Interest Rates Benchmark Reform and Options Markets

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Abstract

We examine the impact of interest rates benchmark reform and upcoming Libor transition on options markets. We address various modelling challenges the transition brings. We specifically focus on the impact of the clearing houses' discounting switch on swaptions, and the consequences of Libor transition on Libor-in-arrears swaps, caps, and range accruals as typical representatives of a very wide range of Libor derivatives.

1 Introduction

Fundamental changes in market structure due to the impending Libor transition, and more generally interest rate benchmark reform, are in the forefront of the minds of market participants. Close attention and voluminous discourse is afforded to various topics such as proposed fallbacks and impact on bilateral trading [6], pre-cessation triggers [3], discounting changes by clearing houses (CCPs) [14], impact on cash versus derivatives markets [4], and many others. The main focus of collective attention so far is firmly on linear markets such as swaps, bonds and loans. Potential impact on non-linear markets has received relatively less attention. In this paper we examine the impact of the benchmark reform on options, or more generally non-linear rates markets and highlight significant challenges for certain product types. We make a number of suggestions how these challenges could be addressed.

We focus on options markets in USD, EUR and GBP as being the most liquid and the furthest along in the benchmark reform. The three markets are reasonably similar, yet exhibit important idiosyncrasies in conventions and benchmark reform approaches. Some of the considerations below are more important for some of these markets and less for the others, and we endeavor to highlight such differences as we go along.

As a small caveat, market developments in the rate benchmark reform space come at a fast pace and while the information here is believed to be accurate at the time of writing, some later developments may not be reflected.

2 Discounting and Swaptions

The impact of the impending CCP discounting switch on swaptions has probably received the most attention as far as non-linear markets are concerned, see e.g. [14] and [2]. Let us quickly review the issue. As part of rates benchmark reform, a number of CCPs (e.g. LCH, CME) have announced that their collateral (so-called PAI, or Price Alignment Interest) rates will switch from legacy overnight rates to their replacements, such as from FedFunds to SOFR for USD and from Eonia to ESTR for EUR, targeting October (USD) and June (EUR) 2020 as the implementation date. Rates used to fund collateral balances directly define rates for discounting cashflows in cleared instruments (see [15]). Values (and risk sensitivities) of cleared interest rate swaps will change on that date. As part of the transition, CCPs have proposed a compensation mechanism to eliminate the potential value transfer.

Unlike swaps, swaptions – options to enter swaps – are not cleared and are governed by bilateral agreements based on ISDA templates. In USD and EUR swaptions typically settle into cleared swaps or, more accurately, into cash amounts determined by referencing cleared swap values[1]. If the CCP discounting switch happens before swaption expiry, the underlying value of the referenced swap will change resulting in a value transfer in a bilateral transaction.

Establishing a market-wide compensation scheme for the value transfer in swaptions is a complex issue, as [14] outlines. ISDA and various regulators are reportedly examining possible options, see e.g. [2], and even some fintechs are making proposals. The markets are, reportedly, skeptical (arguably more in EUR than in USD) that an effective solution can be found or that, at the very least, swaptions traded post discounting switch announcement date would be in scope (see more on this in Section 2.2). Some market participants (in EUR; not so much in USD) have been showing prices on packages of synthetic swap exposures via CCP-settled zero-wide collars (off-market i.e. with strikes at ATM+Y with Y being 50 or even 100 bp) hedged by cleared swaps, positions that should be worth exactly zero if the same discounting applied to all legs. Such a package expresses a view that a cleared swap will be compensated for the value transfer but the swaption position will not be, the difference maximized for deep in-the-money swaps (hence strikes at ATM+50 to 100 bp).

We emphasize that two market wide event dates are important, the date(s) when CCP(s) announced the intent to switch discounting for cleared swaps (September 2019) and the actual discounting switch date (June 2020 for EUR, October 2020 for USD). For a particular swaption two dates are important in our context, the trade date and the expiry date.

[1] The situation in EUR markets is in fact even more subtle as swaptions use ICE screen rate for settlement which at the moment is not CCP-specific and is not linked to DV01 referenced in a contract. We do not explore this important nuance here.

2.1 Compensation for Value Transfer

Leaving the issue of how to best persuade beneficiaries of discounting switch value transfer in swaptions to compensate their counterparts, we consider a simpler question of what the right compensation should be.

The proposal for value transfer compensation for swaps is fairly straightforward, see [5]. As the discounting rate changes, two things happen. The value of a swap changes, and it now has sensitivities (deltas) to the new discounting rate and not the old one. The first one is addressed via a cash payment, and the second, where required, via an exchange of basis swaps. A clearing house, being in the middle of all cleared swaps, facilitates all this activity.

It is tempting to suggest that a similar compensation mechanism for swaptions, i.e. an exchange of a package of cash and basis swaps at swaption expiry is adopted to negate the value and risk transfer. While compellingly simple – the CCP mechanism for swaps is applied to swaptions with no modifications – there are significant issues with this approach. The main one, in our view, is the fact that the swaption expiry may well be 20 or 30 years from now, and it is entirely possible (in fact, expected) that the legacy OIS rates/curves (specifically FedFunds; Eonia has been fixed to be ESTR+8.5 bp which, conceivably, could still be used in the far future) will no longer be available. Moreover, valuing and risk-managing a swaption with such compensation mechanism throughout its life will be a considerable challenge. Since at expiry one would effectively exercise not into a vanilla swap but a basket of a swap, cash, and basis swaps, the swaption will cease to be a "simple" vanilla option anymore.

