So, Computers can Prove Theorems (in Lean), What's Next?

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Harmonic

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- What adjustments are needed to integrate these new sources of formal proofs into libraries like mathlib?

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- **AI systems:**
  - Models that search for proofs, guided by learned heuristics instead of rules.
  - Adapt via learning from unstructured data, or better yet, from experience, to continually improve.
  - Able to solve challenging problems and productively operate autonomously for far longer periods of time.
  - Often more flexible than bespoke automation or hand-written procedures. Can make interacting with a formal system closer to interacting in natural language.
  - May use the above as building blocks, but build higher level abstractions and structure on top of them.

Examples of AI-generated proofs

- Users of systems like Aristotle using it to obtain both novel proofs, and autoformalise existing arguments. Solutions to problems of Erdős via Boris Alexeev and others [1][2].

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```
/-
The Erdos conjecture is true: under the given conditions, every sufficiently large integer is representable.
-/
theorem erdos_conjecture_true (k : ℕ) (d : Fin k → ℕ)
    (h_ge : ∀ i, 2 ≤ d i)
    (h_sum : 1 ≤ ∑ i, (1 : ℚ) / (d i - 1)) :
    ∀ n, ∃ a : Fin k → ℕ,
    (∀ i, ((d i).digits (a i)).toFinset ⊆ {0, 1}) ∧
    n = ∑ i, a i := by
  -- By the properties of the sequence u_seq, every natural number can be represented as a sum of its terms.
  have h_dense : ∀ n : ℕ, ∃ s : Finset ℕ, n = ∑ j ∈ s, u_seq d j := by
    -- Apply Brown's criterion with the given hypotheses.
    apply browns_criterion;
    · apply u_seq_monotone;
      · -- Apply the lemma that states if the sum of reciprocals is at least 1, then k cannot be zero.
        apply k_ne_zero_of_sum_eq_one; assumption;
      · exact fun i => le_trans ( by norm_num ) ( h_ge i );
    · exact?;
    · apply_rules [ u_seq_gap ];
      · aesop;
        norm_num at h_sum;
  -- By definition of $u_seq$, each term in the sum $\sum_{j \in s} u_seq d j$ is of the form $d_i^{e_i}$ for some $i$ and $e_i$.
  have h_terms : ∀ s : Finset ℕ, ∃ a : Fin k → ℕ, (∀ i, ((d i).digits (a i)).toFinset ⊆ {0, 1}) ∧ ∑ j ∈ s, u_seq d j = ∑ i, a i := by
    -- Apply the lemma `digits_of_subset_sum_u_seq` to the set `s`.
    intros s
    apply digits_of_subset_sum_u_seq;
    · rintro rfl; norm_num at h_sum;
    · exact fun i => le_trans ( by norm_num ) ( h_ge i );
  exact fun n => by obtain ⟨ s, hs ⟩ := h_dense n; obtain ⟨ a, ha₁, ha₂ ⟩ := h_terms s; exact ⟨ a, ha₁, hs.trans ha₂ ⟩ ;
```

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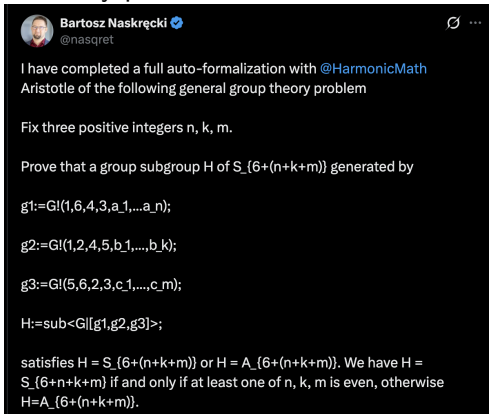

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2605  /-
2606  If n, k, m are all odd, H is the alternating group.
2607  -/
2608  lemma H_eq_alternating_of_all_odd (n k m : ℕ) (hn : 1 < n) (hk : 0 < k) (hm : 0 < m)
2609      (hn_odd : Odd n) (hk_odd : Odd k) (hm_odd : Odd m) : H n k m = alternatingGroup (Fin (N n k m)) := by
2610        -- If n, k, m are all odd, then all generators of H are in the alternating group.
2611        have h_all_in_A : ∀ g ∈ ({g1 n k m, g2 n k m, g3 n k m} : Set (Perm (Fin (N n k m)))), Equiv.Perm.sign g = 1 := by
2612          intro g hg
