V-code: ???

Energy Storage Elements (1): Capacitors

Lecture 10
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Jae W. Lee (jaewlee@snu.ac.kr)
Computer Science and Engineering
Seoul National University

Slide credits*: Prof. Anant Agarwal at MIT*

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Outline

Textbook: 9.1, 9.2 (Capacitors only)

- **Constitutive Laws**
- **Series and Parallel Connections**

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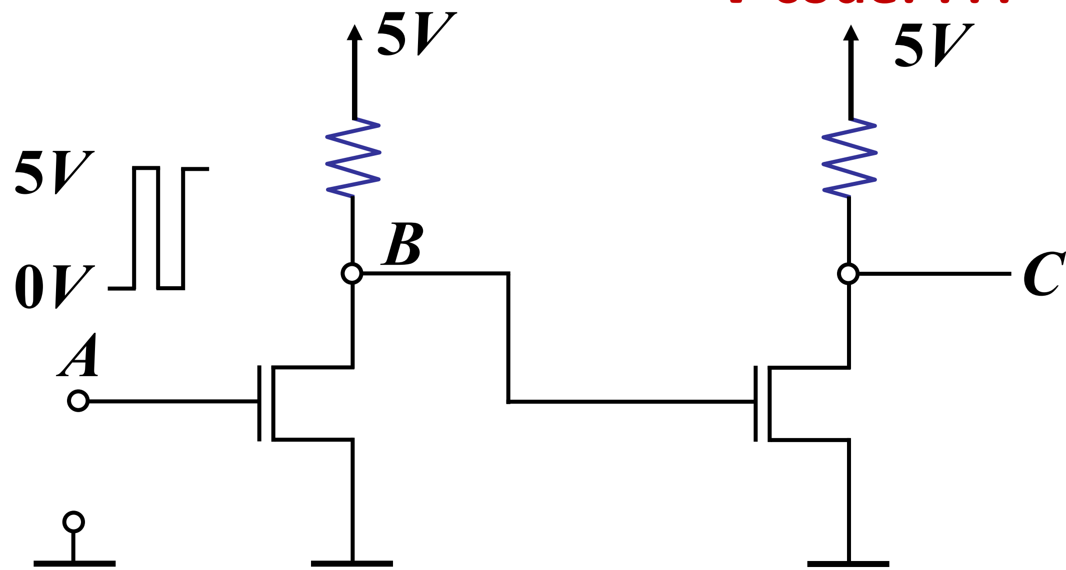
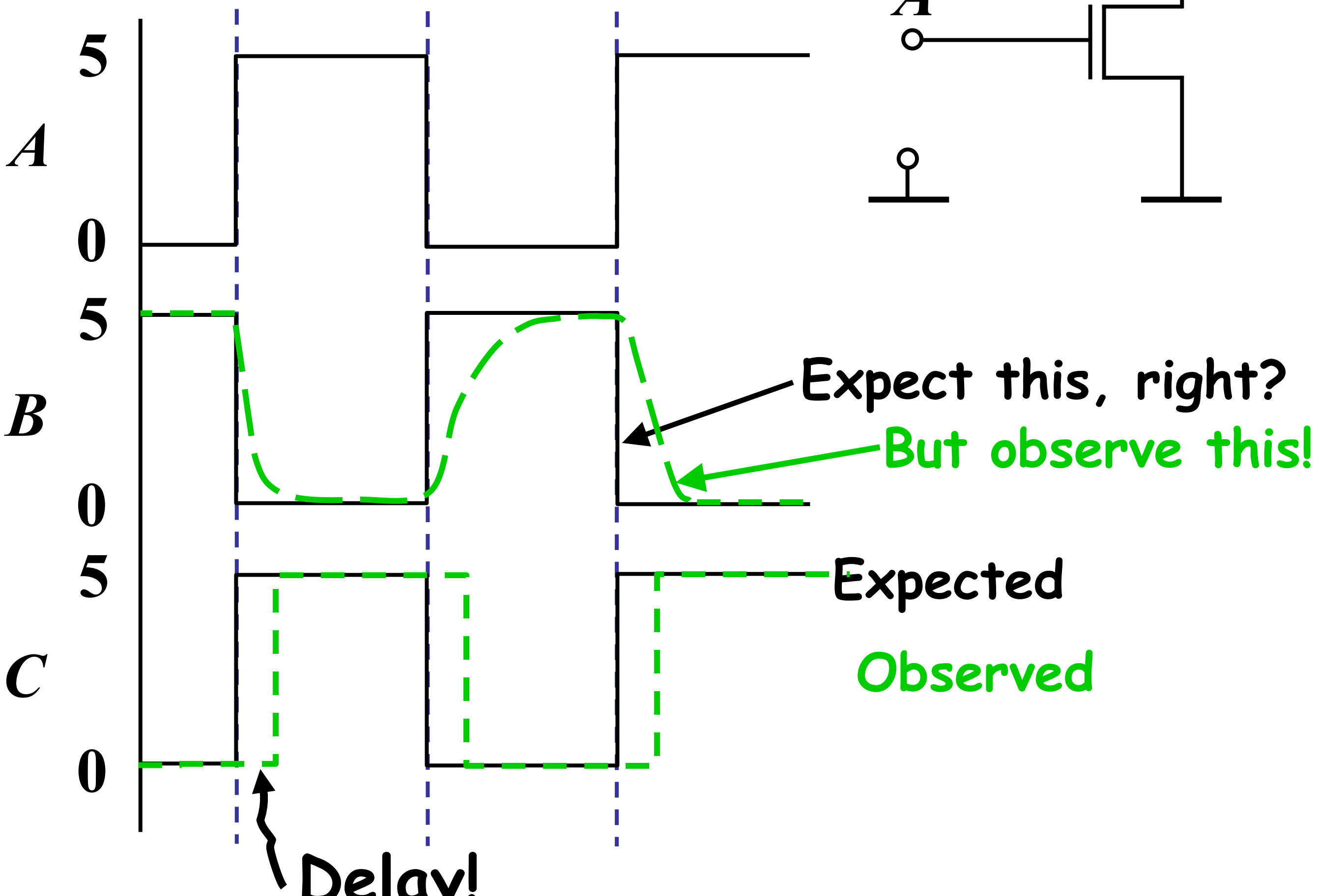
Motivation

