Resistive Networks

Lecture 2
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Jae W. Lee (jaewlee@snu.ac.kr)
Computer Science and Engineering
Seoul National University

***Slide credits**: Prof. Anant Agarwal at MIT*

Review: Lumped Matter Discipline (LMD)

- **Constraints we impose on ourselves to simplify our analysis**
  - Allows us to create the lumped circuit abstraction

$$\frac{\partial \phi_B}{\partial t} = 0$$

Outside elements

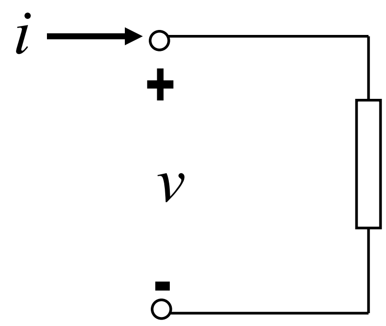
$$\frac{\partial q}{\partial t} = 0$$

Inside elements

wires **resistors** **I/V sources**

Review: Lumped Matter Discipline (LMD)

- LMD allows us to create the lumped circuit abstraction



Lumped circuit element

power consumed by element = vi [Watt]

Review: Kirchhoff's Laws

- **Maxwell's equations simplify to algebraic KVL and KCL under LMD!**

KVL:

$$\sum_j v_j = 0$$

loop

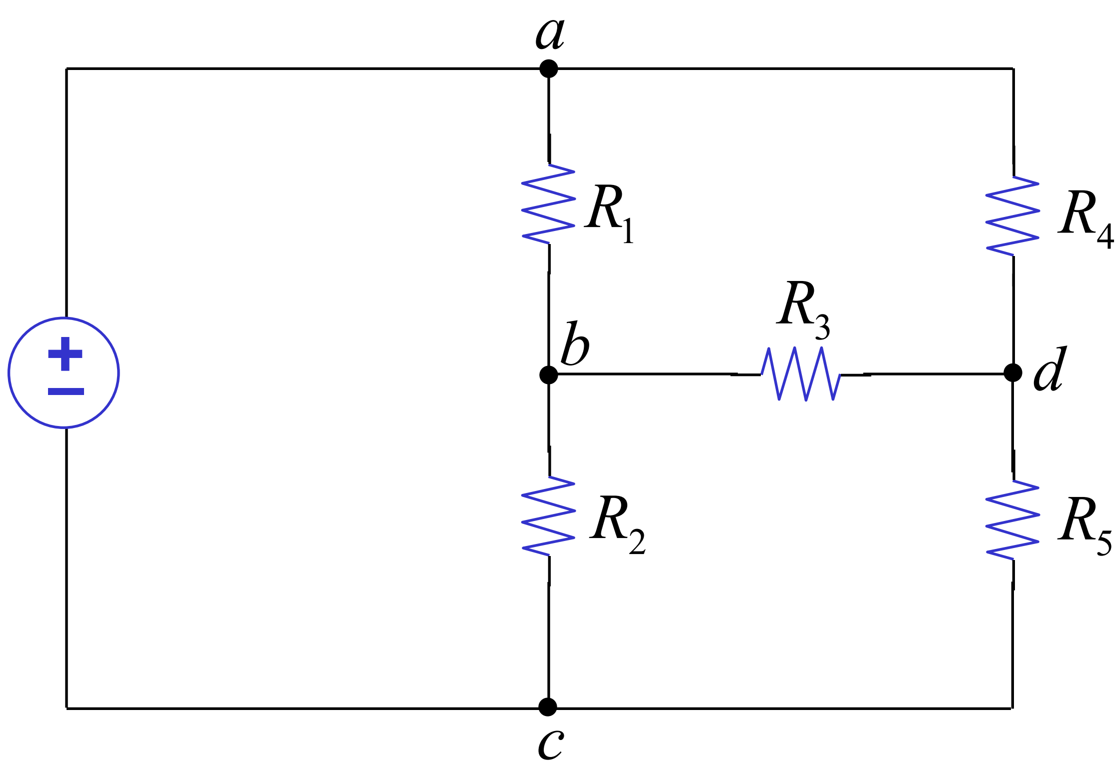
KCL:

$$\sum_j i_j = 0$$

node

Review: Kirchhoff's Laws

- **Example**



$$v_{ca} + v_{ab} + v_{bc} = 0 \quad \textbf{KVL}$$

$$i_{ca} + i_{da} + i_{ba} = 0 \quad \textbf{KCL}$$

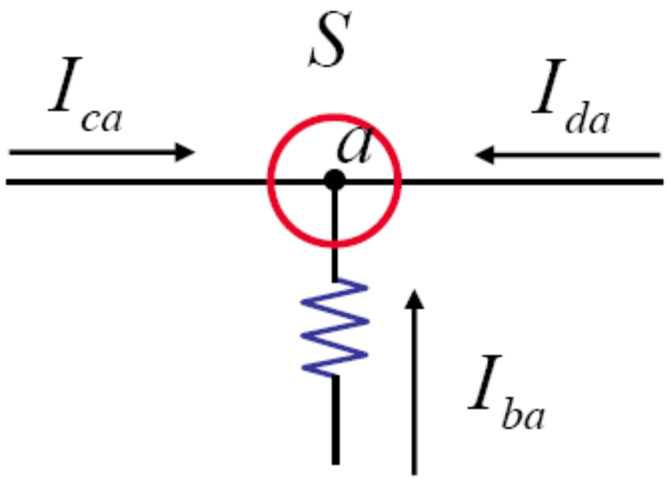
Outline

Textbook: Ch. 2.1-2.6

- **Kirchhoff's Laws: KCL and KVL**
- Circuit Analysis: Basic Method
- Intuitive Method of Circuit Analysis: Series and Parallel Simplification
- Dependent Sources

KCL (Kirchhoff's Current Law)

- The sum of all branch currents flowing into a node is zero – charge conservation.



