Quiz 2

April 19, 2017

1. Prove or give a counter-example of the following claims. (3 pts each)

   A) Consider two random variables X1 and X2. If E[X1·X2] = E[X1]·E[X2], then Var[X1 + X2] = Var[X1] + Var[X2].

   B) If Var[X1 + X2] = Var[X1] + Var[X2], then E[X1·X2] = E[X1]·E[X2].

   C) If X1 and X2 are independent, then Var[X1-X2] = Var[X1] + Var[X2].

   D) Applying the Chebyshev's inequality, prove the weak law of large number.

   If X1, X2, … Xn are independent and identical random variables with mean μ and standard deviation σ, then for any constant $\varepsilon > 0$,

$$\lim_{n\to\infty} \Pr\left(\left|\frac{X1 + X2 + \cdots + Xn}{n} - \mu\right| > \varepsilon\right) = 0$$

2. Assume we roll two fair dice (say A and B) 100 times. Let X and Y be the sums of numbers that appear over the 100 rolls of A and B, respectively. (2, 2, 3 pts)

   A) Compute E[X] and Var[X].

   B) Compute the Markov's bound of Pr(X >= 400).

   C) Compute the Chebyshev's bound of Pr(|X-Y| >= 100).

   D)

3. Assume we toss a fair coin n times. Let Y be the number of heads over n trials. (2, 3, 4 pts)

   A) Compute E[Y] and Var[y]

   B) Compute the bound of Pr(Y >= 3n/4) using Chebyshev's inequality.

   C) Compute the bound of Pr(Y >= 3n/4) using Chernoff's inequality. Your bound should be as tight as possible.