- Example: Inverter delay





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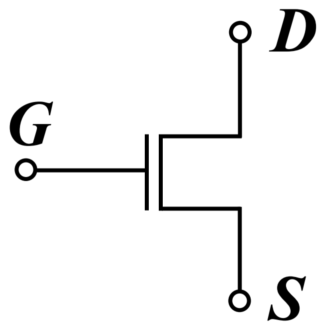
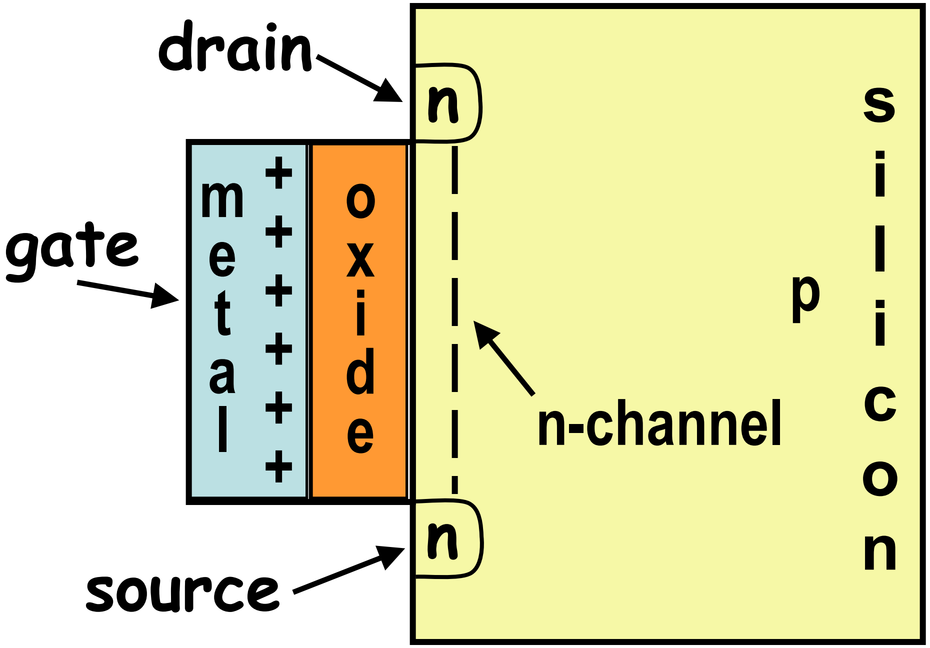
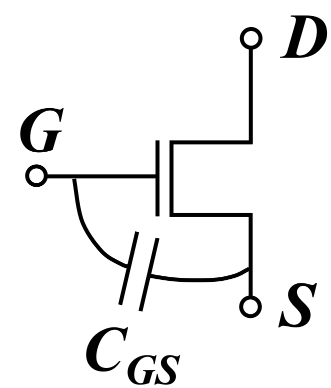
MOSFET Modeling with Capacitor



