M1522.002500 - 양자 컴퓨팅 및 정보의 기초

Introduction to Quantum Computing and Information (001)

(Prof. Taehyun Kim)

Midterm

April. 15, 2019 7:00 PM ~ 9:30 PM

Write the answer under each problem.

You can also use additional answer sheet attached behind.

이름:________________

학번:________________

시험 중 부정 행위는 F 학점 및 징계의 사유가 됩니다.

Total: 110 points

P1	P2	P3	P4	P5
P6	P7	P8	P9	

Problem 1. Solve following problems. (15 points)

(A) Show that $\langle\psi|A|\psi\rangle$ is the expectation value of Hermitian operator A. (10 points)

(B) Show that trace of an operator is preserved even when the basis set is changed under unitary transformation. (5 points)

Problem 2. Assume that qubit A on the Earth and the qubit B on the Moon are entangled. At the same time, another qubit C on the Moon and the fourth qubit D on Mars are also entangled. To be specific, qubit A & B are in state $|\varphi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$ and C & D are in state $|\varphi^+\rangle_{CD} = \frac{1}{\sqrt{2}}(|00\rangle_{CD} + |11\rangle_{CD})$. Now we measure the qubit B & C in Bell basis. <u>After the measurement, show that the qubit A & D form one of the Bell state</u>. (This phenomenon is called entanglement swapping and it can be also understood in terms of quantum teleportation.) (15 points)

Problem 3. Find eigenvalues and corresponding eigenvectors of the given matrix. Also, diagonalize the matrix. (Hint) One of the eigenvalue is 3. (10 points)

$$\begin{pmatrix} 2 & i & 1 \\ -i & 1 & i \\ 1 & -i & 2 \end{pmatrix}$$

(Intentionally left blank for continued answer)

Problem 4. (10 points)

(A) Prove that if two Hermitian operators with finite dimension have the same set of eigenvectors, they commute. (5 points) (Hint) $\mathrm{H} = \sum \lambda_i |e_i\rangle\langle e_i|$.

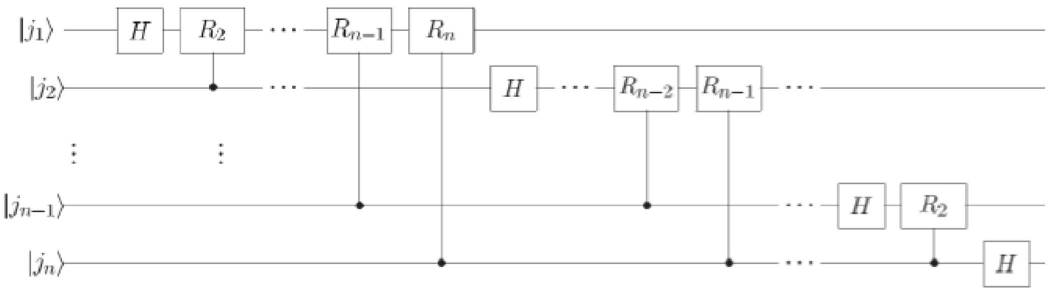
(B) Prove that if they commute and do not have any degeneracy, they have the same set of eigenvectors. (5 points)

Problem 5. Construct a quantum circuit that takes $|0\rangle^{\otimes 3}$ as input state and produces output state $\frac{|001\rangle+|110\rangle}{\sqrt{2}}$. (10 points)

Problem 6. In the class, discrete Fourier transform was defined as $y[n] = \sum_{k=0}^{N-1} x[k]e^{\frac{i2\pi kn}{N}}$ for $0 \leq n \leq N-1$.

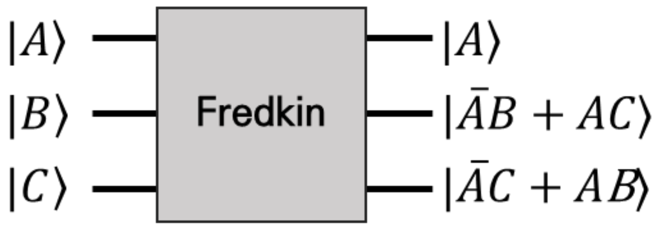
Please find the discrete Fourier transform of signal $x = [1, 0, 2, 0]$. (10 points)

Problem 7. Find the matrix representation of 2-qubit quantum Fourier transform. (Hint) The circuit for n-qubit QFT and the definition of R_k is given as follows. (10 points)

