The Small-Signal Model

Lecture 9
October 18th, 2018

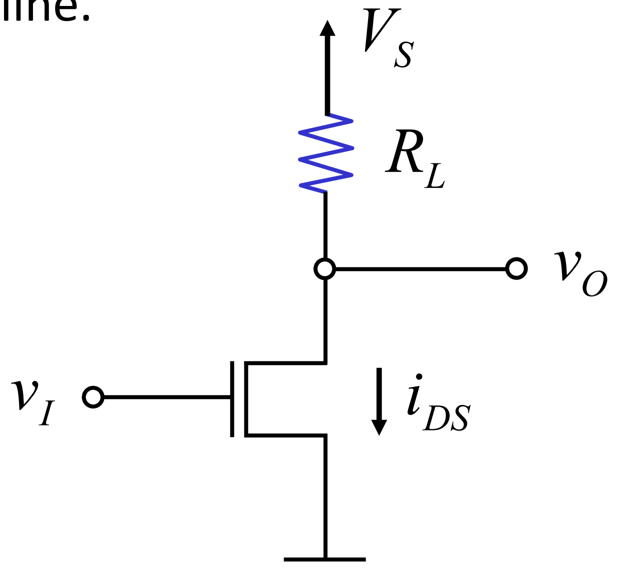
Jae W. Lee (jaewlee@snu.ac.kr)
Computer Science and Engineering
Seoul National University

***Slide credits**: [CS:APP3e] slides from CMU; [COD5e] slides from Elsevier Inc.*

Review: MOSFET Amplifier

- **Saturation discipline — operate MOSFET only in saturation region**

- **Large signal analysis: does two things**
  - 1. Find v_O vs v_I under saturation discipline.
  - 2. Valid v_I , v_O ranges under saturation discipline.



