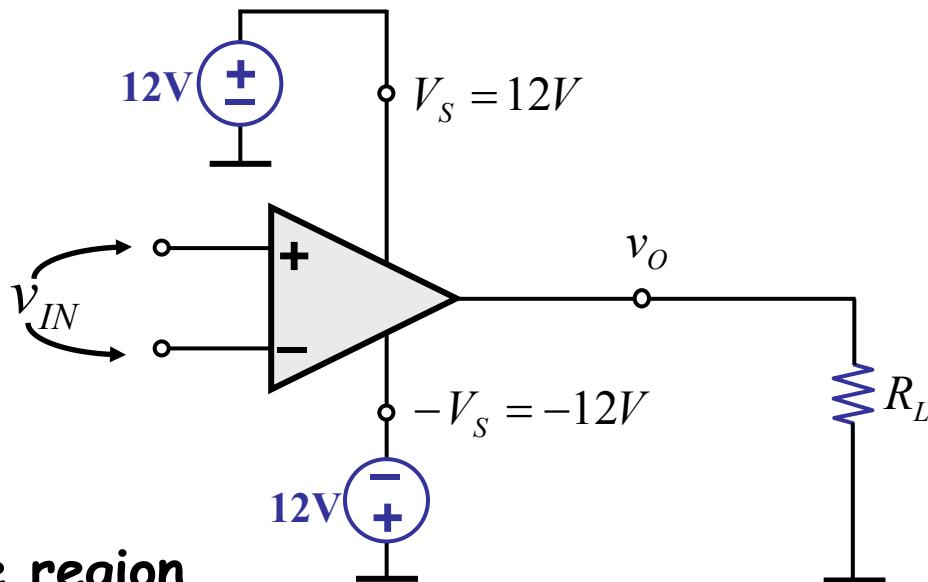


- i.e. ♦ ∞ input resistance
- ♦ 0 output resistance
- ♦ “ A ” virtually ∞
- ♦ No saturation

V-code: ???

Operational Amplifier (Op Amp)

Using it...



$A \sim 10^6$
→ but unreliable,
temp. dependent

(Note: possible confusion with
MOSFET saturation!)

V-code: ???

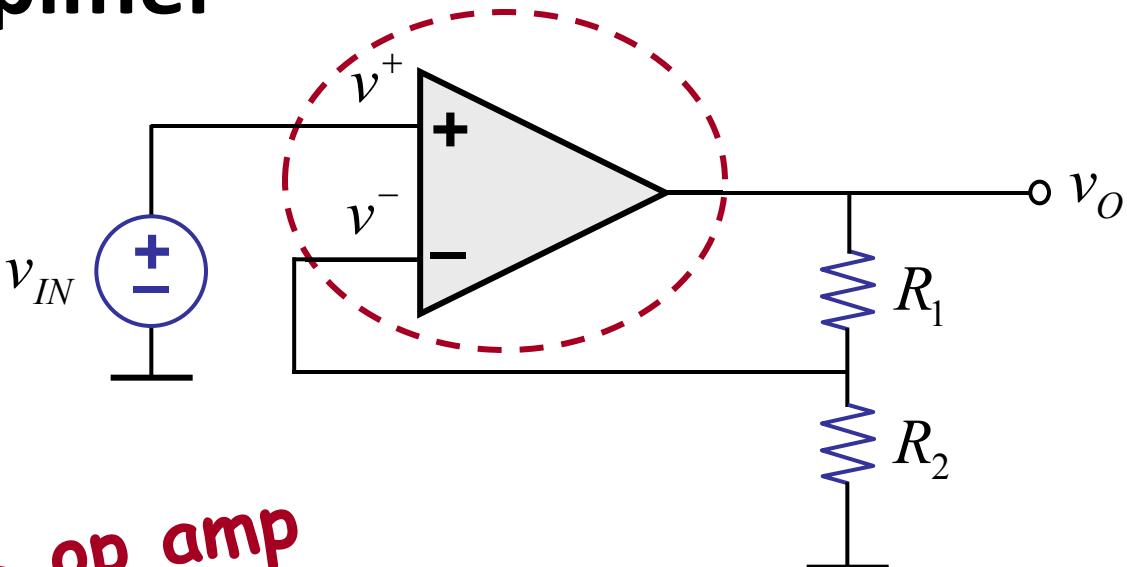
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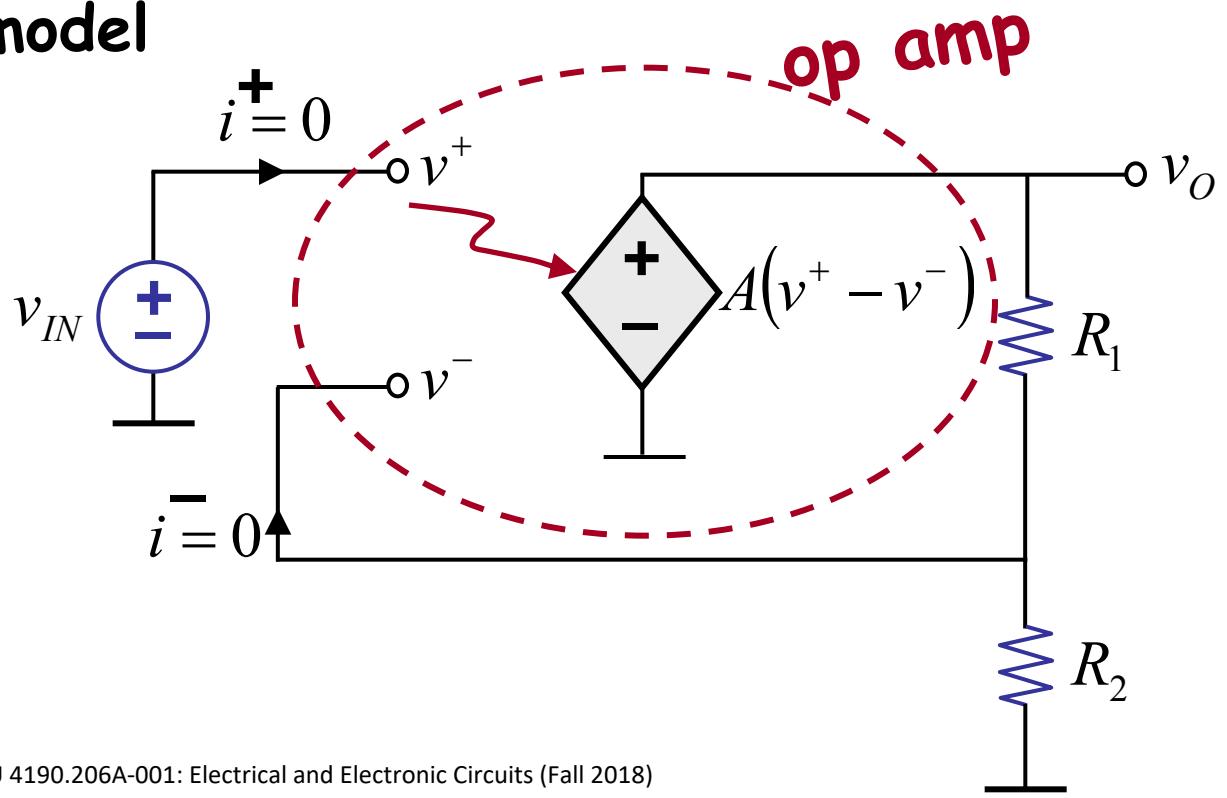
- Device Properties of the Operational Amplifier
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Noninverting Amplifier

V-code: ???



Equivalent circuit
model



Noninverting Amplifier

V-code: ???

Let us analyze the circuit:

Find v_o in terms of v_{IN} , etc.

$$\begin{aligned} v_o &= A(v^+ - v^-) \\ &= A\left(v_{IN} - v_o \frac{R_2}{R_1 + R_2}\right) \end{aligned}$$

$$v_o \left(1 + \frac{AR_2}{R_1 + R_2}\right) = Av_{IN}$$

$$v_o = \frac{Av_{IN}}{1 + \frac{AR_2}{R_1 + R_2}}$$

What happens when
“A” is very large?

