Quiz #2 (CSE 400.001)

Tuesday, March 26, 2002

Name: ______________________ **E-mail:** ______________________

Dept: ______________________ **ID No:** ______________________

1. (10 points) Solve the following differential equation

$$y'' - 2y' + 5y = e^x \cos 2x.$$

$\lambda^2 - 2\lambda + 5 = (\lambda-1)^2 + 2^2 = 0, \quad \lambda = 1 \pm 2i$

$y_h = A e^x \cos 2x + B e^x \sin 2x$ (+1)

$y_p = x e^x (K \cos 2x + M \sin 2x)$ (+2)

$y_p' = (e^x + x e^x)(K \cos 2x + M \sin 2x) + x e^x (-2K \sin 2x + 2M \cos 2x)$ (+1)

$y_p'' = (2e^x + x e^x)(K \cos 2x + M \sin 2x) + (2e^x + 2x e^x)(-4K \sin 2x + 4M \cos 2x) + x e^x (-4K \cos 2x - 4M \sin 2x)$ (+1)

$y_p'' - 2y_p' + 5y_p = e^x (-2K \sin 2x + 2M \cos 2x) = e^x \cos 2x$

$\therefore K = 0, \quad M = \frac{1}{4}$ (+4)

$\therefore y = A e^x \cos 2x + B e^x \sin 2x + \frac{1}{4} x e^x \sin 2x$ (+1)

2. (10 points) Solve the following initial value problem.

$$x^2y'' - 3xy' + 3y = 12x^4e^x, \quad y(1) = 2, \; y'(1) = 24e.$$

$m(m-1) - 3m + 3 = m^2 - 4m + 3 = (m-1)(m-3) = 0$ (+1)

$\therefore y_1 = x, \quad y_2 = x^3$

$W = \begin{vmatrix} x & x^3 \\ 1 & 3x^2 \end{vmatrix} = 2x^3$ (+1)

$y'' - \frac{3}{x}y' + \frac{3}{x^2}y = 12x^2e^x$ (+2)

$y_p = -y_1 \int \frac{y_2}{W} r(x)\,dx + y_2 \int \frac{y_1}{W} r(x)\,dx$ (+2)

$= -x \int \frac{x^3}{2x^3} \cdot 12x^2e^x\,dx + x_3 \int \frac{x}{2x^3} \cdot 12x^2e^x\,dx$

$= -12x \int x^2e^x\,dx + 12x^3 \int e^x\,dx$

$= (-12x)\left[\cancel{x^2e^x} - 2\int xe^x\,dx\right] + \cancel{12x^3e^x}$

$= (24x)\left[xe^x - \int e^x\,dx\right]$

$= 24x^2e^x - 24xe^x$ (+2)

$\begin{cases} y = c_1x + c_2x^3 + 24x^2e^x - 24xe^x \\ y' = c_1 + 3c_2x^2 + 24x^2e^x + 24xe^x - 24e^x \end{cases}$

$\begin{cases} c_1 + c_2 = 2 \\ c_1 + 3c_2 + 24e = 24e \end{cases} \Rightarrow \begin{cases} c_1 + c_2 = 2 \\ c_1 + 3c_2 = 0 \end{cases}$ (+2)

$\therefore c_1 = 3, \; c_2 = -1$

$\therefore y = 3x - x^3 + 24x^2e^x - 24xe^x$

3. (10 points) Solve the following initial value problem

$$y''' + y'' = 10e^x \cos x, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

$$\lambda^3 + \lambda^2 = \lambda^2(\lambda + 1) = 0, \qquad \lambda = 0, -1, \qquad +1$$

$$y_h = c_1 + c_2 x + c_3 e^{-x} \qquad +2$$

$$y_p = e^x (K\cos x + M\sin x) \qquad +1$$

$$y_p' = e^x (K\cos x + M\sin x) + e^x(-K\sin x + M\cos x)$$

$$y_p'' = e^x (\cancel{K\cos x + M\sin x}) + 2e^x(-K\sin x + M\cos x) + e^x(\cancel{-K\cos x - M\sin x})$$

$$y_p''' = 2e^x(-K\sin x + M\cos x) + 2e^x(-K\cos x - M\sin x)$$

+2

$$y_p''' + y_p'' = 4e^x(-K\sin x + M\cos x) + 2e^x(-K\cos x - M\sin x) = 10e^x \cos x$$

$$\begin{cases} -4K - 2M = 0 \\ 4M - 2K = 10 \end{cases} \qquad \therefore K = -1,\ M = 2$$

+2

$$\begin{cases} y = c_1 + c_2 x + c_3 e^{-x} - e^x \cos x + 2e^x \sin x \\ y' = c_2 - c_3 e^{-x} - \cancel{e^x \cos x} + 3e^x \sin x + \cancel{2}e^x \cos x \\ y'' = c_3 e^{-x} + \overset{2}{\cancel{3}}e^x \sin x + 4e^x \cos x - \cancel{e^x \sin x} \end{cases}$$

$$\Rightarrow \begin{cases} c_1 + c_3 - 1 = 0 \\ c_2 - c_3 + 1 = 0 \\ c_3 + 4 = 0 \end{cases} \Rightarrow \begin{cases} c_1 = 5 \\ c_2 = -5 \\ c_3 = -4 \end{cases}$$

$$\therefore y = 5 - 5x - 4e^{-x} - e^x \cos x + 2e^x \sin x$$

+2