$$I_{ca} + I_{ba} + I_{da} = 0$$

KCL: An Illustration

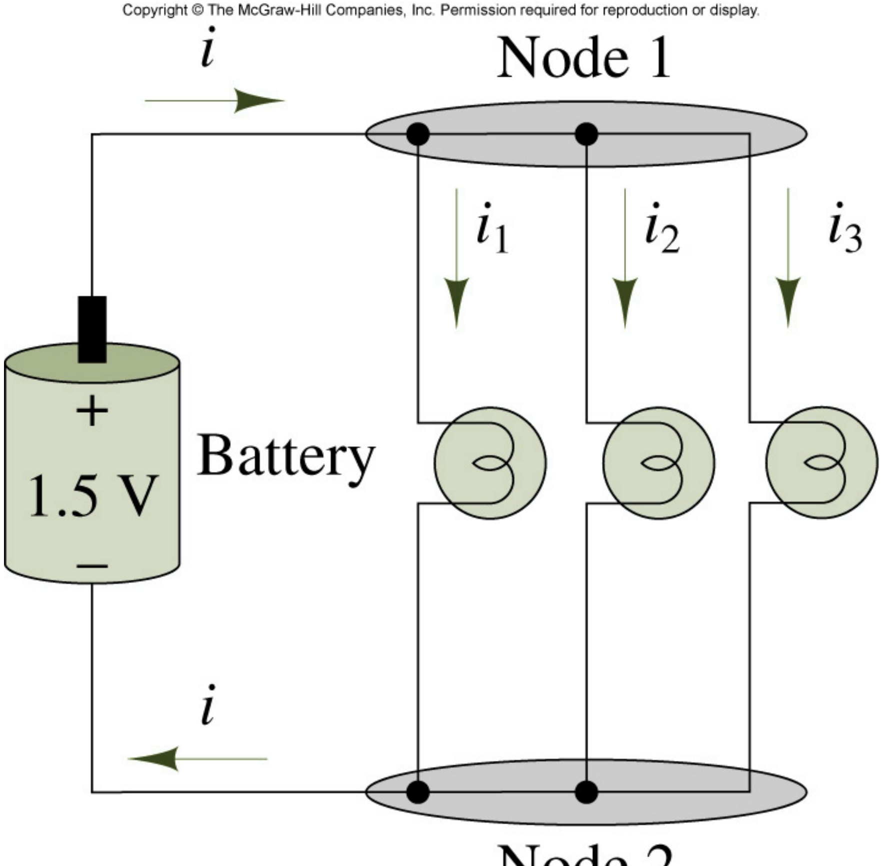
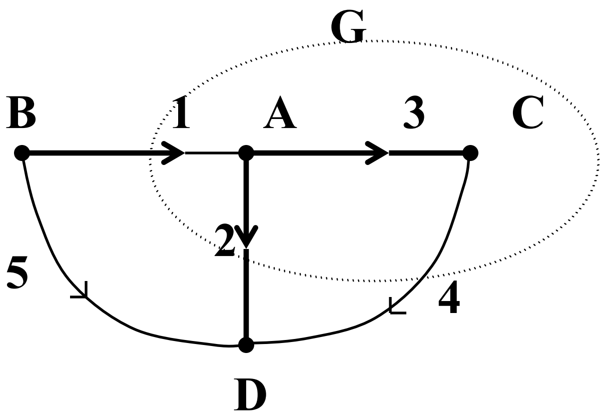


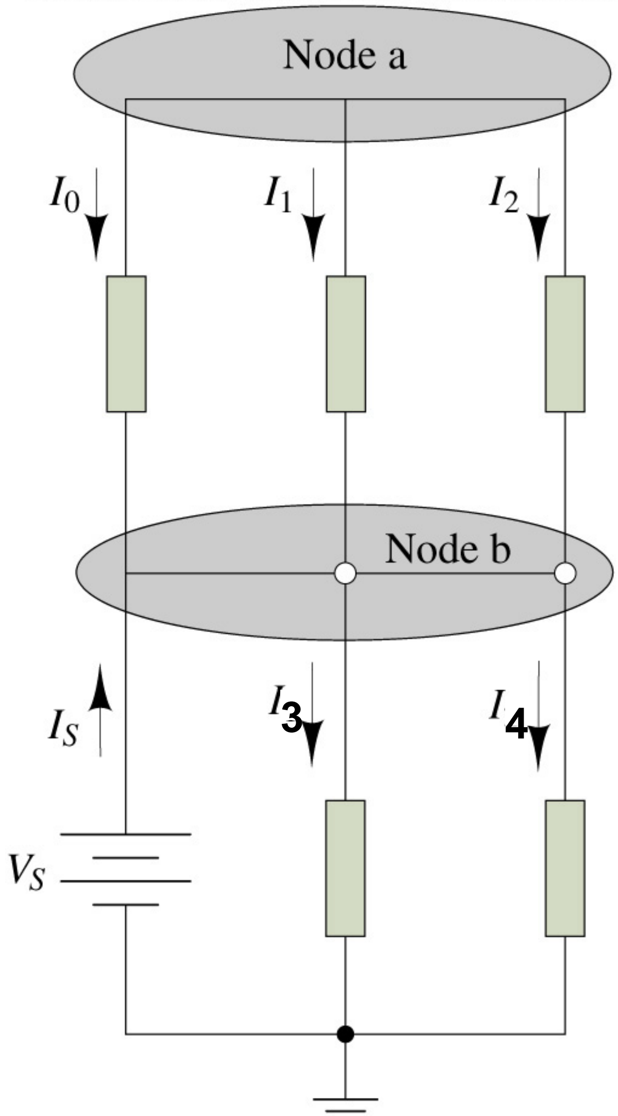
Illustration of KCL at node 1: $-i + i_1 + i_2 + i_3 = 0$



Node: $i_1 - i_2 - i_3 = 0$

G: $i_1 - i_2 - i_4 = 0$

KCL: Demonstration

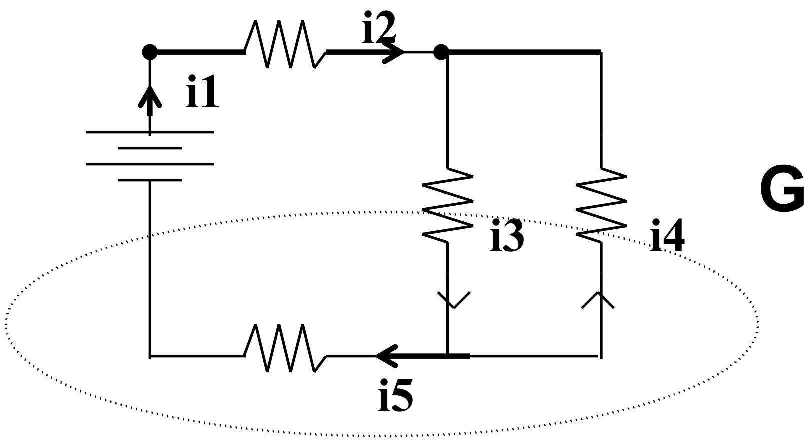


Is = 5A, I1 = 2A,
I2 = -3A, I3 = 1.5 A

Find I0 and I4

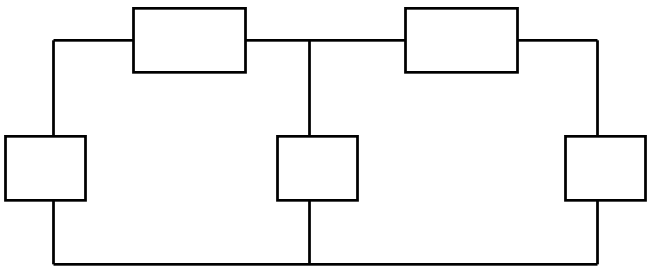
KCL: Example

Write down the KCL equation for G.

