Quiz #3 (CSE 400.001)

Tuesday, April 22, 2003

Name:	E-mail:
Dent.	ID No:

1. (7 points) Use $x_0 = 1.2$ and $x_1 = 1.153$ in solving the following equation by Newton's method

$$x^5 - 2 = 0.$$

How many iterations are necessary to produce the solution to 10D accuracy?

Solution:

$$\frac{f''(s)}{2f'(s)} \approx \frac{f''(x_1)}{2f'(x_1)} = \frac{20x_1^3}{10x_1^4} = \frac{2}{x_1} \approx 1.735$$

$$|\epsilon_{n+1}| \approx 1.735\epsilon_n^2 \approx 1.735^3 \epsilon_{n-1}^4 \approx 1.735^{2^{n+1}-1} \epsilon_0^{2^{n+1}} \le 5 \cdot 10^{-11}$$

$$\epsilon_1 - \epsilon_0 = (\epsilon_1 - s) - (\epsilon_0 - s) = -x_1 + x_0 \approx 0.047$$

$$\epsilon_1 \approx \epsilon_0 + 0.047 \approx -1.735\epsilon_0^2$$

$$1.735\epsilon_0^2 + \epsilon_0 + 0.047 \approx 0$$

$$\epsilon_0 \approx -0.05163$$

$$n = 1$$
: $1.735^3 \cdot 0.05163^4 \approx 3.711 \cdot 10^{-5} > 5 \cdot 10^{-11}$
 $n = 2$: $1.735^7 \cdot 0.05163^8 \approx 2.390 \cdot 10^{-9} > 5 \cdot 10^{-11}$

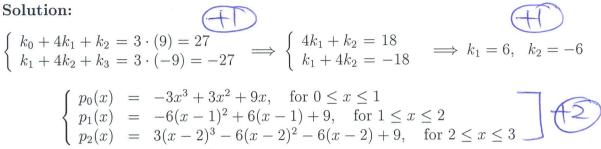
$$n = 3:$$
 $1.735^{15} \cdot 0.05163^{16} < 10^{-16} < 5 \cdot 10^{-11}$

Hence, n = 3 iterations are necessary.

2. (4 points) Interpolate

$$f_0 = f(0) = 0$$
, $f_1 = f(1) = 9$, $f_2 = f(2) = 9$, $f_3 = f(3) = 0$

by the cubic spline satisfying $k_0 = 9$ and $k_3 = -9$.



3. (4 points) Compute the following integral using the Gauss quadrature with n=5.

$$\int_1^5 \frac{2x}{1+x^2} dx$$

Solution:

$$x = 2t + 3 \implies dx = 2dt$$

$$\int_{-1}^{1} \frac{4t + 6}{4t^2 + 12t + 10} \cdot 2dt$$

$$= \int_{-1}^{1} \frac{4t + 6}{2t^2 + 6t + 5} dt$$

$$= \int_{-1}^{1} \frac{4t + 6}{2t^2 + 6t + 5} dt$$