Review: Large Signal Analysis

① **Find** v_O vs v_I

$$v_O = V_S - \frac{K}{2}\left(v_I - 1\right)^2 R_L$$

valid for $v_I \geq V_T$

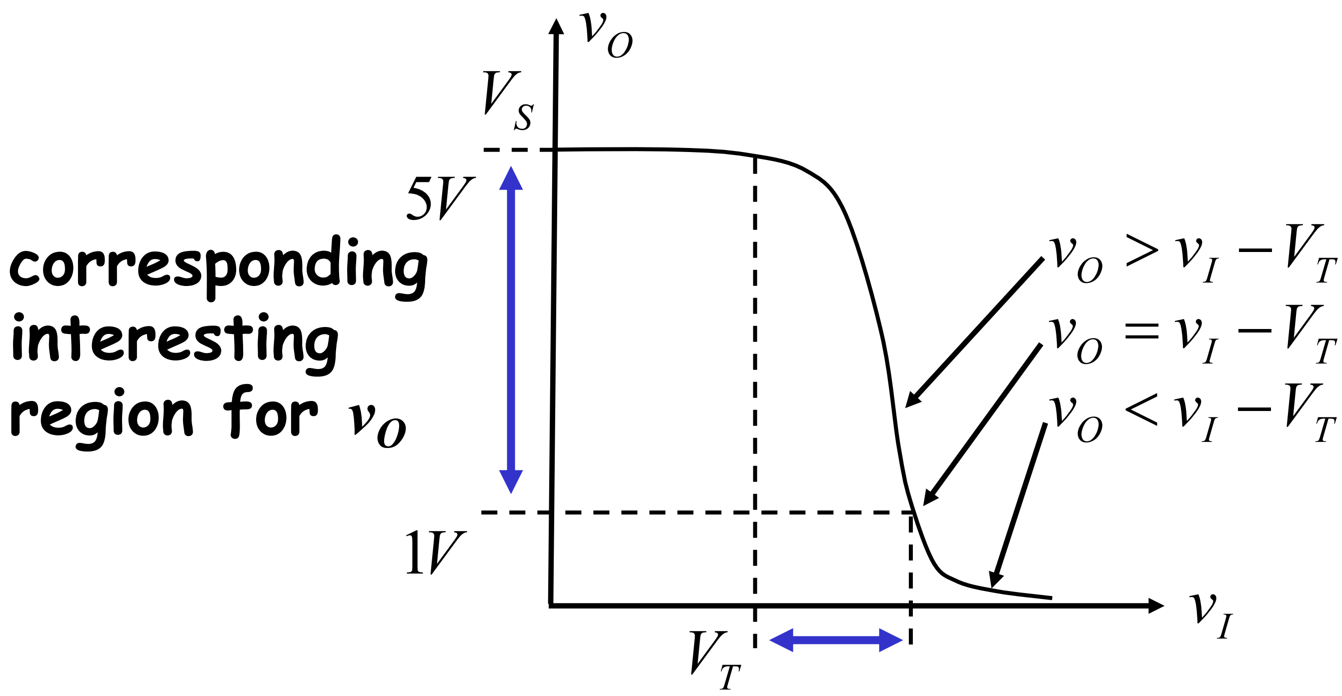
and

$v_O \geq v_I - V_T$

(same as $i_{DS} \leq \frac{K}{2} v_O{}^2$**)**

Review: Large Signal Analysis

② Valid operating ranges

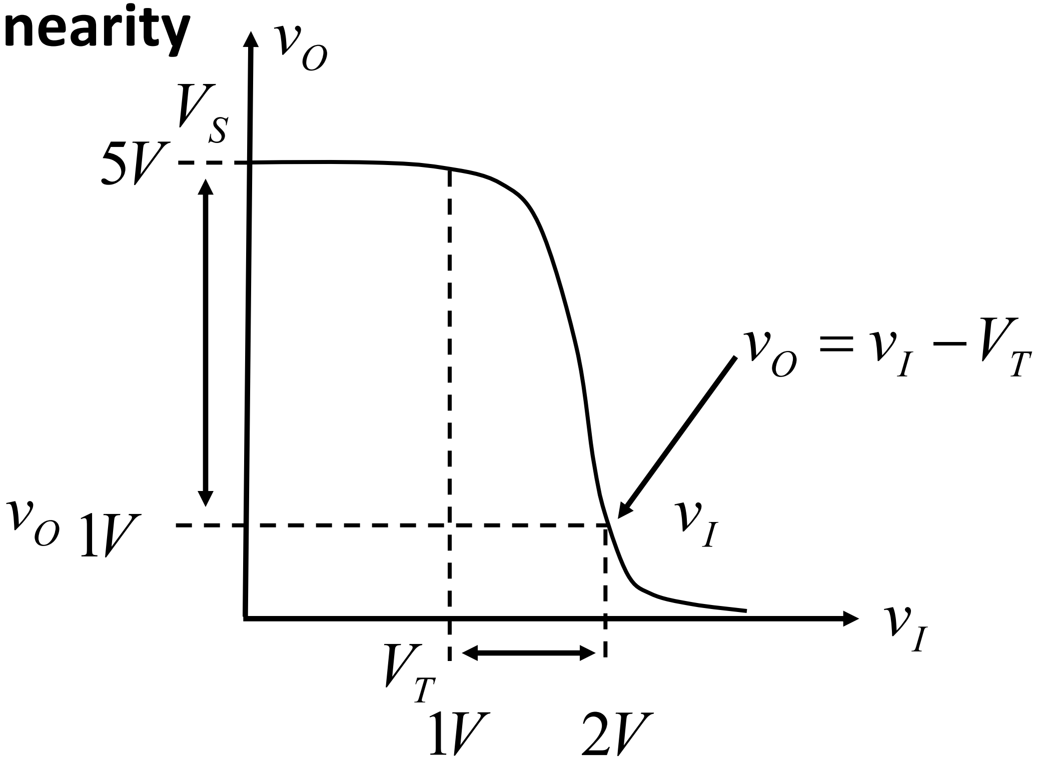


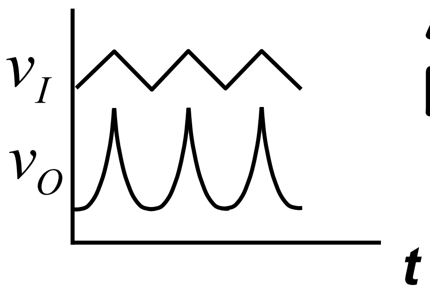
$1V$ $2V$

"interesting" region for v_I .
Saturation discipline satisfied.

Review: Large Signal Analysis

- **Non-linearity**





Amplifies alright,
but distorts

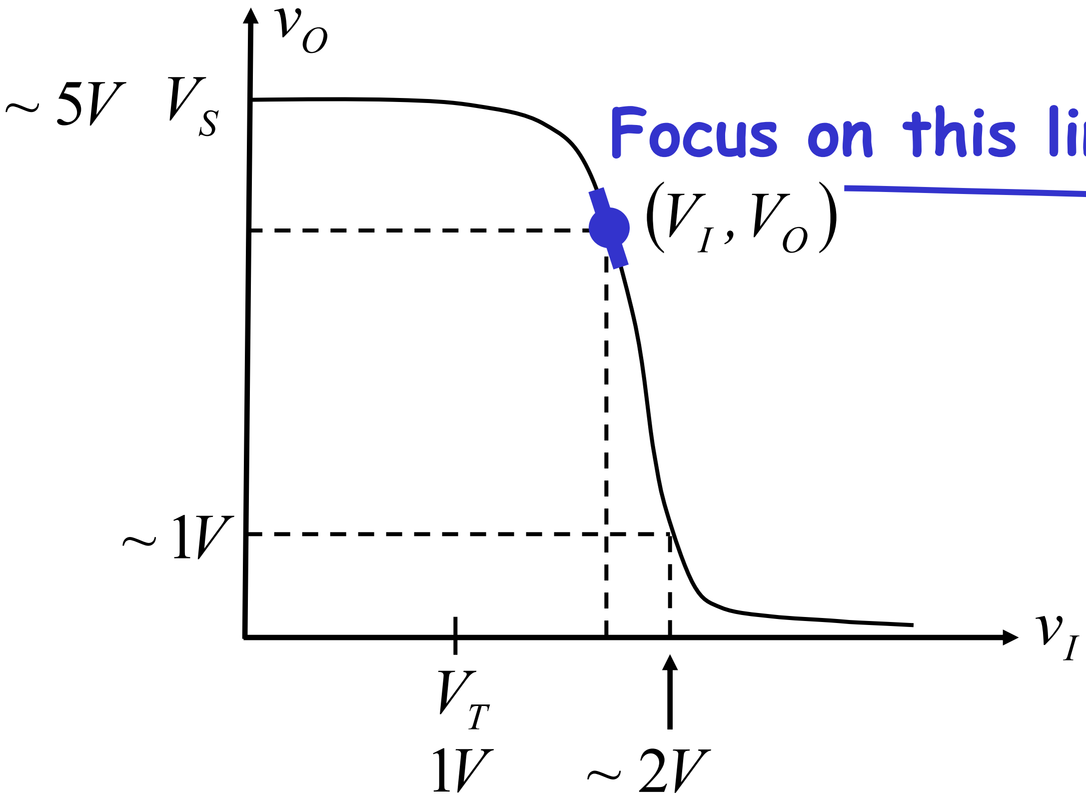
Amp is nonlinear … ☹

Outline

Textbook: 8.1, 8.2

- **Small-Signal Model**
  - Graphical interpretation
  - Mathematical interpretation
  - Small-signal circuit view
- **Small-Signal Circuit Representation**

Small-Signal Model



Focus on this line segment

Insight: But, observe v_I vs v_O about some point (V_I, V_O) … looks quite linear !

