Quiz 3

may 24, 2017

1. Time reversibility of MC (2 pts each)

   A) Let Xn, n=0,…. be an MC with transition probability Pij. Now consider a stochastic process that observe Xn in reversed time. Prove that the time reversed stochastic process is also an MC. Compute the transition probability Qij of the time reversed process.

   B) Definition of time reversibility is as follows.

   An MC is time reversible if $\pi_i P_{ij} = \pi_j P_{ji}$ for all i, j where π_i is the stationary distribution of state i.

   Discuss the meanings of the time reversibility implied in the definition.

   C) Consider a random walk with states 0, 1,…,M and transition probabilities

   $$P_{i,i+1} = \alpha_i = 1 - P_{i,i-1}, i = 1, \dots, M-1$$

   $$P_{0,1} = \alpha_0 = 1 - P_{0,0},$$

   $$P_{M,M} = \alpha_M = 1 - P_{M,M-1}$$

   Argue the random walk process is time reversible. Using the time reversibility, compute the stationary distribution of the random walk.

2. Continuous RV (3 pts each)

   A) Let X and Y be independent, uniform random variables on [0, 1]. Find the density and distribution function for X+Y.

   B) Let X1, X2, …, Xn be a sequence of independent, uniform random variables on [0, 1]. Let Y=max{X1, X2, …, Xn}. Show that the distribution function of Y, F(y) = y^n, 0 <= y <1.

   C) Continuation of B). Compute the probability that Y=Xn. (Hint: Pr[Y = Xn] = 1/n.)

3. Exponential Distribution & Poisson Process (2 pts each)

   A) Consider a computer system where two types of jobs (let them be T1 and T2 job, respectively) ask for CPU time infinitely. The arrival processes of the two jobs follow Poisson distribution with parameter λ_1, λ_2, respectively. Given that the system is waiting for the first job, compute the probability that a T1 job arrives before a T2 job. (Hint: Let X1, X2 be the arrival times of the first T1, and T2 jobs, respectively. Compute Pr[X1 < X2].)

B) Continuation of A). Let Y be the arrival time of the first job regardless of job type (i.e. Y= min{X1, X2}). Derive the distribution (Or density) function of Y.

C) Assume that $\lambda_1 = 0.5\ per\ minute$. Given that we wait for the arrival of T1 job for two minutes. Compute the expected time until the arrivals of three T1 jobs.