Noninverting Amplifier

Let's see... When A is large

$$v_O = \frac{Av_{IN}}{1 + \frac{AR_2}{R_1 + R_2}} \approx \frac{\cancel{Av_{IN}}}{\cancel{AR_2}} \approx v_{IN} \underbrace{\frac{(R_1 + R_2)}{R_2}}_{\text{gain}}$$

Suppose $A = 10^6$

$$R_1 = 9R$$

$$R_2 = R$$

$$\begin{aligned} v_O &= \frac{10^6 \cdot v_{IN}}{1 + \frac{10^6 R}{9R + R}} \\ &= \frac{10^6 \cdot v_{IN}}{1 + 10^6 \cdot \frac{1}{10}} \end{aligned}$$

$$v_O \approx v_{IN} \cdot 10$$

Gain:

- determined by resistor ratio
- insensitive to A , temperature, fab variations

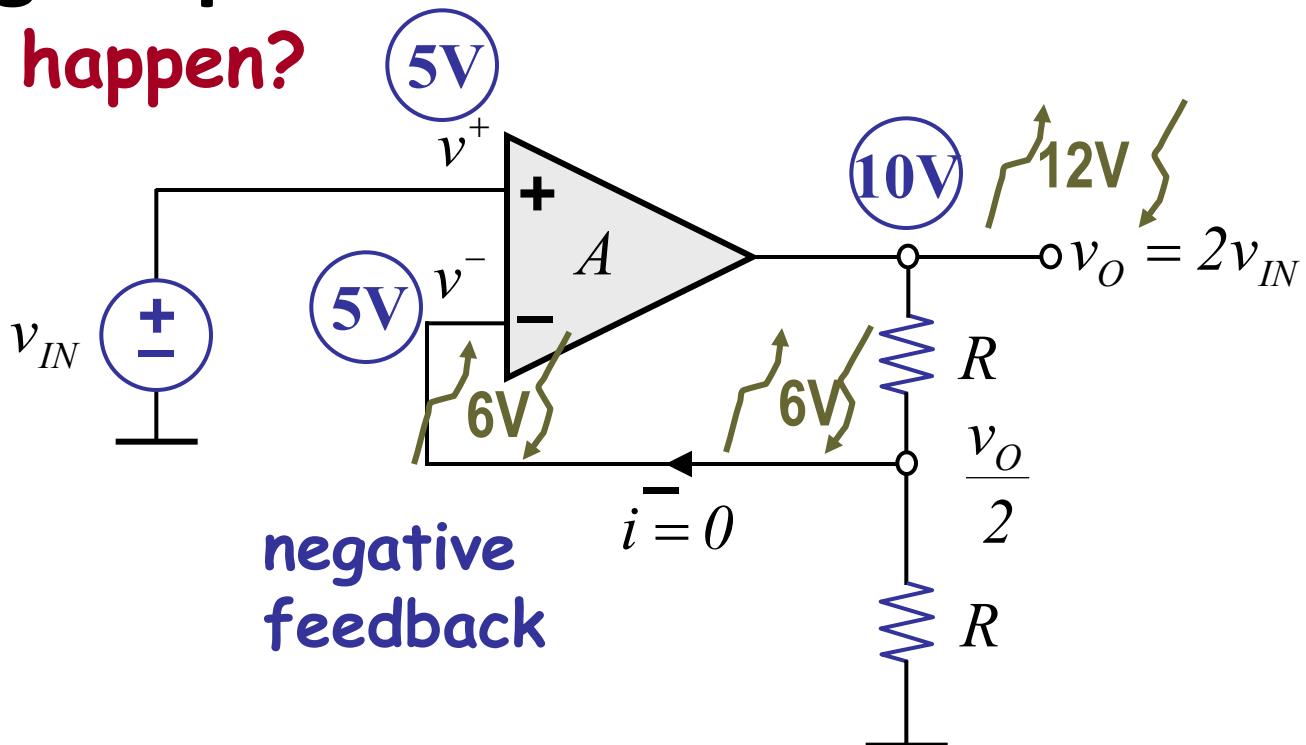
V-code: ???

Noninverting Amplifier

V-code: ???

Why did this happen?

Insight:



e.g. $v_{IN} = 5V$

Suppose I perturb the circuit...

(e.g., force v_O momentarily to $12V$ somehow).

Stable point is when $v^+ \approx v^-$.

Key: neg. feedback \rightarrow portion of output fed to neg input.

e.g. Car antilock brakes
 \rightarrow small corrections.

Noninverting Amplifier

V-code: ???

More op amp insights:

Observe, under negative feedback,

$$v^+ - v^- = \frac{v_o}{A} = \frac{\left(\frac{R_1 + R_2}{R_1} \right) v_{IN}}{A} \rightarrow 0$$

$$v^+ \approx v^-$$

We also know

$$i^+ \approx 0$$

$$i^- \approx 0$$

→ yields an easier analysis method
(under negative feedback).

Noninverting Amplifier

Insightful analysis method under negative feedback

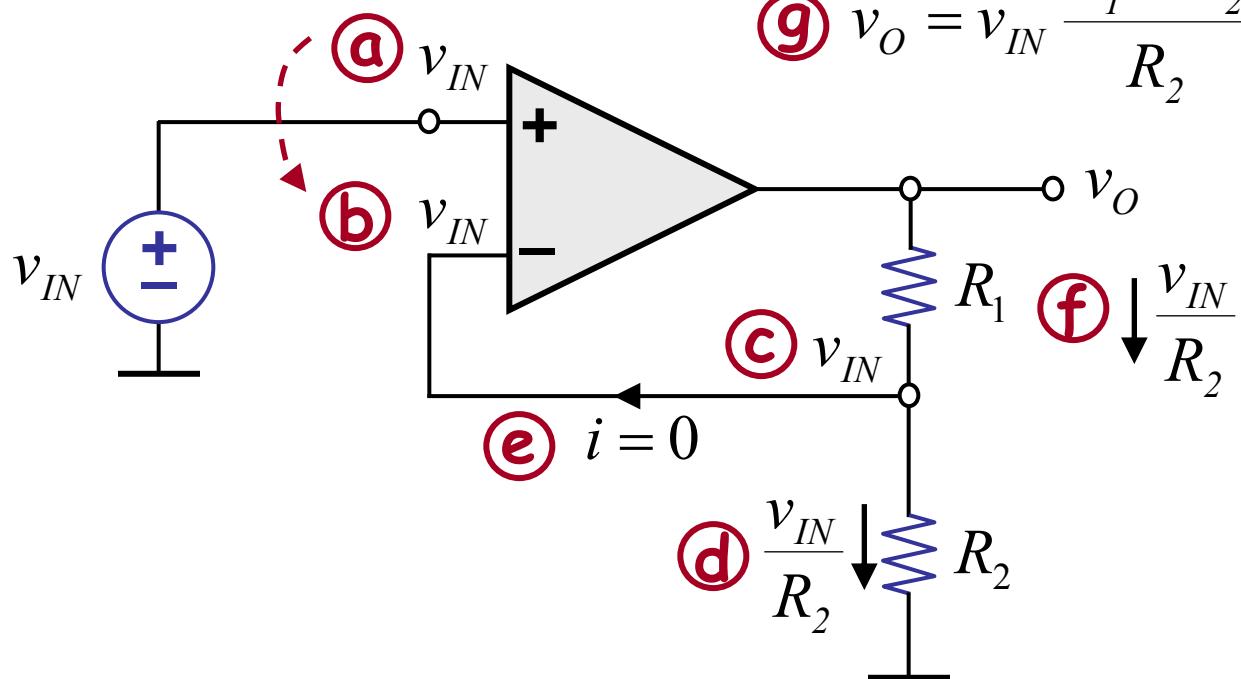
V-code: ???

$$v^+ \approx v^-$$

$$i^+ \approx 0$$

$$i^- \approx 0$$

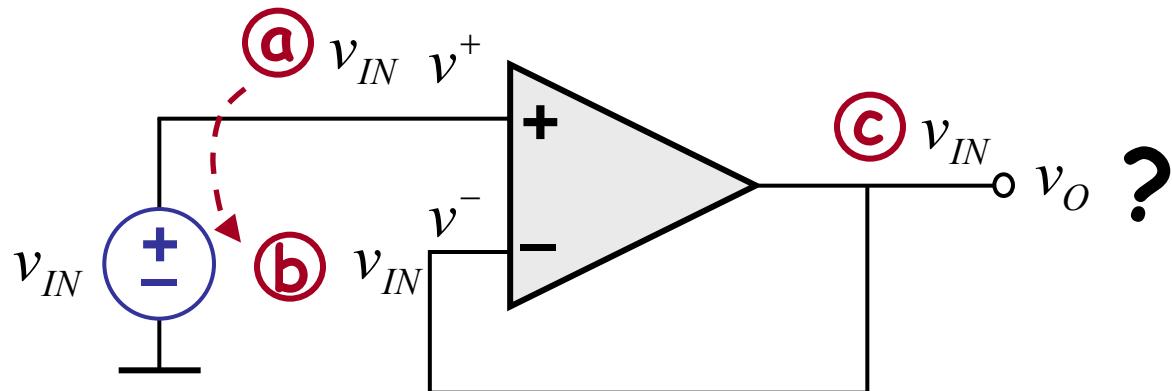
g) $v_O = v_{IN} \frac{R_1 + R_2}{R_2}$



V-code: ???

Special Case: Voltage Follower

Question:



$$v_O \approx v_{IN}$$

or $v_O = v_{IN} \frac{R_1 + R_2}{R_2}$

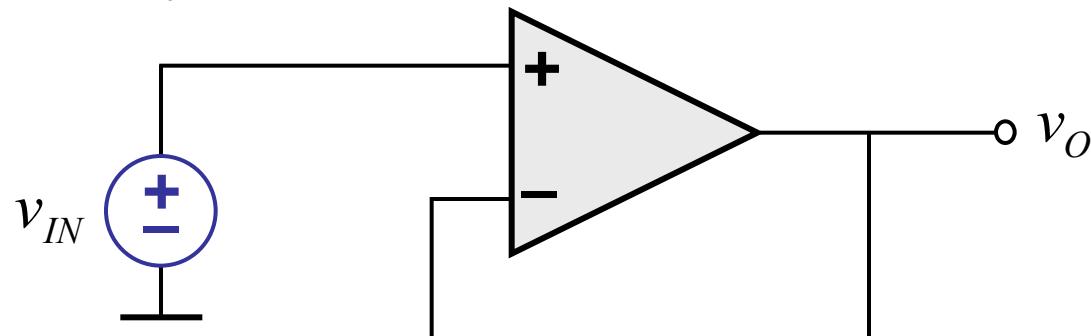
with $R_1 = 0$

$$R_2 = \infty$$

V-code: ???