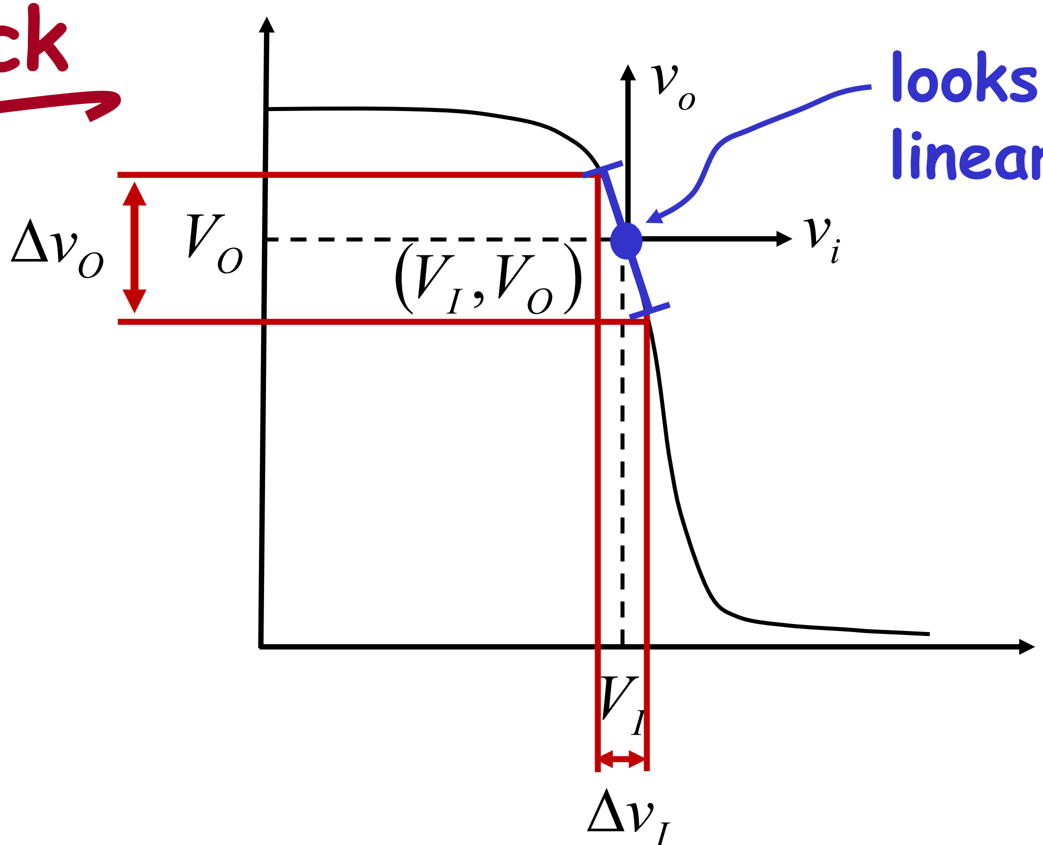
$$v_O = V_S - \frac{K(v_I - V_T)^2}{2} R_L$$

Amp all right, but nonlinear!

Hmmm … So what about our linear amplifier ???

