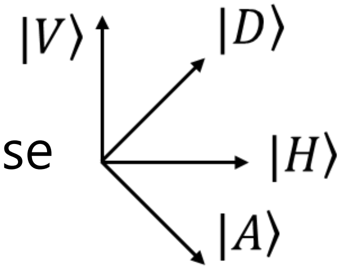
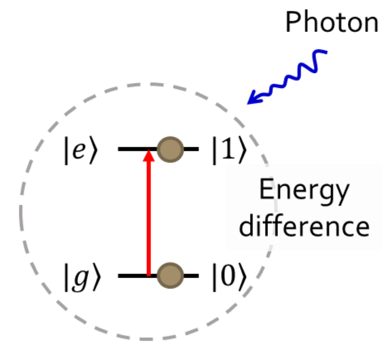
Summary of previous lecture

- Mainly from 2.2.1~2.2.2 of the textbook
- Postulate 1: the state of the particle is represented by a vector $|\psi(t)\rangle$ in a Hilbert space
  - Two-level atom (TLA)
  - Polarization of light, meaning of phase





- Postulate 2: the evolution of a "closed"
  quantum system is described by a unitary transformation
- Postulate 2′ (continuous time version): the time evolution of the sate of a "closed" quantum system is described by Schrödinger equation, $i\hbar\frac{\partial}{\partial t}|\psi\rangle = \mathcal{H}|\psi\rangle$ ➔ $|\psi(t)\rangle = e^{-i\mathcal{H}(t-t_0)/\hbar}|\psi(t_0)\rangle$

Postulate 3

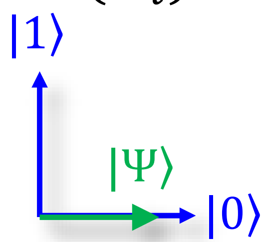
- Postulate 3: Copenhagen interpretation

  If the particle is in a state $|\psi\rangle$, measurement of the variable (corresponding to) Ω will yield one of the eigenvalues ω_i with probability of $P(\omega_i) \propto |\langle\omega_i|\psi\rangle|^2$.

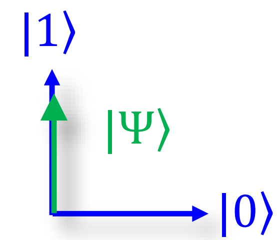
  Then the state of the system will change from $|\psi\rangle$ to $|\omega_i\rangle$ as a result of measurement. ➔ called "collapse"

- Example of $P(\omega_i) \propto |\langle\omega_i|\psi\rangle|^2$

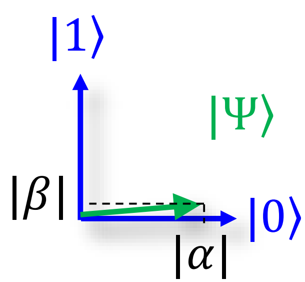
$|\Psi\rangle = |0\rangle$



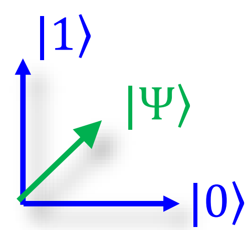
$|\Psi\rangle = |1\rangle$



$|\Psi\rangle = \sqrt{0.9}|0\rangle + \sqrt{0.1}|1\rangle$



$|\Psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$

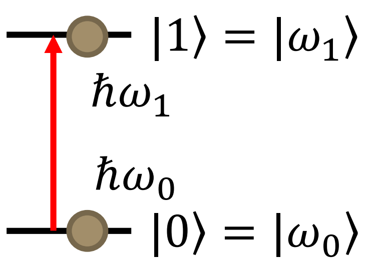


  - Polarization: continuous vs. discrete measurement

Postulate 3

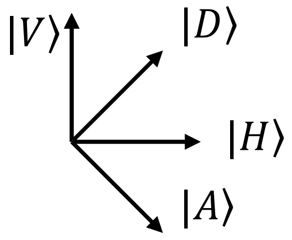
- Why is the probability proportional to $|\langle\omega_i|\psi\rangle|^2$ rather than $\langle\omega_i|\psi\rangle$ or $|\langle\omega_i|\psi\rangle|$?
- Normalization of the state ➔ make the total probability 100%

- What happens if I measure the quantum state of an electron in a TLA when $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$?
  - What do we mean by measuring $|0\rangle$ or $|1\rangle$?
  - ➔ Energy of the states



