

Chapter 1

Introduction

These lecture notes are provided by Prentice Hall
accompanied with “Contemporary Logic Design” 2nd Ed.

Why study logic design?

- **it is the implementation basis for all modern computing devices**
 - building large things from small components
 - provide a model of how a computer works
- **More important reasons**
 - the inherent parallelism in hardware is often our first exposure to parallel computation
 - it offers an interesting counterpoint to software design and is therefore useful in furthering our understanding of computation, in general

What will we learn in this class? (1/2)

- **The basics of logic design**

- Boolean algebra, logic minimization, state, timing, CAD tools

- **The concept of state in digital systems**

- analogous to variables and program counters in software systems

What will we learn in this class? (2/2)

■ **How to specify/simulate/compile/realize our designs**

- ❑ hardware description languages (HDLs)
- ❑ tools to simulate the workings of our designs
- ❑ logic compilers to synthesize the hardware blocks of our designs
- ❑ mapping onto programmable hardware

■ **Contrast with software design**

- ❑ sequential and parallel implementations
- ❑ specify algorithm as well as computing/storage resources it will use

Applications of logic design

- **Conventional computer design**

- CPUs, busses, peripherals

- **Networking and communications**

- phones, modems, routers

- **Embedded products**

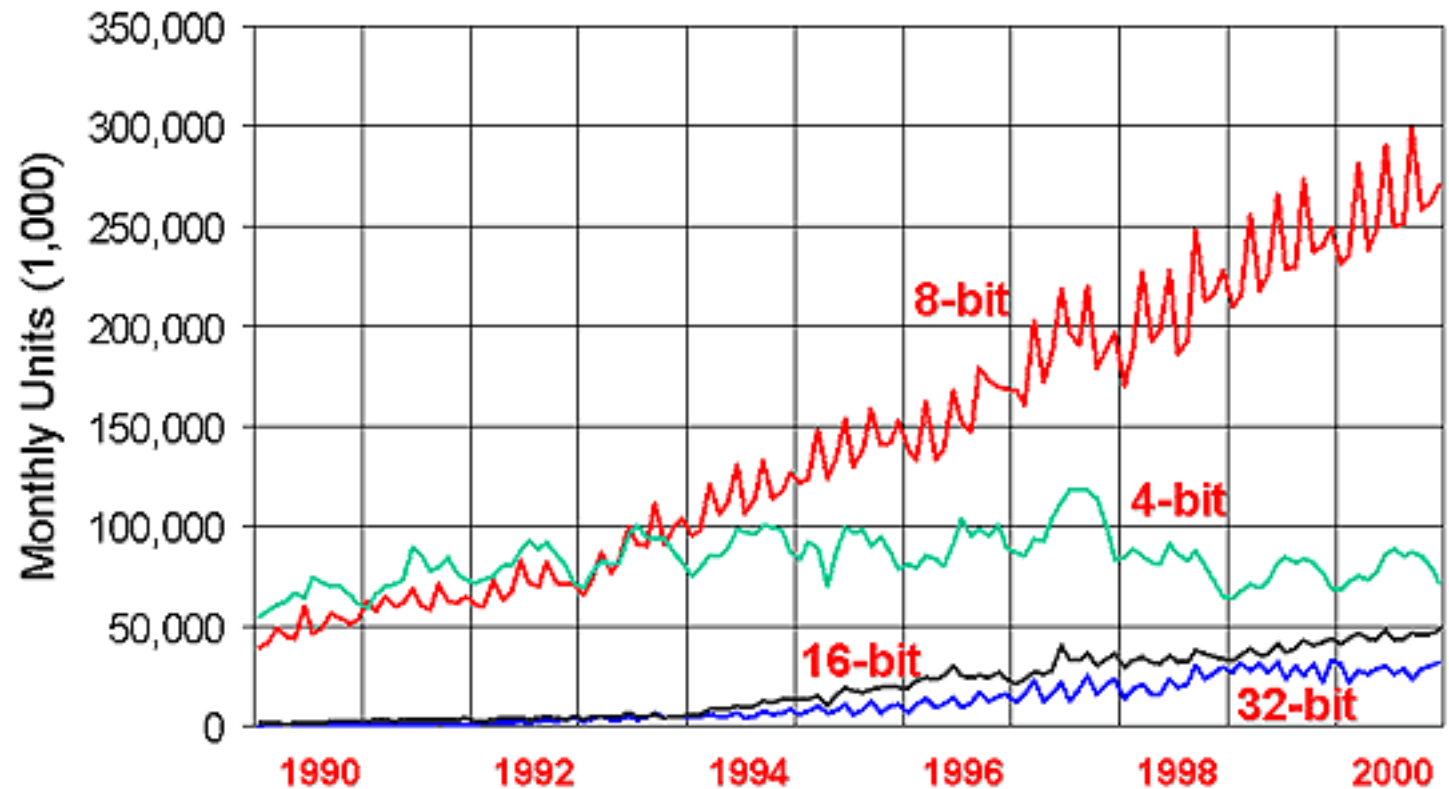
- in cars, toys, appliances, entertainment devices

- **Scientific equipment**

- testing, sensing, reporting

- **The world of computing is much bigger than just PCs!**

Global processor/controller market



Source: WSTS

PC CPUs constitute less than 2% of the market

What is logic design? (1/2)

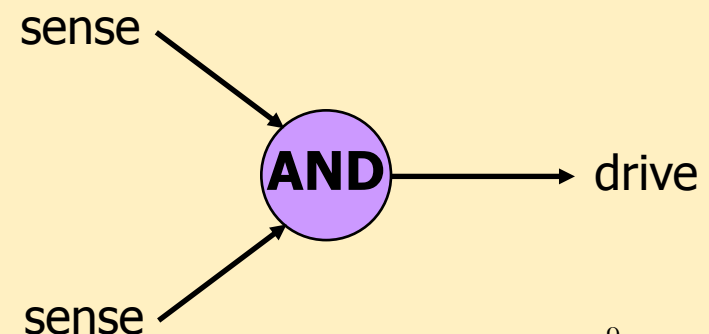
- **Digital hardware consists of components**
 - **Components or building blocks**
 - Switches built from semiconductor transistors
 - Most basic element
 - Higher level circuits such as logic gates and memories
 - **A logic designer should choose the right component to solve logic design problems**
 - **Constraints: size, cost, performance, power consumption**
 - Cost vs. size
- * A circuit is an interconnected collection of switches

What is logic design? (2/2)

- **Each component has**
 - a set of input wires
 - a set of output wires
 - Each wire is set to some analog voltage value
 - But will be interpreted as either 1 or 0 (**digital abstraction**)
- **Transistors react to the voltage levels on the input wires**
 - Switch their state and cause a change in output wires
 - At macro scale, a component that contains transistors reacts to input voltage values
- **Depending on the way a circuit reacts to the input voltages**
 - Combinational logic circuits
 - Sequential logic circuits

What is digital hardware?