It should be clear that cash compensation at the time of the discounting switch (or announcement of discounting switch which is now sadly in the past) is a better option and if fact is one of the proposed solutions in the consultation paper [2]. We argue that a more theoretically-sound mechanism is possible, that we now proceed to outline.

Discounting swaptions generally requires two curves. The curve specified by the bilateral CSA is used for discounting of the swaption payoff from swaption expiry date to today. The CCP discounting curve is used for the underlying swap in payoff calculations. We focus on the CCP discounting switch and remove the bilateral discounting curve from consideration by focusing on forward (to swaption expiry) quantities.

Let T be the expiry date of the swaption. We fix a tenor structure $0 < T = T_1 < \cdots < T_N$, $\tau_i = T_{i+1} - T_i$. Let $L_i(t) = L(t, T_i, T_{i+1})$ be Libor rates, $P_i^f(t) = P^f(t, T, T_i) = P^f(t, T_i)/P^f(t, T)$ the "old" OIS, e.g. FedFunds, forward discount factors, and $P_i^s(t) = P^s(t, T, T_i) = P^s(t, T_i)/P^s(t, T)$ the "new", say SOFR, forward discount factors. Let us denote the forward annuity and the swap rate under the two different discount curves by

$$A^{f,s}(t) = \sum_i \tau_i P_{i+1}^{f,s}(t),$$

$$S^{f,s}(t) = \frac{1}{A^{f,s}(t)} \sum_i \tau_i P_{i+1}^{f,s}(t) L_i(t).$$

The (forward to time T) value of a swaption to exercise into a FedFunds (SOFR) discounted CCP swap (technically its CCP cash value, but the distinction is irrelevant for modelling) with strikes K^f (K^s) and notionals N^f (N^s) is

$$V^{f,s}_{\mathrm{swpt}}(t) = N^{f,s} A^{f,s}(t) \mathrm{E}^{A^{f,s}}_t (S^{f,s}(T) - K^{f,s})^+.$$

Let us suppose a mechanism could be agreed to modify contractual terms of a swaption (notional, strike) due to the discounting switch. The potential value transfer is then given by

$$V^f_{\mathrm{swpt}}(t) - V^s_{\mathrm{swpt}}(t) = N^f A^f(t) \mathrm{E}^{A^f}_t (S^f(T) - K^f)^+ - N^s A^s(t) \mathrm{E}^{A^s}_t (S^s(T) - K^s)^+.$$

Using the same FedFunds annuity measure for both expressions we obtain

$$\begin{aligned} \Delta V_{\mathrm{swpt}} &\triangleq V^f_{\mathrm{swpt}}(t) - V^s_{\mathrm{swpt}}(t) \\ &= N^f A^f(t) \mathrm{E}^{A^f}_t \left((S^f(T) - K^f)^+ - c \frac{A^s(T)}{A^f(T)} (S^s(T) - K^s)^+ \right), \end{aligned}$$

where we have denoted

$$c = \frac{N^s}{N^f}. \tag{1}$$

Our objective is to find the change in contractual terms that would minimize/eliminate value transfer. Clearly the first thing we would want to do is to match the moneyness of the old and the new swaption. To that effect we set

$$K^s = K^f + S^s(t) - S^f(t), \tag{2}$$

i.e. we simply adjust the swaption strike by the difference in swap rates under the old and the new discounting. We do not expect the strike adjustment to be large as discounting has small effect on at-the-money swaps. (In fact is some cases the strike adjustment can be dispensed with altogether.) Let us simplify the notations and set

$$S(T) - K \triangleq S^f(T) - K^f \approx S^s(T) - K^s$$

where the last equality follows from the strike adjustment (2) and some obvious assumptions. With this step completed, the expression for the potential value transfer reduces to

$$\Delta V_{\mathrm{swpt}} = N^f A^f(t) \mathrm{E}^{A^f}_t \left(\left(1 - c \frac{A^s(T)}{A^f(T)}\right) (S(T) - K)^+ \right).$$

Clearly $\frac{A^s(T)}{A^f(T)}$ is a positive martingale under P^{A^f},

$$\frac{A^s(T)}{A^f(T)} = \frac{A^s(t)}{A^f(t)} M(T), \quad M(t) = 1, \quad M(\cdot) \text{ is a } \mathrm{P}^{A^f}\text{-martingale}.$$

Setting the value transfer to zero,

$$0 = A^f(t) \mathrm{E}^{A^f}_t \left(\left(1 - c \frac{A^s(t)}{A^f(t)} M(T)\right) (S(T) - K)^+ \right),$$

we obtain the following condition on c,

$$A^f(t)\left(c\frac{A^s(t)}{A^f(t)}\right)\mathrm{E}_t^{A^f}\left(M(T)(S(T)-K)^+\right) = A^f(t)\mathrm{E}_t^{A^f}\left((S(T)-K)^+\right).$$

The ratio of notionals (1) that is consistent with zero value transfer (under our assumptions) is then

$$c = \frac{A^f(t)}{A^s(t)} \times \frac{\mathrm{E}_t^{A^f}\left((S(T)-K)^+\right)}{\mathrm{E}_t^{A^f}\left(M(T)(S(T)-K)^+\right)}. \tag{3}$$

The factor

$$c \approx \frac{A^f(t)}{A^s(t)} \tag{4}$$

is the dominant one; it specifies that we should replace one FedFunds swaption with c SOFR swaptions, where c is the ratio of the FedFunds and SOFR annuities of the underlying swap or, simply, forward DV01s (forward parallel deltas) of the old and the new swaption.