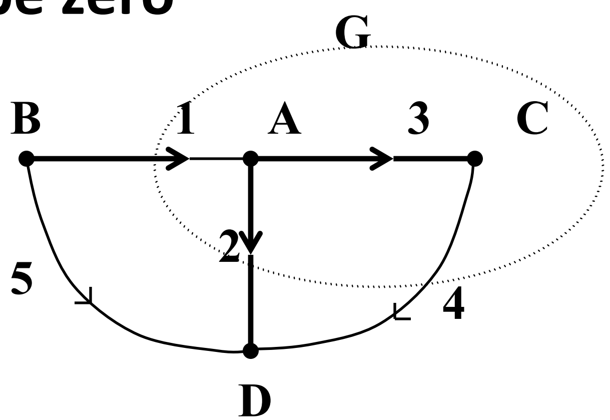


KVL (Kirchhoff's Voltage Law)

- The algebraic sum of the branch voltages around any closed path in a network must be zero



(a)



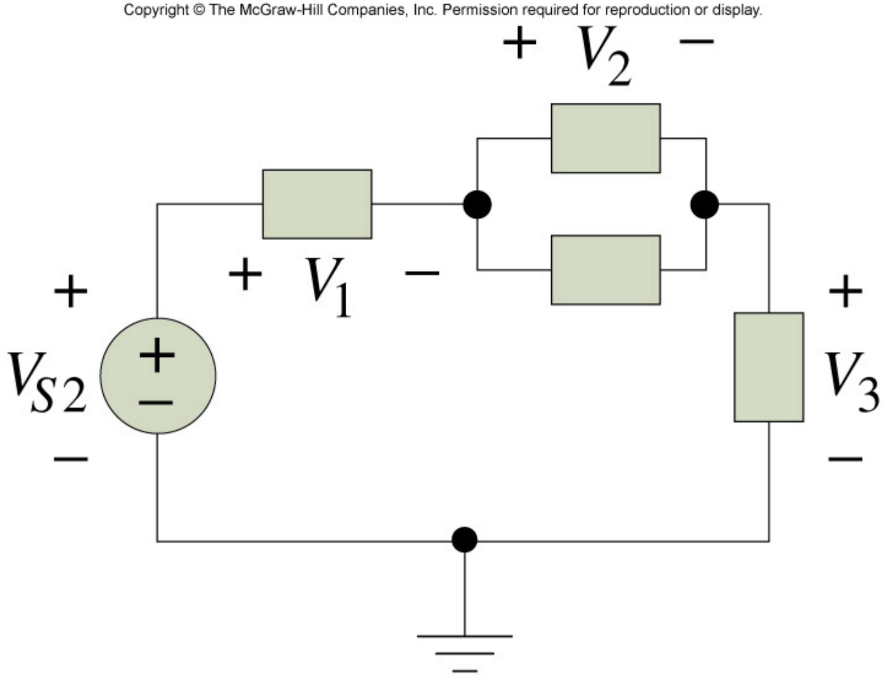
(b)

Node sequence B,A,D,B: $v_{BA} + v_{AD} + v_{DB} = 0$

Using node voltages: $v_{BA} = e_B - e_A, \quad e_D = 0$

(reference node = D, reference voltage = 0)

KVL: Example



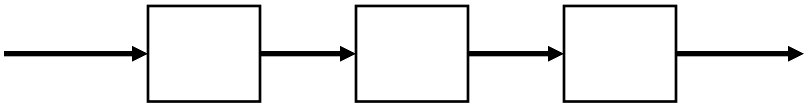
Vs2 = 12 V, V1 = 6V, V3 = 1V

Find V2.

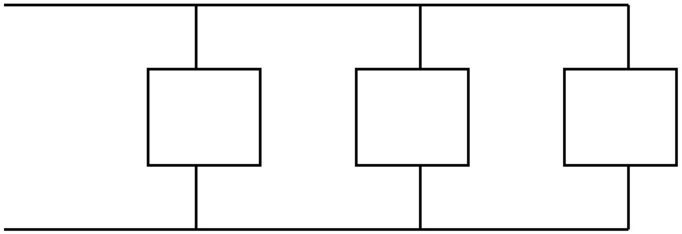
Note: A helpful mnemonic for writing KVL equations is to assign the polarity to a given voltage in accordance with the first sign encountered when traversing the voltage around the loop.

Some Useful Facts

- The branch currents passing through series-connected elements must be the same.



- The voltages across parallel-connected elements must be the same.



Outline

Textbook: Ch. 2.1-2.6

- Kirchhoff's Laws: KCL and KVL
- **Circuit Analysis: Basic Method**
- Intuitive Method of Circuit Analysis: Series and Parallel Simplification
- Dependent Sources

Circuit Analysis: Basic Method

- **Goal: Find all element v's and i's**
  1. write element v-i relationships (from lumped circuit abstraction)
  2. write KCL for all nodes
  3. write KVL for all loops

