Benchmark Reform Goes Non-Linear

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Abstract

We examine the impact of interest rates benchmark reform and upcoming Libor transition on options markets. We address various modelling challenges the transition brings. We specifically focus on the impact of the clearing houses' discounting switch on swaptions, and the consequences of Libor transition on Libor-in-arrears swaps, caps, and range accruals as typical representatives of a very wide range of Libor derivatives.

1 Introduction

Interest rate benchmark reform is currently a focus of intense attention for market participants on such topics as proposed fallbacks and impact on bilateral trading [7], pre-cessation triggers [4], discounting changes by clearing houses (CCPs) [12], impact on cash versus derivatives markets [5], and many others, primarily in linear markets. Potential impact of the reform on non-linear markets has received relatively less attention (with a notable exception of [13] where Libor reform issues for structured notes are considered.) In this paper we aim to close the gap and discuss significant challenges for certain non-linear product types, and how they could be overcome.

We focus on options markets in USD, EUR and GBP as being the most liquid and the furthest along in the benchmark reform. The three markets are reasonably similar, yet exhibit important idiosyncrasies in conventions and benchmark reform approaches. Some of the considerations below are more important for some of these markets and less for the others, and we endeavor to highlight such differences as we go along.

As a small caveat, market developments in the rate benchmark reform space come at a fast pace and while the information here is believed to be accurate at the time of writing, some later developments may not be reflected. The online version of the paper [15] may include more up-to-date information.

2 Discounting and Swaptions

The impact of the impending CCP discounting switch on swaptions has probably received the most attention as far as non-linear markets are concerned, see e.g. [12] and [3]. As part of rates benchmark reform, a number of CCPs (e.g. LCH, CME) have announced that their collateral (so-called PAI, or Price Alignment Interest) rates will switch from legacy overnight rates to their replacements, such as from FedFunds to SOFR for USD and from Eonia to ESTR for EUR, targeting October (USD) and July (EUR) 2020 as the implementation date. Rates used to fund collateral balances directly define rates for discounting cashflows in cleared instruments (see [14]). Values (and risk sensitivities) of cleared interest rate swaps will change on that date. As part of the transition, CCPs have proposed a compensation mechanism to eliminate the potential value transfer.

Unlike swaps, swaptions – options to enter swaps – are not cleared and are governed by bilateral agreements based on ISDA templates. In USD and EUR swaptions typically settle into cleared swaps or, more accurately, into cash amounts determined by referencing cleared swap values[1]. If the CCP discounting switch happens before swaption expiry, the underlying value of the referenced swap will change resulting in a value transfer in a bilateral transaction.

Establishing a market-wide compensation scheme for the value transfer in swaptions is a complex issue, as [12] outlines. ISDA and various regulators are reportedly examining possible options, see e.g. [3]. The markets are, reportedly, skeptical (arguably more in EUR than in USD) that an effective solution can be found or that, at the very least, swaptions traded post discounting switch announcement date would be in scope (see more on this in Section 2.2). Some market participants (in EUR; not so much in USD) have been showing prices on packages of synthetic swap exposures via CCP-settled zero-wide collars (off-market i.e. with strikes at ATM+Y with Y being 50 or even 100 bp) hedged by cleared swaps, positions that should be worth exactly zero if the same discounting applied to all legs. Such a package expresses a view that a cleared swap will be compensated for the value transfer but the swaption position will not be, the difference maximized for deep in-the-money swaps.

We emphasize that two market wide event dates are important, the date(s) when CCP(s) announced the intent to switch discounting for cleared swaps (September 2019) and the actual discounting switch date (July 2020 for EUR, October 2020 for USD). For a particular swaption two dates are important in our context, the trade date and the expiry date.

2.1 Compensation for Value Transfer

The proposal for value transfer compensation for swaps is fairly straightforward, see [6]. The change in swap value is addressed via a cash payment, and the

[1] The situation in EUR markets is in fact even more subtle as swaptions use ICE screen rate for settlement which at the moment is not CCP-specific and is not linked to DV01 referenced in a contract. We do not explore this important nuance here.

change in risk profile, where required, is addressed by an exchange of basis swaps. A clearing house, being in the middle of all cleared swaps, facilitates all this activity.

It is tempting to suggest that a similar compensation mechanism for swaptions, i.e. an exchange of a package of cash and basis swaps at swaption expiry is adopted to negate the value and risk transfer. While compellingly simple – the CCP mechanism for swaps is applied to swaptions with no modifications – there are significant issues with this approach. The main one, in our view, is the fact that the swaption expiry may well be 20 or 30 years from now, and it is entirely possible and, in fact, expected, that the legacy OIS rates/curves (specifically FedFunds; Eonia has been fixed to be ESTR+8.5 bp which, conceivably, could still be used in the far future) will no longer be available. Moreover, valuing and risk-managing a swaption with such compensation mechanism throughout its life will be a considerable challenge. Since at expiry one would effectively exercise not into a vanilla swap but a basket of a swap, cash, and basis swaps, the swaption will cease to be a "simple" vanilla option anymore.