Small-Signal Model

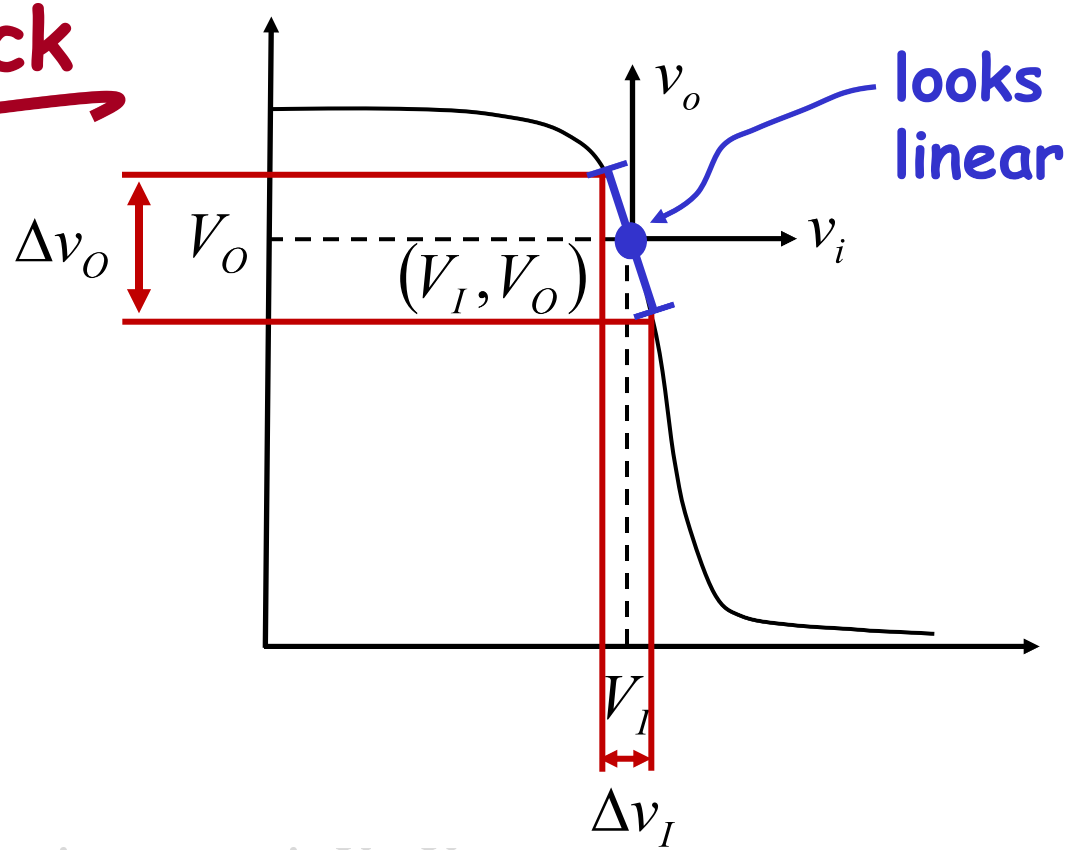
Trick



- Operate amp at V_I, V_O
  → DC "bias" (good choice: midpoint of input operating range)
- Superimpose small signal on top of V_I
- Response to small signal seems to be approximately linear

Small-Signal Model

Trick



- Operate amp at V_I, V_O
  - → DC "bias" (good choice: midpoint of input operating range)
- Superimpose small signal on top of V_I
- Response to small signal seems to be approximately linear

