Seoul National University

Quiz2 Solutions

Date

Name

SCONE Lab.

Exercise 3.18 (Quiz 2)

- Prove $\Pr(X - E[X] \geq t\sigma[X]) \leq 1/(1+t^2)$
  - $\Pr(X - E[X] \geq t\sigma[X]) = \Pr(X - E[X] + z \geq t\sigma[X] + z)$

$$\leq \frac{\mathrm{E}[(X-E[X]+z)^2]}{(t\sigma[X]+z)^2} = \frac{\sigma^2+z^2}{(t\sigma+z)^2}$$

Consider it as a function of z
➔ Find z that minimizes the value

  - $f^{(1)}(z) = 2z(t\sigma + z)^{-2} - 2(\sigma^2 + z^2)\,(t\sigma + z)^{-2}$
  - F(z) is min. at z=σ/t
- Prove $\Pr(|X - E[X]| \geq t\sigma[X]) \leq 2/(1+t^2)$
  - Let Y= $\frac{X-E[X]}{\sigma[X]}$ ➔ E[Y]=0, Var[Y]=1
  - $\Pr(|X - E[X]| \geq t\sigma[X])$ = $\Pr(Y \geq t)$

$$\mathrm{E}[1+Y^2] = (1+y^2)\left(\sum_{y\geq t}\Pr(Y=y) + \sum_{-t\leq y\leq t}\Pr(Y=y) + \sum_{y\leq -t}\Pr(Y=y)\right)$$
$$\geq (1+y^2)\left(\sum_{y\geq t}\Pr(Y=y) + \sum_{y\leq -t}\Pr(Y=y)\right)$$
$$\geq (1+t^2)\left(\sum_{y\geq t}\Pr(Y=y) + \sum_{y\leq -t}\Pr(Y=y)\right)$$
$$= (1+t^2)(\Pr(Y\geq t) + \Pr(Y\leq -t))$$
$$= (1+t^2)(\Pr(|Y|\geq t))$$

- Pr(Y>3 | Y>2) = Pr(Y>3)
  - Pr(Y>3 | Y>2)=$\frac{\Pr((Y>3)\cap(Y>2))}{\Pr(Y>2)}$
    - ➔ $\Pr((Y>3)\cap(Y>2))$= Pr(Y>3)Pr(Y>2)
  - Events Y>3, and Y>2 are independent
  - Suppose we roll a die and define a random variable as follows
    - Y=2 if o o o o are shown
    - Y=3 if o o o o are shown
    - Y=4 if o o o o are shown
  - Goal $\Pr((Y>3)\cap(Y>2))$=1/3, Pr(Y>3)=3/6, Pr(Y>2)=4/6

Y=3,Y=4 events has 4 distinct values

Y=4 event has 3 distinct values

Y=3,Y=4 events has 2 common values