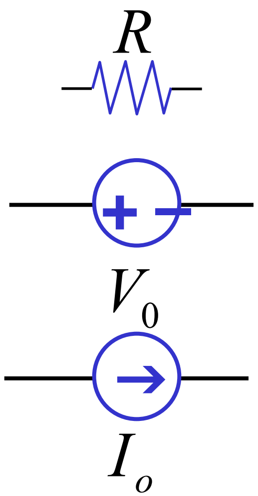
Circuit Analysis: Basic Method

- **Step 1: Element relationships**
  - Example: 3 lumped circuit elements

For R, $V = IR$

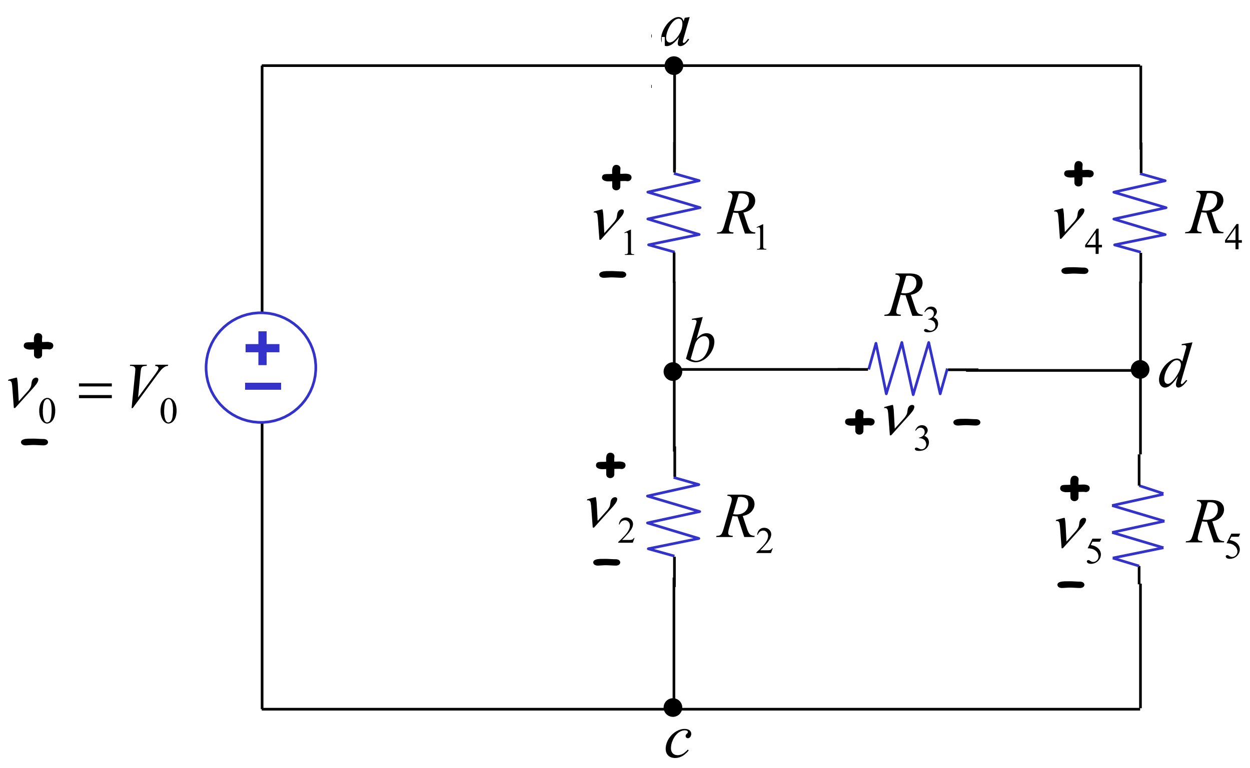
For voltage source, $V = V_0$

For current source, $I = I_0$



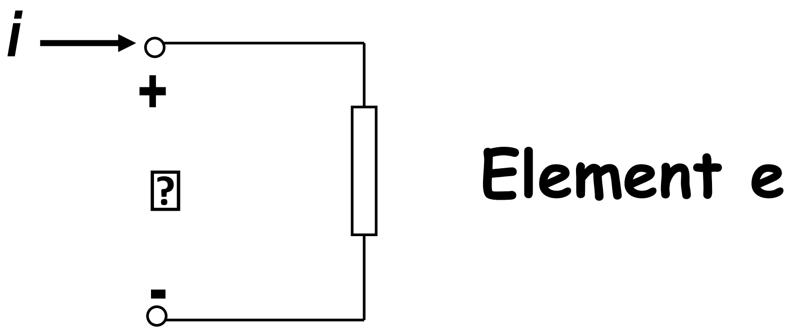
Circuit Analysis: Basic Method

- **KVL, KCL example (Step 2 & 3)**
  - A demo circuit



Circuit Analysis: Basic Method

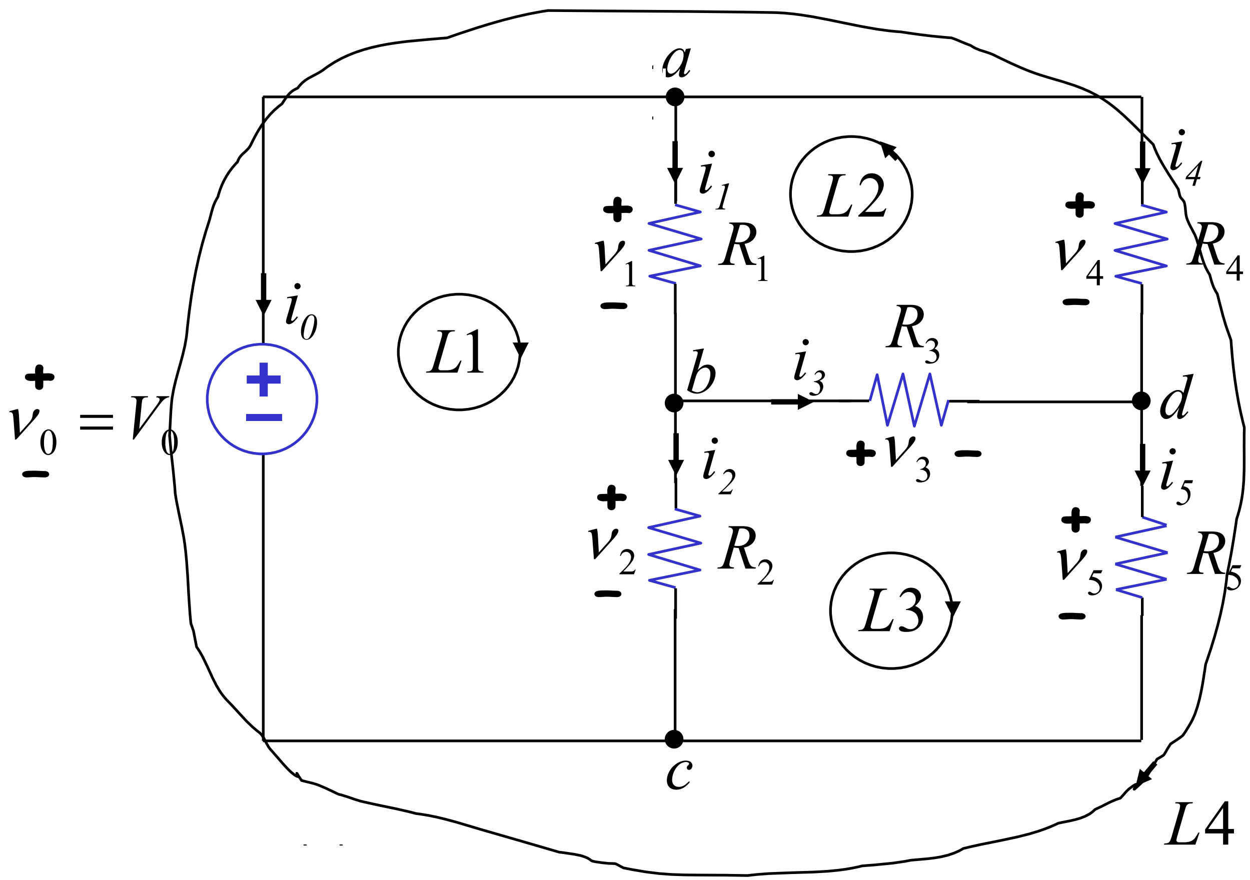
- **Associated variables discipline**
  - Current is taken to be positive going into the positive voltage terminal



