Analysis of Nonlinear Circuits

Lecture 6
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***Slide credits**: Prof. Anant Agarwal at MIT*

Review: Discretize Matters

- **Lumped Matter Discipline (LMD) simplifies circuit analysis.**

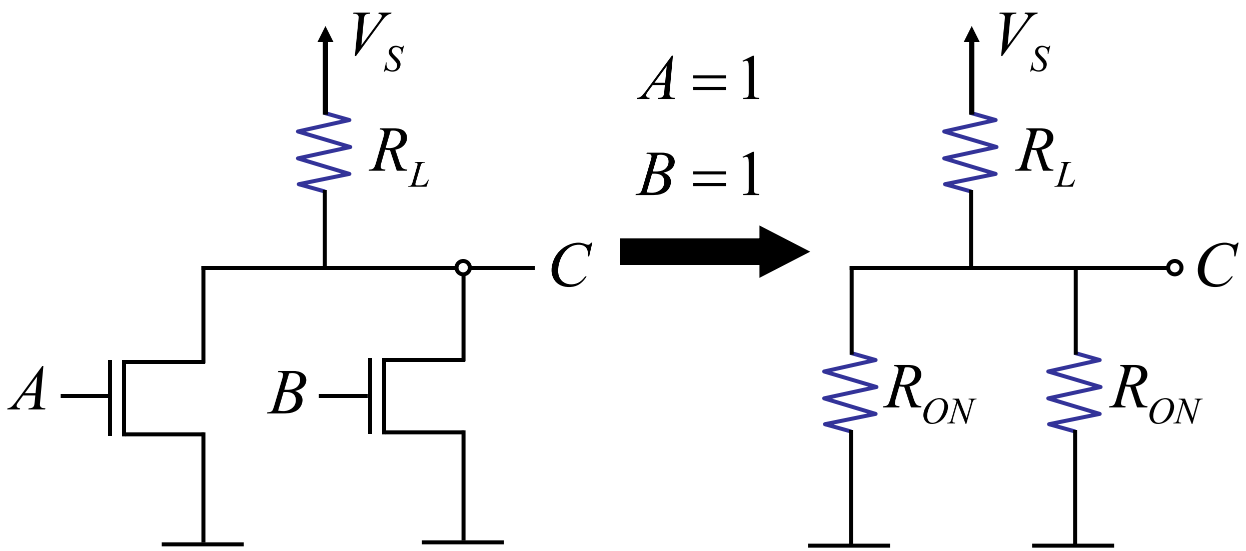
m1 ▶ KVL, KCL, *i-v*
m2 ▶ Composition rules
m3 ▶ Node method
} any circuit

m4 ▶ Superposition
m5 ▶ Thévenin, Norton
} linear circuits

Review: Discretize Value

- Digital abstraction

▶ Subcircuits for given "switch" setting are linear! So, all 5 methods (m1 – m5) can be applied



SR MOSFET Model

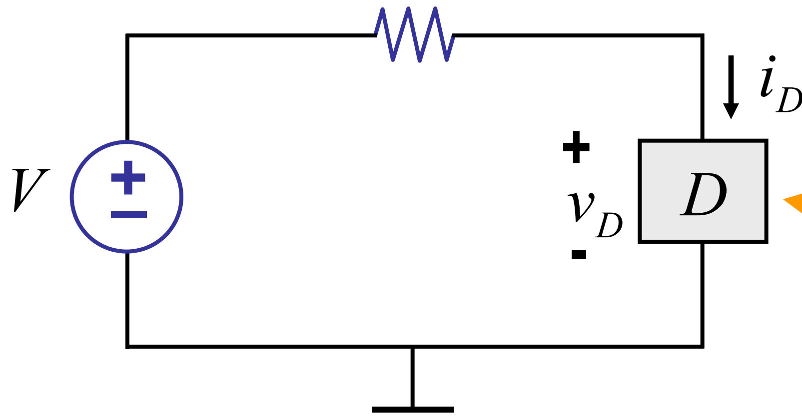
Outline

Textbook: Ch. 4.1, 4.2, 4.3, 4.5

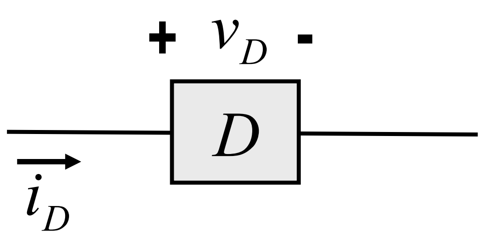
- **Analytical Method (based on m1, m2, m3)**
- **Graphical Method**
- **Incremental Analysis**