It should be clear that cash compensation at the time of the discounting switch (or announcement of discounting switch which is now sadly in the past) is a better option and in fact is one of the proposed solutions in the consultation paper [3]. We argue that a more theoretically-sound mechanism is possible, that we now proceed to outline.

The idea behind the mechanism is fairly simple. If the discounting curve is switched, the interest rate risk of the swaption changes roughly proportionally to the ratio of the forward annuities under the old and the new discount curve. The same happens to the swaption premium if the strike is adjusted in a natural way. Hence, if we replace one unit of the "old" swaption by a number of units of the "new" swaption given by the ratio of annuities, both the PV and the risks of the position will not change, thus leading to a very clean compensation mechanism. We develop this approach more formally below.

Discounting swaptions generally requires two curves. The curve specified by the bilateral CSA is used for discounting of the swaption payoff from swaption expiry date to today. The CCP discounting curve is used for the underlying swap in payoff calculations. We are interested only in the effects of the CCP discounting switch and remove the bilateral discounting curve from consideration by focusing on forward (to swaption expiry) quantities.

Let T be the expiry date of the swaption and $t \leq T$. We fix a tenor structure $0 < T = T_1 < \cdots < T_N$, $\tau_i = T_{i+1} - T_i$. Let $L_i(t) = L(t, T_i, T_{i+1})$ be Libor rates, $P_i^f(t) = P^f(t, T, T_i) = P^f(t, T_i)/P^f(t, T)$ the "old" OIS, e.g. FedFunds, forward discount factors, $P_i^s(t) = P^s(t, T, T_i) = P^s(t, T_i)/P^s(t, T)$ the "new", say SOFR, forward discount factors, and $r^{f,s}(t)$ the relevant short rates. Let us denote the forward annuity and the swap rate under the two different discount curves by

$$A^x(t) = \sum_i \tau_i P_{i+1}^x(t), \quad S^x(t) = \frac{1}{A^x(t)} \sum_i \tau_i P_{i+1}^x(t) L_i(t)$$

where $x \in \{f, s\}$. The forward value of a swaption to exercise into a FedFunds (SOFR) discounted CCP swap (technically its CCP cash value, but the distinction is irrelevant for modelling) with strikes $K^f(K^s)$ and notionals $N^f(N^s)$ is

$$V^x_{\mathrm{swpt}}(t) = N^x A^x(t) \mathrm{E}^{A^x}_t (S^x(T) - K^x)^+, \quad x \in \{f, s\},$$

where $\mathrm{E}^{A^x}_t$is the expected value under the A^s-annuity measure (similar for$\mathrm{E}^{A^f}_t$ used later). Let us suppose a mechanism could be agreed to modify contractual terms of a swaption (notional, strike) due to the discounting switch. The potential (forward) value transfer is then given by

$$\Delta V_{\mathrm{swpt}} \triangleq V^f_{\mathrm{swpt}}(t) - V^s_{\mathrm{swpt}}(t) = N^f A^f(t) \mathrm{E}^{A^f}_t (S^f(T) - K^f)^+ - N^s A^s(t) \mathrm{E}^{A^s}_t (S^s(T) - K^s)^+.$$

Using the same FedFunds annuity measure for both expressions we obtain

$$\Delta V_{\mathrm{swpt}} = N^f A^f(t) \mathrm{E}^{A^f}_t \left((S^f(T) - K^f)^+ - c \frac{A^s(T)}{A^f(T)} (S^s(T) - K^s)^+ \right),$$

where we have denoted

$$c = \frac{N^s}{N^f}. \tag{1}$$

Our objective is to find the change in contractual terms that would minimize/eliminate value transfer. To match the moneyness of the new vs. old swaption we set

$$K^s = K^f + S^s(t) - S^f(t), \tag{2}$$

We do not expect the strike adjustment to be large as discounting has small effect on at-the-money swaps. Let us simplify the notations and set

$$S(T) - K \triangleq S^f(T) - K^f \approx S^s(T) - K^s$$

where the last equality follows from the strike adjustment (2) and the assumption that S^f and S^s move mostly in parallel. With this step completed, the expression for the potential value transfer reduces to

$$\Delta V_{\mathrm{swpt}} = N^f A^f(t) \mathrm{E}^{A^f}_t \left(\left(1 - c \frac{A^s(T)}{A^f(T)}\right) (S(T) - K)^+ \right).$$

Clearly

$$M(T) \triangleq \frac{A^s(T)}{A^f(T)} \frac{A^f(t)}{A^s(t)} \frac{e^{-\int_t^T r^s(u) du}}{e^{-\int_t^T r^f(u) du}}$$

is a positive martingale under P^{A^f} with $M(t) = 1$. Setting the value transfer to zero we obtain the following condition on c,

$$A^f(t) \left(c \frac{A^s(t)}{A^f(t)} \right) \mathrm{E}^{A^f}_t \left(M(T)(S(T) - K)^+ \right) = A^f(t) \mathrm{E}^{A^f}_t \left((S(T) - K)^+ \right).$$