Let's look at this in more detail —

I graphically

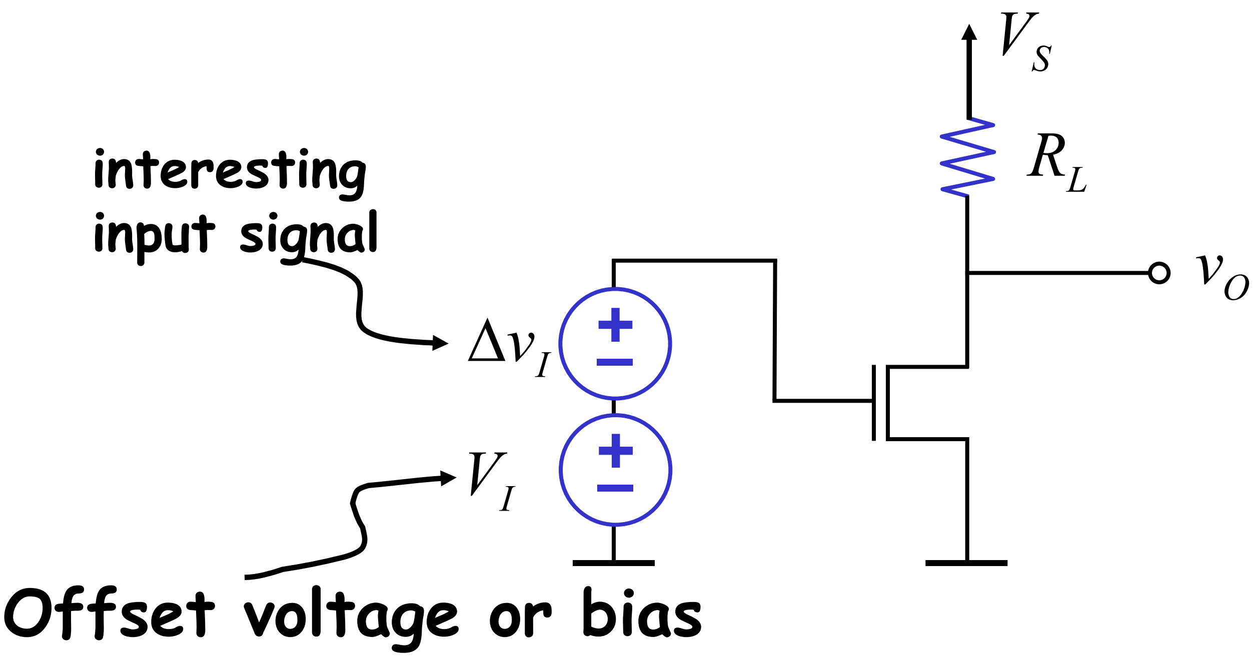
II mathematically

III from a circuit viewpoint

Small-Signal Model

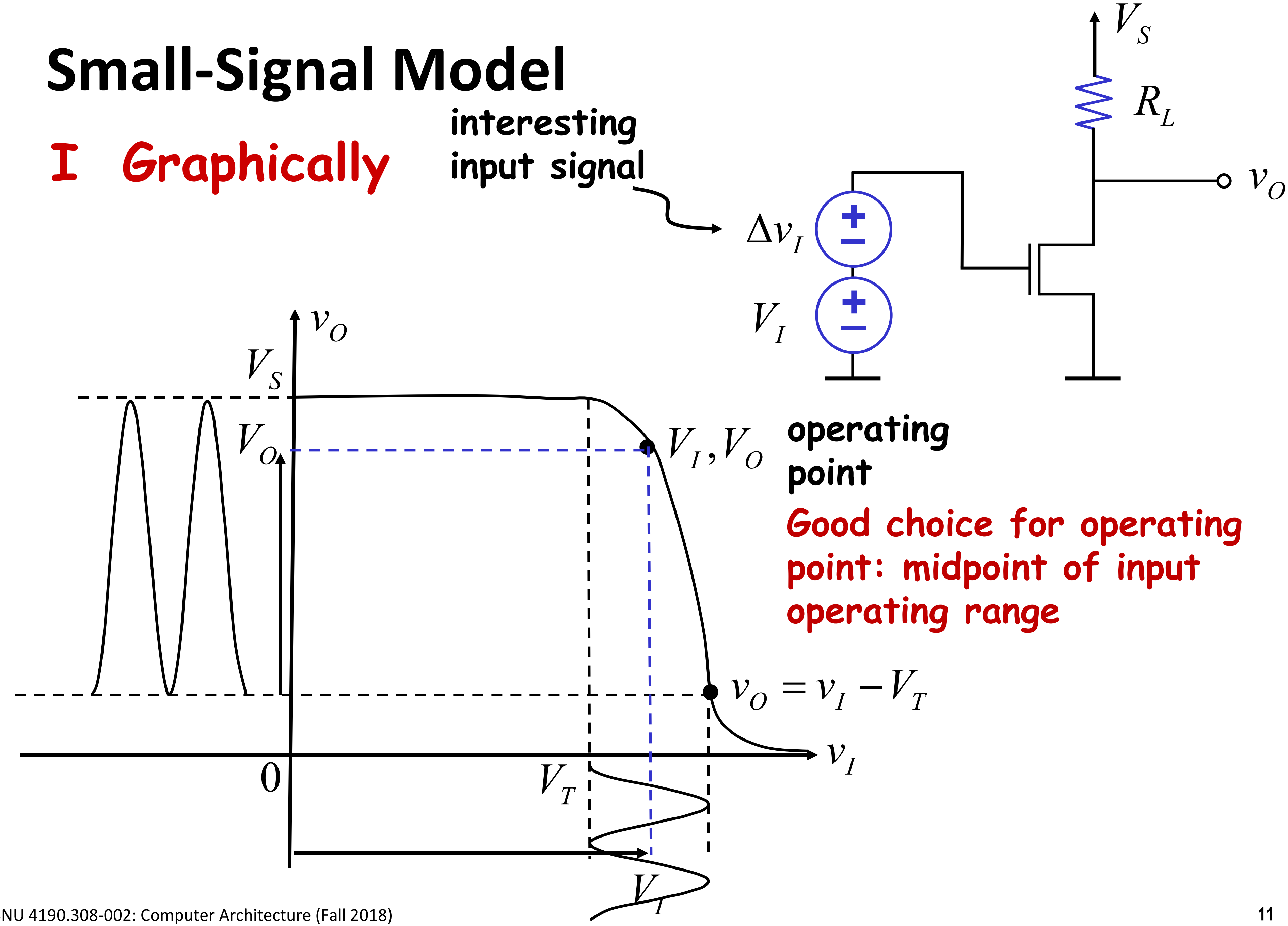
I Graphically

We use a DC bias V_I to "boost" interesting input signal above V_T, and in fact, well above V_T.



Small-Signal Model

I Graphically



Good choice for operating point: midpoint of input operating range

Small-Signal Model aka incremental or linearized model

Notation —

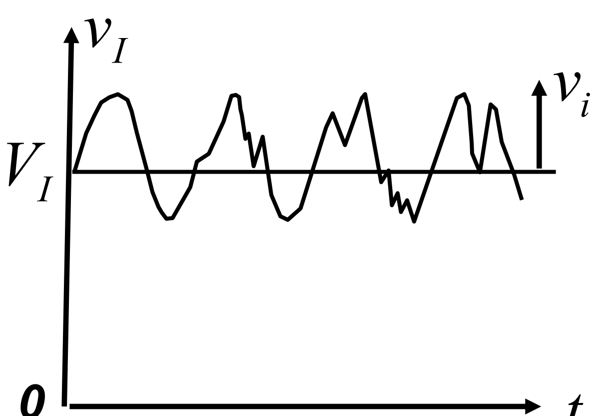
Input: $v_I = V_I + v_i$

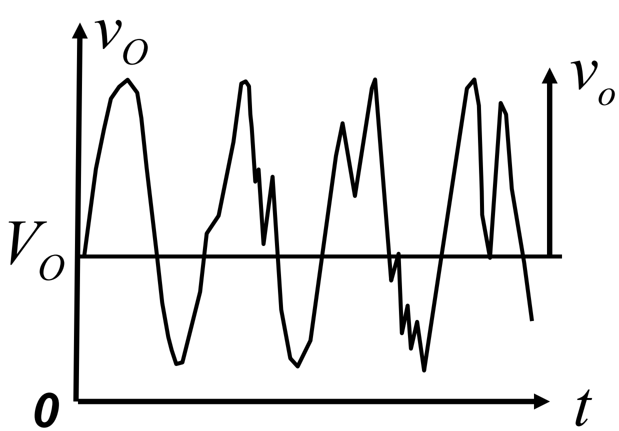
total variable — DC bias — small signal (like Δv_I)

bias voltage aka operating point voltage

Output: $v_O = V_O + v_o$

Graphically,





Small-Signal Model

II Mathematically

$$v_O = V_S - \frac{R_L K}{2}\left(v_I - V_T\right)^2 \qquad \Big| \qquad V_O = V_S - \frac{R_L K}{2}\left(V_I - V_T\right)^2$$

substituting $v_I = V_I + v_i \qquad v_i << V_I$

$$v_O = V_S - \frac{R_L K}{2}\left(\left[V_I + v_i\right] - v_T\right)^2$$

$$= V_S - \frac{R_L K}{2}\left(\left[V_I - V_T\right] + v_i\right)^2$$

$$= V_S - \frac{R_L K}{2}\left(\left[V_I - V_T\right]^2 + 2\left[V_I - v_T\right]v_i + \cancel{v_i^2}\right)$$

$$\cancel{V_O} + v_o = \cancel{V_S - \frac{R_L K}{2}\left(V_I - V_T\right)^2} - R_L K\left(V_I - V_T\right)v_i$$