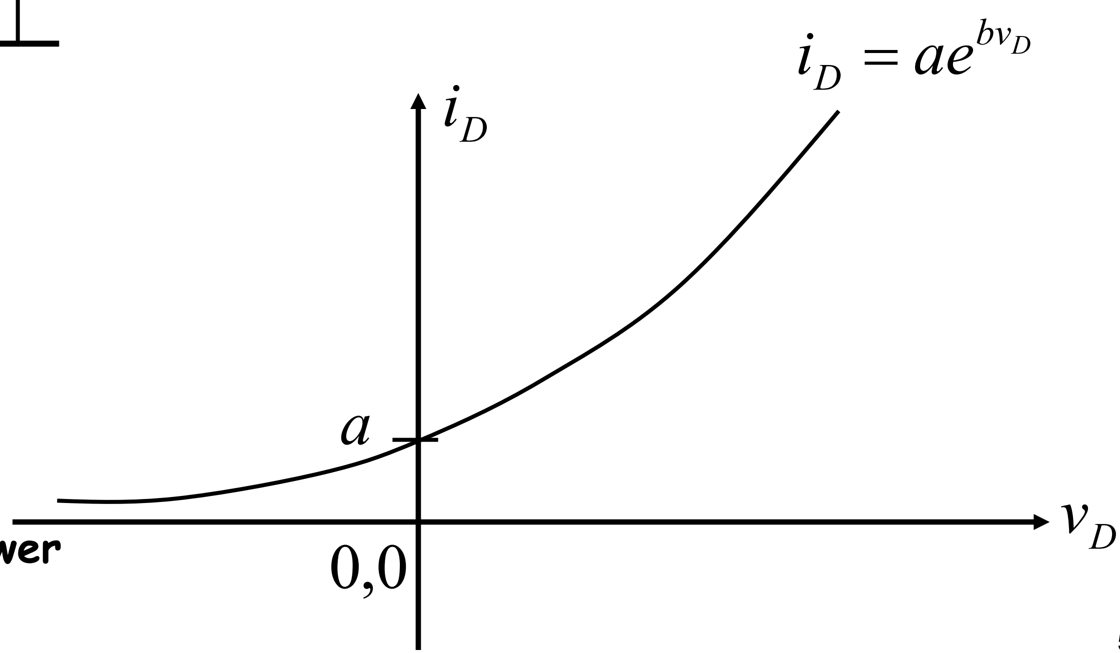
Analytical Method

- **How do we analyze nonlinear circuits, for example:**



Hypothetical nonlinear device





(Curiously, the device supplies power when v_D is negative)

Analytical Method

- **Using the node method!**

(remember the node method applies for linear or nonlinear circuits)