The second factor in (3) is a convexity correction to the basic expression for c in (4) and is driven by the correlation between $M(T)$ and $S(T)$ or, more accurately, $(S(T)-K)^+$ and as such is strike dependent. To first order, it can be ignored. If more precision is required, the following calculation can be employed,

$$\begin{aligned}
\mathrm{E}_t^{A^f}\left(M(T)(S(T)-K)^+\right) &= \mathrm{E}_t^{A^f}\left(\mathrm{E}_t^{A^f}\left(M(T)(S(T)-K)^+ \middle| S(T)\right)\right) \\
&= \mathrm{E}_t^{A^f}\left((S(T)-K)^+\mathrm{E}_t^{A^f}\left(M(T)\middle| S(T)\right)\right) \\
&= \mathrm{E}_t^{A^f}\left((S(T)-K)^+\mu(S(T))\right), \qquad (5)
\end{aligned}$$

where

$$\mu(x) \triangleq \mathrm{E}_t^{A^f}\left(M(T)\middle| S(T)=x\right) \approx \frac{\langle M(T), S(T)\rangle}{\langle S(T), S(T)\rangle}x.$$

The adjustment in (3) vs. (4) can then be calculated by strike replication of the payoff (5) with European swaptions in a way not dissimilar to CMS convexity calculations, see [1].

It could be argued that the convexity correction is not needed as the ratio of discount annuities is only weakly correlated to the swap rate; on the other hand, in the particular case of FedFunds vs. SOFR, the former incorporates some credit risk premium while the latter does not, and $S(\cdot)$ also has certain amount of credit risk premium coming from Libor, so the correlation is probably not zero.

To summarize, we propose the following adjustments to bilateral in-scope CCP-settled swaptions when CCPs change discounting:

- Replace one unit of f swaption with c units of s swaptions, where c is calculated as the ratio of forward annuities (or more accurately including the convexity correction);

- Adjust the strike of the swaption by the difference of the swap rates under FedFunds and SOFR discounting at the time of conversion.

This approach, in our view, enjoys a number of advantages over a simple cash adjustment on the discounting switch date (and certainly over the largely unworkable adjustment at the swaption expiry):

- Deltas to the Libor (projection) curve remain largely unchanged so no extra hedging is required;
- Deltas to discounting curves are comparable in the sense that X units of risk to FedFunds are replaced by X units of risk to SOFR;
- Vegas (volatility sensitivities) remain largely the same, not requiring any additional trading in options to recover the original vega position as would be the case under the cash compensation scheme.

It is fair to note that our approach comes with its own limitations. Arguably, it is more practical to exchange cash as compensation at a point in time than to amend a potentially large number of existing trades. Perhaps a hybrid approach is warranted with market participants agreeing on which route balances their risk management and operational considerations best.

The discounting change is only one of the aspects of the benchmark reform that will affect swaptions markets. Down the line, when Libor ceases to exist, swaptions underlyings will change from Libor to OIS swaps (except in EUR). Potentially, this may lead to other considerations related to value transfer. We do not consider these issues here as there continues to be a fair amount of uncertainty over important details of this transition. At this moment, however, we do not expect a significant valuation impact as the Libor-OIS basis spreads have already largely converged to their proposed fallback levels, see [8], reducing or potentially eliminating future reform-related valuation changes.

2.2 Volatility Smile Impact

Swaptions are quoted in price terms (premiums or, more commonly, forward premiums) in the inter-dealer markets that all dealers use for price discovery. Internally, most dealers represent swaption prices in terms of implied volatilities so that, when inserted into the Black-Scholes (or Normal) model and the values scaled by DV01s, market-observed prices are recovered. Implied volatilities are then used to mark swaption books internally, determine swaption prices offered to clients, and also feed into models for more exotic products such as CMS, Bermudan swaptions, and so on.

Consider a dealer who, pre discounting switch announcement, marks a particular swaption expiring post switch (with notations from Section 2.1) at volatility v^f that corresponds to value $V^f_{\mathrm{swpt}}(t)$, swap rate $S^f(t)$, strike K^f, and annuity $A^f(t)$. The question we consider is where should he mark this volatility post-announcement. There is a range of plausible scenarios.

2.2.1 Expectation of No Value Transfer

Let us consider the case where the market believes that parties to a pre-announcement, or "legacy", swaption will be compensated. In fact let us assume that the market expects compensation along the lines of Section 2.1. As there is no expectation of value transfer, there is no jump in the price $V^f_{\mathrm{swpt}}(t)$. The dealer should continue using existing valuation parameters, i.e. the forward rate $S^f(t)$, strike K^f, FedFunds for discounting the underlying swap (i.e. $A^f(t)$ for annuity), and volatility v^f.

Once the compensation for value transfer occurs, the booking should be changed (i.e. the strike and notional adjusted and the discounting curve upgraded to the new one), but the volatility surface should not be impacted by this event.