Special Case: Voltage Follower

Why is this circuit useful?



$$v_O \approx v_{IN}$$

Buffer: isolate the input and the output circuits

voltage gain = 1

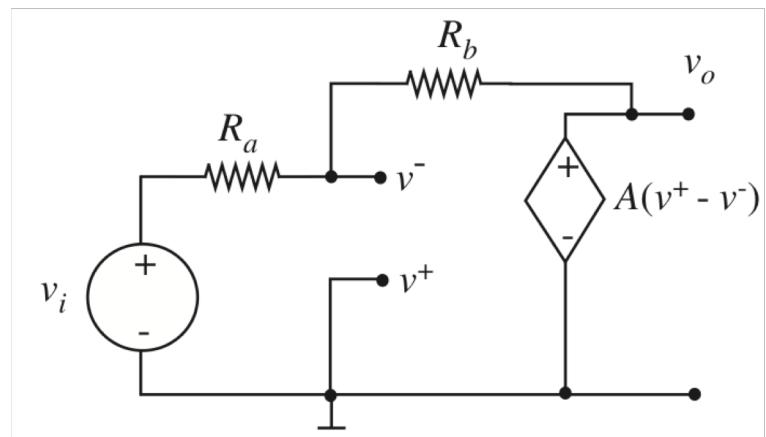
input impedance = ∞

output impedance = 0

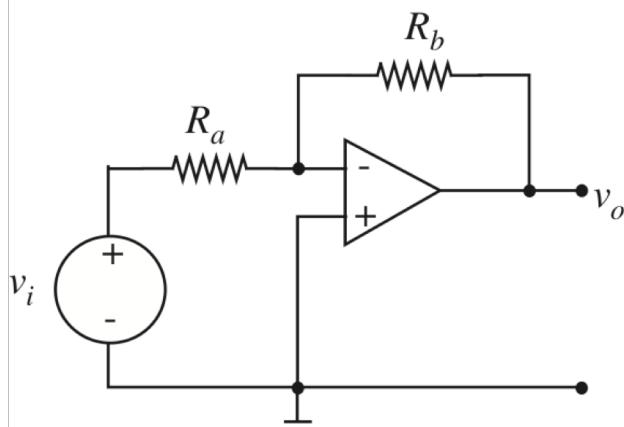
current gain = ∞

power gain = ∞

Inverting Amplifier



Equivalent circuit model



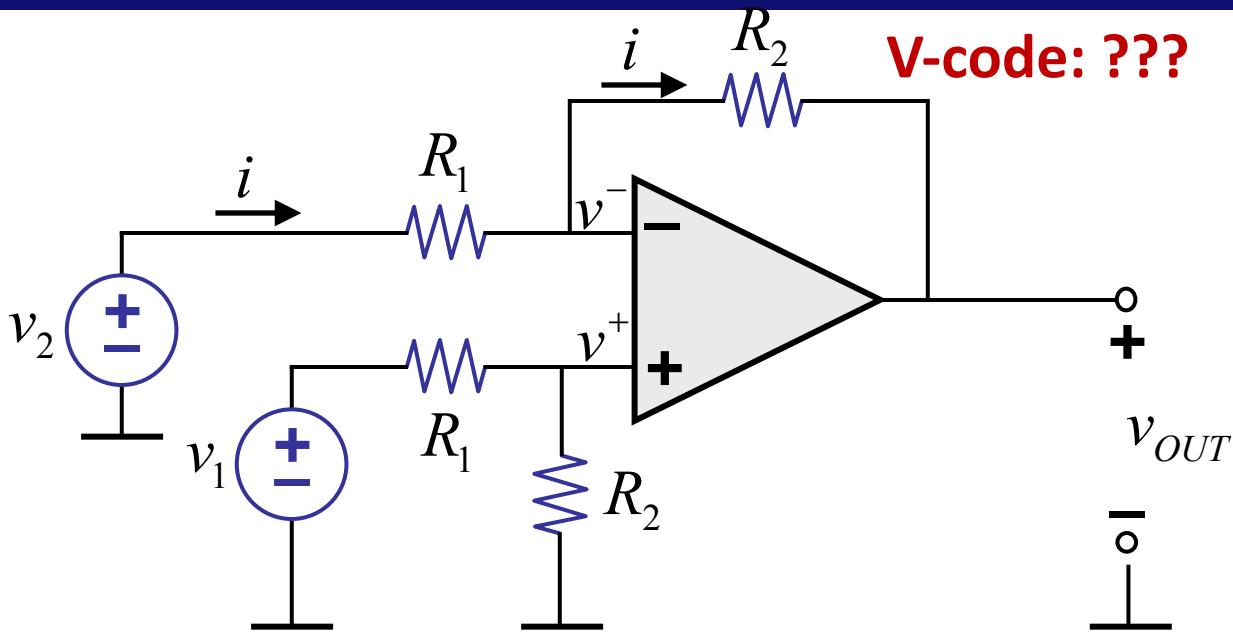
Inverting amplifier (note the input)

$$v_o =$$

V-code: ???

Subtractor

Consider this circuit:



$$v^+ = v_1 \frac{R_2}{R_1 + R_2}$$

$$\approx v^-$$

$$i = \frac{v_2 - v^-}{R_1}$$

$$v_{OUT} = v^- - iR_2$$

$$= v^- - \frac{v_2 - v^-}{R_1} \cdot R_2$$

$$= v^- \left[1 + \frac{R_2}{R_1} \right] - v_2 \frac{R_2}{R_1}$$

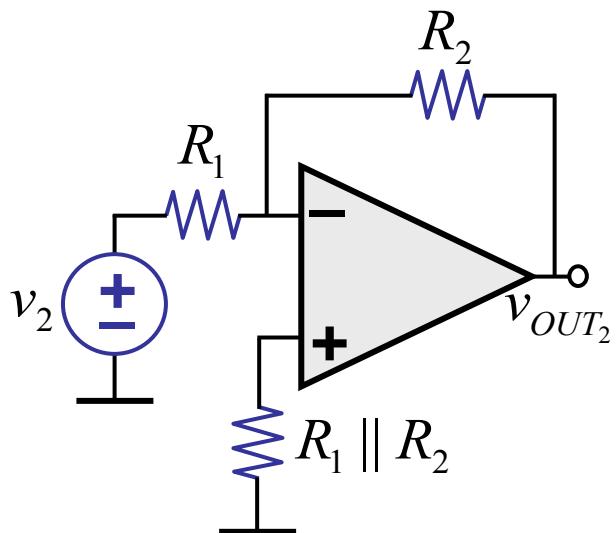
$$= v_1 \frac{R_2}{R_1 + R_2} \cdot \frac{R_1 + R_2}{R_1} - v_2 \frac{R_2}{R_1} = \frac{R_2}{R_1} (v_1 - v_2)$$

subtracts!

Subtractor

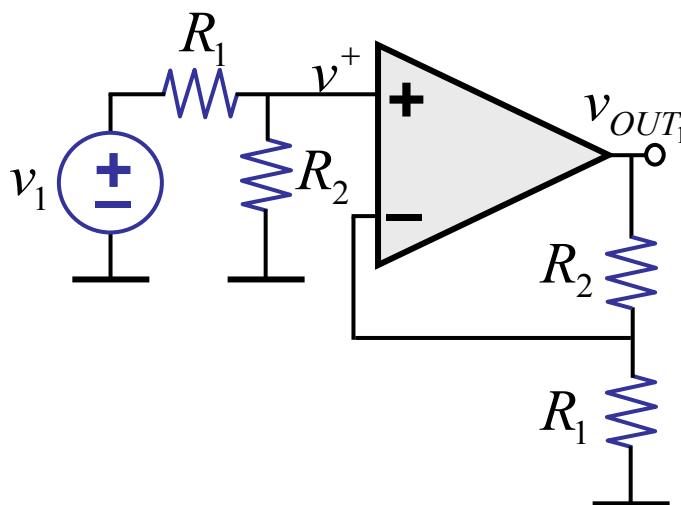
Another way of solving — V-code: ???

$$v_1 \rightarrow 0$$



$$v_{OUT_2} = -\frac{R_2}{R_1} v_2$$

$$v_2 \rightarrow 0$$



$$v_{OUT_1} = v^+ \cdot \frac{R_1 + R_2}{R_1} = \frac{v_1 \cdot R_2}{R_1 + R_2} \cdot \frac{R_1 + R_2}{R_1}$$

$$= v_1 \frac{R_2}{R_1}$$

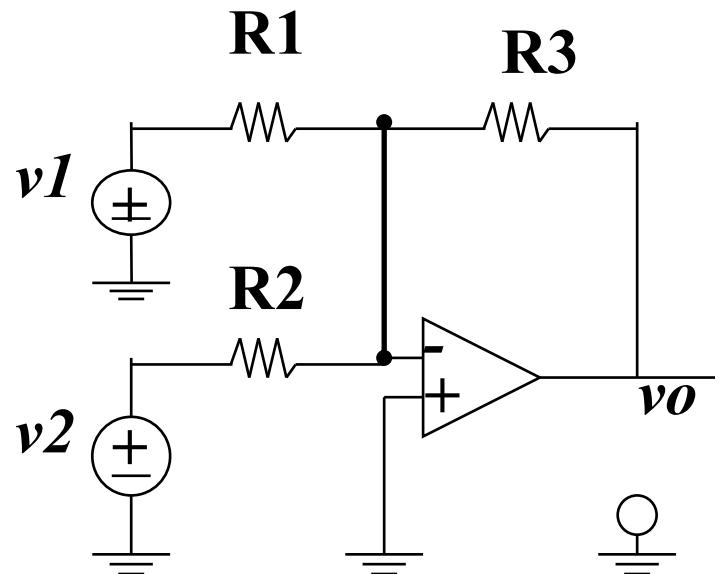
$$v_{OUT} = v_{OUT_1} + v_{OUT_2}$$

$$= \frac{R_2}{R_1} (v_1 - v_2)$$

Still subtracts!