$$\frac{v_D - V}{R} + i_D = 0 \qquad ①$$

$$i_D = ae^{bv_D} \qquad ②$$

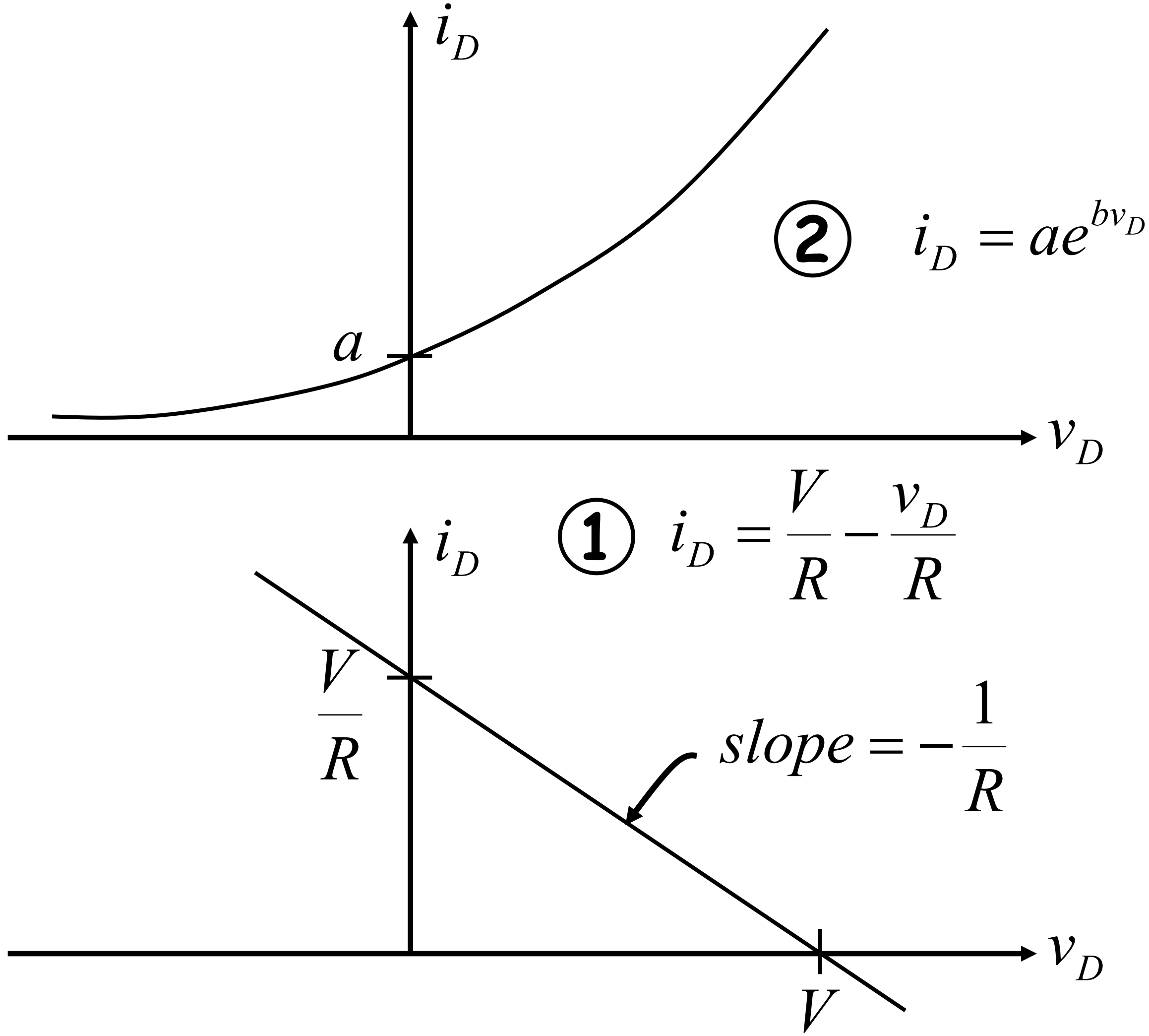
2 unknowns **2 equations**

Solve the equation by

- **trial and error**
- **numerical methods**

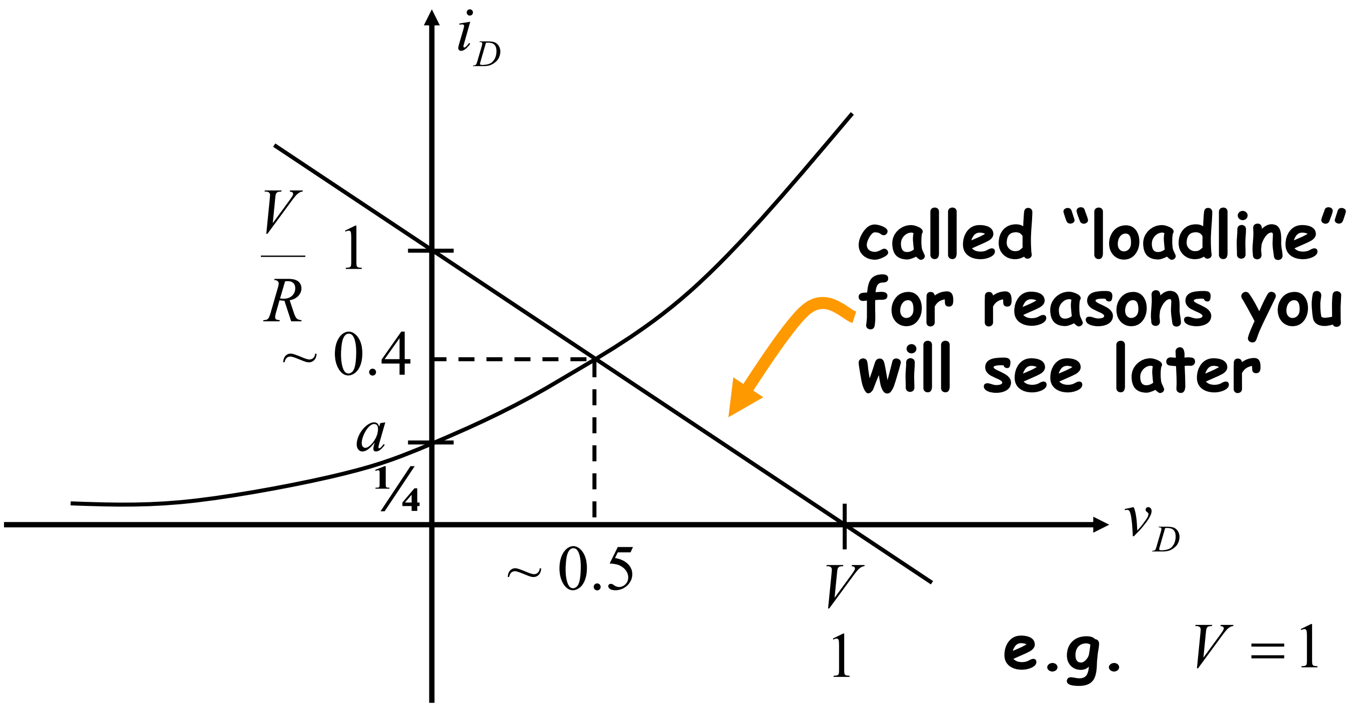
Graphical Method

Note: the solution satisfies equations ① and ②



Graphical Method

Combine the two constraints



e.g.

$V = 1$

$R = 1$

$a = \frac{1}{4}$

$b = 1$

$v_D = 0.5V$

$i_D = 0.4A$

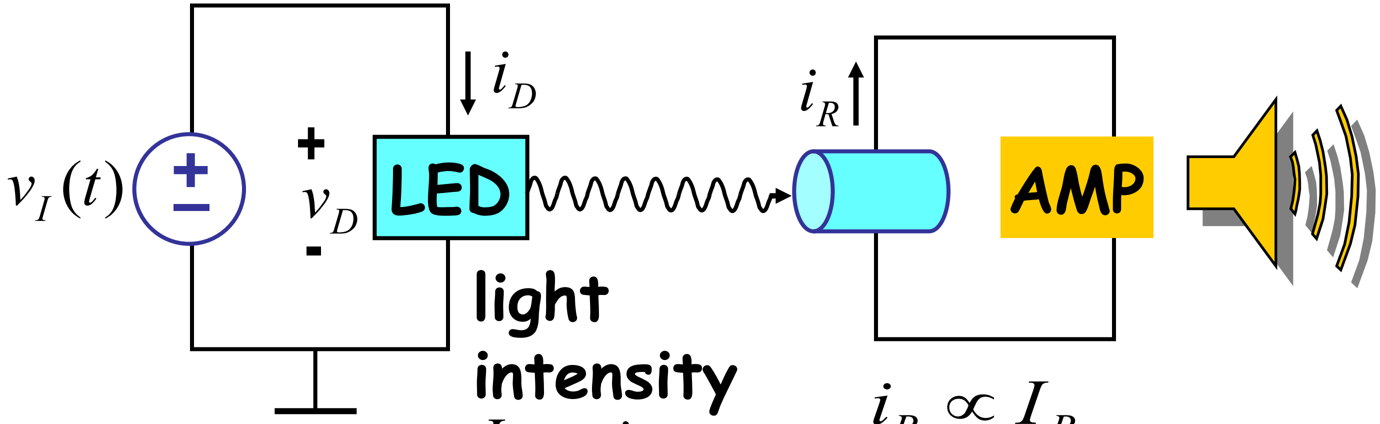
Outline

Textbook: Ch. 4.1, 4.2, 4.3, 4.5

- Analytical Method (based on m1, m2, m3)
- Graphical Method
- **Incremental Analysis**

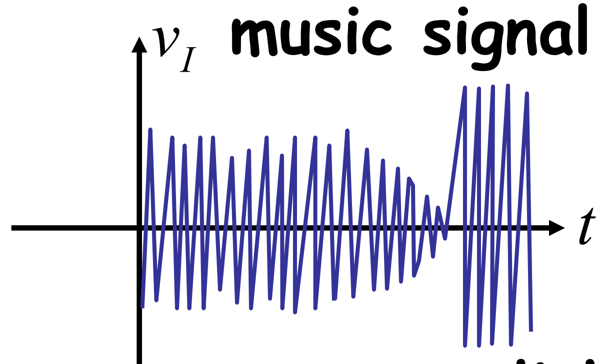
Incremental Analysis

- **Motivation:** music over a light beam



light intensity
$I_D \propto i_D$

$i_R \propto I_R$
light intensity I_R in photoreceiver



LED: Light Emitting Diode

$v_I(t) \longrightarrow i_D(t) \rightsquigarrow$ light $\rightsquigarrow i_R(t) \longrightarrow$ sound