Cash fee paid on the discounting switch date is a an alternative compensation option. Under this scenario, a swaption

$$V^f_{\mathrm{swpt}}(t) = A^f(t)\mathrm{E}^{A^f}_t(S^f(T) - K^f)^+$$

is replaced by a swaption

$$V^s_{\mathrm{swpt}}(t) = A^s(t)\mathrm{E}^{A^s}_t(S^s(T) - K^s)^+$$

plus a cash payment in the amount of $V^f_{\mathrm{swpt}}(t) - V^s_{\mathrm{swpt}}(t)$. As there is no value transfer, keeping the original volatilities with the original trade booking and market data (discounting/projection curves) at the moment of discounting switch announcement is appropriate. On the actual discounting switch/cash fee payment event, volatility should not be affected as long as the other elements are updated, i.e. the fee is paid/received and the discounting curve is switched to the new one.

2.2.2 Expectation of Value Transfer

Now let us consider the scenario where, upon discounting switch announcement, the market assumes that there will be no compensation for value transfer. Prices of swaptions will immediately jump (in this idealized scenario; it will likely take some time for the market to adjust/discover new prices). In particular, as generally $A^s(t) > A^f(t)$ (ESTR is lower than Eonia, while the picture is more mixed with SOFR vs. FedFunds), long swaption positions will generally increase in value.

A "consensus" dealer will switch discounting to SOFR and keep volatilities unchanged. A "contrarian" dealer who insists on continuing to use the legacy FedFunds discounting curve, whether by ignorance, contrarian beliefs, or system limitations, will have to re-mark volatilities higher. This will lead to vega PnL (profit/loss arising from volatility changes). While for the CCP-settled swaptions the PnL will be "real" in the sense of faithfully reflecting market view for uncompensated value transfer, it may not be so real for other derivatives that derive their values from swaption volatilities such as "legacy" pre-November 2018, i.e. IRR, cash-settled European swaptions[2] in EUR, CMS, Bermudan swaptions and other Libor exotics.

The impact of discounting switch announcement on volatilities used by the contrarian dealer is not the same for different strikes. The value of a swaption will change as follows (ignoring impact on the forward swap rate and dropping $t = 0$ from notations)

$$V^f_{\text{swpt}} = A^f \mathrm{E}^{A^f} (S(T) - K)^+ \to V^s_{\text{swpt}} = A^s \mathrm{E}^{A^s} (S(T) - K)^+.$$

The new volatility, $v^s = v^s(K)$ will then be calculated as follows,

$$\begin{aligned} v^s(K) = C^{-1}\left(\frac{V^s_{\text{swpt}}}{A^f}, K\right) = C^{-1}\left(\frac{A^s}{A^f}\mathrm{E}^{A^f}(S(T) - K)^+, K\right) \\ = C^{-1}\left(\frac{A^s}{A^f}C(v^f, K), K\right), \quad (6) \end{aligned}$$

where $C(\sigma, K)$ is the Normal (or Black) option pricing formula with volatility σ and strike K (with expiry and forward rate dependence suppressed), and C^{-1} is the inverse of this function in the first, volatility, argument. It is important to realize that this transformation is strike-dependent, as signified by the notation $v^s = v^s(K)$ in (6). The constant multiplier V^s/V^f has different impact on volatilities at different strikes.

Let us present a simple illustration of the impact just described, where for variety we use representative values from the EUR, rather than USD, swaptions markets. We consider 10y20y swaptions, i.e. swaptions expiring in 10 years into 20 year swaps. Forward swap rate is set to 0.60% (60 basis points, or bps), and we assume that the market has switched to ESTR discounting while the contrarian dealer still uses Eonia. For clarity we use the same normal volatility of 50 bps for all strikes as the market. The DV01 ratio in (6) is estimated to be 1.0087, as the relationship between Eonia and ESTR is already locked down at Eonia = ESTR +8.5 bps.

Figure 1 shows what volatilities a legacy(Eonia)-discounting dealer would have to use, compared to the new(ESTR)-discounting market. The latter are shown as the "baseline" graph. The former are shown as "payers" for payer swaption volatilities, and as "receivers" for receiver swaption volatilities.

Not only there is a pronounced effect on the volatility smile, the volatilities implied for payer (call) and receiver (put) swaptions are worryingly different. This is fairly clear from a brief reflection on (6) and is unequivocally demonstrated in Figure 1. To the extent that high-strike volatilities are typically implied from the prices of OTM payers and low-strike volatilities from OTM receivers, this (arbitrage-inducing) mismatch may not be immediately obvious. Clearly, however, if not recognized, it could lead to significant mis-pricings in swaption and related markets for the legacy-discounting dealer.

[2]In EUR, the IRR vs. the "new" cash-settled zero-wide collars is another segment that is currently used to express views on the probability of compensation for value transfer.