- Find out the measurement operator for TLA
  - $\mathcal{H} = \hbar\omega_0|0\rangle\langle 0| + \hbar\omega_1|1\rangle\langle 1|$

- If polarization measurement device outputs 2 for $|H\rangle$ and -1 for $|V\rangle$, what is the proper operator corresponding to this device?
  - $P = 2|H\rangle\langle H| - |V\rangle\langle V|$



Postulate 3

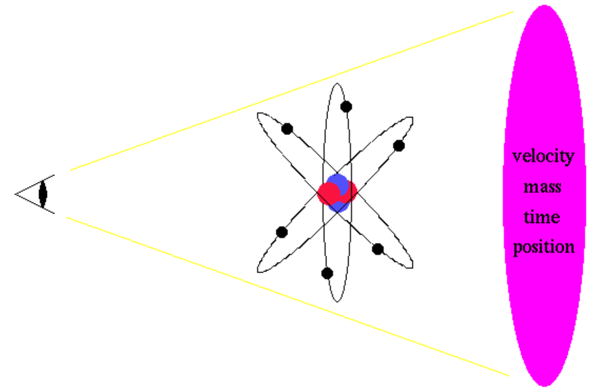
- In the previous explanation, we constructed the measurement operator, but, generally measurement operators are guessed from the corresponding classical models. For example, energy operator, momentum operator, position operator, angular momentum, spin etc.

- Measurement operator should be Hermitian. Why?
  - Measured value should be real value
  - Hermitian requirement for Hamiltonian

    ➔Unitarity of quantum evolution

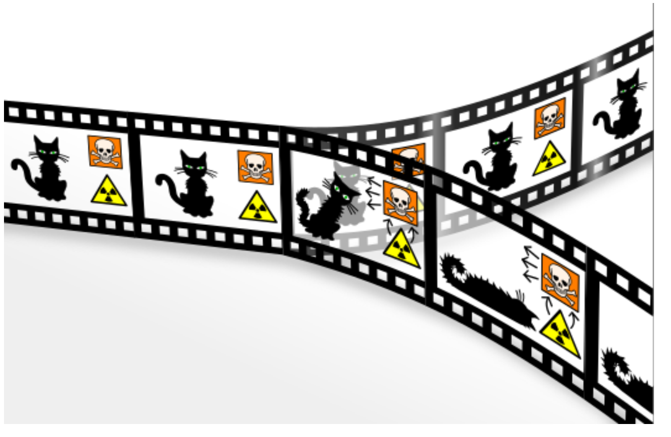
Postulate 3

- Copenhagen interpretation
- Hidden variable theory
  - Einstein, de Broglie, Bohm,…
  - Einstein (1926): "I am convinced God does not play dice."
  - Bohr (1927): "Stop telling God what to do."
- Multiverse theory
  - Or frequently called as many world theory
  - Everett, Deutsch, …



The hidden variables hypothesis assumes that far below the quantum level lies deterministic parameters, unseen to the observer, that control the observed quantum numbers.

From http://abyss.uoregon.edu/~js/21st_century_science/lectures/lec15.html



From https://commons.wikimedia.org/wiki/File:Schroedingers_cat_film.svg

Postulate 3

- When Ω is an operator corresponding to our interested measurement, what is $\langle\psi|\Omega|\psi\rangle$?
  - Expand (or represent) some arbitrary quantum state $|\psi\rangle$ in eigenbasis $(|\omega_1\rangle, |\omega_2\rangle \ldots |\omega_n\rangle)$ of Ω: $|\psi\rangle = \sum_{i=1}^{n} c_i|\omega_i\rangle$
  - $\langle\psi|\Omega|\psi\rangle = \left(\sum_{j=1}^{n} c_j^*\langle\omega_j|\right)\Omega\left(\sum_{i=1}^{n} c_i|\omega_i\rangle\right) = \sum_{j=1}^{n} c_j^*\left(\sum_{i=1}^{n} c_i\langle\omega_j|\Omega|\omega_i\rangle\right) = \sum_{j=1}^{n} c_j^*\left(\sum_{i=1}^{n} c_i\omega_i\delta_{ji}\right) = \sum_{j=1}^{n} c_j^* c_j\omega_j = \sum_{j=1}^{n} \omega_j P(\omega_j)$
  - Expectation value of the measurement: $\langle\psi|\Omega|\psi\rangle$
  - Compare with the expectation value of a function $f(x)$ in a variable x: $< f(x) > = \sum_x f(x)P(x)$