Then power consumed by element e $\Big\} = vi$ **is positive**

Circuit Analysis: Basic Method

- **KVL, KCL example (Step 2 & 3)**
  - A demo circuit



Circuit Analysis: Basic Method

- **Put all steps together!**

$v_0 \ldots v_5, i_0 \ldots i_5$ 12 unknowns

1. Element relationships (v,i) 6 equations

$v_0 = V_0 \longleftarrow$ given $\quad v_3 = i_3 R_3$

$v_1 = i_1 R_1 \quad v_4 = i_4 R_4$

$v_2 = i_2 R_2 \quad v_5 = i_5 R_5$

2. KCL at the nodes 3 independent equations

a: $i_0 + i_1 + i_4 = 0$

b: $i_2 + i_3 - i_1 = 0$

d: $i_5 - i_3 - i_4 = 0$

e: $-i_0 - i_2 - i_5 = 0$ redundant

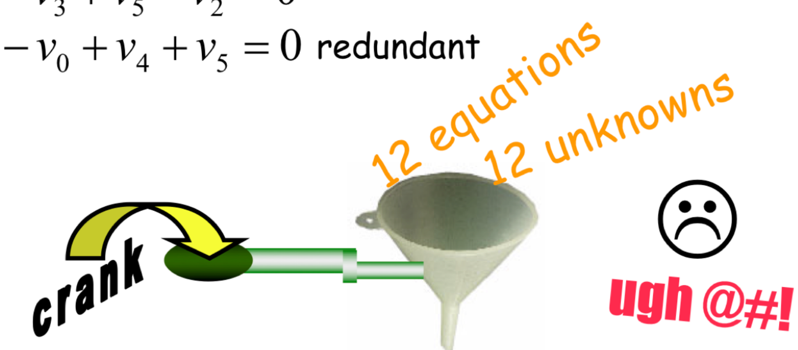
3. KVL for loops 3 independent equations

L1: $-v_0 + v_1 + v_2 = 0$

L2: $v_1 + v_3 - v_4 = 0$

L3: $v_3 + v_5 - v_2 = 0$

L4: $-v_0 + v_4 + v_5 = 0$ redundant



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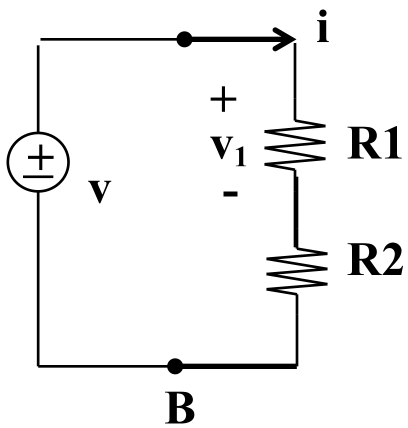
- **Intuitive Method of Circuit Analysis: Series and Parallel Simplification**
- Dependent Sources

Series and Parallel Simplification

- Voltage divider and series resistors



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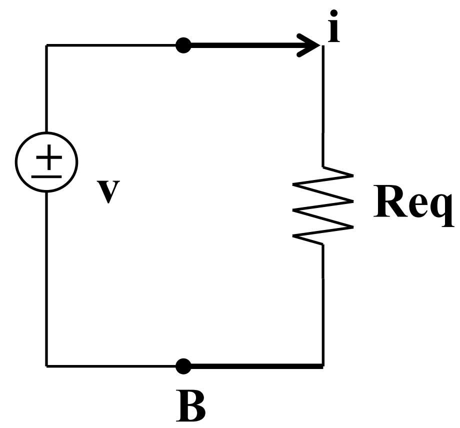


(a) Voltage divider

(b) Equivalent resistance

What does "equivalent" mean?

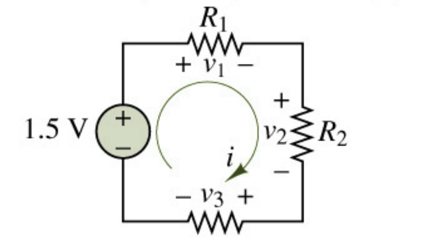
$$v_1 = \frac{R_1}{R_1 + R_2} v$$

Series and Parallel Simplification

- **Voltage divider and series resistors**

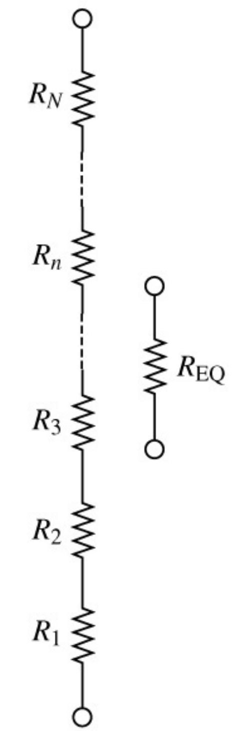
R1 = 10 kΩ, R2 = 6 kΩ,
R3 = 8 kΩ

Find V3.



The current i flows through each of the four series elements. Thus, by KVL,

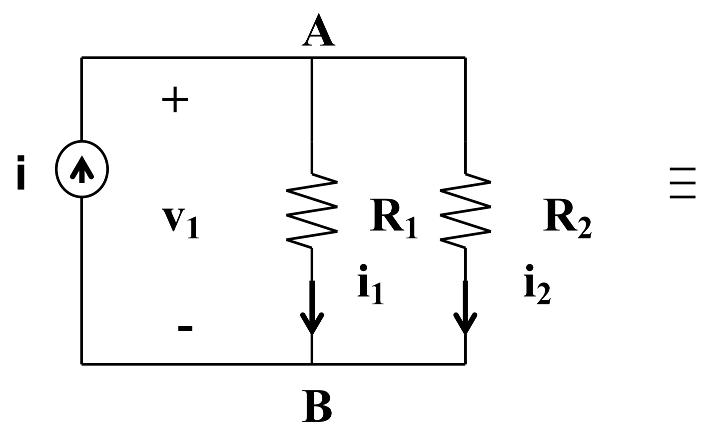
$$1.5 = v_1 + v_2 + v_3$$

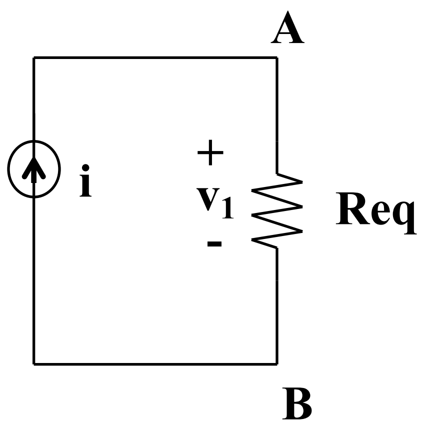


N series resistors are equivalent to a single resistor equal to the sum of the individual resistances.