- **Collection of devices that sense and/or control wires, which carry a digital value (i.e., a physical quantity that can be interpreted as a “0” or “1”)**
 - example: digital logic where voltage $< 0.8\text{v}$ is a “0” and $> 2.0\text{v}$ is a “1”
- **Primitive digital hardware devices**
 - logic computation devices (sense and drive)
 - are two wires both “1” - make another be “1” (AND)
 - is at least one of two wires “1” - make another be “1” (OR)
 - is a wire “1” - then make another be “0” (NOT)
 - memory devices (store)
 - store a value
 - recall a previously stored value

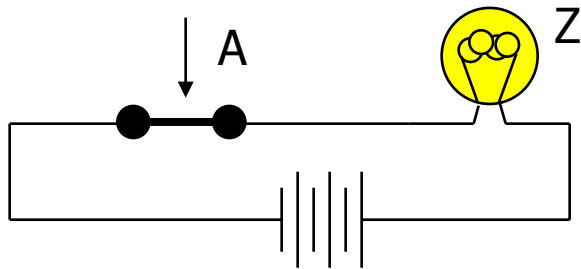


Computation: abstract vs. implementation

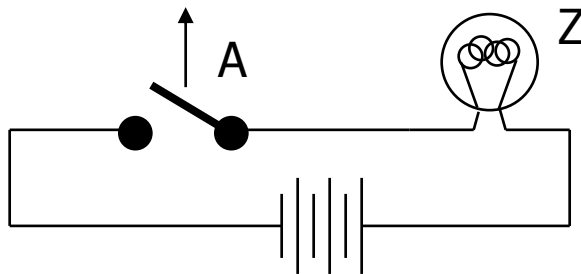
- **Computation has been a mental exercise (paper, programs)**
- **This class is about physically implementing computation using physical devices that use voltages to represent logical values**
- **Basic units of computation are:**
 - representation: "0", "1" on a wire
set of wires (e.g., binary integer)
 - assignment: $x = y$
 - data operations: $x + y - 5$
 - control:
 - sequential statements: $A; B; C$
 - conditionals: $\text{if } x == 1 \text{ then } y$
 - loops: $\text{for } (i = 1 ; i == 10, i++)$
 - procedures: $A; \text{proc}(\dots); B;$
- **We will study how each of these are implemented in hardware and composed into computational structures**

Switches: basic element of physical implementations

■ Implementing a simple circuit



close switch (if A is "1" or asserted)
and turn on light bulb (Z)



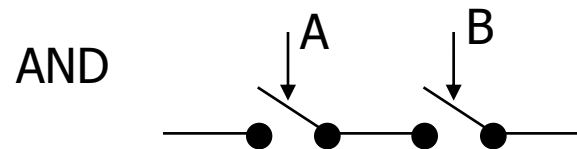
open switch (if A is "0" or unasserted)
and turn off light bulb (Z)

$$Z \equiv A$$

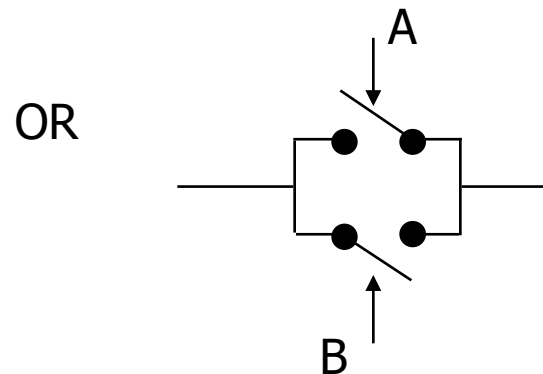
Z and A are equivalent boolean variables

Switches (cont'd)

- Compose switches into more complex ones (Boolean functions):



$$Z \equiv A \text{ and } B$$



$$Z \equiv A \text{ or } B$$

Switching networks

- **Switch settings**

- determine whether or not a conducting path exists to light the light bulb

- **To build larger computations**

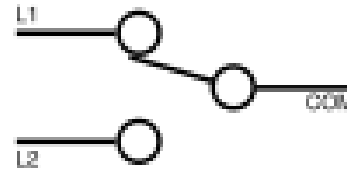
- use a light bulb (output of the network) to set other switches (inputs to another network).

- **Connect together switching networks**

- to construct larger switching networks, i.e., there is a way to connect outputs of one network to the inputs of the next.

A switch?

- A mechanical switch



- A semiconductor switch or transistor

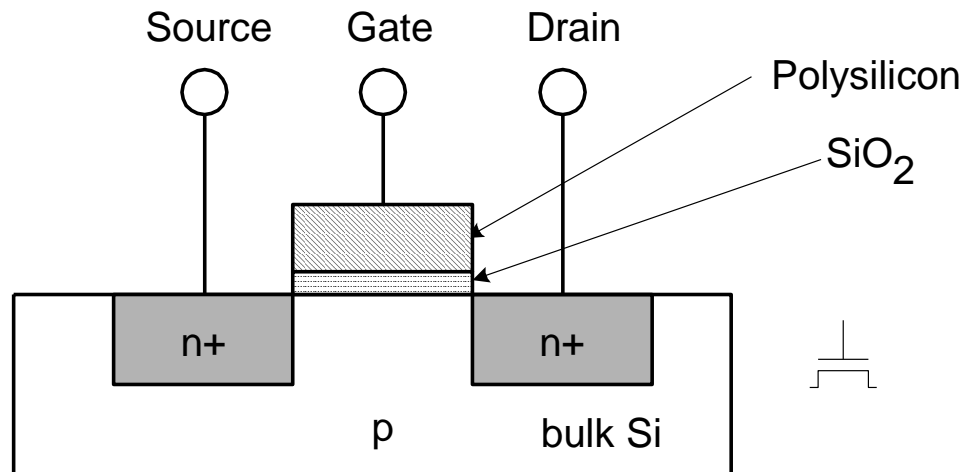
Transistor networks

- **Modern digital systems are designed in CMOS technology**
 - MOS stands for Metal-Oxide on Semiconductor
 - C is for complementary because there are both normally-open and normally-closed switches: nMOS and pMOS
- **MOS transistors act as voltage-controlled switches**

* CMOS: complementary metal-oxide semiconductor

n-channel (or n-type) MOS (nMOS) circuit

- **Three terminals: source-gate-drain (or S-G-D for short)**
- **Three layers: polysilicon (used to be metal) – SiO₂ - substrate**

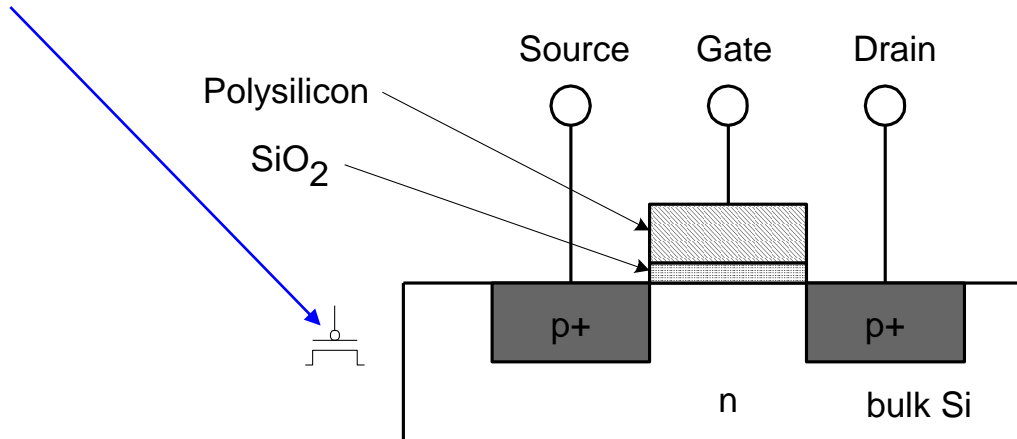


- **If G is at positive voltage, electrons in the substrate will move toward G terminal, which sets up a channel between S and D**
 - And D is at high voltage, current will flow from drain to source
- **Metal is replaced by polysilicon which is more adhesive**