n-channel MOSFET symbol



n-channel MOSFET



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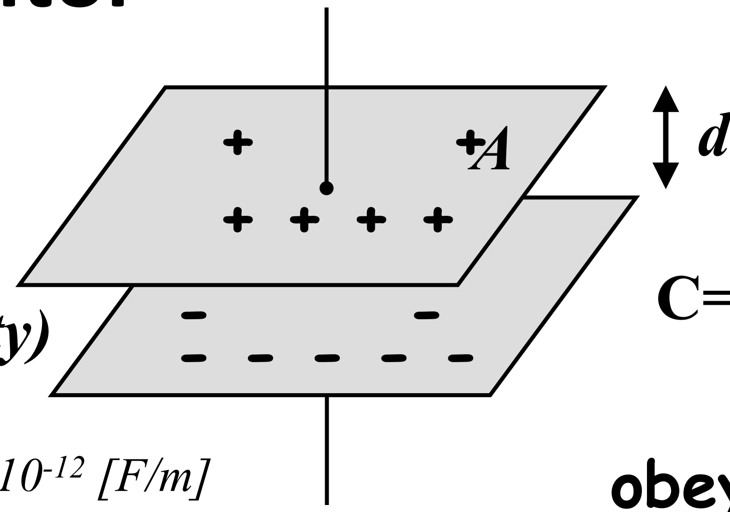
Ideal Linear Capacitor



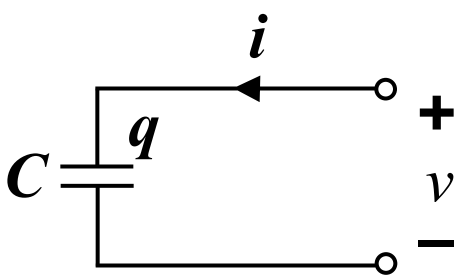
ε (permittivity)

$C=(\varepsilon A)/d$ [F]

*ε of air = 8.854*10^-12 [F/m]*

obeys LMD!
total charge on capacitor

$= +q - q = 0$



$$q = C \, v$$

coulombs farads volts

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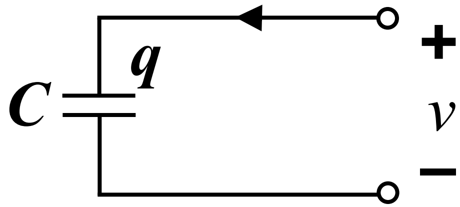
Ideal Linear Capacitor



$$q = C\,v$$

$$v(t) =$$

$$i = \frac{dq}{dt}$$

$$= \frac{d(Cv)}{dt}$$

$$= C\frac{dv}{dt}$$

A capacitor is a memory device
→ history matters!

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Ideal Linear Capacitor

- **Observations from $i(t) = C\, dv/dt$**
  - For DC voltage: Capacitor is an open circuit
  - For i to be finite, v(t) must be continuous
  - Typical units: uF ~ pF

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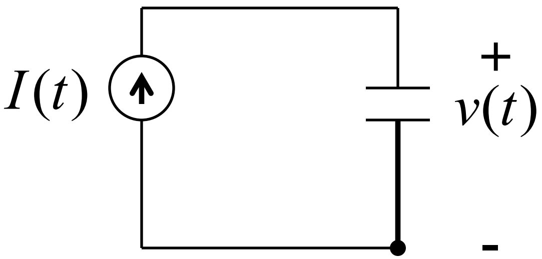
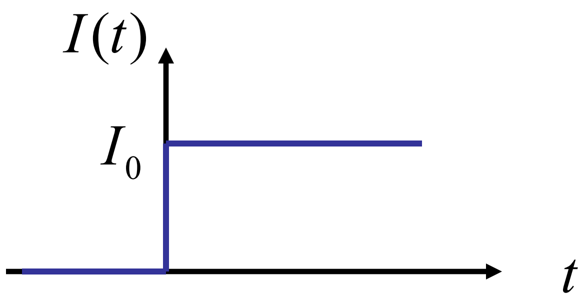
Current Source and Capacitor

- What is ***v(t)***?





