Quiz 1

2017. 3. 15

Explain the details of your solutions as much as possible. Simple answers may not earn credits.

1. [2 pts each] Two persons A and B roll their own dice. A's die is fair, but B uses a biased dice such that Pr(2)=Pr(4)=Pr(6)=1/3 and Pr(1)=Pr(3)=Pr(5)=0.

a) Let Ai and Bi be the result of the i-th roll of dice A and B. Let event Fi be Ai>=Bi. Compute Pr(Fi). Are Ai and Bi independent?

b) Let event Ei be the collection of cases such that Ai+Bi is one of {3, 5, 7, 9}. Compute Pr(Ei).

c) Compute (Fi | Ei). Are Fi and Ei independent?

2) Chain rule [2 pts each]

a) Prove the generalized chain rule.

$$Pr(E1 \cap E2 \cap E3..\cap En)=Pr(E1)Pr(E2|E1)Pr(E3|E1 \cap E2)\ldots Pr(En|E1 \cap E2 \cap .. \cap En-1)$$

b) There are three students whose surname is "Kim" among 15 students. Let their names be AKim, BKim, and CKim. Suppose that 15 students are allocated to three project teams randomly. We may compute the probability that three Kims belong to three different teams using the chain rule. Let E1 be the event that AKim is a member of a certain project team. And E2 be the event that BKim and AKim are in different teams. And E3 be the event that all three Kims belong to three different teams. Now compute the following probabilities.

① Pr(E2|E1) and Pr(E2)

② Pr(E3|E1∩E2) and Pr(E3)

3. Bayes' Theorem. [2 pts each]

Reconsider problem 1. Now we assume that the biased die has the following characteristics: Pr(2)=Pr(4)=Pr(6)=1/4 and Pr(1)=Pr(3)=Pr(5)=1/12. We also do not know which die is biased die and try to detect the biased one by performing experiments.

a) We roll two dice and the first die is 2 and the second die is 5, i.e. (2, 5). Compute the probability that the first die is the biased die.

b) We repeat the experiment twice and the results are (2, 5) and (6, 5), respectively. Compute the probability that the first die is the biased die.

c) Someone claims that instead of using 1, 2, 3, 4, 5, 6 we may use Even and Odd numbers such that biased die has the following properties: Pr(Even)=3/4 and Pr(Odd)=1/4. Using the claim we can replace two events (2, 5) and (6, 5) by (Even, Odd) and (Even, Odd). Is the claim correct?