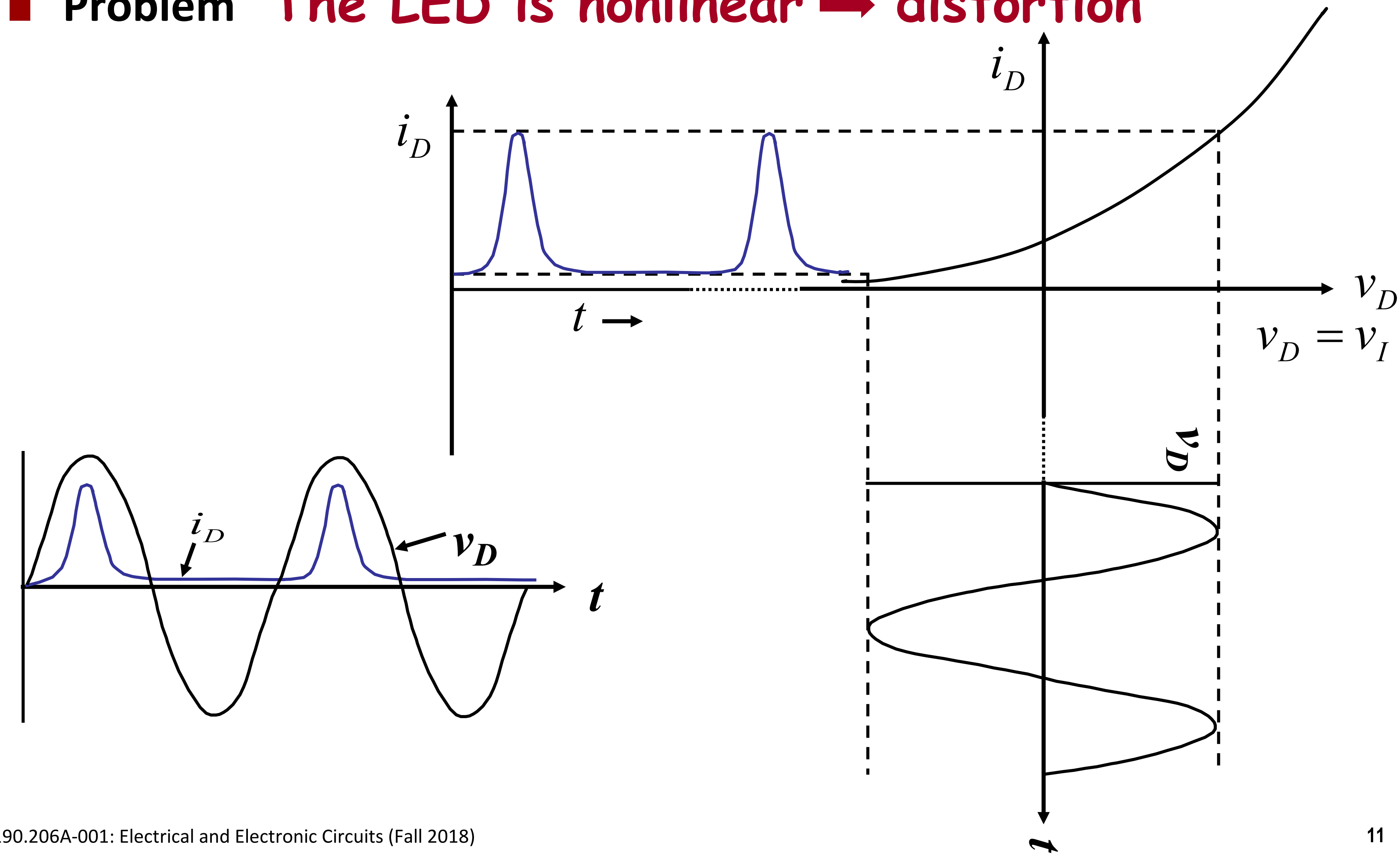
nonlinear

linear

problem! will result in distortion

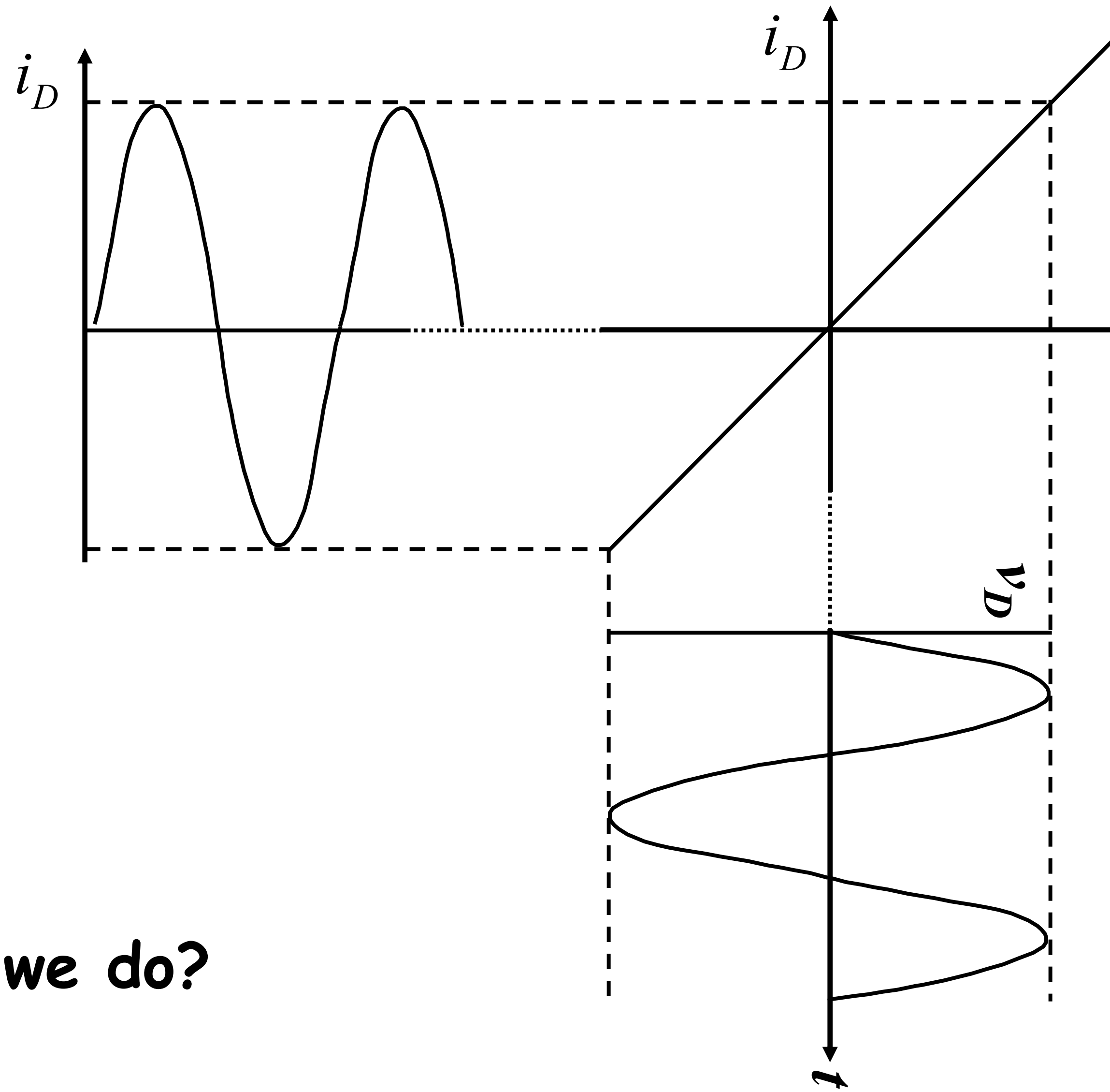
Incremental Analysis

- **Problem** The LED is nonlinear ➞ distortion



Incremental Analysis