V-code: ???

Adder



$$v_o =$$

V-code: ???

Outline

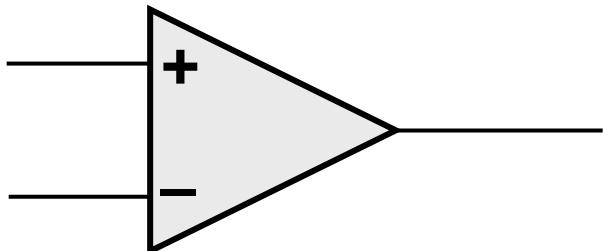
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- Device Properties of the Operational Amplifier
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Review

V-code: ???

■ Operational amplifier abstraction



- ◆ ∞ input resistance
- ◆ 0 output resistance
- ◆ Gain " A " very large

■ Building block for analog systems

■ We will see these examples:

Digital-to-analog converters

Filters

Clock generators

Amplifiers

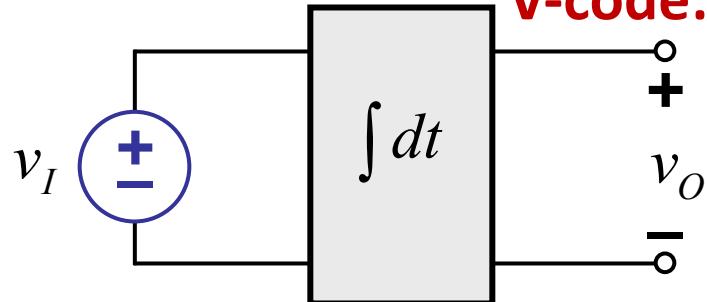
Adders

Integrators & Differentiators

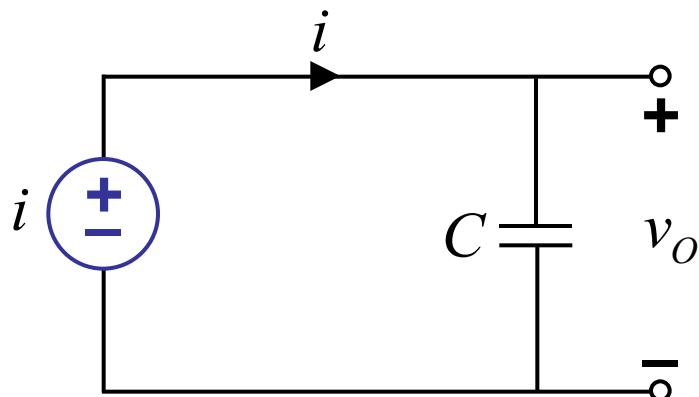
V-code: ???

Integrator

Let's build an integrator...



Let's start with the following insight:



$$v_O = \frac{1}{C} \int_{-\infty}^t i \, dt$$

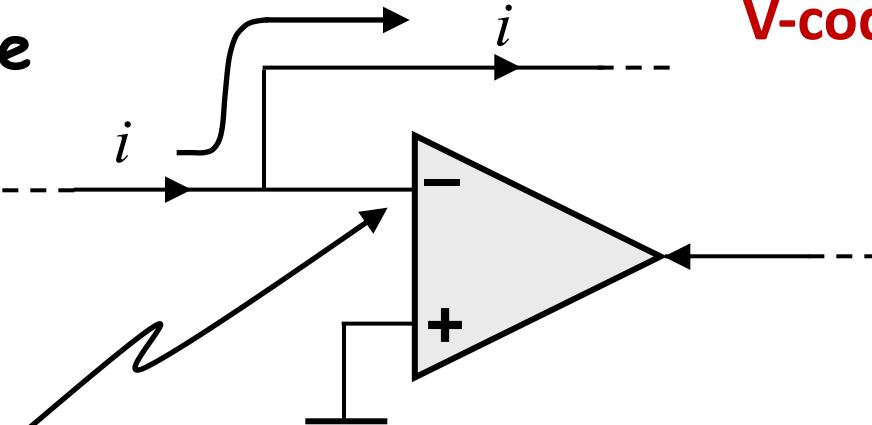
v_O is related to $\int i \, dt$

But we need to somehow convert voltage v_I to current.

Integrator

Using Op Amp!

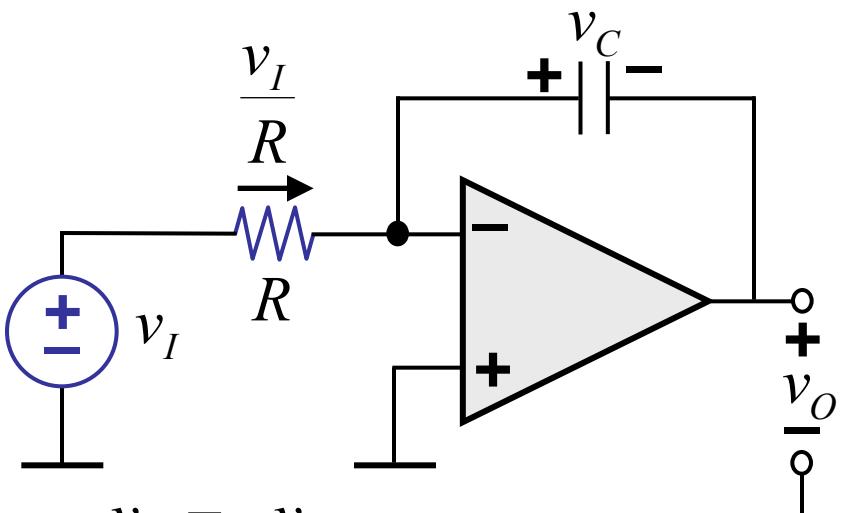
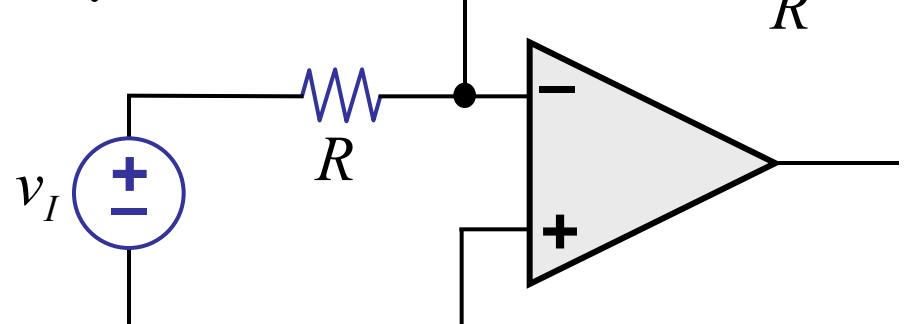
Notice



$v^- \approx 0V$ under negative feedback

$$i = \frac{v_I}{R}$$

so,



$$v_O = -v_C$$

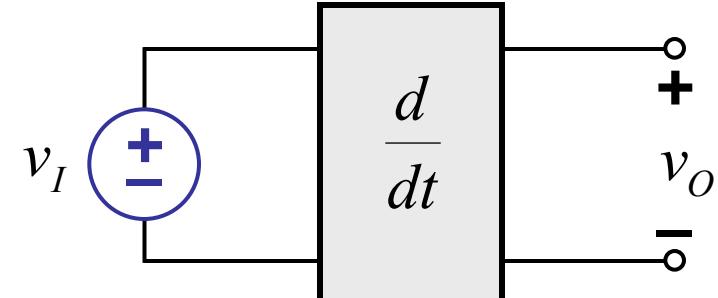
$$v_O = -\frac{1}{C} \int_{-\infty}^t \frac{v_I}{R} dt$$

We have our integrator.

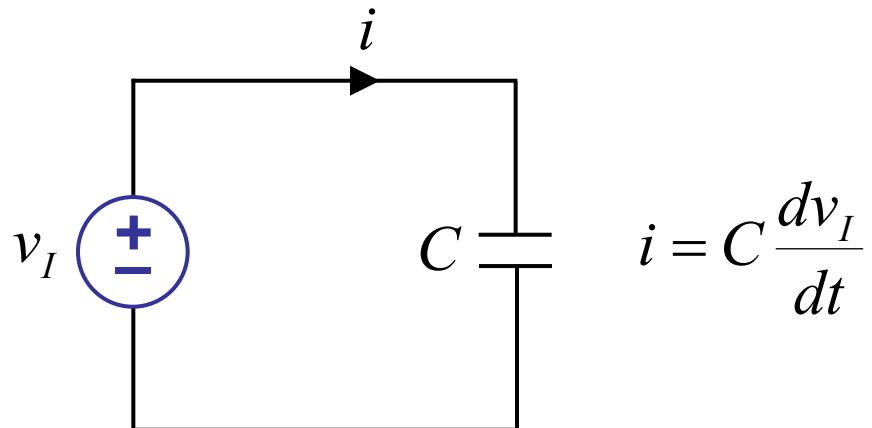
V-code: ???