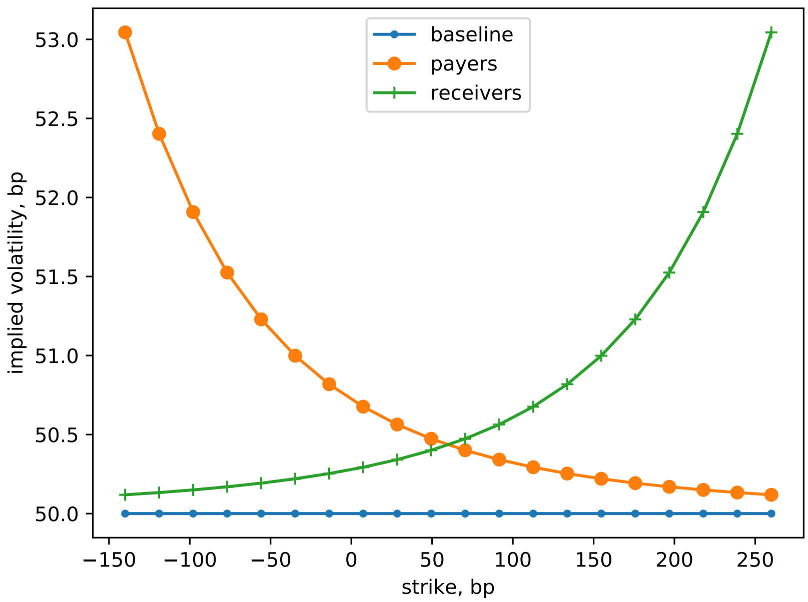

Figure 1: Impact on implied normal volatilities from mismatched discounting. "Baseline" is the baseline volatility. "Payers" is the volatility implied from payer swaptions under mismatched discounting. "Receivers" is the volatility implied from receiver swaptions under mismatched discounting.

2.2.3 Market Segmentation

At the time of writing there is still hope that a market-wide solution to value transfer for swaptions executed before the discount switch announcement date could be found. As mentioned earlier, however, trading activity in swaptions (in EUR) post-announcement seems to indicate that at least some market participants do not believe that this solution, even if found, will apply to swaptions traded post-announcement and expiring after the actual discounting switch. If this scenario, i.e. compensation for legacy swaptions but not for post-announcement ones, becomes widely accepted as the most likely outcome, the swaption market will likely segment. Two swaptions with identical terms (expiry, tenor, strike) traded on different dates may be valued differently and, in particular, may require different volatilities and/or different underlying discount curves. At the very least this development may put yet another strain on internal systems where potential eligibility of swaptions for compensation, perhaps expressed as probabilities of various outcomes, need to be captured and used in pricing and risk management.

The situation is somewhat different in the USD market, at least at the moment, as there appears to exist a stronger impetus and determination to find a market-wide solution to the compensation problem, with a consultation recently launched under the auspices of the Federal Reserve, see [2]. This market sentiment is supported by a seeming absence of "no-compensation" swaptions vs. swaps trades that we discussed earlier.

3 Caps

Interest rate caps present another challenge for the benchmark reform, in this case caused by term Libor rates being replaced by a daily compounded in-arrears overnight rate, as mentioned in e.g. [7], [8], [16]. Here, again, differences between different currencies emerge as EURIBOR is a go-forward term rate and the discussion below is only applicable to USD and GBP and, potentially, more so to GBP, as the US regulators seem somewhat more open to a wider adoption of alternative *term* rates for derivatives.

Continuing with the notations of Section 2.1, let us fix i and simplify notations to $L(t) \triangleq L_i(t) = L(t, T_i, T_i + \tau)$, $T \triangleq T_i$, $\tau \triangleq \tau_i$. The rate $L(t)$ is given by

$$L(t) = \frac{P_i(t) - P_{i+1}(t)}{\tau P_{i+1}(t)}.$$

A caplet is an option that pays $(L(T) - K)^+$ at time $T + \tau$, and its value is given by

$$V_{\mathrm{LibCpl}}(t) = P(t, T + \tau) \mathrm{E}_t^{T+\tau} \left(L(T) - K\right)^+ . \tag{7}$$

Under the standard fallback mechanism (that is yet to be adopted for bilateral markets where caps trade, but assuming it will), a Libor rate is replaced by a compounded in-arrears daily rate with a spread. Under this scenario the payoff in (7) then becomes

$$V_{\mathrm{OisCpl}}(t) = P(t, T + \tau) \mathrm{E}_t^{T+\tau} \left(\frac{1}{\tau} \left(\exp \left(\int_T^{T+\tau} r(s)\ ds \right) - 1 \right) - K' \right)^+ , \tag{8}$$

where we use $r(s)$ for the overnight rate, K' for the fallback-spread-adjusted strike, and replace discrete daily with continuous compounding for notational simplicity.

The fundamental difference between (7) and (8), as noted in [8], is that in the former the rate is fixed at time T, whereas in the latter it continues to stochastically evolve until a later time $T + \tau$, combined with the averaging feature of rates fixed at different times over the time period $[T, T + \tau]$. A simple European option in (7) becomes an Asian option in (8).

Assume for a moment that one-period swaptions are traded. (We will explain the relevance of this thought experiment in a second.) Its value in the Libor world is given by

$$V_{\mathrm{LibSwpt}}(t) = P(t, T + \tau) \mathrm{E}_t^{T+\tau} \left(V_{\mathrm{LibSwap}}(T)\right)^+ , \tag{9}$$

where

$$V_{\text{LibSwap}}(T) = \mathrm{E}_T^{T+\tau}\left(L(T) - K\right) = L(T) - K. \tag{10}$$

Clearly (9)–(10) describe a contract equivalent to (7), and in the Libor world a caplet is equivalent to a one-period swaption. Now consider the post-Libor situation. Here we have

$$V_{\text{OisSwpt}}(t) = P(t, T+\tau)\mathrm{E}_t^{T+\tau}\left(V_{\text{OisSwap}}(T)\right)^+ \tag{11}$$

with

$$\begin{aligned} V_{\text{OisSwap}}(T) = \mathrm{E}_T^{T+\tau}&\left(\frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\ ds\right) - 1\right) - K'\right) \\ &\neq \frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\ ds\right) - 1\right) - K'. \end{aligned} \tag{12}$$