2613          have h_sign_g : Equiv.Perm.sign g = (-1 : ℤ) ^ (n + 3) ∨ Equiv.Perm.sign g = (-1 : ℤ) ^ (k + 3) ∨ Equiv.Perm.sign g = (-1 : ℤ) ^ (m + 3) := by
2614            rcases hg with ( rfl | rfl | rfl ) <;> [ exact Or.inl <| mod_cast sign_g1 n k m <| by linarith; ; exact Or.inr <| Or.inl <| mod_cast sign_g2 n k m <| by linarith; ; exact
2615          rcases h_sign_g with h | h | h <;> norm_cast at h <;> simp_all +decide [ parity_simps ];
2616        refine' le_antisymm _ _;
2617        · exact fun g hg => Subgroup.closure_induction ( fun x hx => by aesop ) ( by aesop ) ( by aesop ) ( by aesop ) hg;
2618        · apply_rules [ alternatingGroup_le_H ]
2619
2620  /-
2621  H is the symmetric group iff at least one of n, k, m is even.
2622  -/
2623  theorem H_eq_top_iff (n k m : ℕ) (hn : 1 < n) (hk : 0 < k) (hm : 0 < m) :
2624      H n k m = ⊤ ↔ (Even n ∨ Even k ∨ Even m) := by
2625        apply Iff.intro;
2626        · contrapose!;
2627          -- If n, k, m are all odd, then H n k m is equal to the alternating group.
2628          intro h_odd
2629          have h_alt : H n k m = alternatingGroup (Fin (N n k m)) := by
2630            exact H_eq_alternating_of_all_odd n k m hn hk hm ( by simpa using h_odd.1 ) ( by simpa using h_odd.2.1 ) ( by simpa using h_odd.2.2 );
2631          simp +decide [ h_alt, Subgroup.eq_top_iff' ];
2632          exact ⟨ Equiv.swap ⟨ 0, by linarith [ show N n k m > 0 from by { unfold N; linarith } ] ⟩ ( ⟨ 1, by linarith [ show N n k m > 1 from by { unfold N; linarith } ] ⟩ ), by simp +de
2633        · exact?
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All except the last of these are from regular users in the Lean community making requests via `aristotle.harmonic.fun`. Throw in your favourite theorems, conjectures and variants (both formally and informally), and just see what happens. It might get proved, or disproved, saving you time either way.

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- For many years, progress came from carefully engineered automation for specific theories.
- Search was present, but heavily constrained and tuned by hand.
- Recent systems use large models to propose proof steps and guide search directly.
- The proof search policy is now learned and auto-updating, rather than hand-specified, it has a lot of mathematics baked in.

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- Instead of training on fixed input/output pairs, we train a model to maximise a reward.
- In mathematics this reward can come from:
  - Reaching a correct numerical answer.
  - Writing code that compiles and produces the right output.
  - Producing a proof that convinces another model / passes a rubric.
  - Producing a Lean proof that solves the goal.
- This turns theorem proving into a game the model learns to play better and better.

A high-level view of Aristotle (the IMO system)

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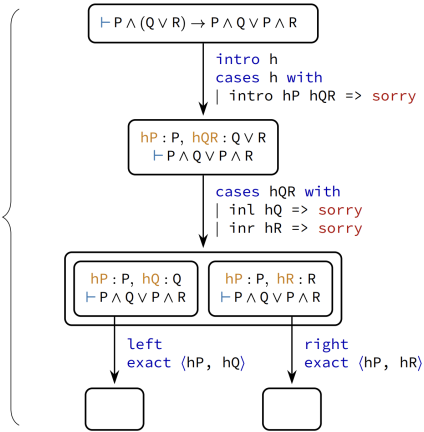
- From interacting with Lean, the model gets precise feedback: goals, errors, and success/failure.
- Details are in the whitepaper (arXiv:2510.01346v2)
- **Two main subsystems:**
  - A Lean proof search system (Monte Carlo graph search over Lean tactics).
  - A lemma-based informal reasoning system that plans, drafts, and formalises lemmas, definitions and other statements.
- A problem is only counted as solved if Aristotle produces a complete, kernel-checked Lean proof, with no gaps or `sorryAx`.

Lean proof search in Aristotle