* n+: heavily doped n-type semiconductor

p-channel (or p-type) MOS (pMOS) circuit

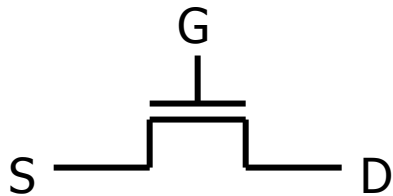
- Three terminals: source-gate-drain (or S-G-D for short)
- Same principle, but reverse doping and voltage
 - Source (V_{ss}) is positive with regard to drain (V_{dd})
- Bubble indicates the inverted behavior



- If G is at positive voltage, the current does not flow
- If G is at ground level, the current flows

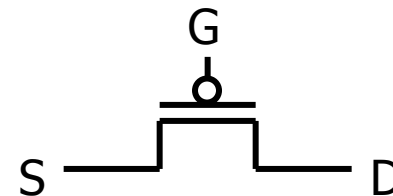
MOS transistors

- **MOS transistors have three terminals: drain, gate, and source**
 - they act as switches in the following way:
if the voltage on the gate terminal is (some amount) higher/lower than the source terminal, then a conducting path will be established between the drain and source terminals



n-channel

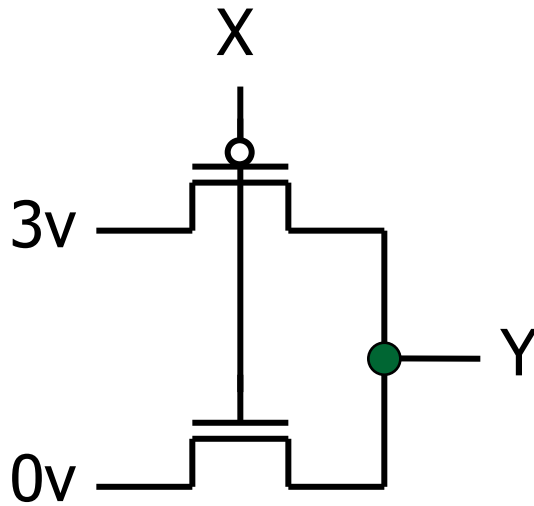
open when voltage at G is low
closed when voltage at G is high



p-channel

closed when voltage at G is low
open when voltage at G is high

MOS networks

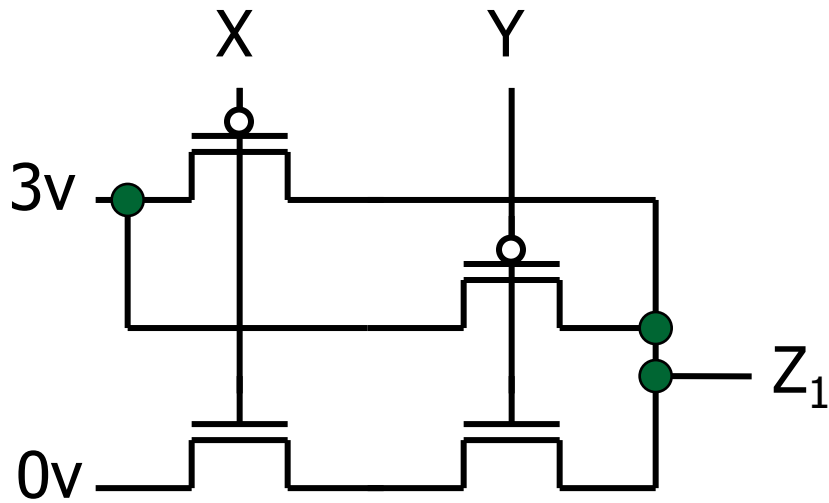


what is the
relationship
between x and y?

x	y
0 volts	
3 volts	

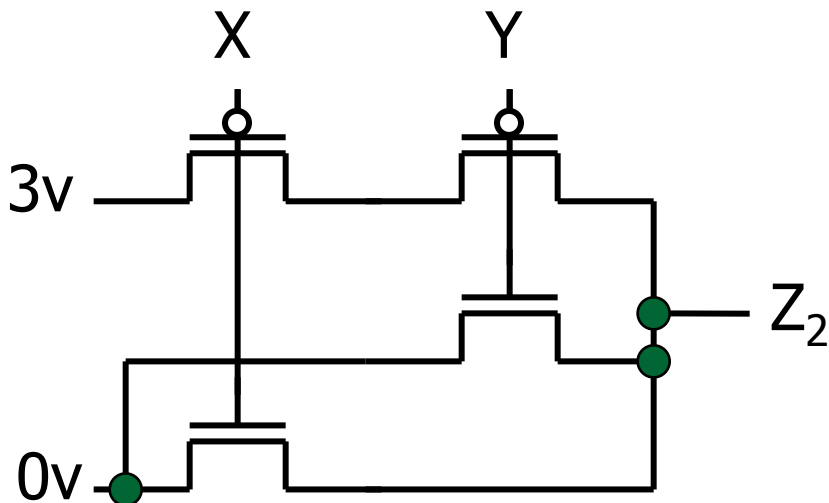
- **A simple component is made up of two transistors**
 - X: input
 - Y: output
 - What is this function?
- **In CMOS circuits, pMos and nMOS are used in pair**

Two input networks



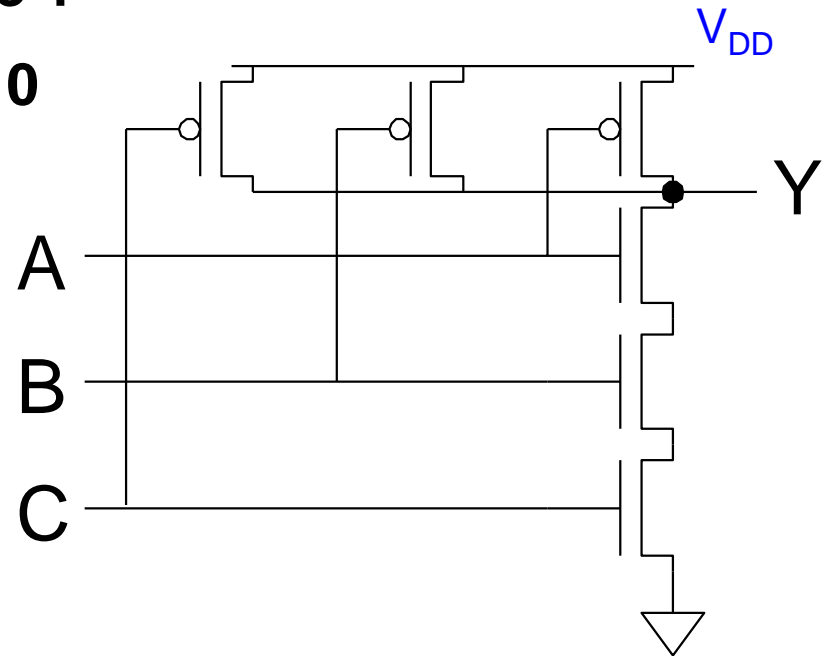
What is the relationship between x , y and z_1/z_2 ?

x	y	z_1	z_2
0 volts	0 volts		
0 volts	3 volts		
3 volts	0 volts		
3 volts	3 volts		



Three input NAND gate

- Y pulls low if ALL inputs are 1
- Y pulls high if ANY input is 0



- In general, the more inputs, the more transistors (TRs)
- In CMOS, a variable requires a pair of TRs

Digital vs. analog

- **Convenient to think of digital systems as having only discrete, digital, input/output values**
- **In reality, real electronic components exhibit continuous, analog behavior**
- **Why do we make the digital abstraction anyway?**
 - switches operate this way
 - easier to think about a small number of discrete values
 - Quantization error, though
- **Why does it work?**
 - does not propagate small errors in values
 - always resets to 0 or 1

Combinational logic symbols

- **Common combinational logic systems have standard symbols called logic gates**