- If only it were linear …

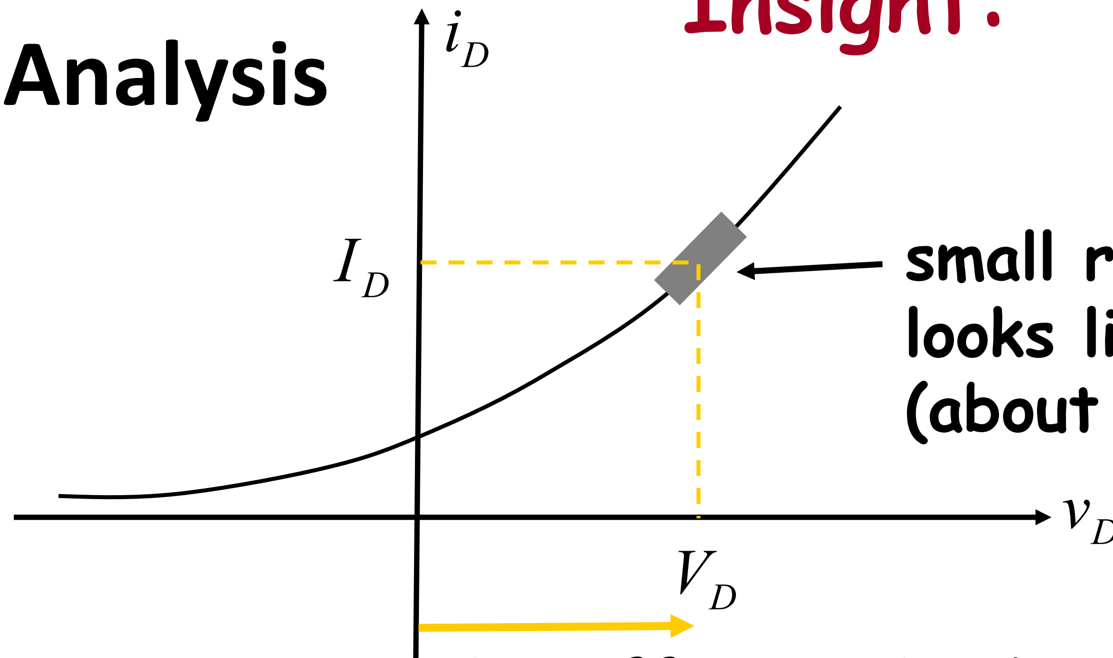


it would've been ok.

What do we do?

Incremental Analysis

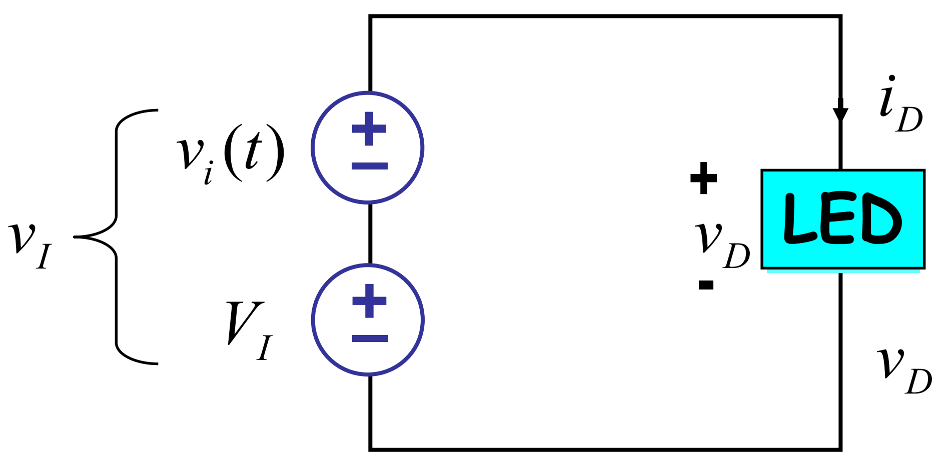
Insight:



small region looks linear (about V_D, I_D)