```
intro h
cases h with
| intro hP hQR =>
  cases hQR with
  | inl hQ =>
    left
    exact ⟨hP, hQ⟩
  | inr hR =>
    right
    exact ⟨hP, hR⟩
```



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- A large transformer serves as both policy (which tactic to try next) and value function (how promising a state is).
- Search is a highly parallel Monte Carlo Graph Search (MCGS) that focuses effort on the hardest remaining subgoals that are likely to help prove the theorem.

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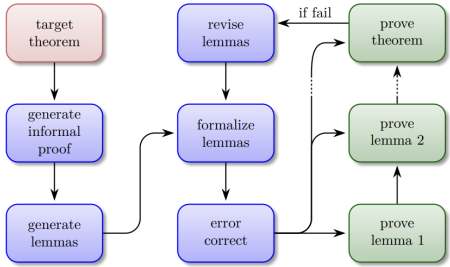
- If the attempt fails, Aristotle revises the lemma list using formal feedback (which lemmas did/did not prove) and iterates.



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- This loop lets informal reasoning explore ideas, while formal verification keeps only the sound ones.

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- The same model serves as a generative policy (tactics to try) and as a value function over states.
- Over time it learns which informal plans and tactic patterns tend to lead to short, successful proofs.

Aristotle at the IMO and beyond

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- Scaling was crucial: a $>$ 200B-parameter model, many parallel lemma pipelines, and multiple refinement iterations per problem.
- Test-time training (TTT) let the model improve on-the-fly from its own search traces for a given problem.

Rethinking our workflows

- Old style of working with an ITP: walk through a file lemma-by-lemma, maybe working backwards from a big goal, or forwards following notes / a paper, refactor and improvise as you go.

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  - Run many of these simultaneously!
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- Think of yourself as managing a team of hard working junior collaborators, who can't do everything but are willing to try anything, and do the majority of the actual coding. And will sometimes surprise you!
- These collaborators will only get better over time, so learning how to use them effectively is a long term investment.

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- With AI assistance, the effort required to formalize a given theory / result drops dramatically.
- Shouldn't think only in terms of speeding up existing plans; it vastly expands the space of projects we can attempt.
- We should be ambitious about what we want `mathlib` and other libraries to contain.

mathlib growth

Mathlib statistics

Counts

Definitions	Theorems	Contributors
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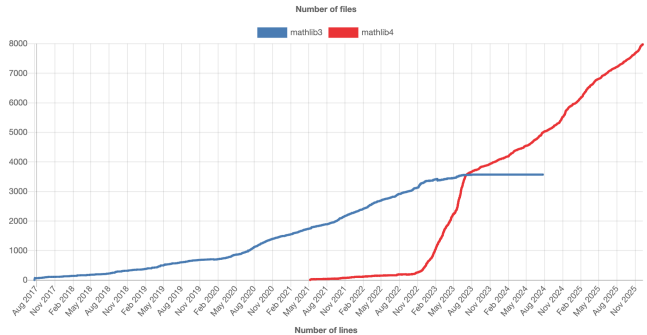
Code growth



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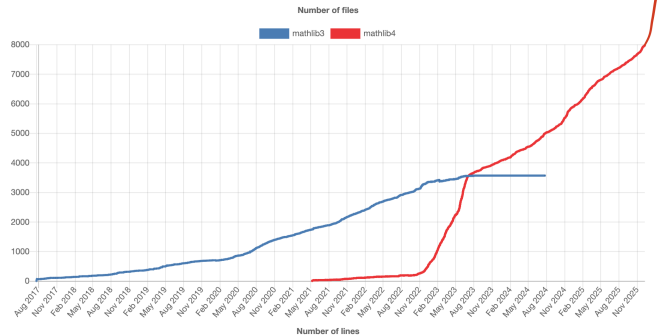
Code growth



