

Noisy Intermediate-Scale Quantum (NISQ) era

- Quantum volume: 양자컴퓨터의 성능에 영향을 끼치는 다양한 요소들의 성능을 종합적으로 고려한 평가지표
- 제어 가능한 큐비트의 개수, 개별 연산의 오류, 큐비트들간의 연결성 등 다양한 요소가 존재함

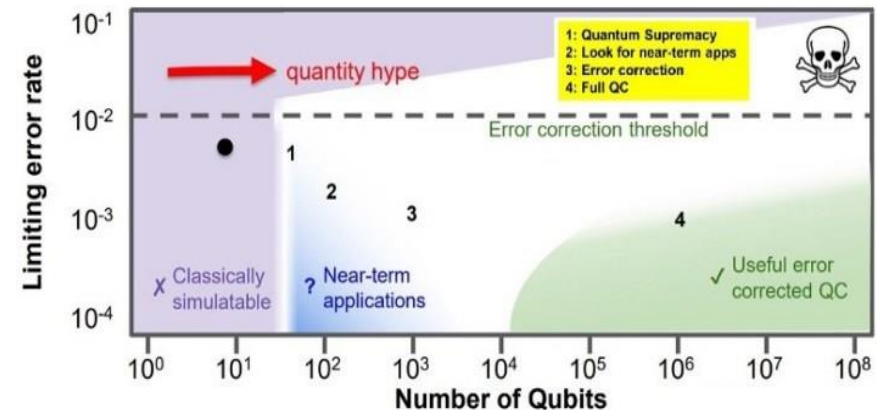
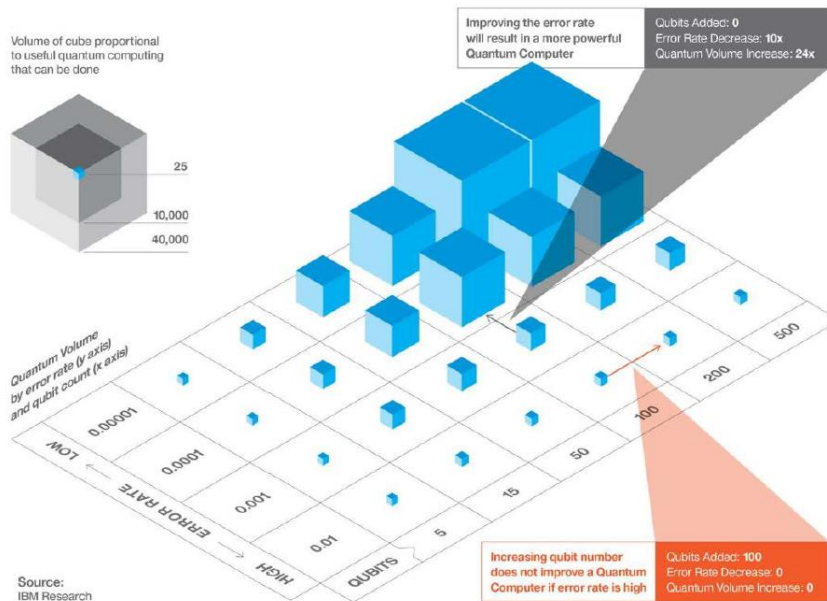


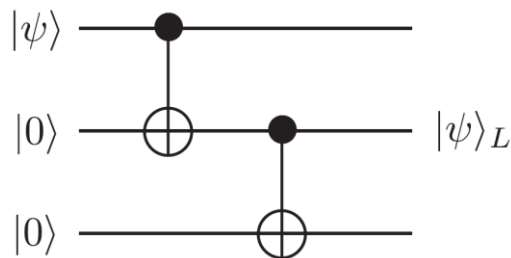
Illustration of the qubit quality vs quantity relationship.
Image credit: John Martinis, Google.

Quantum error correction

- Chap. 10
- Consider the classical communication channel
- The effect of the noise in the channel is to flip the bit being transmitted with probability $p > 0$, while the bit will be transmitted without error with probability of $1 - p$. → Binary symmetric channel.
- Example: majority voting
 - A simple encoding scheme: $0 \rightarrow 000$, $1 \rightarrow 111$
 - The bit string 000 and 111 are referred to as the logical 0 and logical 1.
 - Limitation: the probability that two or more bits are flipped is $3p^2(1 - p) + p^3$. → Error probability is $p_e = 3p^2 - 2p^3$.
 - The code can make the transmission more reliable provided $p_e < p$, which occurs whenever $p < 1/2$.