The ratio of notionals (1) that is consistent with zero value transfer (under our assumptions) is then

$$c = \frac{A^f(t)}{A^s(t)} \times \frac{\mathrm{E}_t^{A^f}\left((S(T)-K)^+\right)}{\mathrm{E}_t^{A^f}\left(M(T)(S(T)-K)^+\right)}. \tag{3}$$

The factor

$$c \approx \frac{A^f(t)}{A^s(t)} \tag{4}$$

is the dominant one as the second factor in (3) is exactly one if correlation between $M(T)$ and $(S(T)-K)^+$ is zero and is generally small otherwise. Expression (4) specifies that we should replace one FedFunds swaption with c SOFR swaptions, where c is the ratio of the FedFunds and SOFR forward annuities of the underlying swap or, simply, forward DV01s (forward parallel deltas) of the old and the new swaption.

The second factor in (3) is a convexity correction to the basic expression for c in (4) and is driven by the correlation between $M(T)$ and $S(T)$ or, more accurately, $(S(T)-K)^+$ and as such is strike dependent. To first order, it can be ignored. If more precision is required, the following calculation can be employed,

$$\begin{aligned}
\mathrm{E}_t^{A^f}\left(M(T)(S(T)-K)^+\right) &= \mathrm{E}_t^{A^f}\left(\mathrm{E}_t^{A^f}\left(M(T)(S(T)-K)^+ \middle| S(T)\right)\right) \\
&= \mathrm{E}_t^{A^f}\left((S(T)-K)^+ \mathrm{E}_t^{A^f}\left(M(T)| S(T)\right)\right) \\
&= \mathrm{E}_t^{A^f}\left((S(T)-K)^+ \mu(S(T))\right),
\end{aligned} \tag{5}$$

where

$$\mu(x) \triangleq \mathrm{E}_t^{A^f}\left(M(T)| S(T) = x\right) \approx \frac{\langle M(T), S(T)\rangle}{\langle S(T), S(T)\rangle} x$$

(this function can be calculated once a particular model of rates evolution is postulated). The adjustment in (3) vs. (4) can then be calculated by strike replication of the payoff (5) with European swaptions in a way not dissimilar to CMS convexity calculations, see [1].

This approach, in our view, enjoys a number of advantages over a simple cash adjustment on the discounting switch date, as deltas to the Libor (projection) curve remain largely unchanged so no extra hedging is required; deltas to discounting curves are comparable in the sense that X units of risk to FedFunds are replaced by X units of risk to SOFR; and vegas (volatility sensitivities) remain largely the same, not requiring any additional trading in options to recover the original vega position as would be the case under the cash compensation scheme. Critically, our approach enjoys consistency with the CCP compensation mechanism for the discounting switch for *swaps* (where it is explicitly designed to replace X units of risk to FedFunds with X units of risk to SOFR), thus maintaining the effectiveness of swaps used as hedges for swaptions.

It is fair to note that our approach comes with its own limitations. Arguably, it is more practical to exchange cash as compensation at a point in time than to amend a potentially large number of existing trades. Perhaps a hybrid approach is warranted with market participants agreeing on which choice is a better balance for their risk management vs. operational considerations.

2.2 Volatility Smile Impact

Swaptions are quoted in (forward) premium terms in the inter-dealer markets that all dealers use for price discovery. Internally, most dealers represent swaption prices as implied volatilities so that, when inserted into the Normal model and scaled by DV01s, market-observed prices are recovered. Implied volatilities are then used to mark swaption books internally, to determine swaption prices offered to clients, and to feed models for more exotic products such as CMS, Bermudan swaptions, and so on.

Consider a dealer who, pre discounting switch announcement, marks a particular swaption expiring post switch (with notations from Section 2.1) at volatility v^f that corresponds to value $V^f_{\text{swpt}}(t)$, swap rate $S^f(t)$, strike K^f, and annuity $A^f(t)$. The question we consider is where should he mark this volatility post-announcement. There is a range of plausible scenarios.

2.2.1 Expectation of No Value Transfer

Let us consider the case where the market believes that parties to a pre - announcement swaption will be compensated, for concreteness assuming Section 2.1 mechanism. As there is no expectation of value transfer, there is no jump in the price $V^f_{\text{swpt}}(t)$ at the discounting switch announcement date. The dealer should continue using the existing valuation parameters, i.e. the forward rate $S^f(t)$, strike K^f, FedFunds for discounting the underlying swap (i.e. $A^f(t)$ for annuity), and volatility v^f.

Once the compensation for value transfer occurs, the booking should be changed with the strike and notional adjusted and the discounting curve upgraded to the new one, but the volatility surface should not be impacted by this event.

As discussed, a cash fee paid on the discounting switch date is a an alternative compensation option. Under this scenario, a swaption

$$V^f_{\text{swpt}}(t) = A^f(t)\mathrm{E}_t^{A^f}(S^f(T) - K^f)^+$$

is replaced by a swaption

$$V^s_{\text{swpt}}(t) = A^s(t)\mathrm{E}_t^{A^s}(S^s(T) - K^s)^+$$

plus a cash payment in the amount of $V^f_{\text{swpt}}(t) - V^s_{\text{swpt}}(t)$. As there is no value transfer, keeping the original volatilities with the original trade booking and market data (discounting/projection curves) at the moment of discounting switch announcement is appropriate. On the actual discounting switch/cash fee payment event, volatility should not be affected as long as the other elements are updated, i.e. the fee is paid/received and the discounting curve is switched to the new one.