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- Current systems already generate correct proofs; the next frontiers are speed and quality, for a reusable library we care about more than corectness.
- We should automate the most common reviewer complaints about the quality of AI-written code as much as possible, to avoid this becoming a bottleneck:
  - Style issues (forgetting dot notation, ...)
  - Unnecessary complexity
  - Missing lemmas, that should be extracted from proofs
  - Poor naming
- A great intermediate project for this week: metaprogramming and linting tools that enforce some of these rules automatically.

Computers as calculators

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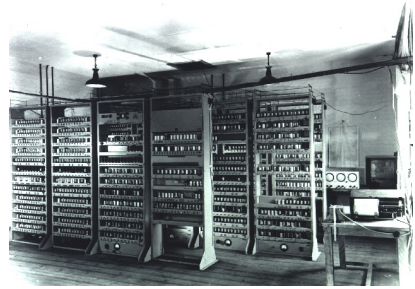
- Example: Birch–Swinnerton-Dyer's conjecture was informed by extensive computations on an early mainframe in Cambridge.
- These experiments changed what could be conjectured and tested numerically, but access was scarce: mainframes were expensive, shared, and required institutional infrastructure.
- This was a transformative tool, but not something every working mathematician could use, a very slow rollout. Not every field of mathematics was affected by this, and not every working mathematician uses computer calculation.



EDSAC at Cambridge.

The internet, arXiv, and TEX

- The second wave of automation was the combination of TEX, electronic archives, and the internet.
- Communication and dissemination sped up dramatically:
  - A paper can be written one week and read by a study group on the other side of the world the next.
  - Preprints, mailing lists, and online seminars reshaped how results spread.
  - You can email the author, ask on MathOverflow, watch talks by experts
  - Google search, Google scholar, are now essential tools for finding relevant research.
- This changed workflows: faster feedback loops, more collaboration, and broader access to cutting-edge work.
- It also normalised machine-assisted typesetting and distribution as part of everyday mathematics.

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- What happens to mathematics when computers can routinely outplay us at proof search?
- Will it become less interesting, as some predicted for chess?
- Or will it become more accessible and more widely practised?

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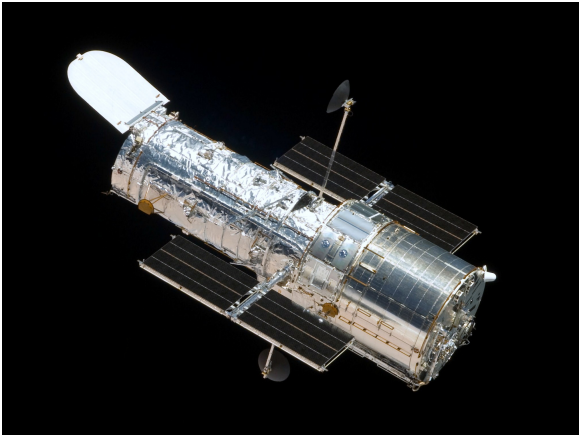
- As models can produce convincing proofs faster than we can read them, we need machine-checkable guarantees either way.
- That means: whatever front-end we use (chat, LaTeX, diagrams), we will still want a formal proof object underneath.

Which analogy



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- Experiment with new workflows and interfaces: outlines and informal instructions first, then AI-assisted proof search at scale, where in the loop do humans want to be?
- And most importantly: decide, as a community, what kind of future for (formal) mathematics we want.