- Buffer, NOT



- AND, NAND



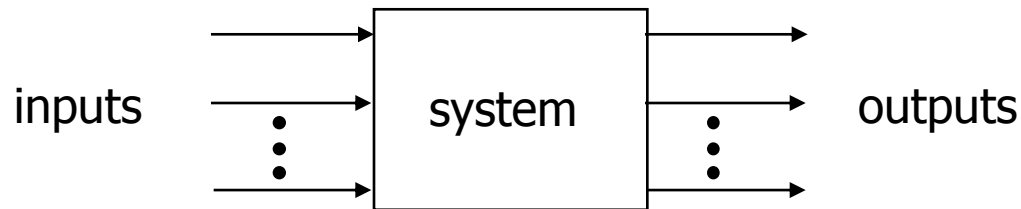
- OR, NOR



easy to implement
with CMOS transistors
(the switches we have
available and use most)

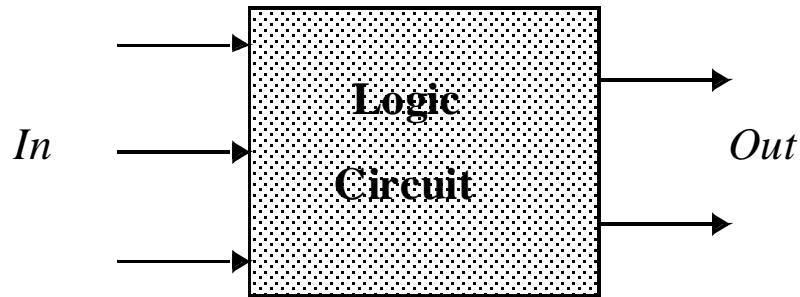
Combinational vs. sequential digital circuits

- A simple model of a digital system is a unit with inputs and outputs:



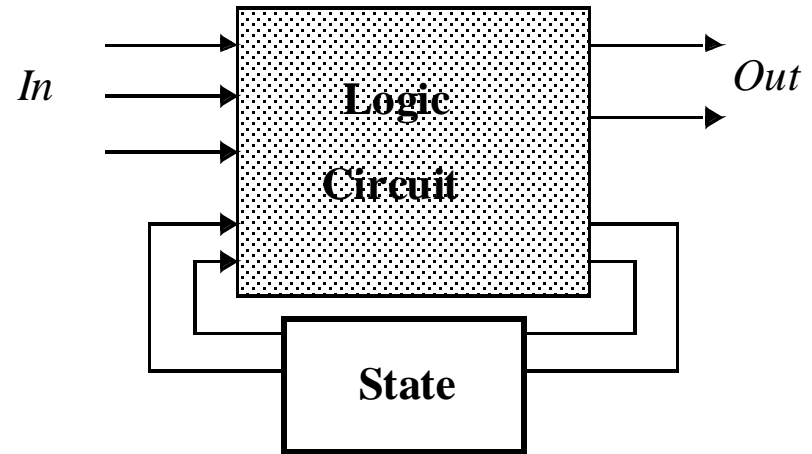
- **Combinational means "memory-less"**
 - a digital circuit is combinational if its output values only depend on its (current) input values
- **Sequential systems**
 - exhibit behaviors (output values) that depend not only on the current input values, but also on previous input values

Combinational vs. sequential digital circuits



(a) Combinational

$$\text{Output} = f(\text{In})$$



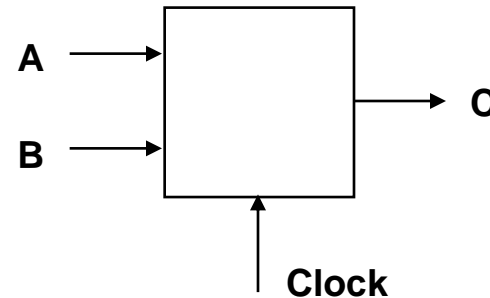
(b) Sequential

$$\text{Output} = f(\text{In}, \text{Previous In})$$

Example of combinational and sequential logic

■ Combinational:

- ❑ input A, B
- ❑ wait for clock edge
- ❑ observe C
- ❑ wait for another clock edge
- ❑ observe C again: will stay the same



■ Sequential:

- ❑ input A, B
- ❑ wait for clock edge
- ❑ observe C
- ❑ wait for another clock edge
- ❑ observe C again: may be different

Intermediate Summary

■ Some we've seen already

- ❑ digital interpretation of analog values
- ❑ transistors as switches
- ❑ switches as logic gates
- ❑ use of a clock to realize a synchronous sequential circuit

■ Some others we will see

- ❑ truth tables and Boolean algebra to represent combinational logic
- ❑ encoding of signals with more than two logical values into binary form
- ❑ state diagrams to represent sequential logic
- ❑ hardware description languages to represent digital logic
- ❑ waveforms to represent temporal behavior

An example

- **Calendar subsystem: number of days in a month (to control watch display)**
 - Combinational logic
 - used in controlling the display of a wrist-watch LCD screen
 - inputs: month, leap year flag
 - outputs: number of days

Implementation in software

```
integer number_of_days ( month, leap_year_flag)
{
    switch (month) {
        case 1: return (31);
        case 2: if (leap_year_flag == 1) then return (29)
                else return (28);
        case 3: return (31);
        ...
        case 12: return (31);
        default: return (0);
    }
}
```

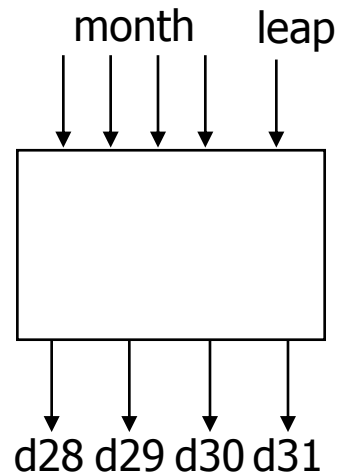
Implementation as a combinational digital system

■ Encoding:

- how many bits for each input/output?
- binary number for month
- four wires for 28, 29, 30, and 31

■ Behavior:

- combinational
- truth table specification



month	leap	d28	d29	d30	d31
0000	—	—	—	—	—
0001	—	0	0	0	1
0010	0	1	0	0	0
0010	1	0	1	0	0
0011	—	0	0	0	1
0100	—	0	0	1	0
0101	—	0	0	0	1
0110	—	0	0	1	0
0111	—	0	0	0	1
1000	—	0	0	0	1
1001	—	0	0	1	0
1010	—	0	0	0	1
1011	—	0	0	1	0
1100	—	0	0	0	1
1101	—	—	—	—	—
111—	—	—	—	—	—

Don't
care

Combinational example (cont'd)

■ Truth-table to logic to switches to gates

- $d_{28} = 1$ when month=0010 and leap=0
- $d_{28} = m_8' \cdot m_4' \cdot m_2 \cdot m_1' \cdot \text{leap}'$
- $d_{31} = 1$ when month=0001 or month=0011 or ... month=1100
- $d_{31} = (m_8' \cdot m_4' \cdot m_2' \cdot m_1) + (m_8' \cdot m_4' \cdot m_2 \cdot m_1) + \dots (m_8 \cdot m_4 \cdot m_2' \cdot m_1')$
- $d_{31} =$ can we simplify more?