2.2.2 Expectation of Value Transfer

Now let us consider the scenario where, upon discounting switch announcement, the market assumes that there will be no compensation for value transfer. Prices of swaptions will then immediately jump. In particular, as generally $A^s(t) > A^f(t)$, long swaption positions will generally increase in value.

A "consensus" dealer will switch discounting to SOFR and keep volatilities unchanged. A "contrarian" dealer who insists on continuing to use the legacy FedFunds discounting curve, whether by ignorance, contrarian beliefs, or system limitations, will have to re-mark volatilities higher, leading to vega PnL. While for the CCP-settled swaptions the PnL will be "real" in the sense of faithfully reflecting market view for uncompensated value transfer, it may not be so real for other derivatives that derive their values from swaption volatilities such as pre-November 2018, i.e. IRR, cash-settled European swaptions[2] in EUR, CMS, Bermudan swaptions and other Libor exotics.

The impact of discounting switch announcement on volatilities used by the contrarian dealer is different for different strikes. The value of a swaption will change as follows (we drop the time variable t for brevity),

$$V^f_{\text{swpt}} = A^f \mathrm{E}^{A^f}(S(T) - K)^+ \to V^s_{\text{swpt}} = A^s \mathrm{E}^{A^s}(S(T) - K)^+.$$

The new volatility, $v^s = v^s(K)$ will then be calculated as follows,

$$v^s(K) = C^{-1}\left(\frac{V^s_{\text{swpt}}}{A^f}, K\right) = C^{-1}\left(\frac{A^s}{A^f} C(v^f, K), K\right), \tag{6}$$

where $C(\sigma, K)$ is the Normal option pricing formula with volatility σ and strike K, and C^{-1} is the inverse of this function in the first, volatility, argument. It is important to realize that this transformation is strike-dependent as constant multiplier A^s/A^f has different impact on volatilities at different strikes.

Using representative EUR values, we consider 10y20y swaptions as an example. The forward swap rate is 60 basis points (bps), and we assume that the market has switched to ESTR discounting while the contrarian dealer still uses Eonia. For emphasis we assume the market uses the same normal volatility of 50 bps for all strikes. The DV01 ratio in (6) is estimated to be 1.0087, as the relationship between Eonia and ESTR is already locked down at Eonia = ESTR +8.5 bps.

Figure 1 shows what volatilities a legacy (Eonia)-discounting dealer would have to use, compared to the new (ESTR)-discounting market. The latter are shown as the "baseline" graph. The former are shown as "payers" for payer swaption volatilities, and as "receivers" for receiver swaption volatilities.

Not only there is a pronounced effect on the volatility smile, but also the volatilities implied for payer (call) and receiver (put) swaptions are worryingly different. This is fairly clear from a brief reflection on (6) and is demonstrated in Figure 1. To the extent that high-strike volatilities are typically implied

[2] In EUR, the IRR vs. the "new" cash-settled zero-wide collars is another segment that is currently used to express views on the probability of compensation for value transfer.

from the prices of OTM payers and low-strike volatilities from OTM receivers, this (arbitrage-inducing) mismatch may not be immediately obvious. Clearly, however, if not recognized, it could lead to significant mis-pricings in swaption and related markets for the legacy-discounting dealer.

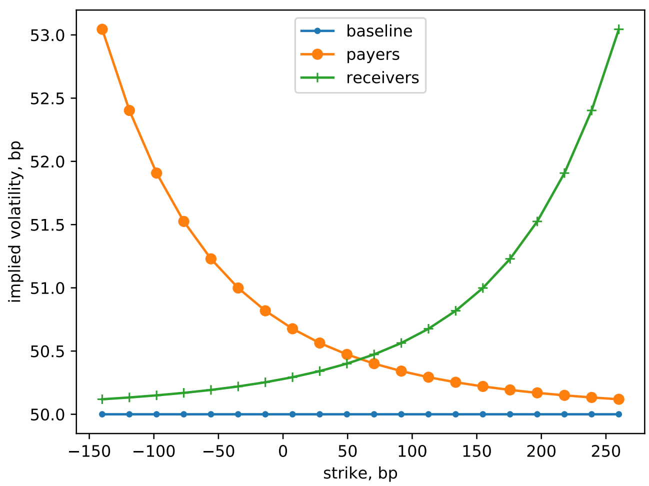

Figure 1: Impact on implied normal volatilities from mismatched discounting. "Baseline" is the baseline volatility. "Payers" is the volatility implied from payer swaptions under mismatched discounting. "Receivers" is the volatility implied from receiver swaptions under mismatched discounting.

3 Caps

Interest rate caps present another challenge for the benchmark reform, in this case caused by term Libor rates being replaced by a daily compounded in-arrears overnight rate, as mentioned in e.g. [8], [9], [16].