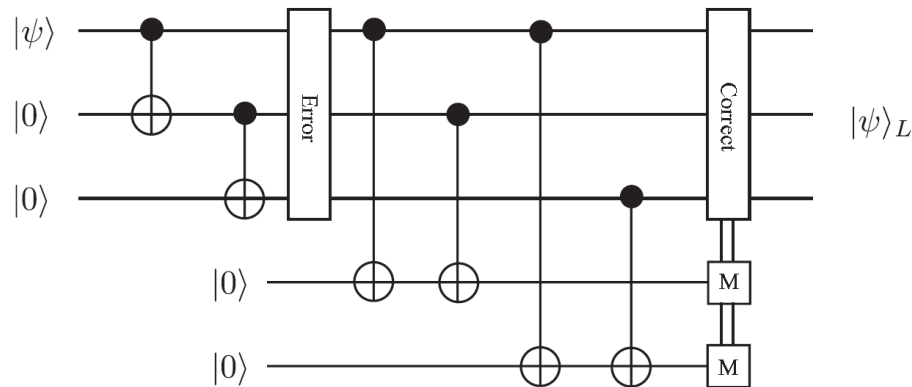
Quantum error correction

- Difference of quantum error-correction compared to the classical error correction
 - No cloning
 - Errors are continuous
 - Measurement destroys quantum information
- Quantum error model
 - **Bit flip channel:** if we send qubits through a channel, then that channel flips the qubits with probability p .
 - $\rightarrow |\psi\rangle$ state is taken to $X|\psi\rangle$ with probability of p . X is sometimes called as bit flip operator.
- Encoding: $a|0\rangle + b|1\rangle \rightarrow a|0_L\rangle + b|1_L\rangle \equiv a|000\rangle + b|111\rangle$



Quantum error correction

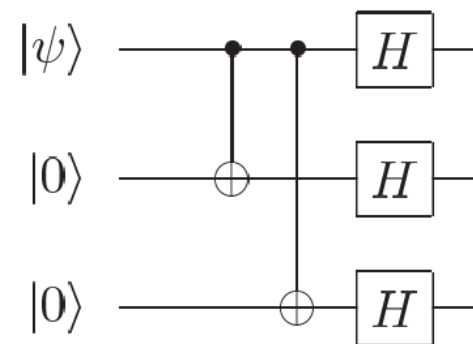
- Recovery
 - Based on the measured syndrome, apply the bit flip operator X on the corresponding qubit



- Errors occur in continuous fashion and probabilistically. Is the previous complete bit flip model valid?
 - Before the syndrome measurement, it is superposition of no-error case and error case. However, once the error syndrome is measured, the quantum state becomes pure state again except the multi-qubit errors.

Quantum error correction

- Quantum error model
 - **Phase flip channel:** if we send qubits through a channel, then that channel flips the phase between $|0\rangle$ and $|1\rangle$ with probability p .
 - $\rightarrow |\psi\rangle$ state is taken to $Z|\psi\rangle$ with probability of p . Z is sometimes called as phase flip operator.
- Suppose we work in the qubit basis $|+\rangle \equiv (|0\rangle + |1\rangle)/\sqrt{2}$, $|-\rangle \equiv (|0\rangle - |1\rangle)/\sqrt{2}$. In this basis, phase flip operator Z flips between $|+\rangle$ and $|-\rangle$.
- Encoding: $|0_L\rangle \equiv |+++\rangle$, $|1_L\rangle \equiv |--\rangle$
- By checking the parity in $|+\rangle$ and $|-\rangle$ basis, phase-flipped qubit can be identified if only one qubit is affected.





The Shor code

- Combination of the three qubit phase flip and bit flip codes.
- First encode the qubit using the phase flip code:
 $|0\rangle \rightarrow |+++\rangle, |1\rangle \rightarrow |--\rangle$
- Next, encode each of these qubits using the three qubit bit flip code:

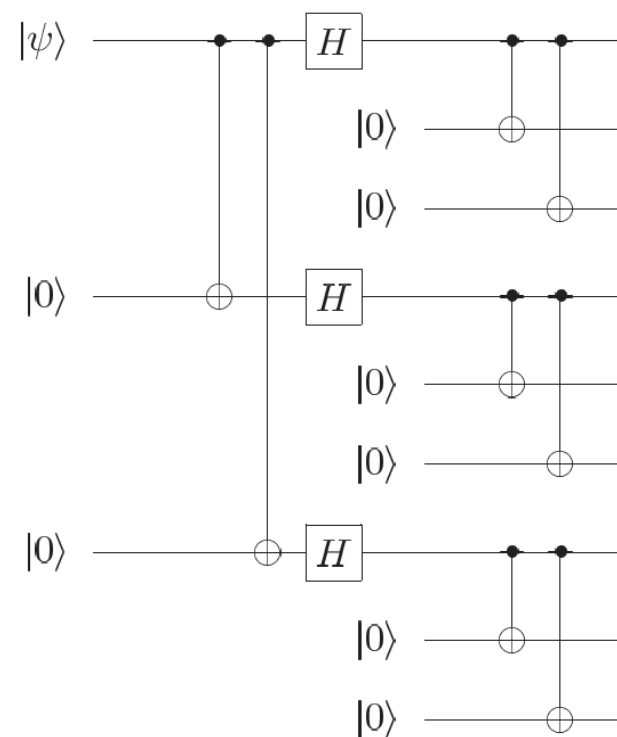
$$|+\rangle = \frac{(|0\rangle+|1\rangle)}{\sqrt{2}} \rightarrow \frac{(|000\rangle+|111\rangle)}{\sqrt{2}} \text{ and } |-\rangle = \frac{(|0\rangle-|1\rangle)}{\sqrt{2}} \rightarrow \frac{(|000\rangle-|111\rangle)}{\sqrt{2}}$$

- Overall,

- $|0\rangle \rightarrow |0_L\rangle \equiv \frac{(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)}{2\sqrt{2}}$

- $|1\rangle \rightarrow |1_L\rangle \equiv \frac{(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)}{2\sqrt{2}}$

- Called concatenation of code
- The Shor code (or sometimes called as 9-qubit code) can detect and correct single qubit error.