symbol
for and

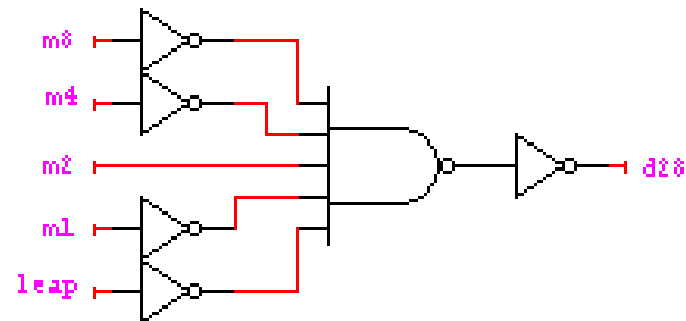
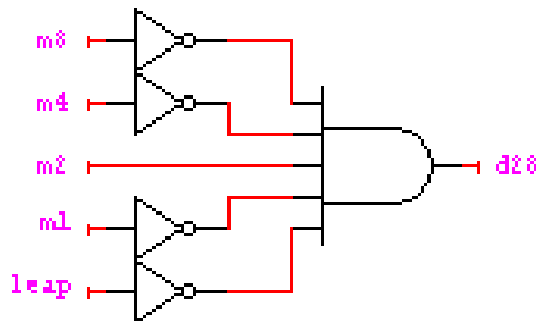
symbol
for or

symbol
for not

month	leap	d28	d29	d30	d31
0001	—	0	0	0	1
0010	0	1	0	0	0
0010	1	0	1	0	0
0011	—	0	0	0	1
0100	—	0	0	1	0
...					
1100	—	0	0	0	1
1101	—	—	—	—	—
111—	—	—	—	—	—
0000	—	—	—	—	—

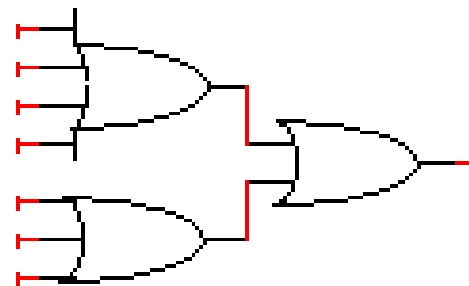
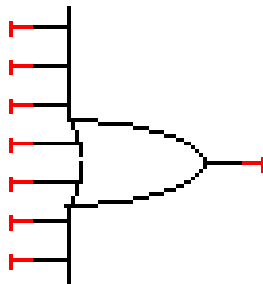
Combinational example (cont'd)

- $d28 = m8' \cdot m4' \cdot m2 \cdot m1' \cdot leap'$
- $d29 = m8' \cdot m4' \cdot m2 \cdot m1' \cdot leap$
- $d30 = (m8' \cdot m4 \cdot m2' \cdot m1') + (m8' \cdot m4 \cdot m2 \cdot m1') + (m8 \cdot m4' \cdot m2' \cdot m1) + (m8 \cdot m4' \cdot m2 \cdot m1)$
 $= (m8' \cdot m4 \cdot m1') + (m8 \cdot m4' \cdot m1)$
- $d31 = (m8' \cdot m4' \cdot m2' \cdot m1) + (m8' \cdot m4' \cdot m2 \cdot m1) + (m8' \cdot m4 \cdot m2' \cdot m1) + (m8' \cdot m4 \cdot m2 \cdot m1) + (m8 \cdot m4' \cdot m2' \cdot m1') + (m8 \cdot m4' \cdot m2 \cdot m1') + (m8 \cdot m4 \cdot m2' \cdot m1')$



Combinational example (cont'd)

- $d28 = m8' \cdot m4' \cdot m2 \cdot m1' \cdot \text{leap}'$
- $d29 = m8' \cdot m4' \cdot m2 \cdot m1' \cdot \text{leap}$
- $d30 = (m8' \cdot m4 \cdot m2' \cdot m1') + (m8' \cdot m4 \cdot m2 \cdot m1') + (m8 \cdot m4' \cdot m2' \cdot m1) + (m8 \cdot m4' \cdot m2 \cdot m1)$
- $d31 = (m8' \cdot m4' \cdot m2' \cdot m1) + (m8' \cdot m4' \cdot m2 \cdot m1) + (m8' \cdot m4 \cdot m2' \cdot m1) + (m8' \cdot m4 \cdot m2 \cdot m1) + (m8 \cdot m4' \cdot m2' \cdot m1') + (m8 \cdot m4' \cdot m2 \cdot m1') + (m8 \cdot m4 \cdot m2' \cdot m1')$



Summary of Combinational Circuit Design

- **Step 1: Block Diagram (Specify Inputs and Outputs)**
- **Step 2: Truth Table**
- **Step 3: Implementation**

Another example (Door combination lock)

- **punch in 3 values in sequence and the door opens; if there is an error the lock must be reset; once the door opens the lock must be reset**
 - Sequential logic
 - inputs: sequence of input values, reset
 - Numeric number: 4 wires
 - outputs: door open/close
 - memory: must remember combination
 - or always have it available as an input

Implementation in software

```
integer combination_lock ( ) {  
    integer v1, v2, v3;  
    integer error = 0;  
    static integer c[3] = 3, 4, 2;  
  
    while (!new_value( ));  
    v1 = read_value( );  
    if (v1 != c[1]) then error = 1;  
  
    while (!new_value( ));  
    v2 = read_value( );  
    if (v2 != c[2]) then error = 1;  
  
    while (!new_value( ));  
    v3 = read_value( );  
    if (v2 != c[3]) then error = 1;  
  
    if (error == 1) then return(0); else return (1);  
}
```

Array index starts from 1

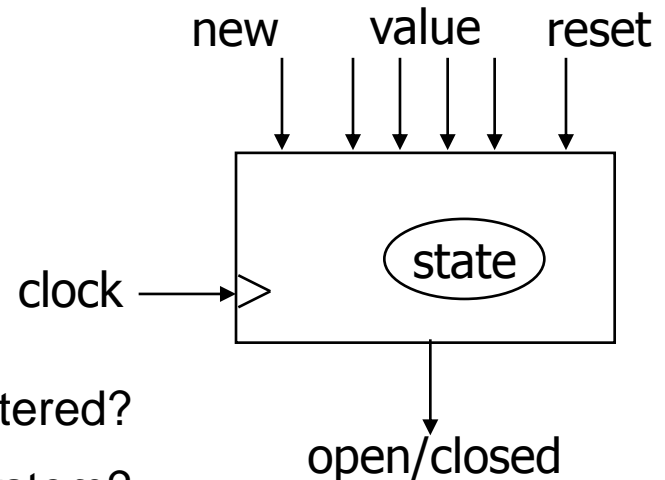
Implementation as a sequential digital system

■ Encoding:

- ❑ how many bits per input value?
- ❑ how many values in sequence?
- ❑ how do we know a new input value is entered?
- ❑ how do we represent the states of the system?