Let us fix i and simplify notations to $L(t) \triangleq L_i(t) = L(t, T_i, T_i + \tau)$, $T \triangleq T_i$, $\tau \triangleq \tau_i$. The rate $L(t)$ is given by

$$L(t) = \frac{P_i(t) - P_{i+1}(t)}{\tau P_{i+1}(t)}.$$

A caplet is an option that pays $(L(T) - K)^+$ at time $T + \tau$, and its value is given by

$$V_{\mathrm{LibCpl}}(t) = P(t, T + \tau)\mathrm{E}_t^{T+\tau} (L(T) - K)^+ . \tag{7}$$

Under the standard fallback mechanism, a Libor rate would be replaced by a compounded in-arrears daily rate with a spread, with the payoff in (7) becoming

$$V_{\mathrm{OisCpl}}(t) = P(t, T+\tau)\mathrm{E}_t^{T+\tau}\left(\frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\ ds\right) - 1\right) - K'\right)^+, \quad (8)$$

where we use $r(s)$ for the overnight rate, K' for the fallback-spread-adjusted strike, and replace discrete daily with continuous compounding for notational simplicity.

The fundamental difference between (7) and (8), as noted in [9], is that in the former the rate is fixed at time T, whereas in the latter it continues to stochastically evolve until a later time $T + \tau$, combined with the averaging feature of rates fixed at different times over the time period $[T, T + \tau]$. A simple European option in (7) becomes an Asian option in (8).

Assume for a moment that one-period swaptions are traded. Their value in the Libor world are given by

$$V_{\mathrm{LibSwpt}}(t) = P(t, T+\tau)\mathrm{E}_t^{T+\tau}\left(V_{\mathrm{LibSwap}}(T)\right)^+, \quad (9)$$

where

$$V_{\mathrm{LibSwap}}(T) = \mathrm{E}_T^{T+\tau}\left(L(T) - K\right) = L(T) - K. \quad (10)$$

Clearly (9)–(10) describe a contract equivalent to (7), and in the Libor world a caplet is equivalent to a one-period swaption. In the post-Libor situation we have

$$V_{\mathrm{OisSwpt}}(t) = P(t, T+\tau)\mathrm{E}_t^{T+\tau}\left(V_{\mathrm{OisSwap}}(T)\right)^+ \quad (11)$$

with

$$\begin{aligned} V_{\mathrm{OisSwap}}(T) &= \mathrm{E}_T^{T+\tau}\left(\frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\ ds\right) - 1\right) - K'\right) \\ &\neq \frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\ ds\right) - 1\right) - K'. \quad (12) \end{aligned}$$

In fact, combining (11) and (12) we obtain

$$\begin{aligned} &V_{\mathrm{OisSwpt}}(t) \\ &= P(t, T+\tau)\mathrm{E}_t^{T+\tau}\left(\mathrm{E}_T^{T+\tau}\left(\frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\ ds\right) - 1\right)\right) - K'\right)^+. \quad (13) \end{aligned}$$

In contrast to (8), part of the expectation value operator moves inside the $\max(\cdot, 0)$ operator. Jensen's inequality (as also noted in [10]) implies

$$V_{\mathrm{OisCpl}}(t) > V_{\mathrm{OisSwpt}}(t). \quad (14)$$

Let us denote

$$R(t) \triangleq R(t, T, T+\tau) = \mathrm{E}_t^{T+\tau}\left(\frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\ ds\right) - 1\right)\right), \quad (15)$$

where $t \in [0, T+\tau]$, i.e. t is allowed to be larger than T (as in [10]). For $t \leq T$ this is called the forward term one-period compounded OIS swap rate and is defined as the break-even rate on a one-period forward starting OIS swap observed at time t. With this notation we have

$$V_{\text{OisSwap}}(T) = \mathrm{E}_T^{T+\tau}\left(R(T) - K'\right)$$

which is the equivalent of (10) in post-Libor world. We note that the caplet value (8) is then

$$V_{\text{OisCpl}}(t) = P(t, T+\tau)\mathrm{E}_t^{T+\tau}\left(R(T+\tau) - K'\right)^+ .$$

3.1 Market Impact

A standard interest rate swap post Libor transition looks essentially like a swap before transition as far as interest rate risk is concerned. Caps, however, change their character significantly, and may no longer be suitable for market participants who have them on the books as, for example, a loan hedge. Sensitivity to volatility looks different, and the complexity of valuation is different as well, potentially affecting liquidity and the ability to exit positions.

There is also, of course, the issue of value transfer. A cap versus a matching tenor swaption, for example a one-year forward starting cap on three-month Libor versus a one-year swaption with a matching first expiry, was at times a popular hedge fund trade. Originally designed as a bet on Libor/swap rate correlations, under the fallback protocol not only will it likely change its value, but will also acquire different risk characteristics. A similar divergence may occur between caplets and exchange-traded options on Eurodollar futures.