Series and Parallel Simplification

- **Current divider and parallel resistors**





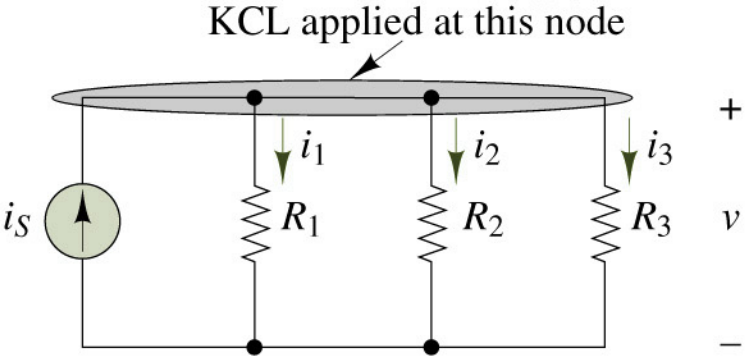
(a) Current divider

(b) Equivalent resistance

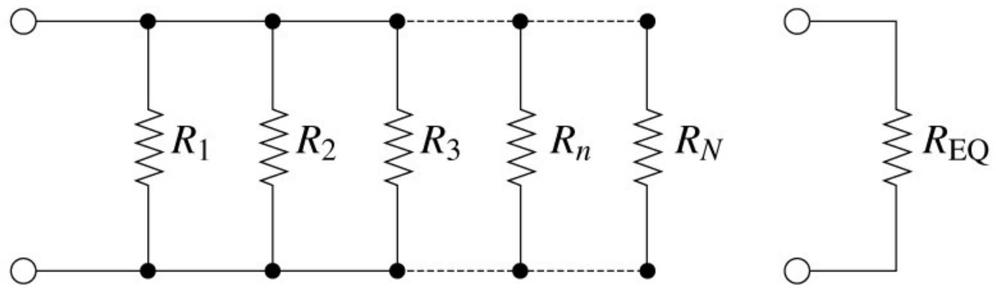
$$i_1 = \frac{G_1}{G_1 + G_2} i = \frac{R_2}{R_1 + R_2} i$$

Series and Parallel Simplification

- **Current divider and parallel resistors**



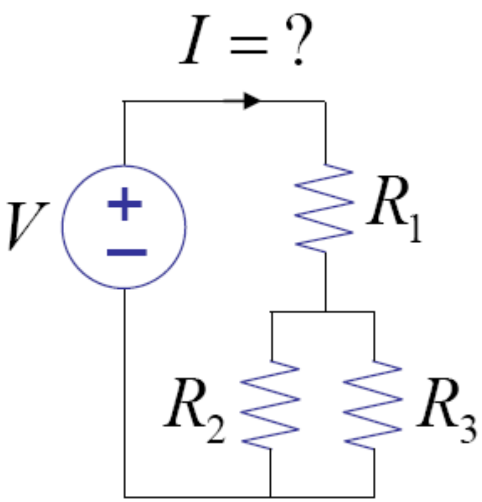
The voltage v appears across each parallel element; by KCL, $i_S = i_1 + i_2 + i_3$



N resistors in parallel are equivalent to a single equivalent resistor with resistance equal to the inverse of the sum of the inverse resistances.