Classical linear error correction code

- Classical linear error correction code (section 10.4.1)
 - Multiplication operation and all other arithmetic operations are done modulo 2.
 - Also note that bitwise-addition modulo 2 is equivalent to bitwise-XOR.
 - Terminology: codeword, generator matrix, parity check matrix
 - A linear code C encoding k bits of information into n bit code space is specified by an n by k generator matrix G whose entries are all elements of \mathbb{Z}_2 , that is, zeroes and ones. $\rightarrow [n, k]$ code
 - k –bits message y is encoded as $x = Gy$, where the message y is treated as a column vector.
 - Example
 - $G = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \rightarrow G[0] = [0,0,0]^T, G[1] = [1,1,1]^T \rightarrow$ called as $[3, 1]$ code
 - $G = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$
 - $\rightarrow G[0,0] = [0,0,0,0,0,0]^T, G[0,1] = [0,0,0,1,1,1]^T, G[1,0] = [1,1,1,0,0,0]^T, G[1,1] = [1,1,1,1,1,1]^T \rightarrow$ called as $[6,2]$ code

Classical linear error correction code

- Parity check matrix H
 - A $[n, k]$ code is defined to consist of all n -element vectors x over \mathbb{Z}_2 such that $Hx = 0$ where H is an $n - k$ by n matrix known as the parity check matrix, with entries all zeroes and ones.
 - Example
 - $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow Hx = 0$ only for $x = [0, 0, 0]^T$ and $x = [1, 1, 1]^T$
 - For other x 's with one bit different from $x = [0, 0, 0]^T$ and $x = [1, 1, 1]^T$, Hx will calculate the position.
 - Suppose that we encode the message y as $x = Gy$, but an error e due to noise corrupts x giving the corrupted codeword $x' = x + e$ (+ means bitwise modulo 2). Because $Hx = 0$ for all codewords, $Hx' = He$ which is called as the error syndrome.
 - $HG = 0$
 - Hamming distance $d(x, y)$: the number of places at which x and y differ.
 - To correct up to t bits, the distance between the closest codewords should be at least $2t + 1$.
 - $[n, k, d]$ code

Classical linear error correction code

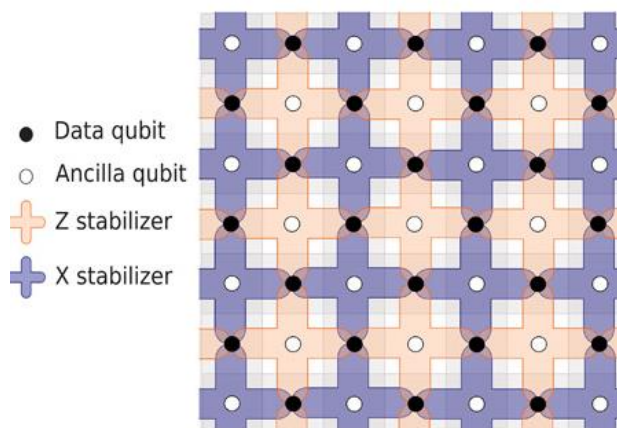
- Hamming code
 - Suppose $r \geq 2$ is an integer and let H be the matrix whose columns are all $2^r - 1$ bit strings of length r which are not identically 0. This parity check matrix defines a $[2^r - 1, 2^r - r - 1]$ linear code known as Hamming code.
 - Example $r = 3 \rightarrow [7,4]$ code

- $H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$

- $G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

Quantum error correction code

- CSS code: Calderbank-Shor-Steane code
 - Originally based on Hamming code [7,4]
 - $|0\rangle_L = |0000000\rangle + |1010101\rangle + |0110011\rangle + |1100110\rangle + |0001111\rangle + |1011010\rangle + |0111100\rangle + |1101001\rangle$
 - $|1\rangle_L = |1111111\rangle + |0101010\rangle + |1001100\rangle + |0011001\rangle + |1110000\rangle + |0100101\rangle + |1000111\rangle + |0010110\rangle$
 - Can correct both bit-flip and phase-flip error
- Quantum error correction code generally can be defined in terms of stabilizer code. (Section 10.5)
- Surface code



Overview of quantum computer architecture

- 아직까지 확정된 시스템 계층 구조는 없음
- 물리적 시스템에 따라 구체적인 구현에서 차이가 존재함