Differentiator

Now, let's build a differentiator...



Let's start with the following insights:



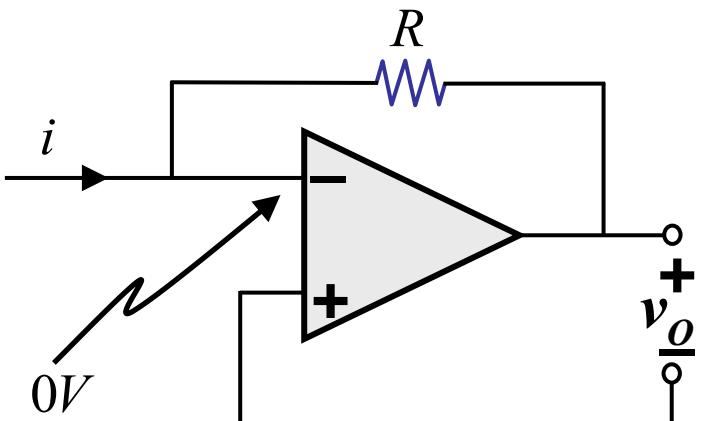
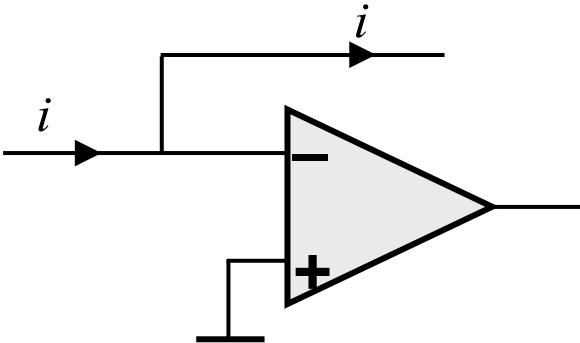
i is related to $\frac{dv_I}{dt}$

But we need to somehow convert current to voltage.

V-code: ???

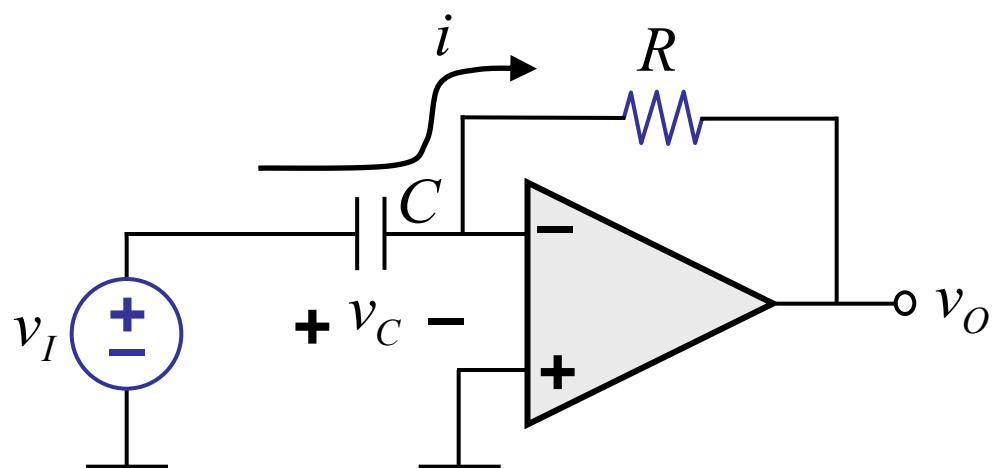
Differentiator

Recall



$$v_o = -iR$$

current to voltage



$$v_I = v_C$$

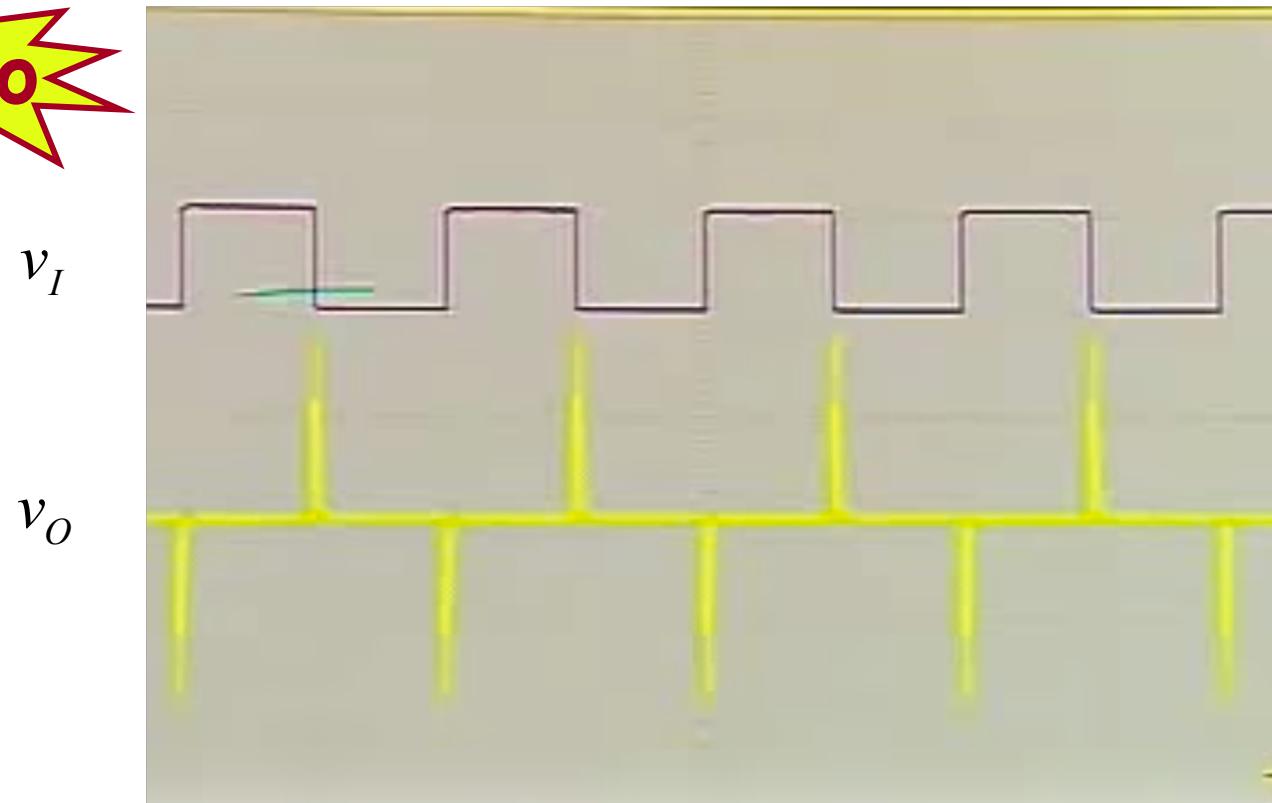
$$i = C \frac{dv_I}{dt}$$

$$v_o = -RC \frac{dv_I}{dt}$$

V-code: ???

Differentiator

Demo



Source: <https://www.youtube.com/watch?v=2SwT6JnfCq8&itct=CAoQpDAYCCITCK6-sL2M2dcCFRajAwodSm0ApjlHcmVsYXRlZEi0y462romQuccB>

V-code: ???