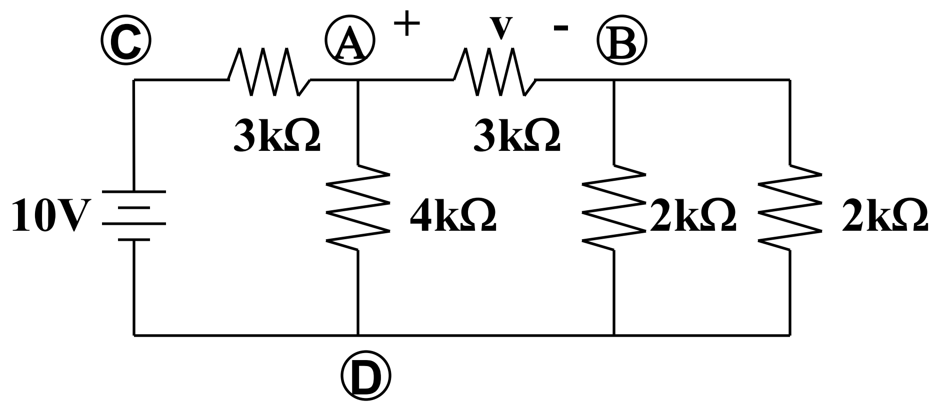
Series and Parallel Simplification

- **Example 1**



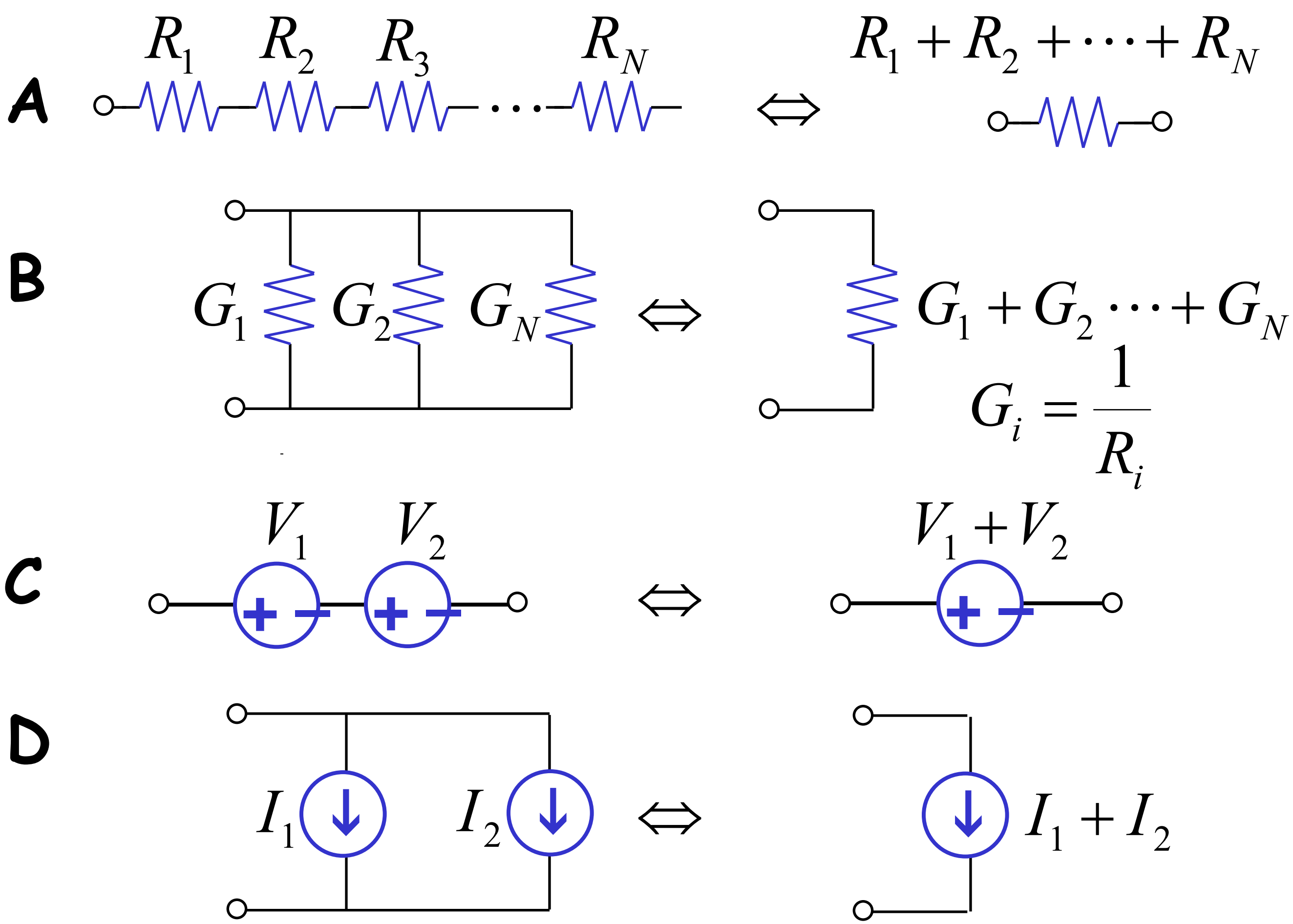
Example 2

Find the voltage (v) between nodes A and B.



Series and Parallel Simplification

- **Element combination rules**



Outline

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- Kirchhoff's Laws: KCL and KVL
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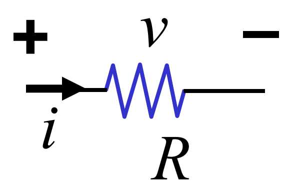
- **Dependent Sources**

Dependent Sources

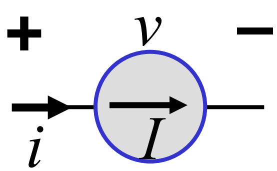
- Seen previously… **independent sources!**

Resistor



$$i = \frac{v}{R}$$

Independent
Current source

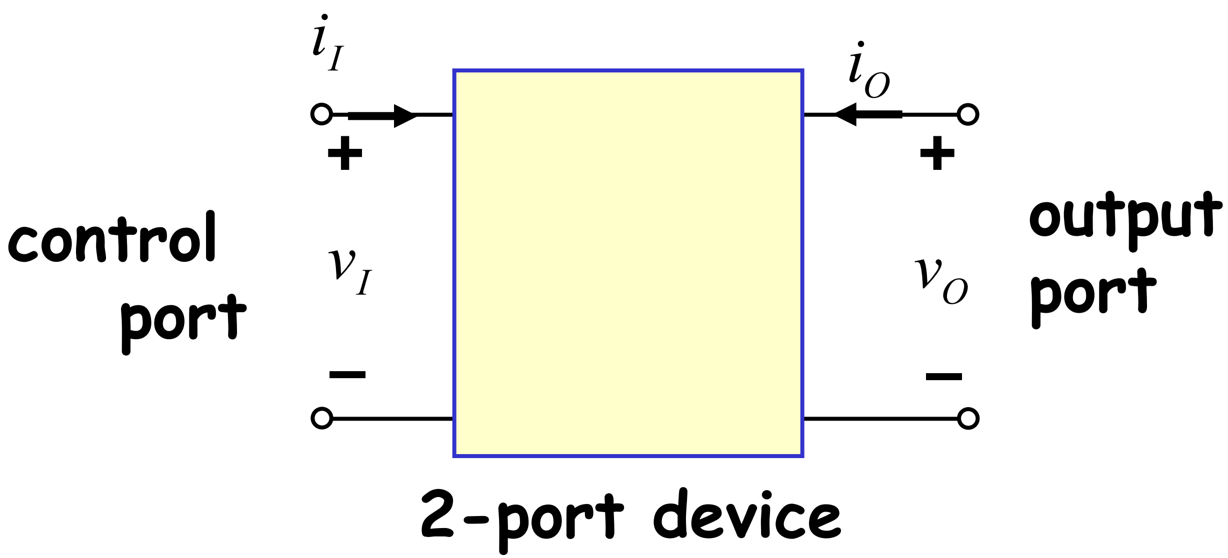


$$i = I$$

2-terminal 1-port devices

Dependent Sources

- **New type of device: Dependent source**

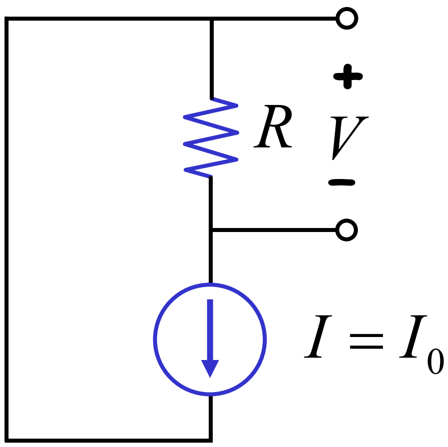


- e.g., Voltage Controlled Current Source (VCCS)
- Current at output port is a function of voltage at the input port