■ Behavior:

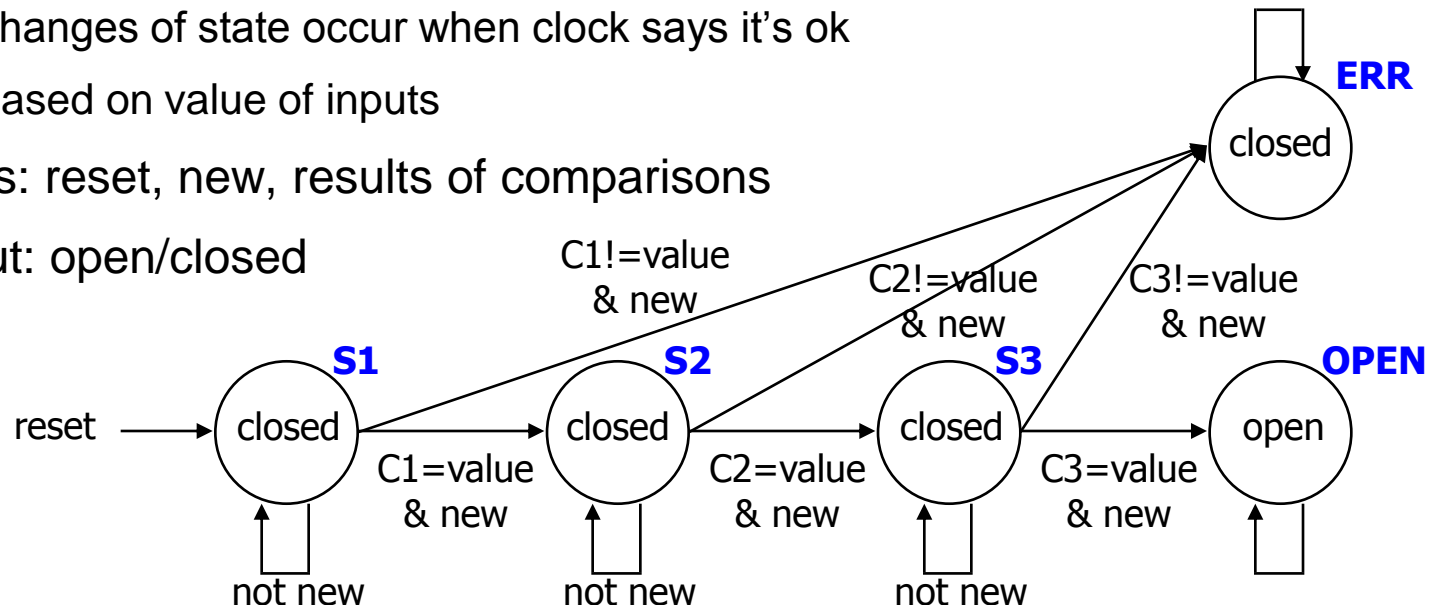
- ❑ clock wire tells us when it's ok to look at inputs
(i.e., they have settled after change)
- ❑ sequential: sequence of values must be entered
- ❑ sequential: remember if an error occurred
- ❑ finite-state specification



Sequential example (cont'd): abstract control

■ Finite-state diagram

- states: 5 states
 - represent point in execution of machine
 - each state has inputs and outputs
- transitions: 6 from state to state, 5 self transitions, 1 global
 - changes of state occur when clock says it's ok
 - based on value of inputs
- inputs: reset, new, results of comparisons
- output: open/closed



Sequential example (cont'd): data-path vs. control

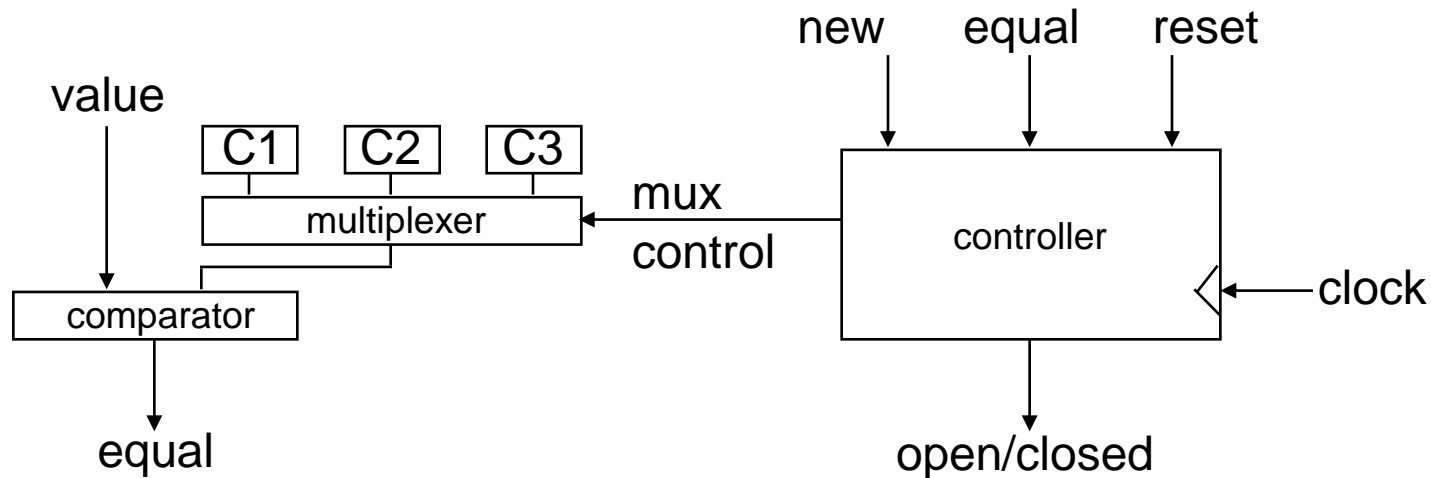
■ Internal structure

□ data-path

- storage for combination
- comparators

□ control

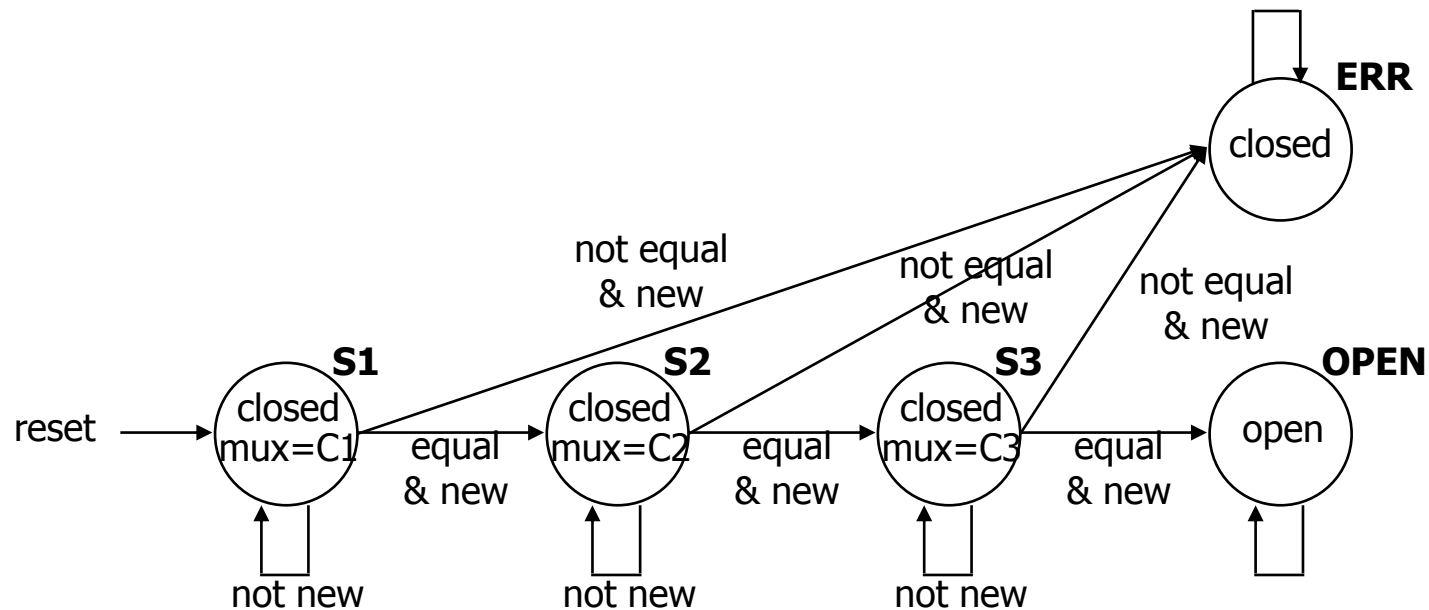
- finite-state machine controller
- control for data-path
- state changes controlled by clock