Outline

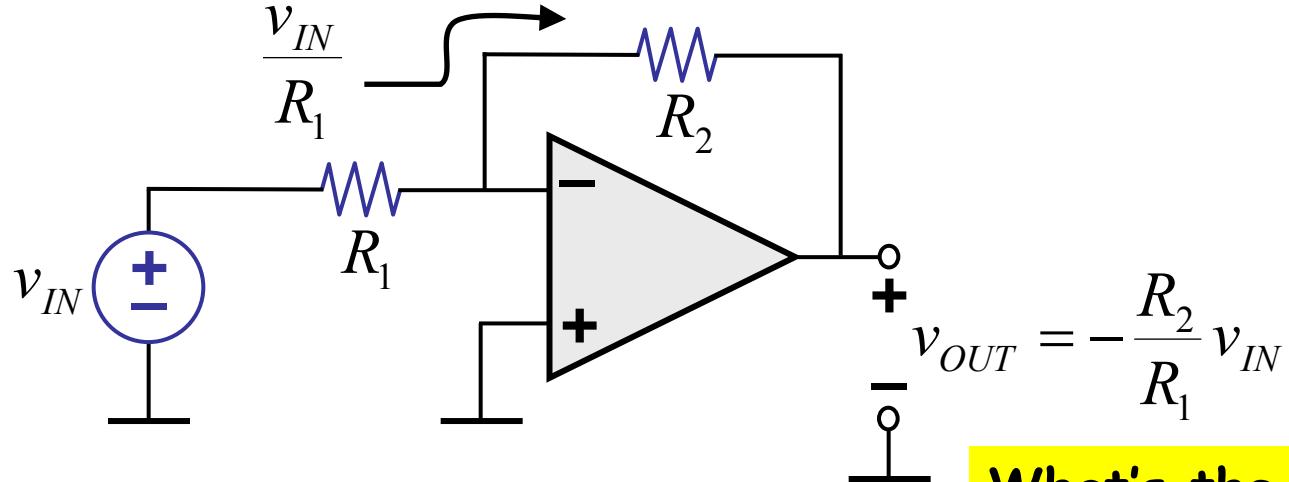
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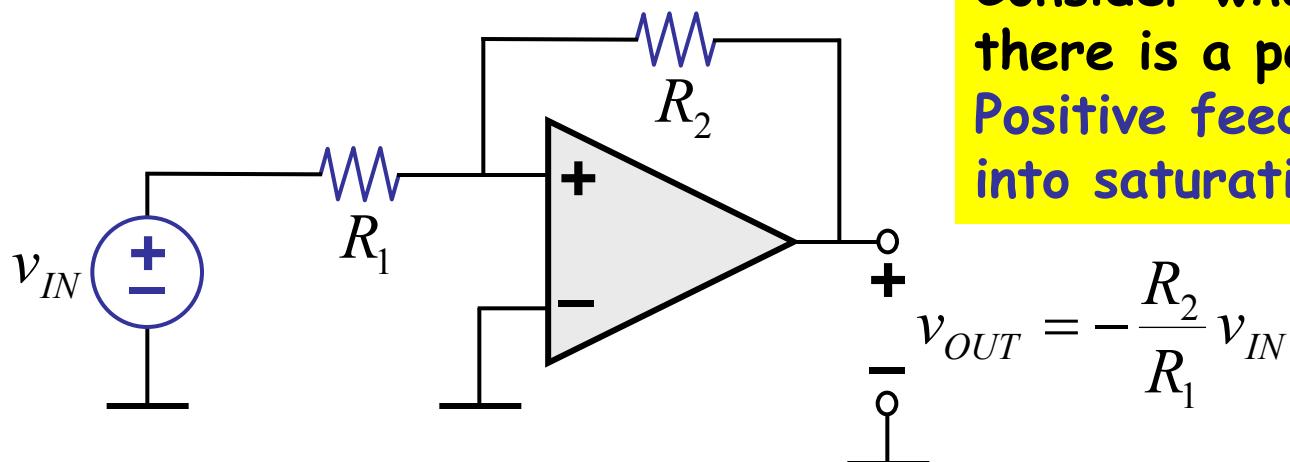
V-code: ???

Negative vs. Positive Feedback

Consider this circuit — *negative feedback*



and this — *positive feedback*



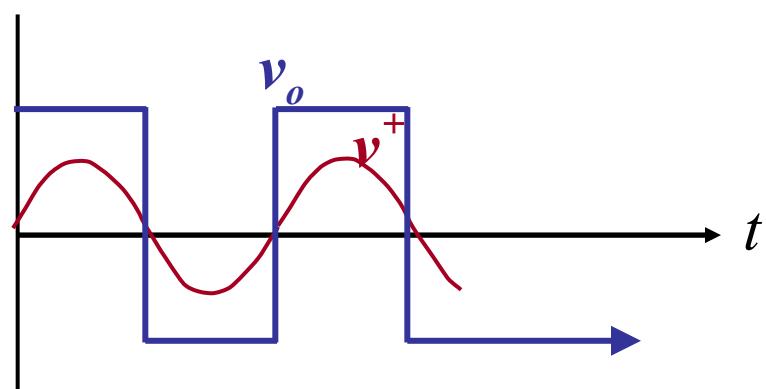
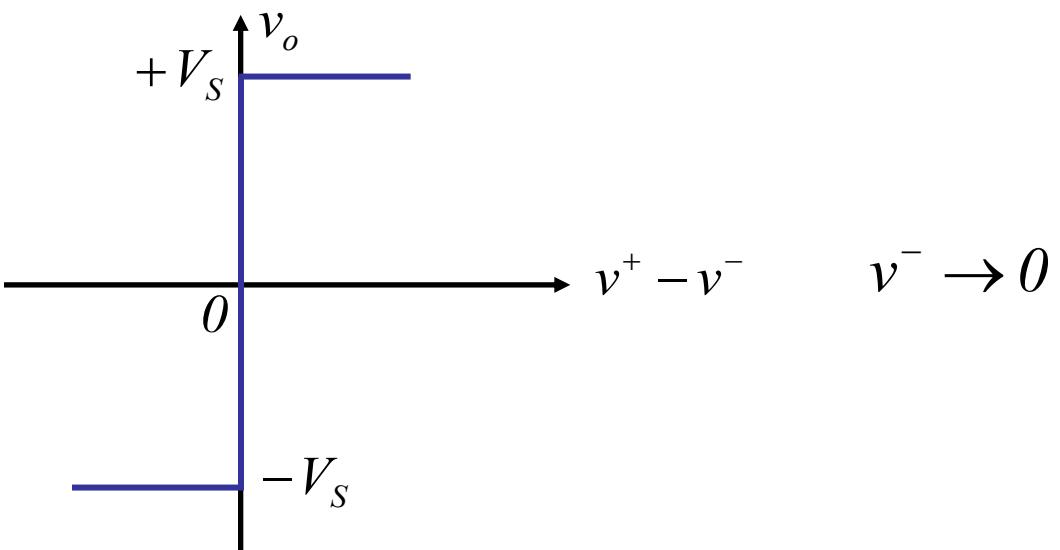
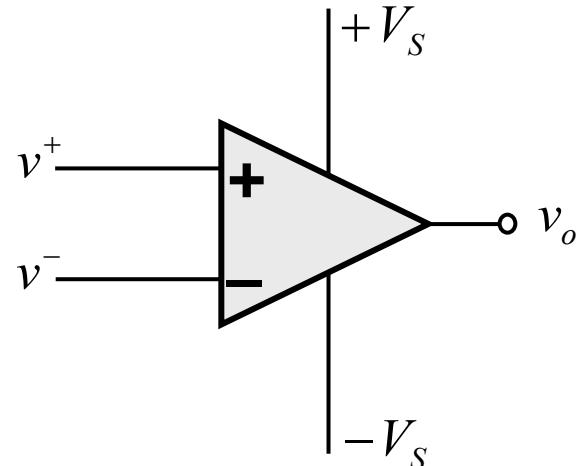
What's the difference?

Consider what happens when there is a perturbation...
Positive feedback drives op amp into saturation: $v_{OUT} \rightarrow \pm V_S$

V-code: ???

Comparator

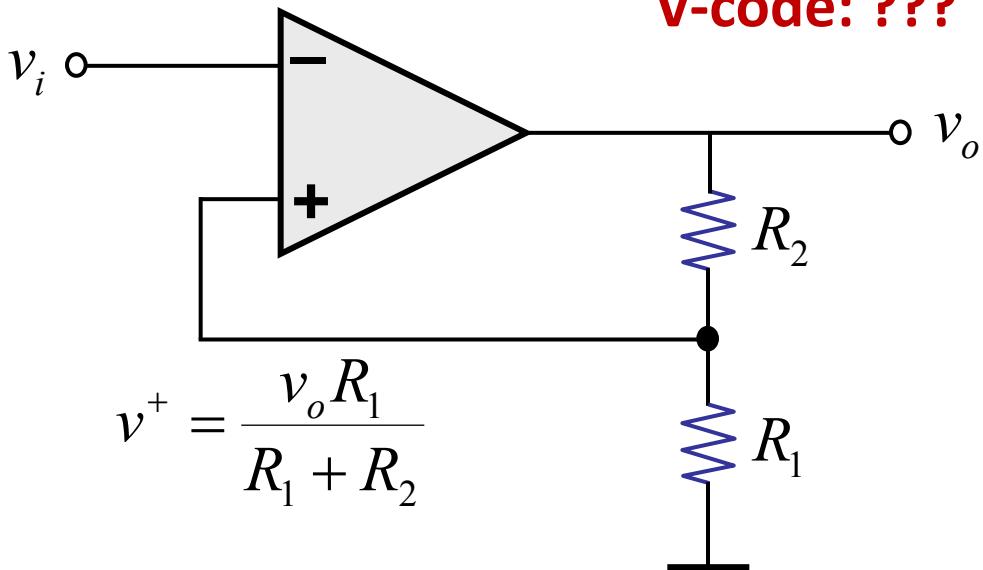
One use for instability: Build on the basic op amp as a comparator



V-code: ???