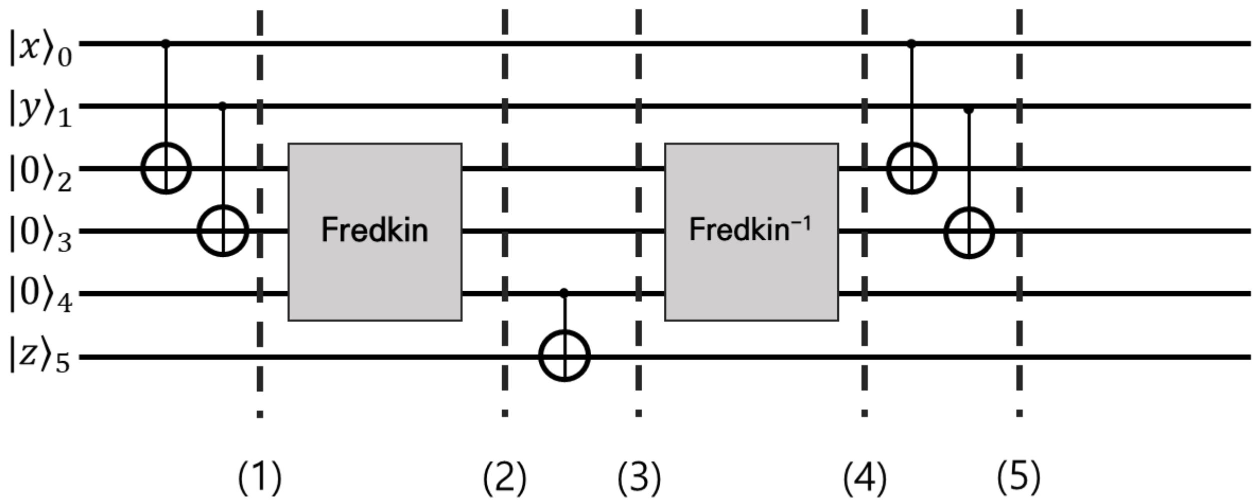


$$R_k = \begin{bmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^k} \end{bmatrix}$$

Problem 8. We want to make an oracle that implements $|x, y, z\rangle \rightarrow |x, y, (z \oplus xy)\rangle$, while uncomputing auxiliary qubits. The input x, y form an arbitrary 2-qubit state. In other words, $\alpha|00\rangle_{01} + \beta|01\rangle_{01} + \gamma|10\rangle_{01} + \delta|11\rangle_{01}$. Show that the uncomputation scheme given in the lecture works by explicitly writing the state at (2) ~ (5). To see how to write the answer, you can refer to the state at (1). Also, after (5), write the state in $|V\rangle_{015}|W\rangle_{234}$ form to show that the auxiliary qubits 2-4 are correctly uncomputed. (15 points)



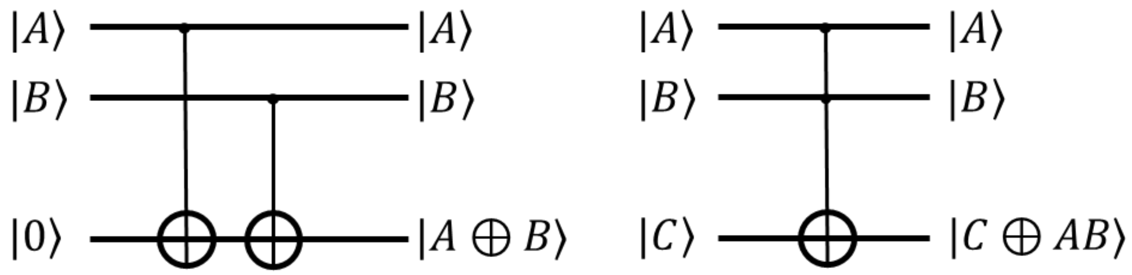
$$|(1)\rangle_{012345} = \alpha|00000z\rangle + \beta|01010z\rangle + \gamma|10100z\rangle + \delta|11110z\rangle$$



(Intentionally left blank for continued answer)

Problem 9. We have a function with following condition for input bit string $x = x_1x_2x_3$ and output bit string $y = y_1y_2y_3$, $h: \{0,1\}^3 \rightarrow \{0,1\}^3, h(x) = y_1y_2y_3, y_1 = x_2 \oplus x_3, y_2 = x_3 \oplus x_1, y_3 = x_1 \oplus x_2$. Follow the instructions. (15 points)

(A) Construct a reversible oracle $f(\mathrm{x}) = \begin{cases} 1 \; h(x) = 6 \\ 0 \; h(x) \neq 6 \end{cases}$ based on function h. You can use up to 3 auxiliary qubits for intermediate computation and 1 additional qubit for oracle output. You can use controlled-controlled NOT gate and XOR based on CNOT given below. (10 points)



(B) Now, we attempt to find the answer to the oracle by using Grover's algorithm. Estimate how many times we need to perform Grover operation. (5 points)

(Additional answer sheet)