* Multiplexer (MUX)

Sequential example (cont'd): FSM

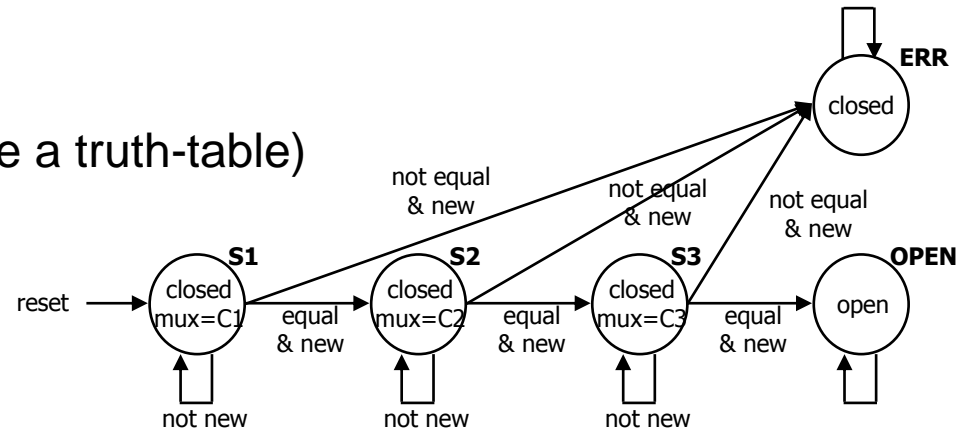
- **Finite-state machine (FSM)**
 - refine state diagram to include internal structure



Sequential example (cont'd): FSM

■ Finite-state machine

- generate state table (much like a truth-table)



reset	new	equal	state	next state	mux	open/closed
1	—	—	—	S1	C1	closed
0	0	—	S1	S1	C1	closed
0	1	0	S1	ERR	—	closed
0	1	1	S1	S2	C2	closed
0	0	—	S2	S2	C2	closed
0	1	0	S2	ERR	—	closed
0	1	1	S2	S3	C3	closed
0	0	—	S3	S3	C3	closed
0	1	0	S3	ERR	—	closed
0	1	1	S3	OPEN	—	open
0	—	—	OPEN	OPEN	—	open
0	—	—	ERR	ERR	—	closed

* state is not input, but internal variable

Sequential example (cont'd): encoding

■ Encode state table

- state can be: S1, S2, S3, OPEN, or ERR
 - needs at least 3 bits to encode: 000, 001, 010, 011, 100
 - and as many as 5: 00001, 00010, 00100, 01000, 10000
 - choose 4 bits: 0001, 0010, 0100, 1000, 0000
- output mux can be: C1, C2, or C3
 - needs 2 to 3 bits to encode
 - choose 3 bits: 001, 010, 100
- output open/closed can be: open or closed
 - needs 1 or 2 bits to encode
 - choose 1 bits: 1, 0

Sequential example (cont'd): encoding

■ Encode state table

- state can be: S1, S2, S3, OPEN, or ERR
 - choose 4 bits: 0001, 0010, 0100, 1000, 0000
- output mux can be: C1, C2, or C3
 - choose 3 bits: 001, 010, 100
- output open/closed can be: open or closed
 - choose 1 bits: 1, 0

	reset	new	equal	state	next state	mux	open/closed	
external input	1	—	—	—	0001	001	0	
	0	0	—	0001	0001	001	0	
	0	1	0	0001	0000	—	0	good choice of encoding!
	0	1	1	0001	0010	010	0	
	0	0	—	0010	0010	010	0	
	0	1	0	0010	0000	—	0	mux is identical to last 3 bits of next state
	0	1	1	0010	0100	100	0	
	0	0	—	0100	0100	100	0	
	0	1	0	0100	0000	—	0	
	0	1	1	0100	1000	—	1	open/closed is identical to first bit of next state
	0	—	—	1000	1000	—	1	
	0	—	—	0000	0000	—	0	

Internal input

Internal output

system's or external output

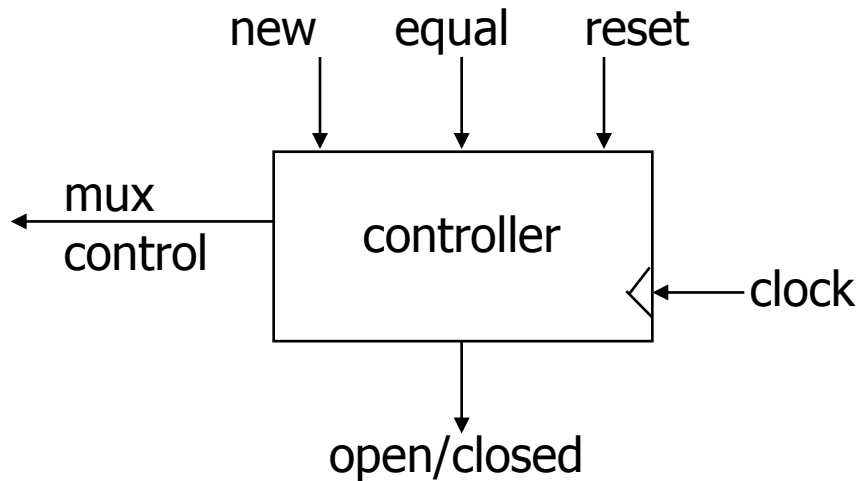
good choice of encoding!

mux is identical to last 3 bits of next state

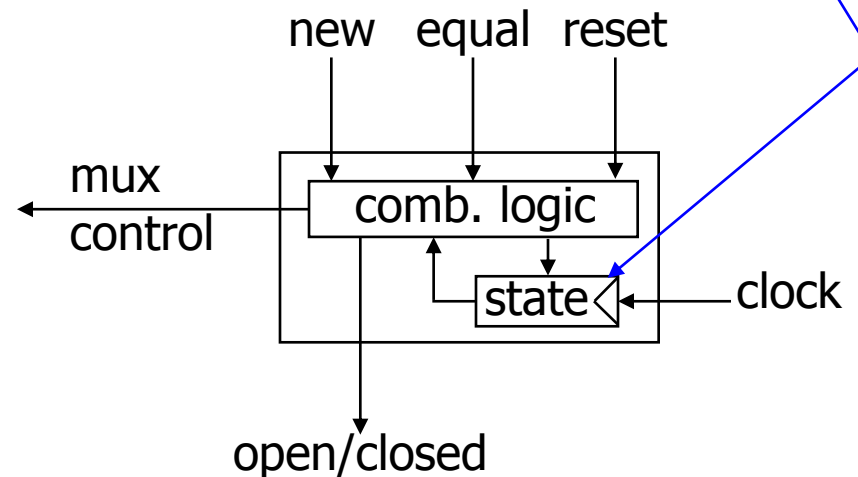
open/closed is identical to first bit of next state

