First Midterm

April 3, 2017

1. Prove the following claims or give a counter-example (3 pts each)

   A) Two events $\bar{E}, \bar{F}$ are independent iff E and F are independent.

   B) H1 and H2 are independent events and H2 and H3 are also independent. Then, H1 and H3 are independent.

   C) Consider two ransom variables X and Y. If X and Y are independent, then E[XY]=E[X]E[Y].

   D) If E[XY]=E[X]E[Y], then random variables X and Y are independent.

   E) $\mathrm{E}[X^2] \geq E[X]^2$

2. Bayses' Theorem (4, 2 pts)

   A) There are two coins, one fair and another biased such that Pr(Heads)= 1/3 = 1-Pr(Tails). Assume that we know one coin is biased with the above property. We select one coin randomly and flips the coin several times to determine if the selected coin is fair or biased.

   Assume that the first flip is Heads (let the event be H), compute the following probabilities

   – Pr(H | Fair Coin), Pr(H | Biased Coin)

   – Pr(Fair Coin | H)

   Assume that first two flips are Heads and Heads. Compute the following probabilities.

   – Pr(HH | Fair Coin), Pr(HH | Biased Coin)

   – Pr(Fair Coin | HH)

   B) A medical company touts its new test for a certain genetic disorder. The false negative rate is small: if you have the disorder, the probability that the test returns a positive result is 0.999. The false positive rate is also small: if you do not have the disorder, the probability that the test returns a positive result is only 0.005. Assume that 2% of the population has the disorder. If a person chosen uniformly from the population is tested and the result comes back positive, what is the probability that the person has the disorder?

3. Geometric distribution & Others (2, 3, 2 pts)

   A) Geometric random variable is the number of trials until a certain event occurs. Let X be a geometric r.v. with parameter p. Applying the conditional expectation, derive E[X]=1/p.

   B) Now, consider a r.v., X, that represents the number of trials until a certain event occur n times (Geometric distribution is a case where n=1). Assuming that the event occurs with probability p, compute Pr(X=n+k). Also, derive an equation that show the expectation of X given the result of the first trial.

   C) Let X1, X2,...Xn be independent r.v. with Pr(Xi=0) = p =1-Pr(Xi=1). Let Y=max{X1, X2, ...Xn}. Compute the probability of r.v. Y and its expectation and variance.

4. MGF (2 pts each)

   A) Let X and Y are independent binomial r.v.s with B(n, p) and B(m, p). Derive the MGF of X and Y, respectively

   B) Derive the relation between MGF of X+Y and MGFs of X and Y and show that X+Y~B(n+m,p)