3.2 Volatility Adjustment

Let us estimate the difference in value of a caplet vs. a matching single-period swaption in the post-Libor world. For a very simple estimate, let us assume that $r(t)$ follows a Brownian motion with constant volatility σ, and let us approximate

$$R(T+\tau) = \frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\, ds\right) - 1\right) \approx \widetilde{R}(T+\tau) = \frac{1}{\tau}\int_T^{T+\tau} r(s)\, ds. \quad (16)$$

To estimate the variance of $\widetilde{R}(T+\tau)$, we recall (see [15]) that for the standard Brownian motion $W(\cdot)$ for $T \geq 0$

$$\mathrm{E}\left(\left(\int_T^{T+\tau} W(s)\, ds\right)^2\right) = \tau^2\left(T + \frac{\tau}{3}\right).$$

Thus $\mathrm{Var}\left(\widetilde{R}(T+\tau)\right) = \sigma^2(T+\tau/3)$. At the same time $\widetilde{R}(T) = r(T)$ under the Brownian motion assumption and the approximation (16), so that

$\mathrm{Var}\left(\widetilde{R}(T)\right) = \sigma^2 T$. The ratio of volatilities is then given by

$$\frac{\mathrm{Vol}\left(R(T+\tau)\right)}{\mathrm{Vol}\left(R(T)\right)} \approx \frac{\mathrm{Vol}\left(\widetilde{R}(T+\tau)\right)}{\mathrm{Vol}\left(\widetilde{R}(T)\right)} = \sqrt{1+\frac{\tau}{3T}} \tag{17}$$

(see also [11]). It is always larger than 1 as already discussed, and could be significantly larger than 1 for shorter-dated options i.e. smaller T.

3.3 Skew Adjustment

The impact of rate averaging on volatility is likely to be the dominant effect for the volatility smile, but ultimately it is also important to quantify how averaging affects other smile parameters. We leave the full treatment of this question for future research, but consider a simple example here to highlight the magnitude of the likely impact, and to also introduce a novel quantitative technique for dealing with averaged rates that is likely to be important in the post-Libor world.

We simplify (16) further by using the trapezoidal rule as in

$$\widetilde{R}(T+\tau) \approx \widehat{R}(T+\tau), \quad \widehat{R}(t) = \mathrm{E}_t\left(w_1 r(T_1) + w_2 r(T_2)\right),$$

where $T_1 = T$, $T_2 = T+\tau$, and $w_1 = w_2 = 0.5$ for the trapezoidal rule, but are left generic with the requirement that $w_1 + w_2 = 1$ for now.

We define the skew for the set of options on $\widehat{R}(T+\tau) = w_1 r(T_1) + w_2 r(T_2)$ as $\overline{\beta}$ in the shifted-lognormal process

$$dY(t) = \overline{\lambda}(\overline{\beta}Y(t) + (1-\overline{\beta})\widehat{R}_0)\; d\overline{W}(t), \quad \widehat{R}_0 = \widehat{R}(0),$$

such that

$$\mathrm{E}\left(Y(T)-K\right)^+ \approx \mathrm{E}\left(\widehat{R}(T+\tau)-K\right)^+ = \mathrm{E}\left(w_1 r(T_1) + w_2 r(T_2) - K\right)^+ \;\; \forall K. \tag{18}$$

For inspiration we recall the derivation of the basket skew from [2], as covered in [1, Section A.4]. The options on the right-hand side of (18) look like options on a basket except the underlying(s) are observed at different times. We deal with this complication via time change, a technique that we believe can be generally applied to derive other volatility parameters for options on time averages. We define

$$X_1(t) = r(t), \quad X_2(t) = r(tT_2/T_1),$$

so that the right-hand side of (18) is given by

$$\mathrm{E}\left(w_1 X_1(T) + w_2 X_2(T) - K\right)^+,$$

which now looks indeed like a basket option and [1, Section A.4] can be applied. Assuming shifted-lognormal process for the overnight rate

$$dr(t) = \sigma(\beta r(t) + (1-\beta) r_0)\; dW(t),$$

time-change calculations lead to

$$dX_i(t) = \sigma_i(\beta X_i(t) + (1-\beta)r_0)\ dW_i(t), \quad i = 1, 2,$$

where

$$\sigma_1 = \sigma, \quad \sigma_2 = (T_2/T_1)^{1/2}\,\sigma, \quad W_1(t) = W(t), \quad W_2(t) = (T_1/T_2)^{1/2}\,W(tT_2/T_1),$$

and

$$\rho \triangleq \mathrm{corr}(W_1(t), W_2(t)) = (T_1/T_2)^{1/2}\,.$$

Applying [1, Proposition A.4.1] we obtain, after some manipulations, that

$$\overline{\beta} = \beta \frac{w_1\,(w_1 + w_2)^2 + w_2\,(w_1 + w_2\,(T_2/T_1))^2}{\left(w_1^2 + 2w_1 w_2 + w_2^2\,(T_2/T_1)\right)^2}, \tag{19}$$

which is our main result for the skew of (the discrete approximation to) an Asian option. For $w_1 = 1$ ($w_2 = 0$) or $w_1 = 0$ ($w_2 = 1$) we naturally obtain $\overline{\beta} = \beta$, and for $w_1 = w_2 = 0.5$ we obtain

$$\overline{\beta} = \beta \frac{10 + 4\,(T_2/T_1) + 2\,(T_2/T_1)^2}{9 + 6\,(T_2/T_1) + (T_2/T_1)^2}.$$