In fact, combining (11) and (12) we obtain

$$\begin{aligned} &V_{\text{OisSwpt}}(t) \\ &= P(t, T+\tau)\mathrm{E}_t^{T+\tau}\left(\mathrm{E}_T^{T+\tau}\left(\frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\ ds\right) - 1\right)\right) - K'\right)^+. \end{aligned} \tag{13}$$

In contrast to (8), part of the expectation value operator moves inside the $\max(\cdot, 0)$ operator. Jensen's inequality (as also noted in [12]) implies

$$V_{\text{OisCpl}}(t) \geq V_{\text{OisSwpt}}(t). \tag{14}$$

Let us denote (as introduced in [12])

$$R(t) \triangleq R(t, T, T+\tau) = \mathrm{E}_t^{T+\tau}\left(\frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\ ds\right) - 1\right)\right), \tag{15}$$

where $t \in [0, T+\tau]$, i.e. t is allowed to be larger than T (as in [12]). It is trivially a martingale in $\mathrm{P}^{T+\tau}$ so can easily be used as the underlying in the standard "vanilla", such as Black-Scholes or SABR, modelling. For $t \leq T$ this is called the (forward) term one-period compounded OIS swap rate and is defined as the break-even rate on a one-period forward starting OIS swap observed at time t. With this notation we have

$$_{\text{OisSwap}}(T) = \mathrm{E}_T^{T+\tau}\left(R(T) - K'\right)$$

which is the equivalent of (10) in post-Libor world. We note that the caplet value (8) is then

$$V_{\text{OisCpl}}(t) = P(t, T+\tau)\mathrm{E}_t^{T+\tau}\left(R(T+\tau) - K'\right)^+.$$

3.1 Market Impact

There are a number of implications of the differences just explained that are worth exploring. Whereas a standard interest rate swap post Libor transition looks essentially like a swap before transition as far as interest rate risk is concerned, caps change their character significantly, and may no longer be suitable for market participants who have them on the books as, for example, a loan hedge. Sensitivity to volatility looks different, and the complexity of valuation is different as well, potentially affecting liquidity and the ability to exit positions.

There is also, of course, the issue of value transfer. Before the talk of Libor transition began, one could enter into a long Libor caplet hedged by a short matching single period swaption at zero cost. While single-period swaptions are not really traded, a cap versus a matching tenor swaption, for example a one-year forward starting cap on three-month Libor versus a one-year swaption with a matching first expiry, was at times a popular hedge fund trade. Originally designed as a bet on Libor/swap rate correlations, under the fallback protocol not only will it likely change its value, but will also acquire different risk characteristics.

[9] proposes a different fallback mechanism for caps that would keep them aligned with single-period swaptions – we discuss it in more detail later in the context of other products.

A similar divergence may occur between caplets and exchange-traded options on Eurodollar futures.

3.2 Volatility Adjustment

Let us estimate the difference in value of a caplet vs. a matching single-period swaption in the post-Libor world. For a very simple estimate, let us assume that $r(t)$ follows a Brownian motion with constant volatility σ, and let us approximate

$$R(T+\tau) = \frac{1}{\tau}\left(\exp\left(\int_T^{T+\tau} r(s)\ ds\right) - 1\right) \approx \frac{1}{\tau}\int_T^{T+\tau} r(s)\ ds. \qquad (16)$$

To estimate the variance of $R(T+\tau)$, we recall (see Appendix) that for the standard Brownian motion $W(\cdot)$ for $T \geq 0$

$$\mathrm{E}\left(\left(\int_T^{T+\tau} W(s)\ ds\right)^2\right) = \tau^2\left(T + \frac{\tau}{3}\right).$$

Thus

$$\mathrm{Var}\left(R(T+\tau)\right) = \sigma^2\left(T + \frac{\tau}{3}\right).$$

At the same time

$$R(T) = r(T)$$

under the Brownian motion assumption and the approximation (16), so that

$$\mathrm{Var}\left(R(T)\right) = \sigma^2 T.$$

The ratio of volatilities is then given by (see also [13])

$$\frac{\mathrm{Vol}\left(R(T+\tau)\right)}{\mathrm{Vol}\left(R(T)\right)} = \sqrt{1 + \frac{\tau}{3T}}. \tag{17}$$

It is always larger than 1 as already discussed, and could be significantly larger than 1 for shorter-dated options i.e. smaller T.

3.3 Considerations for Modelling

Modeling caps post-Libor transition is more complicated as the option becomes Asian. In this regard, [8] derives valuation formulas in a one-factor Gaussian model (see also [10], [11]), and of course the seminal extension of the Libor market model for overnight rates from [12] can be used. Both, however, are likely to be deemed impractical by traders for what would still be likely considered a vanilla product. The Gaussian model is likely to be ruled out on the basis of not supporting the smile natively, and the LMM because of computation cost (and also difficulties in controlling the smile). Hence, an adaptation of a vanilla model such as SABR will likely be required.

Complicating matters somewhat is the uncertainty that surrounds fallback mechanisms as applied to caps. As we have seen, a straightforward application of the standard fallback makes caps materially different from what they were/are now. It is possible that the industry will converge on a different solution for caps, e.g. replacing them with (strips of) single period swaptions – essentially creating instruments linked to term risk-free rates (as swap rates on single-period OIS swaps essentially are), without calling them as such.