From ✸,

$$v_o = -R_L \underbrace{K\left(V_I - V_T\right)}_{g_m} v_i$$

related to V_I

Small-Signal Model

II Mathematically

(From previous page…)

$$v_o = -R_L \underbrace{K\left(V_I - V_T\right)}_{g_m} v_i$$

g_m related to V_I

$$v_o = -g_m R_L \ v_i$$

For a given DC operating point voltage V_I, $V_I - V_T$ is constant. So,

$$v_o = -A \ v_i$$

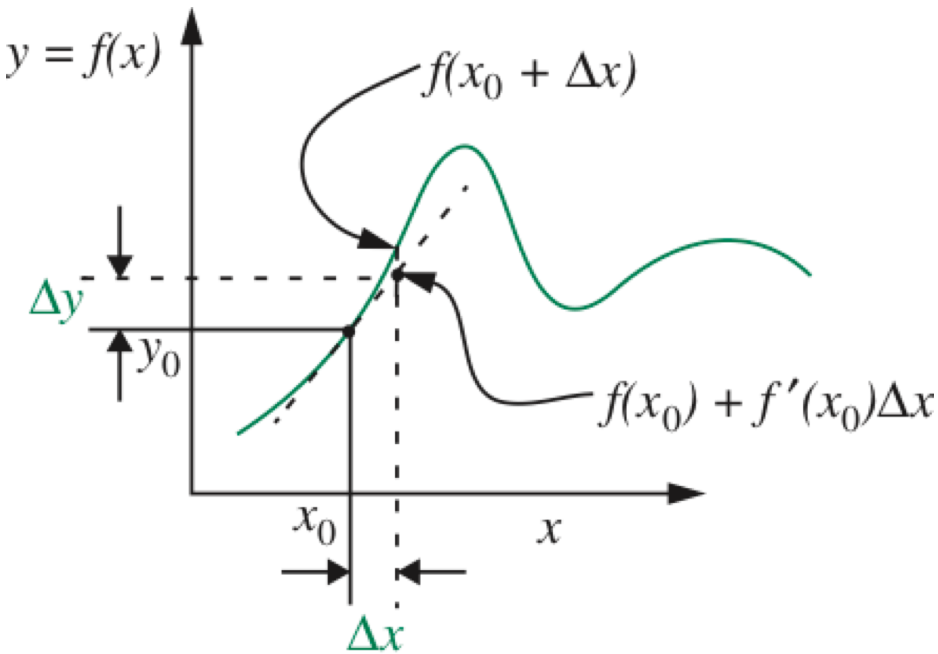
constant w.r.t. v_i

In other words, our circuit behaves like a linear amplifier for small signals

Small-Signal Model

II Mathematically (alternative way)

For small Δx near x_0, $f(x_0 + \Delta x) \cong f(x_0)+f'(x_0)\,\Delta x$



Small-Signal Model

II Mathematically (alternative way)

$$v_O = V_S - \frac{R_L K}{2}\left(v_I - V_T\right)^2$$

$$v_o = \underbrace{\left.\frac{d\left[V_S - \frac{R_L K}{2}\left(v_I - V_T\right)^2\right]}{dv_I}\right|_{v_I = V_I}}_{\text{slope at } V_I} \cdot v_i$$

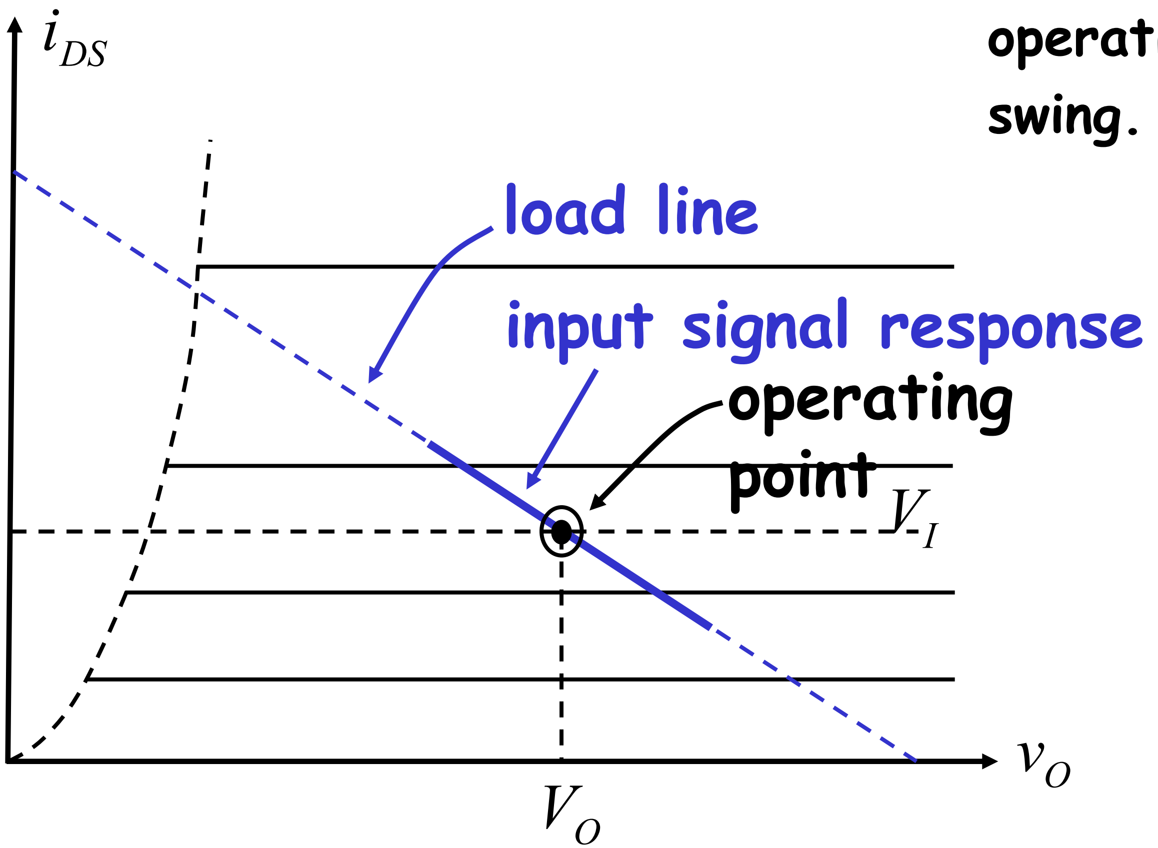
$$v_o = -R_L K\left(V_I - V_T\right) \cdot v_i$$