Specifically, for $T_2/T_1 = 2$ (e.g. $T = 1$, $\tau = 1$),

$$\overline{\beta} = \frac{26}{25}\beta = 1.04b,$$

so the skew is adjusted by 4% relative. While not large, it is noticeable, as we show in Figure 2, where we also demonstrate the quality of our approximation. For $T_2/T_1 = 10$, the the case of a rather short expiry, $\overline{\beta} \approx 1.48b$, i.e. almost 50% relative adjustment to the skew.

3.4 Considerations for Modelling

Modeling caps post-Libor transition is more complicated as the option becomes Asian. In this regard, [9] derives valuation formulas in a one-factor Gaussian model, and of course the seminal extension of the Libor market model for overnight rates from [10] can be used. Both, however, are likely to be deemed impractical by traders for what would still be likely considered a vanilla product. Hence, an adaptation of a vanilla model such as SABR will likely be required.

It is possible that both types of contracts, Asian-style OIS caplets and European style single-period swaptions, will be traded in the future. This possibility should be one of the considerations of any future vanilla risk management framework. In particular the model should be flexible enough so that the two types of contracts can be marked at different levels of volatility and, potentially, of other volatility smile parameters. In particular, it seems unlikely that a simple scaling of swaption volatility along the lines of (17), or skew as in (19), would be sufficient to match both markets.

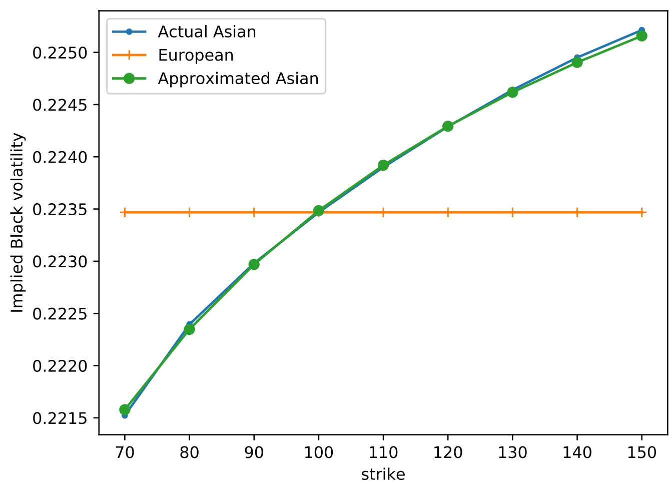

Figure 2: Asian (two-element basket) option skew approximation from Section 3.3. Overnight rate $r(\cdot)$ follows a shifted-lognormal process with spot 100, volatility 20% and skew 100%. Weights $w_1 = w_2 = 0.5$. $T = 1$, $\tau = 1$. "Actual" is calculated by a numerical scheme. "European" uses 100% skew. "Approximated Asian" uses our approximation.

The relations (17), (19) do, however, help as they express volatilities and skews of different contracts in a comparable way. Let us assume we mark single-period OIS swaptions with expiry T and tenor τ with parameters, in the standard SABR parameterization, σ_{swpt}, α_{swpt}, β_{swpt}, ρ_{swpt}. It is natural then to use a SABR model for caplets as well. We would then mark σ_{cpl}, β_{cpl} as we see fit and, perhaps, in the first iteration of the model use the same values for other parameters, $\alpha_{\text{cpl}} = \alpha_{\text{swpt}}$, $\rho_{\text{cpl}} = \rho_{\text{swpt}}$. Before applying the SABR model to a caplet we would first transform σ_{cpl}, β_{cpl} into an approximation to the "Asian volatility and skew" using (17), (19) by setting

$$\sigma_{\text{asian}} = \sigma_{\text{cpl}} \sqrt{1 + \tau/(3T)}, \quad \beta_{\text{asian}} = \beta_{\text{cpl}} \frac{10 + 4\,(1 + \tau/T) + 2\,(1 + \tau/T)^2}{9 + 6\,(1 + \tau/T) + (1 + \tau/T)^2} \tag{20}$$

for $T \geq 0$. We would use the set of parameters

$$\sigma_{\text{asian}}, \alpha_{\text{cpl}}, \beta_{\text{asian}}, \rho_{\text{cpl}}$$

in the SABR formula to calculate (8) with the strike K' and the forward set to $R(0)$ (or the exact daily compounding version thereof). The scalings (20), while not strictly necessary, express caplets versus swaptions, and caplets of different expiries, in normalized units and thus help in hedging and marking activities.

4 Disappearing Species?

Interest rate caps change quite significantly under the standard Libor fallback. Some of the other products may be affected even more and disappear altogether.