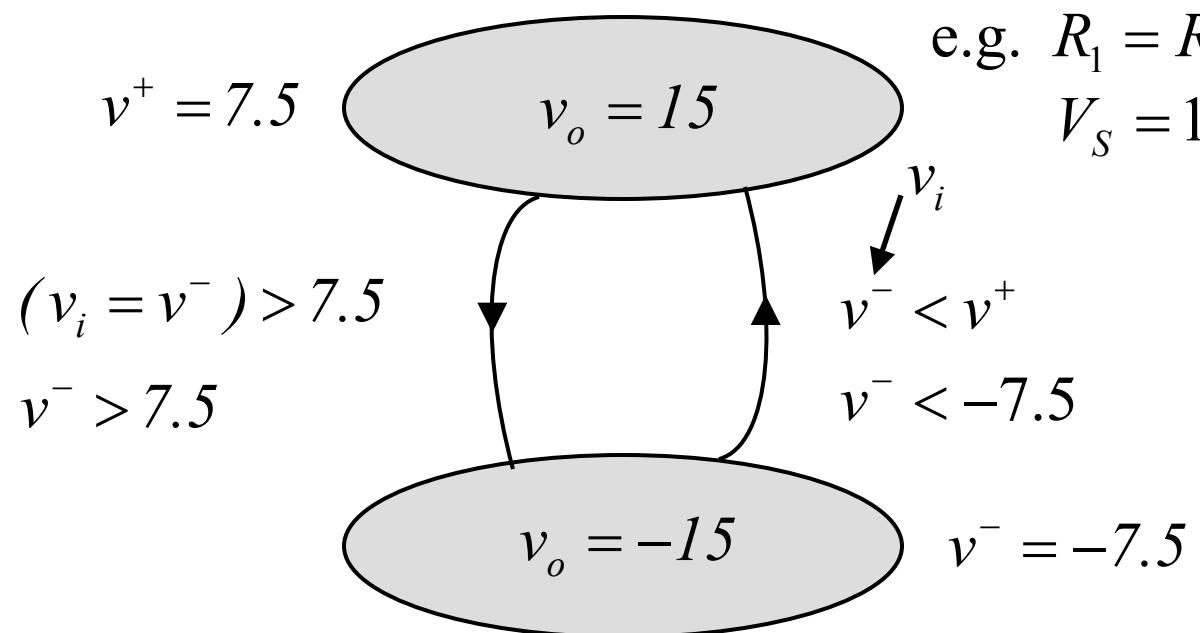
Comparator

Now, use positive feedback



$$v^+ = \frac{v_o R_1}{R_1 + R_2}$$

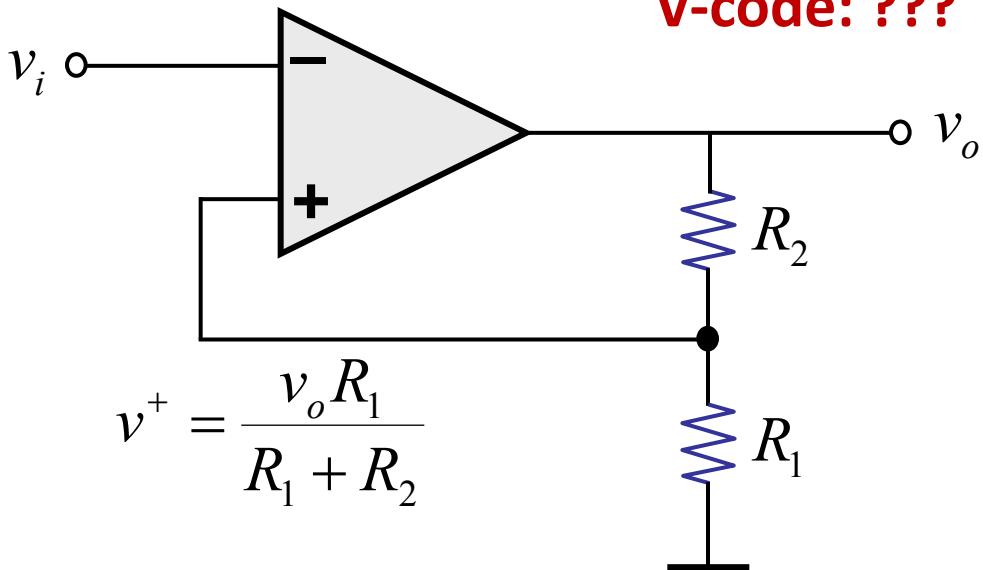
e.g. $R_1 = R_2$
 $V_S = 15$



V-code: ???

Comparator

Now, use positive feedback



$$v^+ = \frac{V_S R_1}{R_1 + R_2}$$

$$(v_i = v^-) > v^+$$

$$v^- > 7.5$$

$$v_o = +V_S$$

15

$$v_i$$

$$v^- < v^+$$

$$\text{e.g. } R_1 = R_2$$

$$V_S = 15$$

$$v_o = -V_S$$

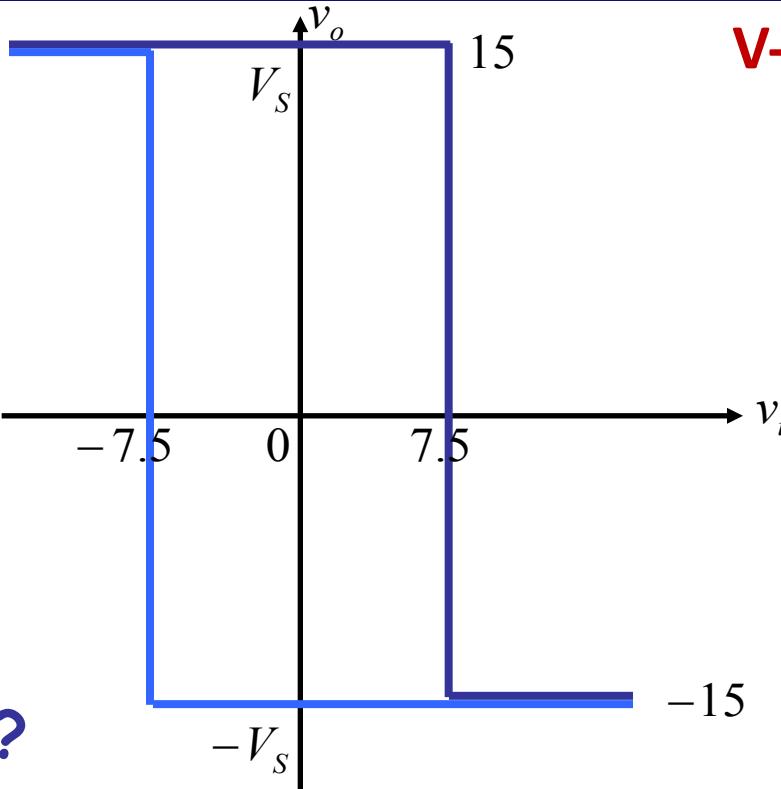
-15

$$v^- < -7.5$$

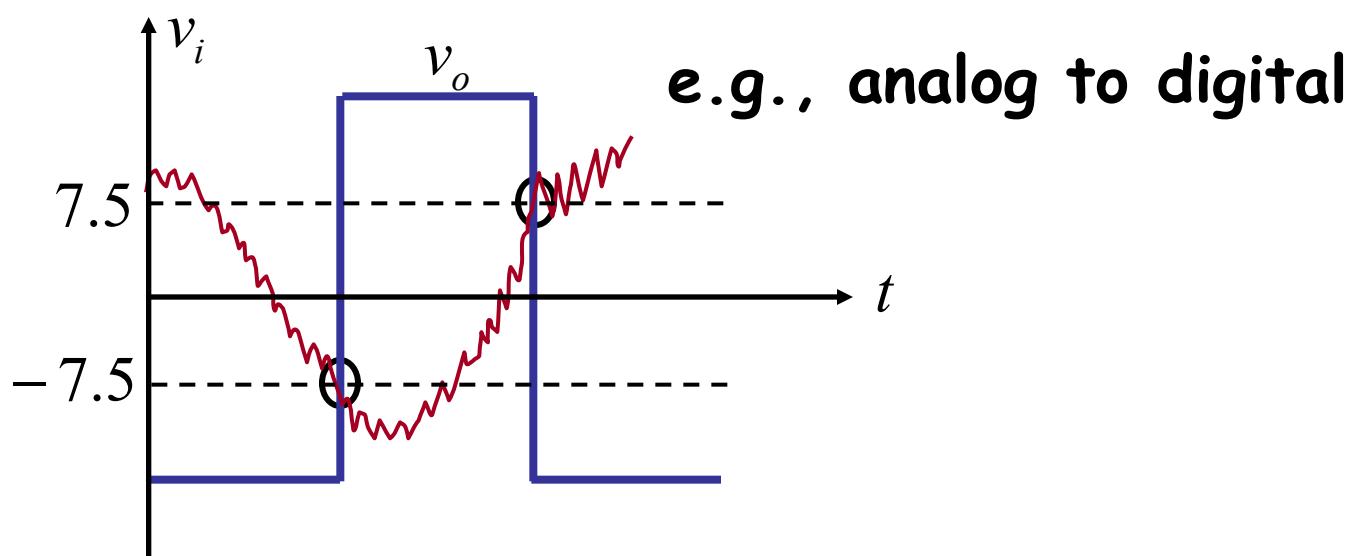
$$v^- = \frac{-V_S R_1}{R_1 + R_2}$$

V-code: ???