While the future is uncertain, it is possible that both types of contracts, Asian-style OIS caplets and European style single-period swaptions, will be traded in the future. This possibility should be one of the considerations of any future vanilla risk management framework. In particular the model should be flexible enough so that the two types of contracts can be marked at different levels of volatility and, potentially, of other volatility smile parameters. In particular, it seems unlikely that a simple scaling of swaption volatility along the lines of (17) would be sufficient to match both markets.

The relation (17) does, however, have a purpose as it expresses volatilities of different contracts in a comparable way. Let us assume we mark single-period OIS swaptions with expiry T and tenor τ with parameters (in the standard SABR parameterization) σ_{swpt}, α_{swpt}, β_{swpt}, ρ_{swpt}. It is natural then to use a SABR model for caplets as well. We would then mark σ_{cpl} as we see fit and, perhaps, in the first iteration of the model use the same smile parameters

$$\alpha_{\mathrm{cpl}} = \alpha_{\mathrm{swpt}}, \quad \beta_{\mathrm{cpl}} = \beta_{\mathrm{swpt}}, \quad \rho_{\mathrm{cpl}} = \rho_{\mathrm{swpt}}.$$

Before applying the SABR model to a caplet we would first transform σ_{cpl} into an approximation to the "Asian volatility" using (17) by setting

$$\sigma_{\text{asian}} = \sigma_{\text{cpl}} \sqrt{1 + \frac{1}{3} \frac{\tau}{T}} \tag{18}$$

for $T \geq 0$ (the case $-\tau < T < 0$ should obviously be treated somewhat differently – we omit obvious details). Finally, we would use the set of parameters

$$\sigma_{\text{asian}}, \alpha_{\text{cpl}}, \beta_{\text{cpl}}, \rho_{\text{cpl}}$$

in the SABR formula to calculate (8) with the strike K' and the forward set to

$$R(0) = \mathrm{E}^{T+\tau} \left(\frac{1}{\tau} \left(\exp \left(\int_T^{T+\tau} r(s)\ ds \right) - 1 \right) \right)$$

(or the exact daily compounding version thereof). We remind the reader that while this expression seems to indicate that some convexity corrections are required because of the difference of the payment and fixing times, it is in fact these quantities that are bootstrapped from the (eventually liquidly traded) compounded OIS in-arrears swaps, so in fact are a direct market input.

The scaling (18), while not strictly necessary, allows traders to think of caplets versus swaptions, and caplets of different expiries, in normalized units, as they do not need to worry about the translation of caplet volatilities to the same units as swaptions volatilities and the like. We have not performed a similar re-scaling for other SABR parameters as we consider their impact to be of second-order that will possibly come into effect later as the markets develop (if, in fact, they develop as we envisage). When we do have liquid caplet and single-period swaption markets, the question of how to adjust other SABR parameters for Asian features may need to be tackled.

4 Disappearing Species?

Interest rate caps change quite significantly under the standard Libor fallback. Some of the products may be affected even more and disappear altogether. Let us look at some examples.

4.1 Libor-In-Arrears

In a Libor-In-Arrears (LIA, see [1]) swap, the floating leg pays a Libor rate as soon as it is fixed, unlike a Libor rate in the standard swap paid at the end of the accrual period. The fallback for Libor, the in-arrears compounded overnight rate, works well for a standard swap but not for an in-arrears swap. The Libor replacement rate will simply not be known at the beginning of the accrual period. So while caps get transformed into meaningful, although different from the original, contracts, LIA swaps simply do not work under the standard fallback. This is similar to the situation with FRAs as discussed in [7]. Legal recourse of counterparties under a contract that cannot be fulfilled due to time being unidirectional is a fascinating topic that space limitations prevent us from exploring. Instead we look at how this dilemma could be prevented by modifying LIA swaps accordingly. There are two angles that we can think of, and we consider them in turn.

Clearly a term version of the overnight rate – a fixed break-even rate one would pay on a one-period swap against the compounded in-arrears overnight rate such as (15) – would be a near perfect substitute for a Libor rate in an LIA swap, as well as for many other Libor-linked contracts such as FRAs. Regulators so far have discouraged reliance on the potential existence of such rates for derivatives fallback. It can, however, be created synthetically. A practical suggestion in this spirit is proposed in [9]. Alternatively, before Libor disappears, an LIA swap can be restructured into a contract that, on each fixing date, observes a prevailing swap rate on a single-period swap vs. the daily compounded rate, e.g. the upcoming ICE RFR swap rate[3] on the shortest supported tenor, and pays an amount linked to that. It is not however clear that such a rate would be reliably observable, or regulators would not actively discourage contracts with de-facto links to term rates.

Let us now consider what can be done with overnight compounded rates. The prevailing commercial rationale for customers to trade LIA, rather than standard, swaps is that on a receiver swap (i.e. the client receives fixed, pays Libor), the client would pay Libor-based payments sooner than in a normal swap and the dealer may offer a (marginally) higher fixed rate for that. Moreover, dealers receiving Libor in arrears benefit from convexity (see [1]) and may share some of that by paying an even higher fixed rate to the client.