DC offset or DC bias

Trick:



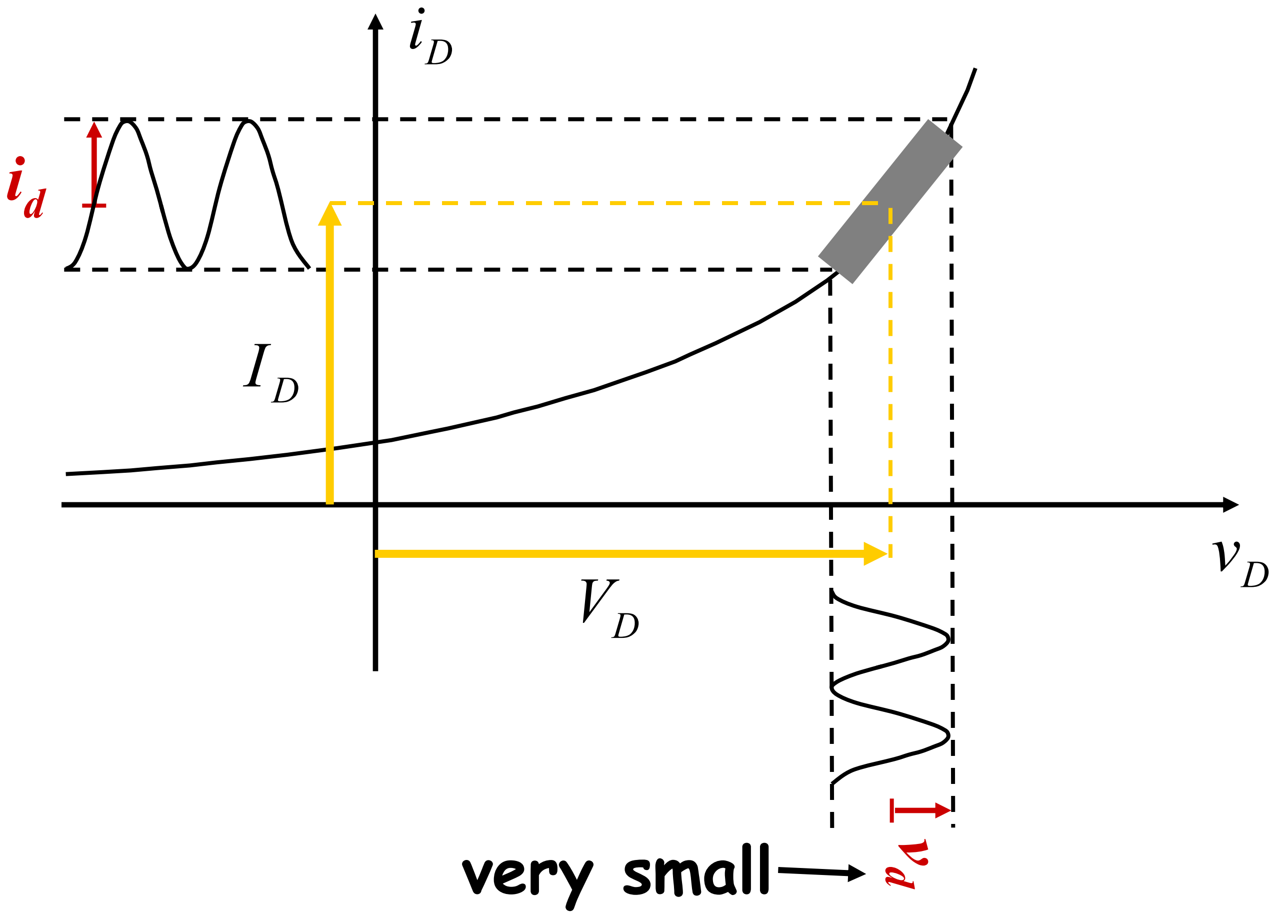
$$i_D = I_D + i_d$$

$$v_D = V_D + v_d$$

V_I v_i

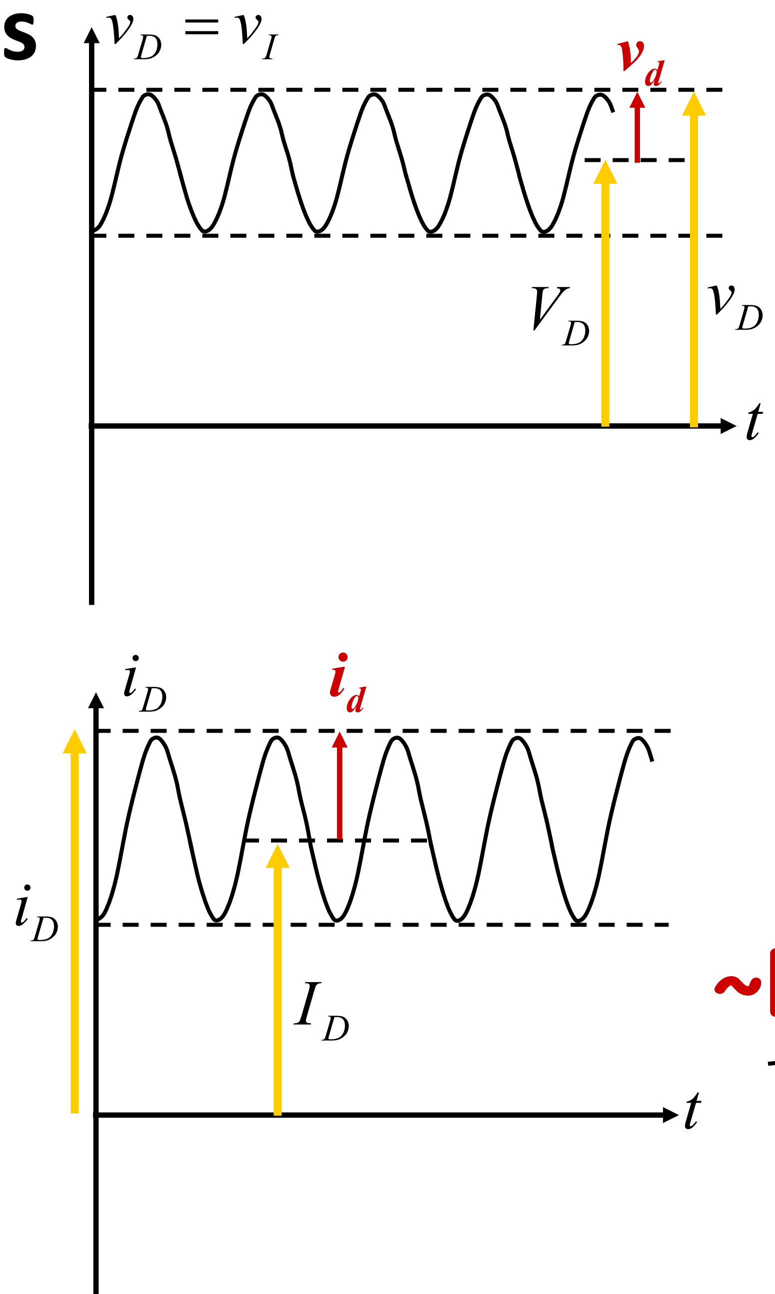
Incremental Analysis

Result



Incremental Analysis

Result



~linear!

Incremental Analysis

- **The incremental method (or small signal method)**

1. Operate at some DC offset or bias point V_D, I_D.
2. Superimpose small signal v_d (music) on top of V_D.
3. Response i_d to small signal v_d is approximately linear.

Notation: $i_D = I_D + i_d$

total variable — DC offset — small superimposed signal

Incremental Analysis

- **What does this mean mathematically?**
  - Or, why is the small signal response linear?

nonlinear

$$i_D = f(v_D)$$

large DC

We replaced

$$v_D = V_D + \underbrace{\Delta v_D}_{v_d}$$

increment from V_D

using Taylor's Expansion to expand $f(v_D)$ near $v_D=V_D$:

$$i_D = f(V_D) \quad + \quad \left.\frac{df(v_D)}{dv_D}\right|_{v_D=V_D} \cdot \Delta v_D$$

neglect higher order terms because Δv_D is small

$$+ \quad \cancel{\frac{1}{2!}\left.\frac{d^2 f(v_D)}{dv_D^{\,2}}\right|_{v_D=V_D}} \cdot \Delta v_D^{\,2} + \cdots$$

Incremental Analysis

- **What does this mean mathematically? (Cont'd)**

$$\underbrace{i_D \approx f(V_D)}_{\substack{\text{constant} \\ \text{w.r.t. } \Delta v_D}} + \underbrace{\left.\frac{df(v_D)}{dv_D}\right|_{v_D=V_D}}_{\substack{\text{constant w.r.t. } \Delta v_D \\ \text{slope at } V_D,\ I_D}} \cdot \Delta v_D$$

We can write

$$: I_D + \Delta i_D \approx f(V_D) + \left.\frac{df(v_D)}{dv_D}\right|_{v_D=V_D} \cdot \Delta v_D$$

Incremental Analysis

- What does this mean mathematically? (Cont'd)

equating DC and time-varying parts,

$$I_D = f(V_D) \longrightarrow \text{operating point}$$

$$\Delta i_D = \underbrace{\left.\frac{d f(v_D)}{d v_D}\right|_{v_D = V_D}}_{\text{constant w.r.t. } \Delta v_D} \cdot \Delta v_D$$

so, $\underline{\Delta i_D \propto \Delta v_D}$

By notation,

$$\Delta i_D = i_d$$
$$\Delta v_D = v_d$$

Incremental Analysis

- **What does this mean mathematically? (Cont'd)**

In our example,

$$i_D = a\,e^{bv_D}$$

From ⓧ : $I_D + i_d \approx a\,e^{bV_D} + a\,e^{bV_D} \cdot b \cdot v_d$

Equate DC and incremental terms,

$$\boxed{I_D = a\,e^{bV_D}}$$ → operating point

aka bias pt.

aka DC offset

$$i_d = a\,e^{bV_D}\,b \cdot v_d$$

$$i_d = \underbrace{I_D \cdot b}_{\text{constant}} \cdot v_d$$ → small signal behavior