Comparator hysteresis



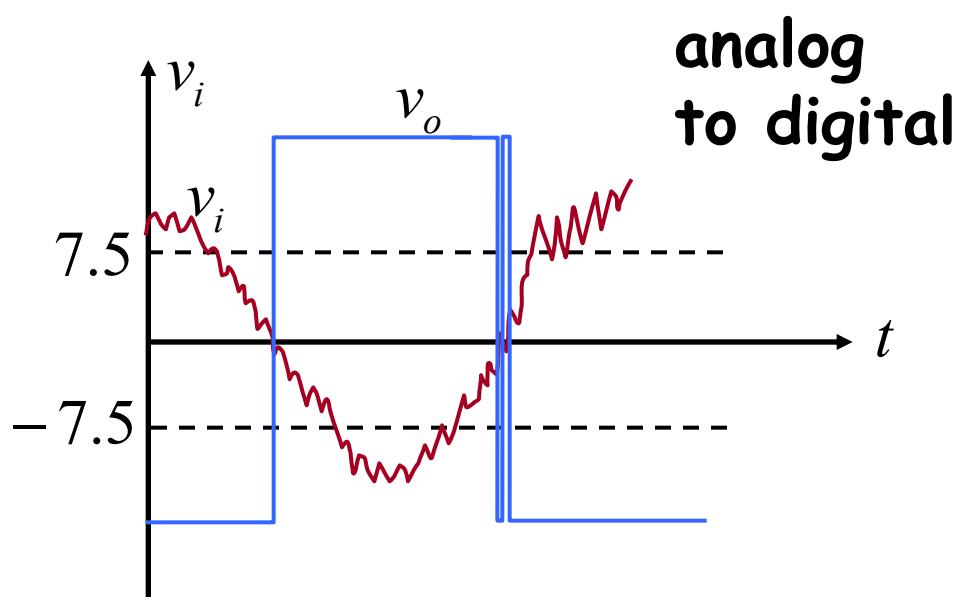
Why is hysteresis useful?



V-code: ???

Comparator

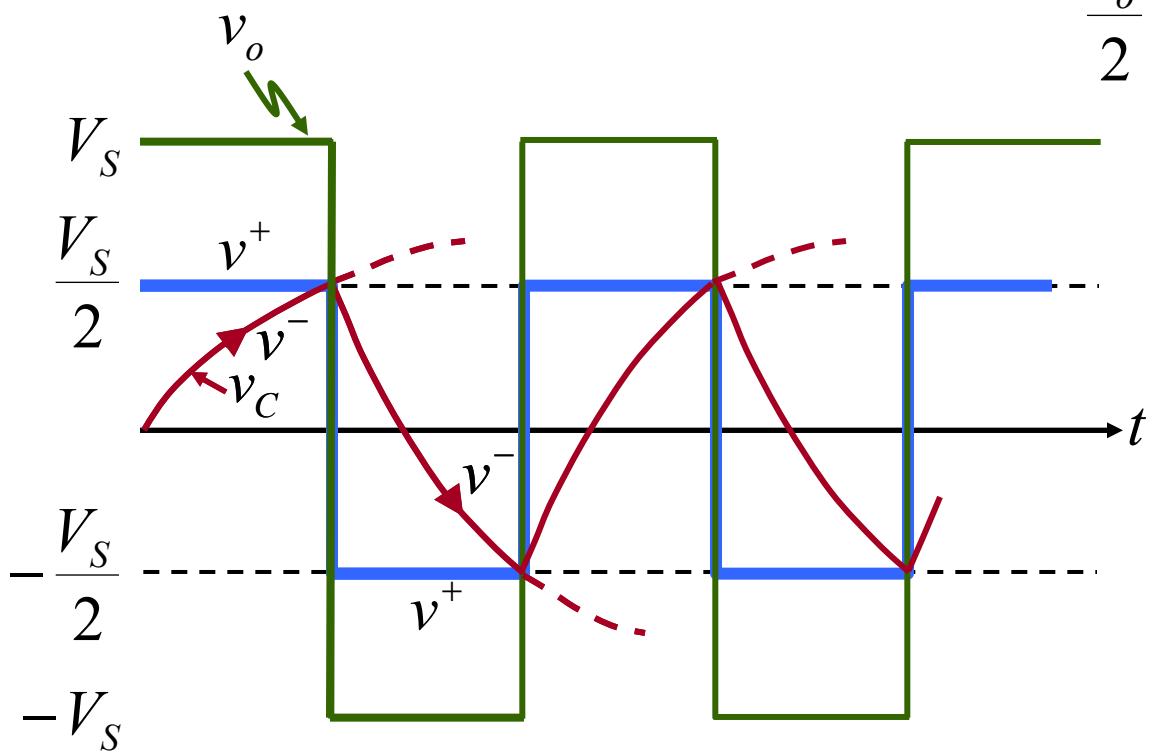
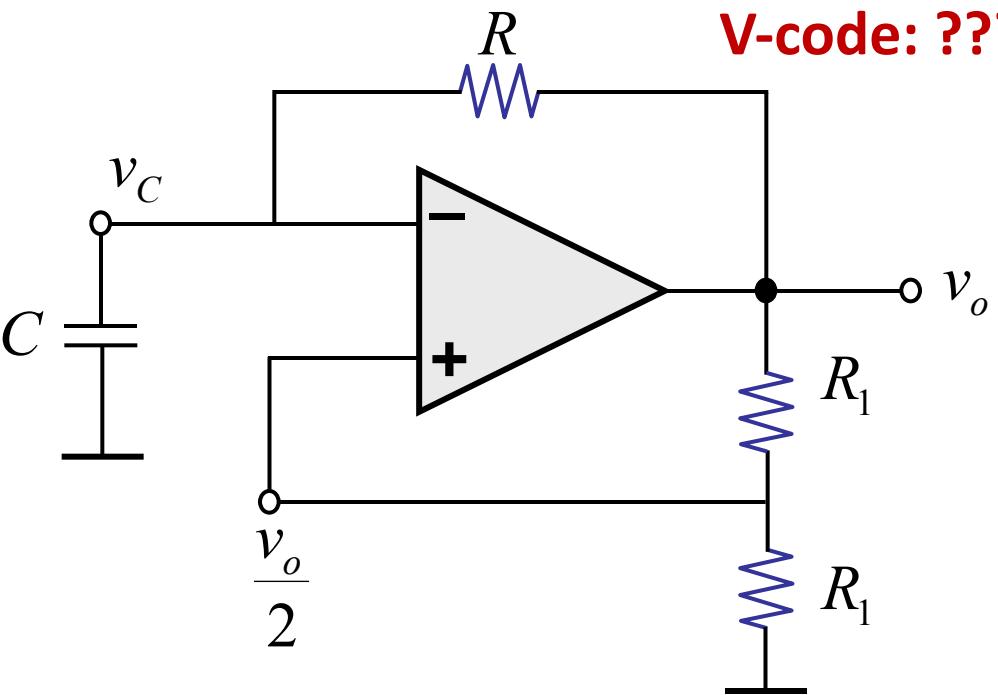
Without hysteresis



Oscillator

Can create a clock

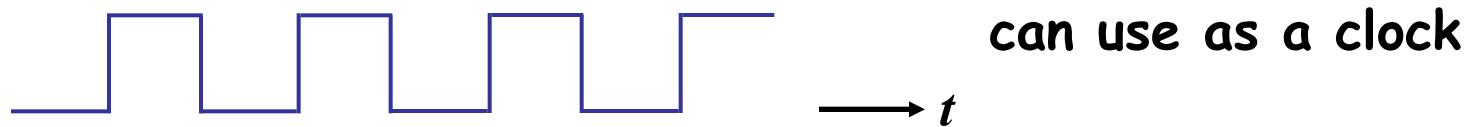
V-code: ???



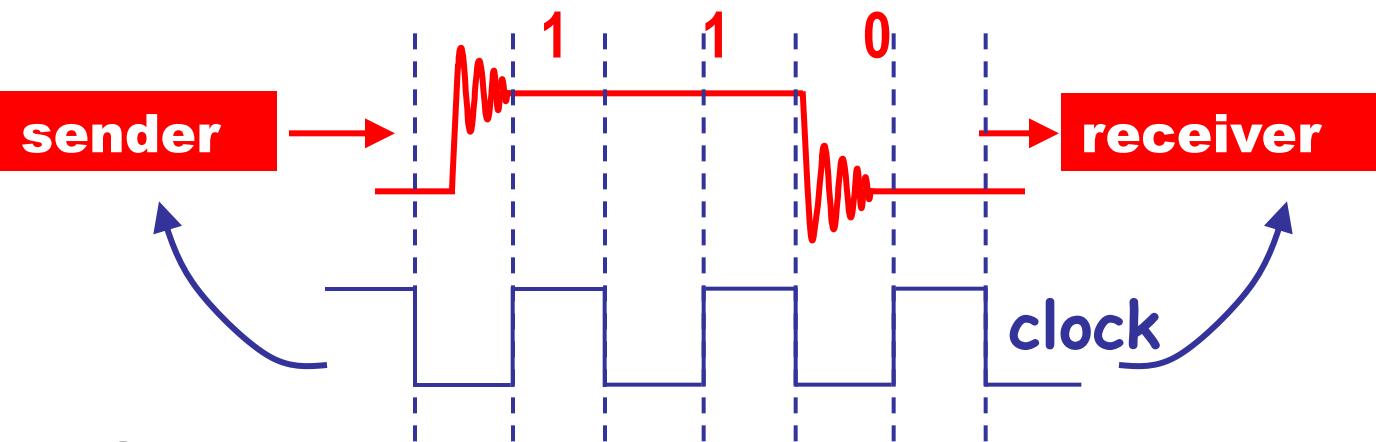
Assume $v_o = V_S$ at $t = 0$
 $v_C = 0$

Oscillator Clocks in Digital Systems V-code: ???

- We built an oscillator using an op amp.



- Why do we use a clock in a digital system?



(a) 1,1,0?

(b) When is the signal valid?

common timebase -- when to "look" at a signal
(e.g. whenever the clock is high)

→ Discretization of time
one bit of information associated with
an interval of time (cycle)