Quiz #2 (EngMath I) [Wednesday, Sept. 28, 2016]

Name: ____________ Dept: ____________ ID No: ________

1. (15 points) Find a particular solution of the following differential equation:

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{3x}$$

$y_p = y_{p_1} + y_{p_2}$. (+1)

$y_{p_1} = c_1 x + c_2$, $y_{p_1}' = c_1$, $y_{p_2}'' = 0$ (+1)

$y_{p_2} = (Ax^2 + Bx)e^{3x}$ (+4)

$y_{p_2}' = (3Ax^2 + 3Bx + 2Ax + B)e^{3x}$ (+4)

$y_{p_2}'' = (9Ax^2 + 9Bx + 12Ax + 6B + 2A)e^{3x}$

$y_{p_1}'' - 2y_{p_1}' = -3c_1 x - 2c_1 - 3c_2 = 4x - 5$

$\therefore c_1 = -\frac{4}{3}$, $c_2 = \frac{23}{9}$ (+1)

$y_{p_2}'' - 2y_{p_2}' - 3y_{p_2} = (8Ax + 2A + 4B)e^{3x} = 6xe^{3x}$

$\therefore A = \frac{3}{4}$, $B = -\frac{3}{8}$ (+2)

$\therefore y_p = y_{p_1} + y_{p_2} = -\frac{4}{3}x + \frac{23}{9} + \left(\frac{3}{4}x^2 - \frac{3}{8}x\right)e^{3x}$ (+1)

Check: $y_p' = -\frac{4}{3} + \left(\frac{9}{4}x^2 - \frac{9}{8}x + \frac{6}{4}x - \frac{3}{8}\right)e^{3x}$

$y_p'' = \left(\frac{27}{4}x^2 + \frac{9}{8}x + \frac{9}{2}x - \frac{6}{8}\right)e^{3x}$ (+1)

$y_p'' - 2y_p' - 3y_p = 6xe^{3x}$

2. (15 points) Solve the following initial value problem:

$$4x^2y'' + y = 16x^2\sqrt{x}, \ (x > 0), \quad y(1) = 1, \ y'(1) = 3.$$

$4m(m-1)+1=0, \ (2m-1)^2=0, \ m=\frac{1}{2}$ (+1)

$y_1=\sqrt{x}, \quad y_2=\sqrt{x}\ln x$ (+2)

$$W = \begin{vmatrix} \sqrt{x} & \sqrt{x}\ln x \\ \frac{1}{2}\frac{1}{\sqrt{x}} & \frac{1}{2}\frac{1}{\sqrt{x}}\ln x + \frac{1}{\sqrt{x}} \end{vmatrix} = 1$$ (+2)

$y'' + \frac{1}{4x^2}y = 4\sqrt{x} = r(x)$ (+2)

$y_p = -\sqrt{x}\int \sqrt{x}\ln x \cdot 4\sqrt{x}\,dx + \sqrt{x}\ln x \int \sqrt{x}\cdot 4\sqrt{x}\,dx$

$= -4\sqrt{x}\int x\ln x\,dx + 4\sqrt{x}\ln x\int x\,dx$ (+4)

$= x^2\sqrt{x}$

$y = y_h + y_p = C_1\sqrt{x} + C_2\sqrt{x}\ln x + x^2\sqrt{x}$

$y(1) = C_1 + 1 = 1 \Rightarrow C_1 = 0$ (+1)

$y' = \frac{1}{2}C_2 x^{-\frac{1}{2}}\ln x + C_2\cdot x^{-\frac{1}{2}} + \frac{5}{2}x^{\frac{3}{2}}$

$y'(1) = C_2 + \frac{5}{2} = 3 \Rightarrow C_2 = \frac{1}{2}$ (+1)

$\therefore y = \frac{1}{2}\sqrt{x}\ln x + x^{\frac{5}{2}}$ (+1)

Check: $y' = \frac{1}{4}x^{-\frac{1}{2}}\ln x + \frac{1}{2}\cdot x^{-\frac{1}{2}} + \frac{5}{2}x^{\frac{3}{2}}$

$y'' = -\frac{1}{8}x^{-\frac{3}{2}}\ln x + \cancel{\frac{1}{4}x^{-\frac{3}{2}}} - \cancel{\frac{1}{4}x^{-\frac{3}{2}}} + \frac{15}{4}x^{\frac{1}{2}}$ (+1)

$4x^2y'' + y = -\frac{1}{2}\sqrt{x}\ln x + 15x^{\frac{5}{2}} + \frac{1}{2}\sqrt{x}\ln x + x^{\frac{5}{2}}$

$= 16x^{\frac{5}{2}}$

$y(1) = 1$

$y'(1) = \frac{1}{2} + \frac{5}{2} = 3$