$$g_m = K\left(V_I - V_T\right)$$

$$A = -g_m R_L \quad \text{amp gain}$$

Small-Signal Model

- How to choose the bias point?



1. Gain component $g_m \propto V_I$
2. v_i gets big → distortion. So bias carefully
3. Input valid operating range.

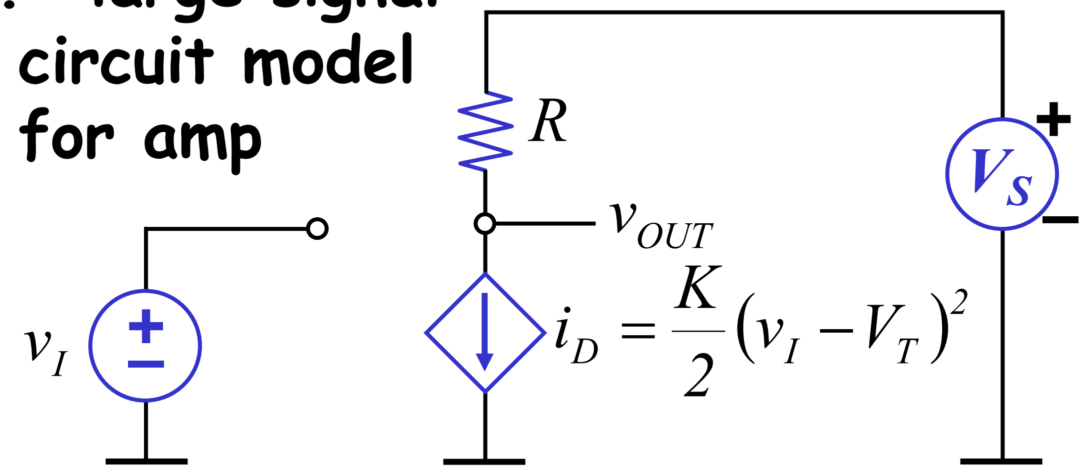
Bias at midpoint of input operating range for maximum swing.

Small-Signal Model

III The Small Signal Circuit View

We can derive small circuit equivalent models for our devices, and thereby conduct small signal analysis directly on circuits

e.g. large signal circuit model for amp



①

We can replace large signal models with small signal circuit models.

Outline

Textbook: 8.1, 8.2

- Small-Signal Model
  - Graphical interpretation
  - Mathematical interpretation
  - Small-signal circuit view
- **Small-Signal Circuit Representation**

Small-Signal Circuit Representation

- Small-signal circuit analysis

① Find operating point using DC bias inputs using large signal model.

② Develop small signal (linearized) models for elements.

③ Replace original elements with small signal models.

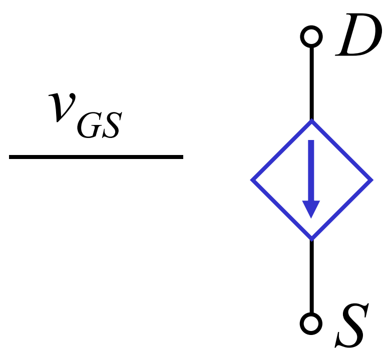
Analyze resulting linearized circuit…

Key: Can use superposition and other linear circuit tools with linearized circuit!

Small-Signal Circuit Representation

Ⓐ **MOSFET**

large signal



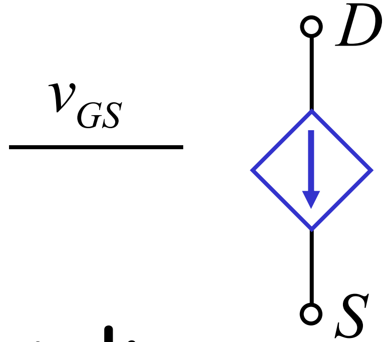
$$i_{DS} = \frac{K}{2}\left(v_{GS} - V_T\right)^2$$

Small signal?