Dependent Sources: Examples

Example 1: Find V

independent
current
source

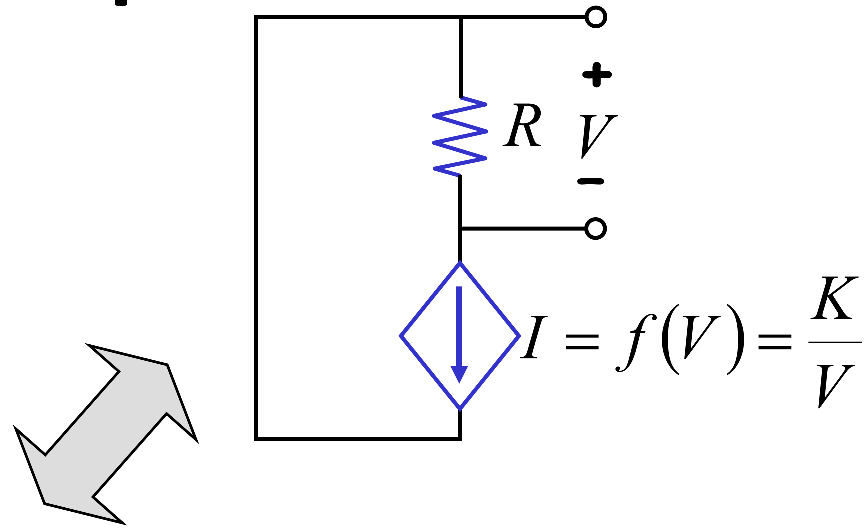


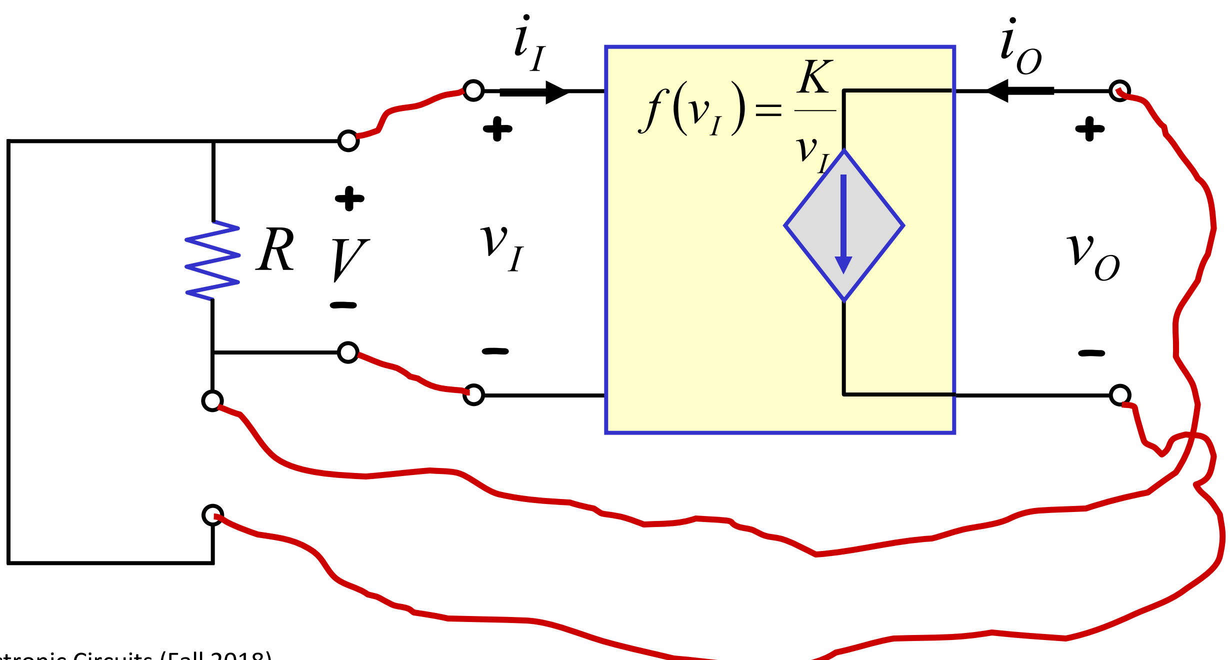
$$V = I_0 R$$

Dependent Sources: Examples

Example 2: Find V

voltage
controlled
current
source

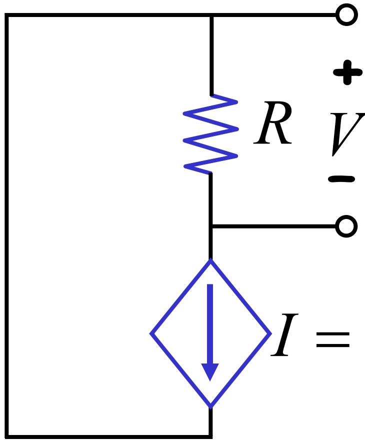




Dependent Sources: Examples

Example 2: Find V

voltage
controlled
current
source



$$I = f(V) = \frac{K}{V}$$

e.g. $K = 10^{-3}\ Amp{\cdot}Volt$

$R = 1k\Omega$

$$V = IR = \frac{K}{V} R$$

or $$V^2 = KR$$

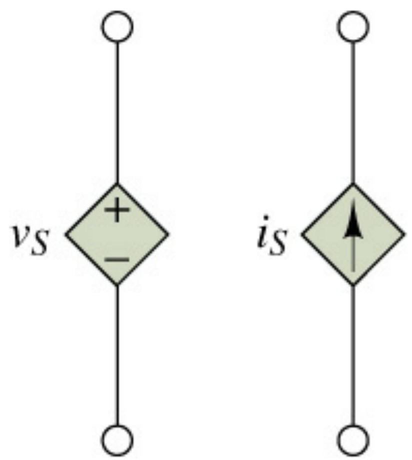
or $$V = \sqrt{KR}$$

$$= \sqrt{10^{-3} \cdot 10^{3}}$$

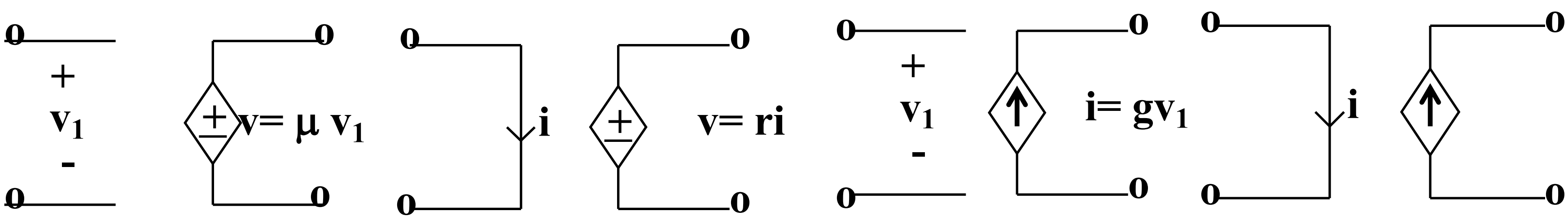
$$= 1\ Volt$$

Dependent Sources

- **Symbols for (other) dependent sources**
  - We will heavily use them to model amplifiers (in Chapter 7)!



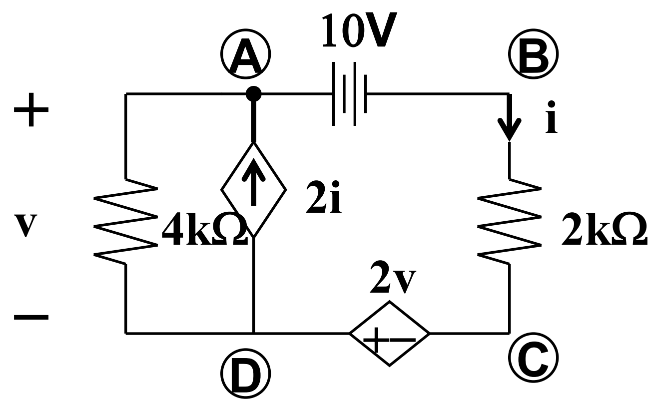
Source type	Relationship
Voltage controlled voltage source (VCVS)	$v_S = \mu v_x$
Current controlled voltage source (CCVS)	$v_S = r i_x$
Voltage controlled current source (VCCS)	$i_S = g v_x$
Current controlled current source (CCCS)	$i_S = \beta i_x$



(a)VCVS (b) CCVS (c) VCCS (d) CCCS

Dependent Sources: Another Example

Find v. How much power does 4 kΩ resistor consume?



Now, the voltage source is changed from 10V to 5V.
How much power does 4kΩ resistor consume?