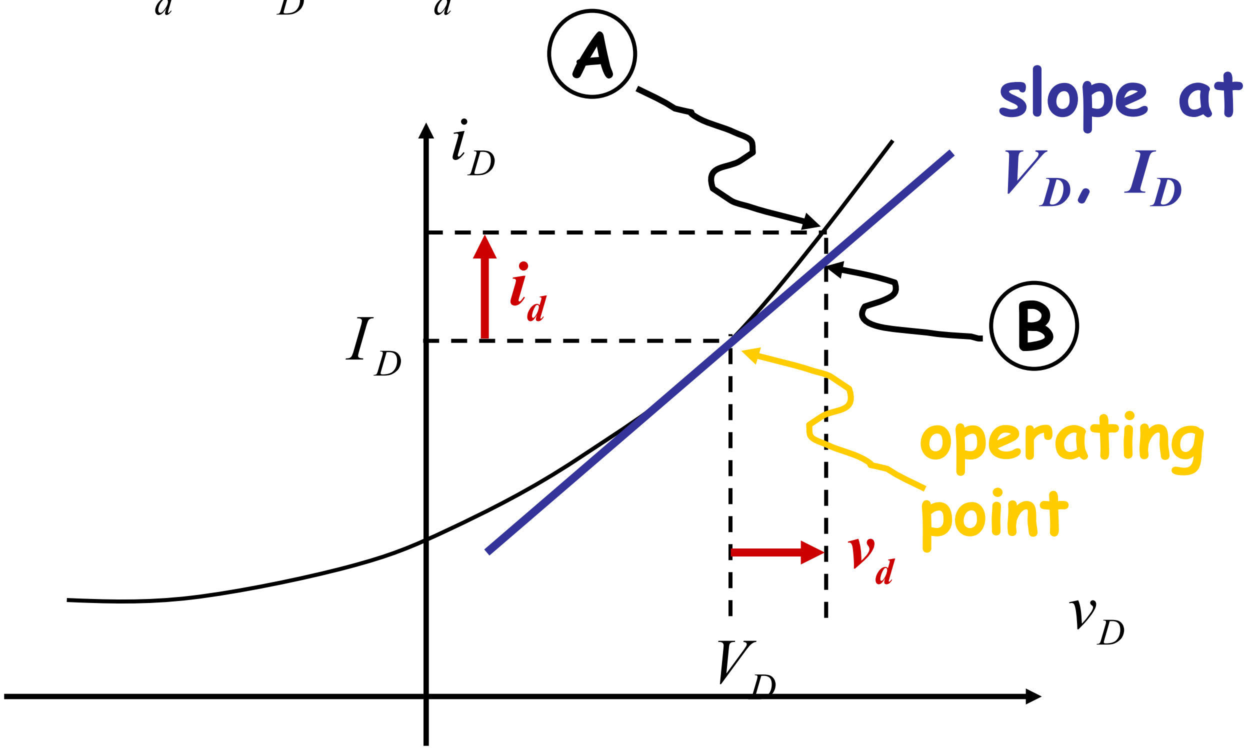
constant → linear!

Incremental Analysis

- **Graphical interpretation**

$$I_D = a\, e^{bV_D} \longrightarrow \text{operating point}$$

$$i_d = I_D \cdot b \cdot v_d$$

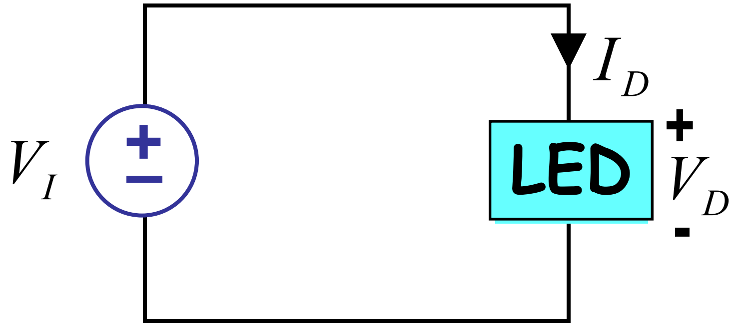


We are approximating A with B!

Incremental Analysis

We saw the small signal → graphically, mathematically, now, circuit

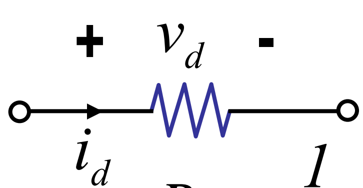
Large signal circuit:



$$\boxed{I_D = a\, e^{bV_D}}$$

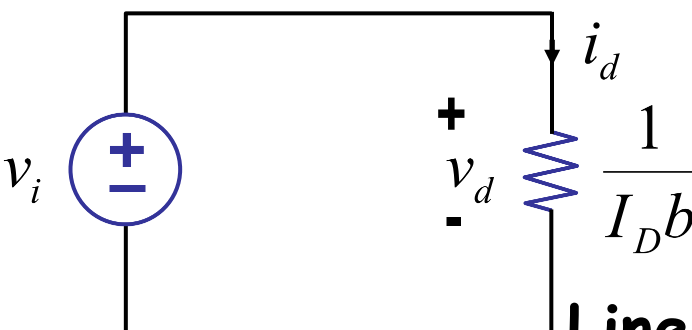
Small signal reponse: $i_d = I_D\, b\, v_d$

behaves like:



$$R = \frac{1}{I_D\, b}$$

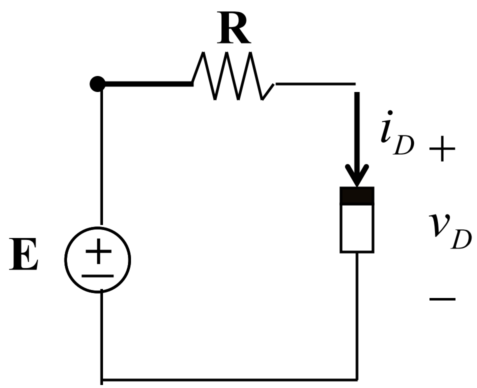
small signal circuit:



Linear!

Incremental Analysis

- **Example: Draw the small signal circuit**



$$i_D = \begin{cases} K{v_D}^2 & for\ v_D > 0 \\ 0 & for\ v_D \le 0 \end{cases}$$

$$K = 1\, mA/V^2$$

$$V_D = 1V$$

Conclusion

- **Nonlinear elements**
  - e.g., (light emitting) diodes
- **Solution methods**
  - Analytical method
  - Graphical method
  - Piecewise linear method (not covered, Chapter 4.4)
  - Incremental analysis
- **Two types of non-linear circuits**
  - Digital circuit: assume the operating mode of the device
    - e.g., logic gates
  - Analog circuit: bias + incremental analysis (small signal analysis)
    - e.g., amplifiers