Small-Signal Circuit Representation

A MOSFET

large signal

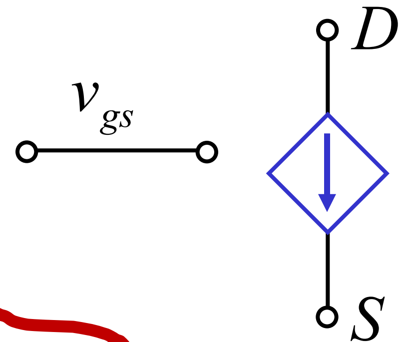


$$i_{DS} = \frac{K}{2}\left(v_{GS} - V_T\right)^2$$

Small signal:

$$i_{DS} = \frac{K}{2}\left(v_{GS} - V_T\right)^2$$

small signal



$$i_{ds} = K\left(V_{GS} - V_T\right)v_{gs}$$

$$i_{ds} = g_m v_{gs}$$

$$i_{ds} = \frac{\partial}{\partial v_{GS}}\left[\frac{K}{2}\left(v_{GS} - V_T\right)^2\right]\Bigg|_{v_{GS}=V_{GS}} \cdot v_{gs}$$

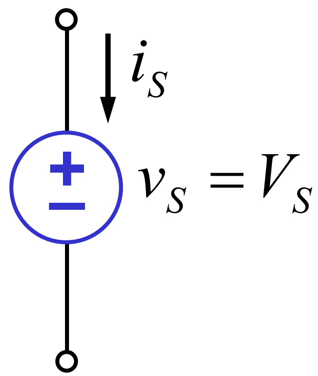
$$i_{ds} = \underbrace{K\left(V_{GS} - V_T\right)}_{g_m} \cdot v_{gs}$$

⇒ i_{ds} is linear in v_{gs}!

Small-Signal Circuit Representation

Ⓑ DC Supply V_S

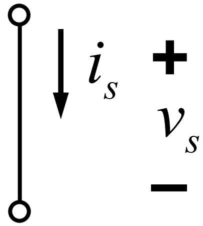
large signal



$$v_S = V_S$$

Small signal

$$v_s = \left.\frac{\partial V_S}{\partial i_S}\right|_{i_S = I_S} \cdot i_s$$



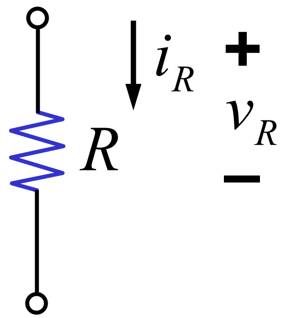
$$v_s = 0$$

DC source behaves as short to small signals.

Small-Signal Circuit Representation

C Similarly, R

large signal

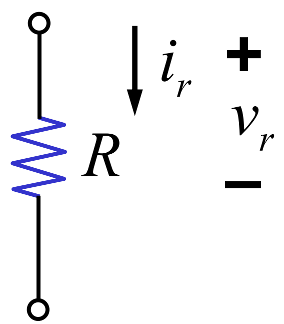


$$v_R = R\, i_R$$

$$v_r = \left.\frac{\partial (R i_R)}{\partial i_R}\right|_{i_R = I_R} \cdot i_r$$

$$v_r = R \cdot i_r$$

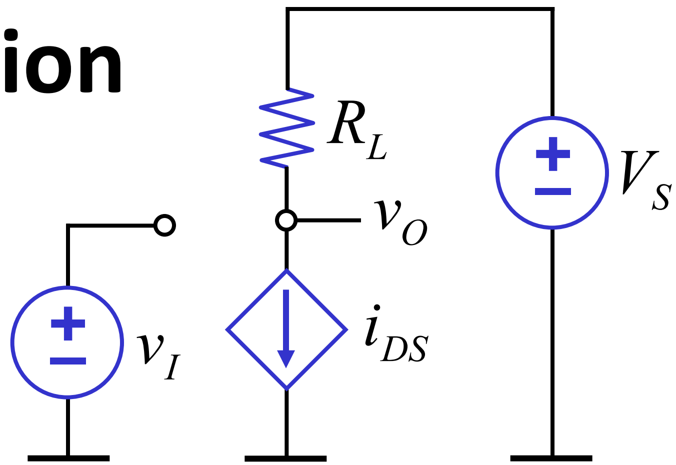
small signal



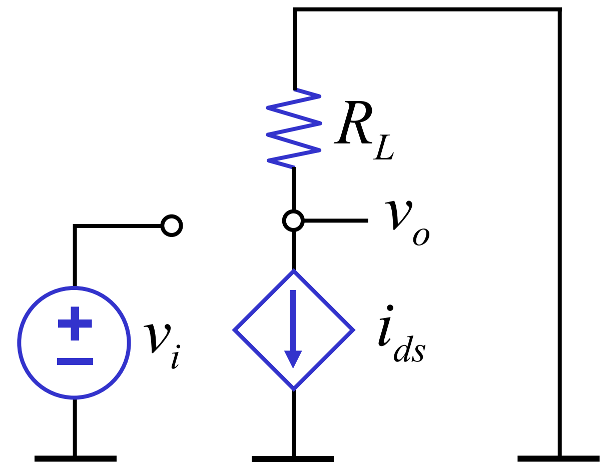
Small-Signal Circuit Representation

- Amplifier example

Large signal



Small signal



Notice, first we need to find operating point voltages/currents.

Get these from a large signal analysis.

$$i_{DS} = \frac{K}{2}(v_I - V_T)^2$$

$$v_O = V_S - \frac{K}{2}(v_I - V_T)^2 R_L$$

$$i_{ds} = K(V_I - V_T) \cdot v_i$$

$$i_{ds} R_L + v_o = 0$$

$$v_o = -i_{ds} R_L$$

$$v_o = -K(V_I - V_T) R_L \cdot v_i$$

$$= -g_m R_L \cdot v_i$$

Small-Signal Circuit Representation

- **Example 8.1: Derive a small-signal model**