Sequential example (cont'd): controller implementation

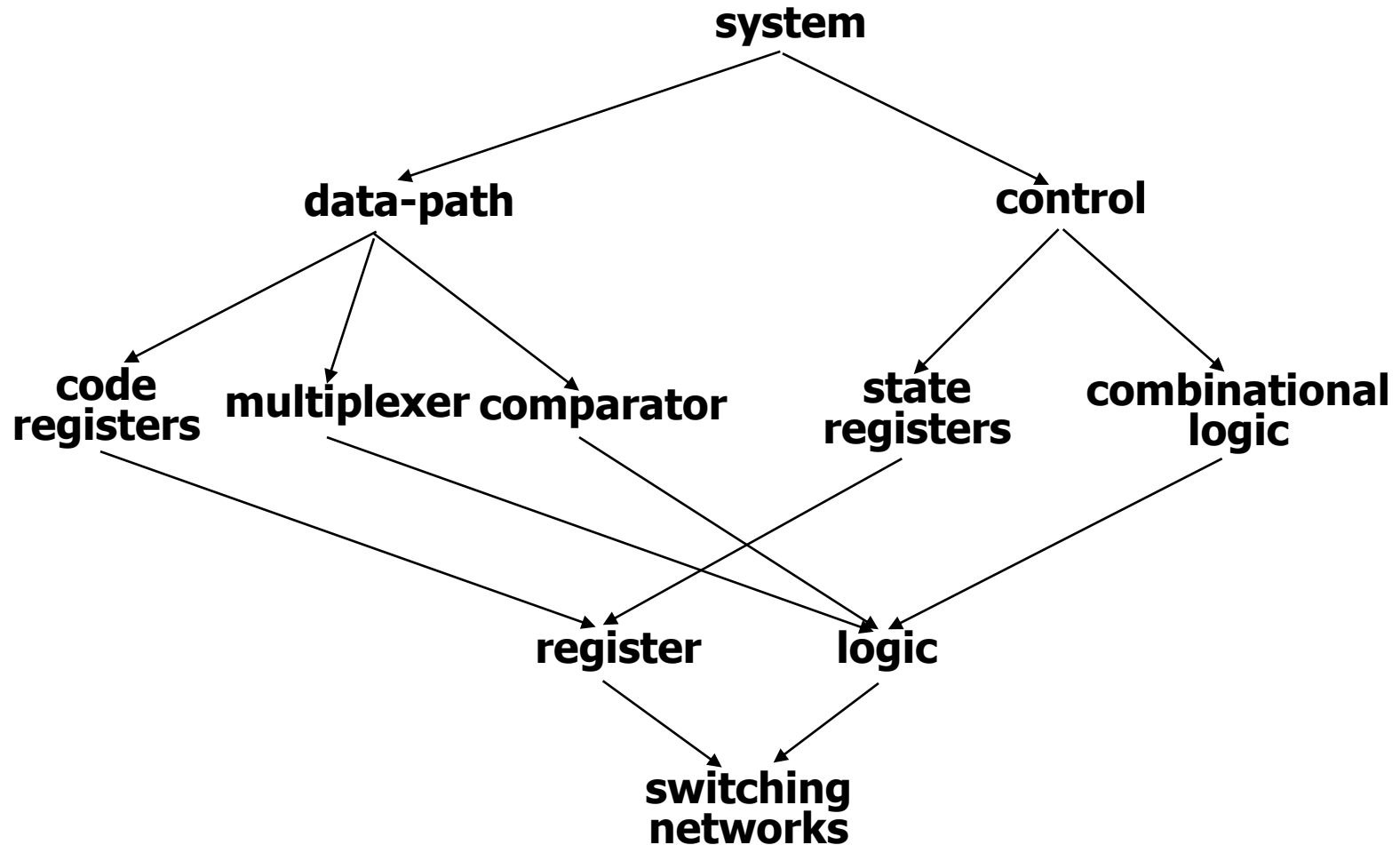
■ Implementation of the controller



special circuit element, called a register, for remembering inputs when told to by clock



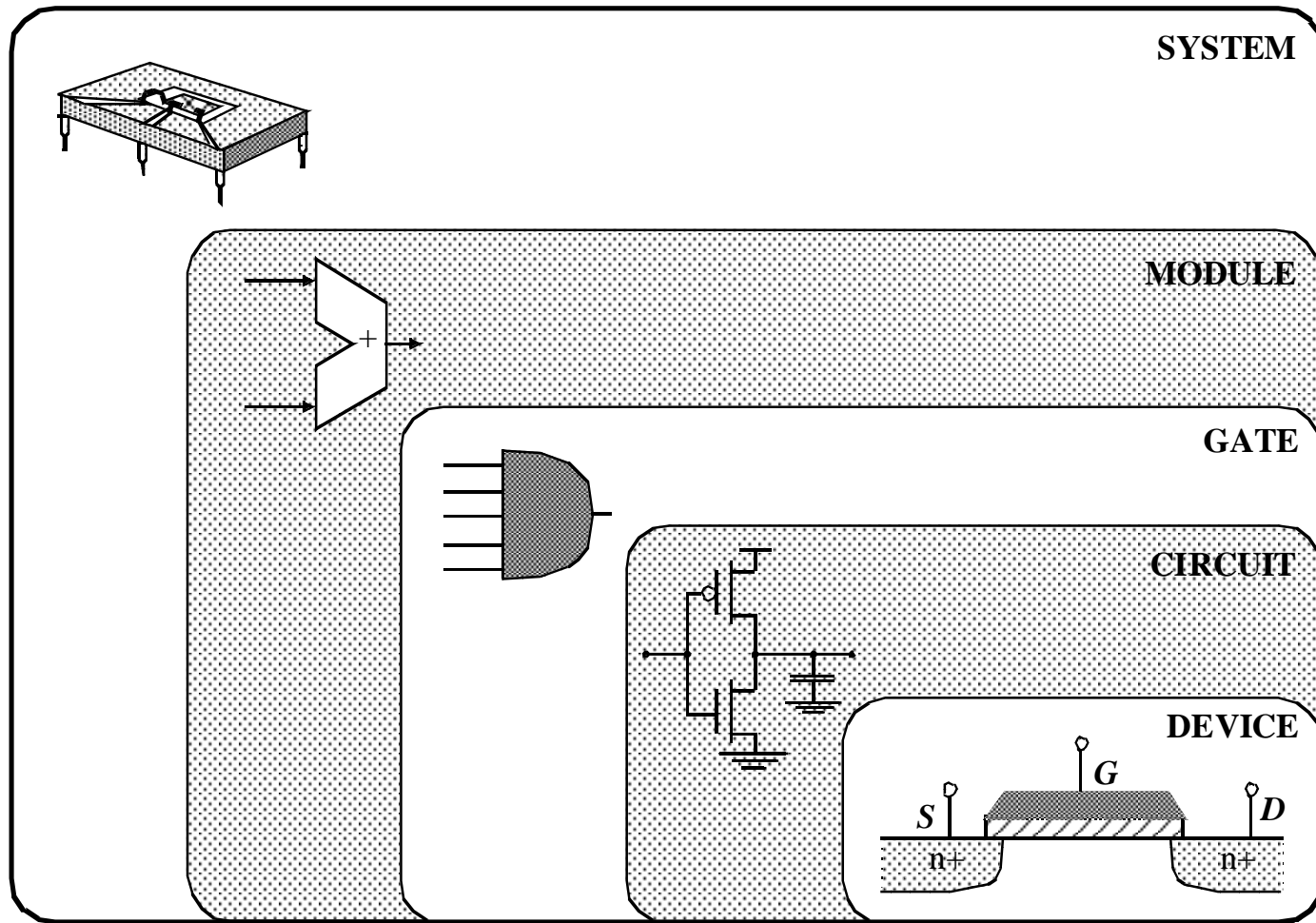
Sequential Circuit Design hierarchy



Summary of Sequential Circuit Design

- **Step 1: Block diagram (Specify Inputs and Outputs)**
- **Step 2: State diagram**
- **Step 3: Decomposition into data part and control part**
- **Step 4: FSM for control part**
- **Step 5: Implementation**

Terminologies



Summary

- **That was what the entire course is about**
 - converting solutions for problems into combinational and sequential networks effectively organizing the design hierarchically
 - doing so with a modern set of design tools that lets us handle large designs effectively
 - taking advantage of optimization opportunities