Step input

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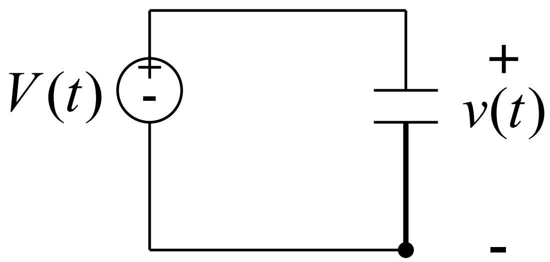
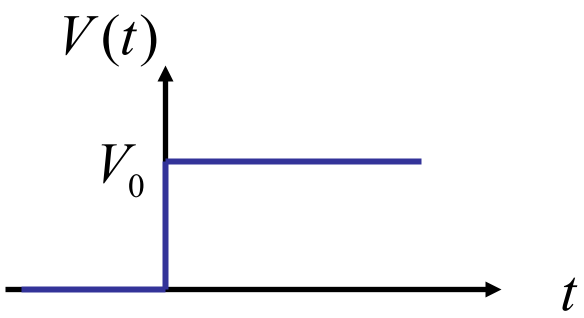
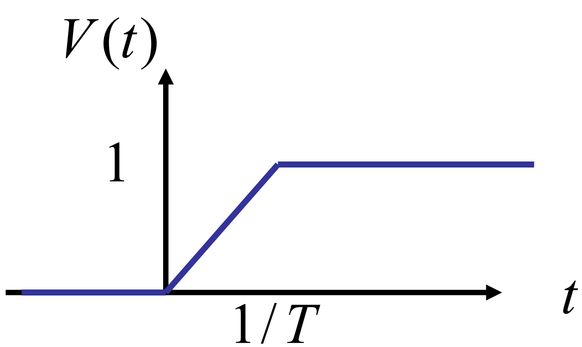
Voltage Source and Capacitor

- **Voltage across a capacitor must be continuous!**





Step input



Ramp input

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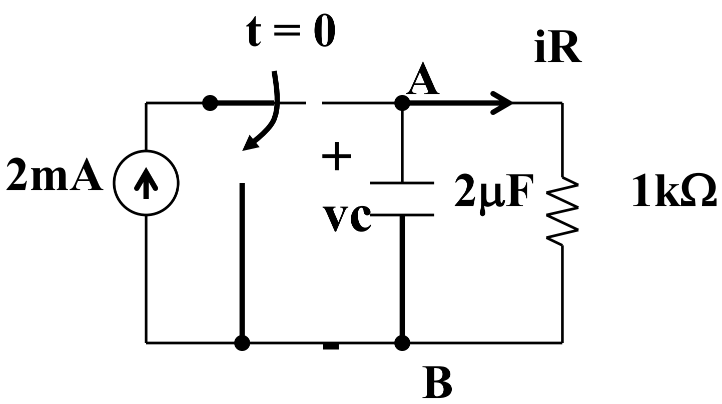
Capacitor Voltage is Continuous



$$i_R(0-) = 1mA$$

$$\frac{dv_C}{dt}(0-) = ? \quad \frac{dv_C}{dt}(0+) = ?$$

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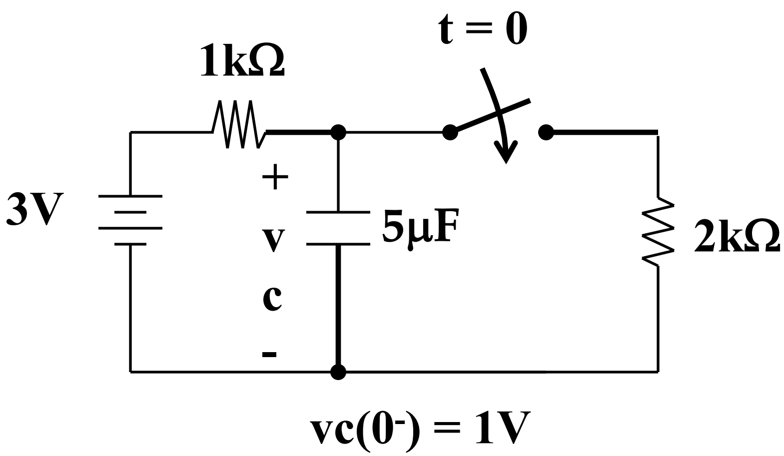
Exercise

- **Switch is closed at t=0. Find** $\frac{dv_C}{dt}(0+)$ **.**



V-code: ???