4.1 Libor-In-Arrears

In a Libor-In-Arrears (LIA, see [1]) swap, the floating leg pays a Libor rate as soon as it is fixed. The fallback for Libor, the in-arrears compounded overnight rate, works well for a standard swap but not for an in-arrears swap as the fallback rate will simply not be known at the beginning of the accrual period. So while caps get transformed into meaningful, although different from the original, contracts, LIA swaps simply do not work under the standard fallback, similar to FRAs (see [8]). Let us look at how this issue could be prevented by modifying LIA swaps accordingly.

Clearly a term version of the risk-free rate – a fixed break-even rate one would pay on a one-period swap against the compounded in-arrears overnight rate such as (15) – would be a near perfect substitute for a Libor rate in an LIA swap, as well as for many other Libor-linked contracts such as FRAs. Regulators so far have discouraged reliance on the potential existence of such rates for derivatives fallback. So let us consider what can be done with overnight compounded rates. The prevailing commercial rationale for customers to trade LIA, rather than standard, swaps is that on a receiver swap the client would pay Libor-based payments sooner than in a normal swap, and also the dealer would benefit from LIA convexity, so he would pay a higher fixed rate.

Libor rate $L(t) = L(t, T, T + \tau)$, if paid in-arrears, is worth at time t (see [1])

$$V_{\mathrm{LibLis}}(t) = P(0,T)\mathrm{E}_t^T L(T) = P(0, T + \tau)\mathrm{E}_t^{T+\tau} \left(L(T)\left(1 + \tau L(T)\right)\right).$$

A payment of Libor at time T is financially equivalent to a payment of $L(T)\left(1 + \tau L(T)\right)$ at time $T + \tau$. The latter is much more amendable to the proposed standard fallback, and we can simply replace a LIA swap with a contract that pays $\tilde{R}(1 + \tau\tilde{R})$ at time $T + \tau$, where $\tilde{R} = R(T + \tau) + F$ as defined by (15), with F being the fallback spread. Note that a similar fix works for an FRA. It is fair to note that while this fix is theoretically "perfect", operational considerations such as payment timings and the need to change underlying legal documentation may make this approach not practically feasible (see also).

4.2 Range Accruals

Things get more complicated still with range accruals, a popular Libor exotic often embedded in structured notes, sometimes in callable form (see [1]). A basic range accrual (RA) coupon pays

$$\int_T^{T+\delta} 1_{\{L_0(t)>a\}} dt \tag{21}$$

at time $T+\delta$, where $L_0(t) \triangleq L(t,t,t+\tau)$ is the current time-t (signified by the subscript 0) Libor fixing, and a is a lower barrier. As with LIA, a direct replacement of $L_0(t)$ with the standard fallback compounded overnight rate does not work as it is not known at time t, and hence the payment cannot be made at $T+\delta$. Let us see what happens if we directly replace $L_0(t)$ in (21) with $R_0(t) \triangleq R(t+\tau,t,t+\tau)$, where the subscript 0 signifies that the rate starts compounding immediately on the observation date. Ignoring the adjustment of the barrier a for the Libor-OIS fallback spread, the payoff becomes

$$\int_T^{T+\delta} 1_{\{R_0(t)>a\}} dt,$$

and is fully known only at time $T+\delta+\tau$, and not at time $T+\delta$ for the original contract. Therefore it cannot be paid earlier than $T+\delta+\tau$; to preserve the economics of discounting we can compound it by the rate from $T+\delta$ to $T+\delta+\tau$, thus replacing (21) with

$$(1+\tau R_0\,(T+\delta)) \int_T^{T+\delta} 1_{\{R_0(t)>a\}} dt \tag{22}$$

paid at $T+\delta+\tau$.

Paying a certain amount on a given date is economically equivalent to paying a properly un-discounted value at a later date; operationally, however, these two payments are not the same. For example, an investor in a structured note with an embedded range accrual rightfully expects coupon and principal payments on the agreed-upon dates and not on some future dates, and may be reluctant to accept changes. This is yet another manifestation of the practical challenges of the benchmark reform implementation for structured products.

This is not the end of the story, however. The same issue that we had with Libor caps turning "Asian" under the fallback is present here as well. Whereas an option that pays $1_{\{L_0(t)>a\}}$ is a European-style digital option and can be valued directly from the caplet smile for time t, an option that pays $1_{\{R_0(t)>a\}}$ is an option on an average rate with the observations for the average covering the time interval $[t, t+\tau]$. The underlying rate will have higher volatility but the impact of that on the digital option value can be in either direction. Hence, even if we go into the trouble of restructuring range accrual swaps with clients from paying (21) to (22), there will still be potentially significant value transfer and risk impact that would need to be dealt with.

5 Conclusions

We explored a number of areas where rates benchmark reform affects non-linear rates markets in significant ways. While by no means exhaustive, even the cases we consider demonstrate the sheer amount of further considerations and work required to bring Libor termination to a successful conclusion. We have made a number of practical suggestions on how the biggest challenges could be overcome. It is clear however that replacing term Libor rates with compounded overnight rates in all possible use cases in non-linear markets remains a formidable challenge.

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Disclaimer

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