Let us briefly recall LIA convexity. Libor rate $L(t) = L(t, T, T + \tau)$, if paid in-arrears, is worth at time t (see [1])

$$V_{\mathrm{LibLis}}(t) = P(0,T)\mathrm{E}_t^T L(T) = P(0,T+\tau)\mathrm{E}_t^{T+\tau}\left(L(T)\left(1+\tau L(T)\right)\right).$$

A payment of Libor at time T is financially equivalent to a payment of $L(T)\left(1 + \tau L(T)\right)$ at time $T + \tau$. The latter is much more amendable to the proposed standard fallback, and we can simply replace an LIA swap with a contract that pays

$$\tilde{R}\left(1 + \tau\tilde{R}\right)$$

at time $T + \tau$, where $\tilde{R} = R(T + \tau) + F$ as defined by (15), with F being the fallback spread. Note that a similar fix works for an FRA.

4.2 Range Accruals

Things get more complicated still with range accruals, a popular Libor exotic often embedded in structured notes, sometimes in callable form (see [1]). A

[3] The so-called screen swap rate produced by ICE from cleared (Libor at the moment) swap quotes and currently used for settling swaptions in EUR. ICE is reportedly planning to introduce RFR-based swap rates in April 2020.

basic range accrual (RA) coupon pays (a daily discretized version of)

$$\int_T^{T+\delta} 1_{\{L_0(t)>a\}} dt \tag{19}$$

at time $T+\delta$, where $L_0(t) \triangleq L(t,t,t+\tau)$ is the current time-t (signified by the subscript 0) Libor fixing, and a is a lower barrier. We note that many other versions of range accruals exist, see [1].

As with LIA, a direct replacement of $L_0(t)$ with the standard fallback compounded overnight rate does not work as it is not known at time t, and hence the payment cannot be made at $T+\delta$. There are also more subtle issues that we will come to in a moment.

Clearly, the RA contract would also benefit from the existence of term risk-free rates. In the absence of the official term fixings, they could possibly be simulated by referencing the breakeven fixed rate on a one-period swap vs. compounded daily rate such as the ICE rate as mentioned in Section 4.1.

Let us see what happens if we directly replace $L_0(t)$ in (19) with $R_0(t) \triangleq R(t+\tau,t,t+\tau)$, where the subscript 0 signifies that the rate starts compounding immediately on the observation date. Ignoring the adjustment of the barrier a for the Libor-OIS fallback spread, the payoff becomes

$$\int_T^{T+\delta} 1_{\{R_0(t)>a\}} dt,$$

and is fully known only at time $T+\delta+\tau$, and not at time $T+\delta$ for the original contract. Therefore it cannot be paid earlier than $T+\delta+\tau$; to preserve the economics of discounting we can compound it by the rate from $T+\delta$ to $T+\delta+\tau$, thus replacing (19) with

$$\left(1+\tau R_0\left(T+\delta\right)\right) \int_T^{T+\delta} 1_{\{R_0(t)>a\}} dt \tag{20}$$

paid at $T+\delta+\tau$.

A note of caution is in order here. While paying a certain amount on a given date is economically equivalent to paying a properly un-discounted value at a later date, operationally these two payments are not the same. For example, an investor in a structured note with an embedded range accrual rightfully expects coupon and principal payments on the agreed-upon dates and not on some future dates, and may be reluctant to accept changes. This is yet another manifestation of the practical challenges of the benchmark reform implementation for structured products.

This is not the end of the story, however. The same issue that we had with Libor caps turning "Asian" under the fallback is present here as well. Whereas an option that pays $1_{\{L_0(t)>a\}}$ is a European-style digital option and can be valued directly from the caplet smile for time t, an option that pays $1_{\{R_0(t)>a\}}$ is an option on an average rate with the observations for the average covering the time interval $[t,t+\tau]$. The underlying rate will have higher volatility but the impact of that on the digital option value can be in either direction. Hence, even if we go into the trouble of restructuring range accrual swaps with clients from paying (19) to (20), there will still be potentially significant value transfer and risk impact that would need to be dealt with.

5 Conclusions

We explored a number of areas where rates benchmark reform affects non-linear rates markets in significant ways. While by no means exhaustive, even the cases considered demonstrate the sheer amount of further considerations and work required to bring Libor termination to a successful conclusion. We have made a number of practical suggestions on how the biggest challenges could be overcome. It is clear however that replacing term Libor rates with compounded overnight rates in all possible use cases in non-linear markets remains a formidable challenge.

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Disclaimer

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A Appendix

For $T \geq 0$,

$$\begin{aligned}
\mathrm{E}\left(\left(\int_T^{T+\tau} W(s)\ ds\right)^2\right) &= \int_T^{T+\tau} ds \int_T^{T+\tau} du\ \mathrm{E}\left(W(s)W(u)\right) \\
&= \int_T^{T+\tau} ds \int_T^{T+\tau} du\ s \wedge u \\
&= 2\int_T^{T+\tau} ds \int_s^{T+\tau} s\ du \\
&= 2\int_T^{T+\tau} s(T+\tau-s) ds \\
&= \tau^2 \left(T + \frac{\tau}{3}\right).
\end{aligned}$$

The formula is easily generalized for all possible values of T, i.e. for all $T \geq -\tau$:

$$\mathrm{E}\left(\left(\int_T^{T+\tau} W(s)\ ds\right)^2\right) = \frac{1}{3}\left(T-\tau\right)^3 - \left(T^+\right)^2\left(T+\tau-\frac{2}{3}T^+\right),$$

where $T^+ = \max(T, 0)$.