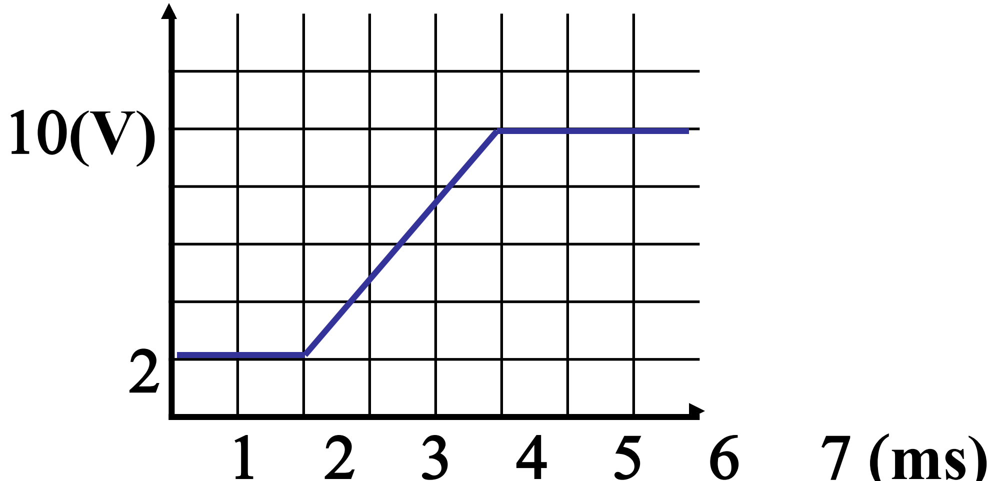
Energy Storage-Memory Device

- **Stored energy = $\omega_E(t)$**

$$\frac{d\omega_E(t)}{dt} = i(t)v(t)$$

$$d\omega_E(t) = v(t)\big(i(t)dt\big) = v(t)dq(t)$$

$$\omega_E = \int_0^q v\, dx = \frac{q^2(t)}{2C} = \frac{Cv(t)^2}{2}$$



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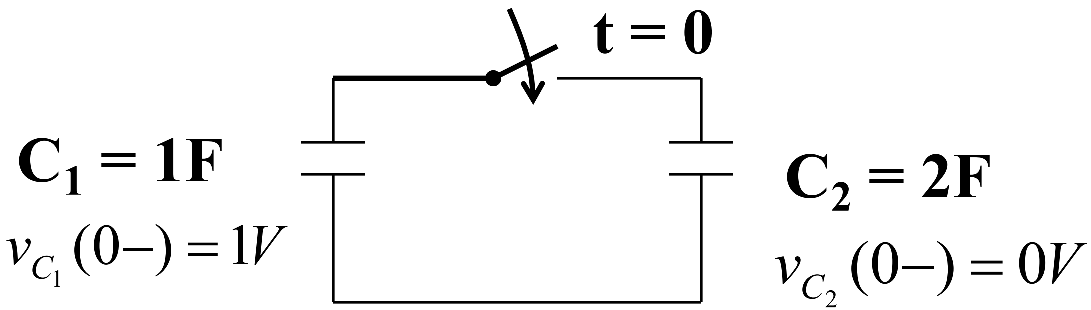
Capacitor Loop: Charge Conservation



Question: Is energy conserved?

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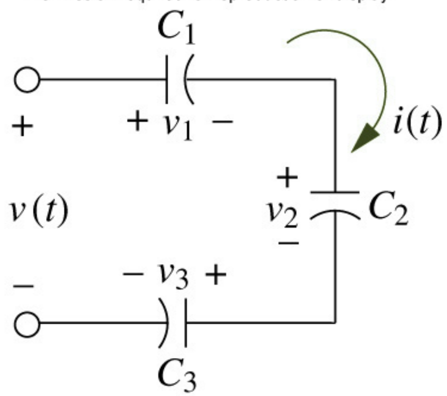
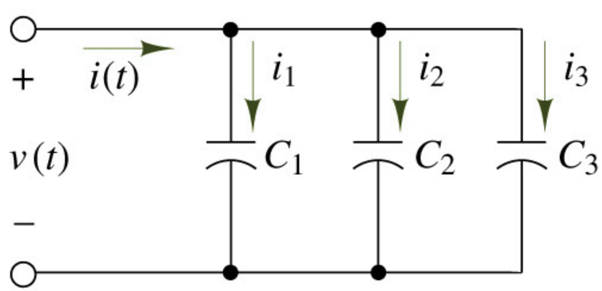
Combining Capacitors in a Circuit



$$C_{EQ} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Capacitances in series combine like resistors in parallel



$$C_{EQ} = C_1 + C_2 + C_3$$

Capacitances in parallel add

Series connection:

Parallel connection: