

The sTeX3 Manual *

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Abstract

sTeX is a collection of L^AT_EX packages that allow to markup documents semantically without leaving the document format.

Running ‘pdflatex’ over sTeX-annotated documents formats them into normal-looking PDF. But sTeX also comes with a conversion pipeline into semantically annotated HTML5, which can host semantic added-value services that make the documents active (i.e. interactive and user-adaptive) and essentially turning L^AT_EX into a document format for (mathematical) knowledge management (MKM).

sTeX augments L^AT_EX with

- *semantic macros* that denote and distinguish between mathematical concepts, operators, etc. independent of their notational presentation,
- a powerful *module system* that allows for authoring and importing individual fragments containing document text and/or semantic macros, independent of – and without hard coding – directory paths relative to the current document, and
- a mechanism for exporting sTeX documents to (modular) XHTML, preserving all the semantic information for semantically informed knowledge management services.

This is the user manual for the sTeX package and associated software. It is primarily directed at end-users who want to use sTeX to author semantically enriched documents. For the full documentation, see [the sTeX documentation](#).

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Boxes like this one contain implementation details that are mostly relevant for more advanced use cases, might be useful to know when debugging, or might be good to know to better understand how something works. They can easily be skipped on a first read.



Boxes like this one explain how some \LaTeX concept relates to the MMT/OMDoc system, philosophy or language; see [MMT; Koh06] for introductions.

Chapter 1

What is sTeX?

Formal systems for mathematics (such as interactive theorem provers) have the potential to significantly increase both the accessibility of published knowledge, as well as the confidence in its veracity, by rendering the precise semantics of statements machine actionable. This allows for a plurality of added-value services, from semantic search up to verification and automated theorem proving. Unfortunately, their usefulness is hidden behind severe barriers to accessibility; primarily related to their surface languages reminiscent of programming languages and very unlike informal standards of presentation.

sTeX minimizes this gap between informal and formal mathematics by integrating formal methods into established and widespread authoring workflows, primarily L^AT_EX, via non-intrusive semantic annotations of arbitrary informal document fragments. That way formal knowledge management services become available for informal documents, accessible via an IDE for authors and via generated *active* documents for readers, while remaining fully compatible with existing authoring workflows and publishing systems.

Additionally, an extensible library of reusable document fragments is being developed, that serve as reference targets for global disambiguation, intermediaries for content exchange between systems and other services.

Every component of the system is designed modularly and extensibly, and thus lay the groundwork for a potential full integration of interactive theorem proving systems into established informal document authoring workflows.

The general sTeX workflow combines functionalities provided by several pieces of software:

- The sTeX package collection to use semantic annotations in L^AT_EX documents,
- RuSTeX [] to convert `tex` sources to (semantically enriched) `xhtml`,
- The MMT system [MMT], that extracts semantic information from the thus generated `xhtml` and provides semantically informed added value services.

Chapter 2

Quickstart

2.1 Setup

There are two ways of using sTeX: as a

1. way of writing L^AT_EX more modularly (object-oriented Math) for creating PDF documents or
2. foundation for authoring active documents in HTML5 instrumented with knowledge management services.

Both are legitimate and useful. The first requires a significantly smaller tool-chain, so we describe it first. The second requires a much more substantial (and experimental) toolchain of knowledge management systems. Both workflows profit from an integrated development environment (IDE), which (also) automates setup as far as possible (see [subsection 2.1.4](#)).

2.1.1 Minimal Setup for the PDF-only Workflow

In the best of all worlds, there is no setup, as you already have a new version of T_EXLive on your system as a L^AT_EX enthusiast. If not now is the time to install it; see [\[TL\]](#). You can usually update T_EXLive via a package manager or the T_EXLive manager **tlmgr**.

Alternatively, you can install sTeX from CTAN, the Comprehensive T_EX Archive Network; see [\[ST\]](#) for details.

2.1.2 GIT-based Setup for the sTeX Development Version

If you want use the latest and greatest sTeX packages, you can that have not even been released to CTAN, then you can directly clone them from the sTeX development repository [\[sTeX\]](#) by the following commandline instructions:

```
cd <stexdir>
git clone https://github.com/slatex/sTeX.git
```

and keep it updated by pulling updates via `git pull` in the cloned sTeX directory. Then update your TEXINPUTS environment variable, e.g. by placing the following line in your `.bashrc`:

¹NEW PART: MK: reorganized, we do not need the full MKM tool chain

```
export TEXINPUTS="$(TEXINPUTS):<sTeXDIR>//:"
```

2.1.3 sTeX Archives (Manual Setup)

Writing semantically annotated sTeX becomes much easier, if we can use well-designed libraries of already annotated content. sTeX provides such libraries as sTeX archives – i.e. GIT repositories at <https://gl.mathhub.info> – most prominently the SMGLoM libraries at <https://gl.mathhub.info/smgloom>.

To do so, we set up a **local MathHub** by creating a MathHub directory `<mhdir>`. Every sTeX archive as an **archive path** `<apath>` and a name `<archive>`. We can clone the sTeX archive by the following command-line instructions:

```
cd <mhdir>/<apath>
git clone https://gl.mathhub.info/smgloom/<archive>.git
```

Note that sTeX archives often depend on other archives, thus you should be prepared to clone these as well – e.g. if `pdflatex` reports missing files. To make sure that sTeX too knows where to find its archives, we need to set a global system variable `MATHHUB`, that points to your local MathHub-directory (see [section 3.2](#)).

```
export MATHHUB="<mhdir>"
```

2.1.4 The sTeX IDE

We are currently working on an sTeX IDE as an sTeX plugin for VScode; see [\[Sla\]](#). It will feature a setup procedure that automates the setup described above (and below). For additional functionality see the (now obsolete) plugin for sTeX 1 [\[SLS; Stb\]](#).

2.1.5 Manual Setup for Active Documents and Knowledge Management Services

Foregoing on the sTeX IDE, we will need several additional (on top of the minimal setup above) pieces of software; namely:

- **The Mmt System** available [here](#)². We recommend following the setup routine documented [here](#).

Following the setup routine (Step 3) will entail designating a **MathHub**-directory on your local file system, where the MMT system will look for sTeX/MMT content archives.

- **sTeX Archives** If we only care about L^AT_EX and generating pdfs, we do not technically need MMT at all; however, we still need the `MATHHUB` system variable to be set. Furthermore, MMT can make downloading content archives we might want to use significantly easier, since it makes sure that all dependencies of (often highly interrelated) sTeX archives are cloned as well.

Once set up, we can run `mmt` in a shell and download an archive along with all of its dependencies like this: `lmh install <name-of-repository>`, or a whole *group* of archives; for example, `lmh install smgloom` will download all `smgloom` archives.

- **RuSTeX** The MMT system will also set up RuSTeX for you, which is used to generate (semantically annotated) `xhtml` from tex sources. In lieu of using MMT, you can also download and use RuSTeX directly [here](#).

²EdNOTE: For now, we require the `sTeX`-branch, requiring manually compiling the MMT sources

2.2 A First \LaTeX Document

Having set everything up, we can write a first \LaTeX document. As an example, we will use the `smglom/calculus` and `smglom/arithmetics` archives, which should be present in the designated MathHub-folder, and write a small fragment defining the *geometric series*:

TODO: use some \LaTeX -archive instead of `smglom`, use a convergence-notion that includes the limit, mark-up the theorem properly

```

1 \documentclass{article}
2 \usepackage{stex,xcolor,stexthm}
3
4 \begin{document}
5 \begin{smodule}{GeometricSeries}
6   \importmodule[smglom/calculus]{series}
7   \importmodule[smglom/arithmetics]{realarith}
8
9   \symdef{geometricSeries}[name=geometric-series]{\comp{S}}
10
11   \begin{sdefinition}[for=geometricSeries]
12     The \definame{geometricSeries} is the \symname{?series}
13     \[\defeq{\geometricSeries}{\definiens{
14       \infinitesum{\svar{n}}{1}{
15         \realdivide[frac]{1}{
16           \realpower{2}{\svar{n}}
17         }
18       }}
19     \].\]
20   \end{sdefinition}
21
22   \begin{sassertion}[name=geometricSeriesConverges,type=theorem]
23     The \symname{geometricSeries} \symname{converges} towards $1$.
24   \end{sassertion}
25 \end{smodule}
26 \end{document}

```

Compiling this document with `pdflatex` should yield the output

Definition 0.1. The **geometric series** is the **series**

$$S := \sum_{n=1}^{\infty} \frac{1}{2^n}.$$

Theorem 0.2. The **geometric series converges** towards 1.

Move your cursor over the various highlighted parts of the document – depending on your pdf viewer, this should yield some interesting (but possibly for now cryptic) information.

Remark 2.2.1:

Note that all of the highlighting, tooltips, coloring and the environment headers come from `stexthm` – by default, the amount of additional packages loaded is kept to a minimum and all the presentations can be customized, see [chapter 6](#).

Let's investigate this document in detail to understand the respective parts of the \TeX markup infrastructure:

```
\begin{smodule}{GeometricSeries}
...
\end{smodule}
```

smodule First, we open a new *module* called `GeometricSeries`. The main purpose of the `smodule` environment is to group the contents and associate it with a *globally unique* identifier (URI), which is computed from the name `GeometricSeries` and the document context. (Depending on your pdf viewer), the URI should pop up in a tooltip if you hover over the word `geometric series`.

```
\importmodule[smglom/calculus]{series}
\importmodule[smglom/arithmetics]{realarith}
```

\importmodule Next, we *import* two modules – `series` from the \TeX archive `smglom/calculus`, and `realarith` from the \TeX archive `smglom/arithmetics`. If we investigate these archives, we find the files `series.en.tex` and `realarith.en.tex` (respectively) in their respective source-folders, which contain the statements `\begin{smodule}{series}` and `\begin{smodule}{realarith}` (respectively).

The `\importmodule`-statements make all \TeX symbols and associated semantic macros (e.g. `\infinitesum`, `\realdive`, `\realpower`) in the imported module available to the current module `GeometricSeries`. The module `GeometricSeries` “exports” all of these symbols to all modules imports it via an `\importmodule{GeometricSeries}` instruction. Additionally it exports the local symbol `\geometricSeries`.

\usemodule If we only want to *use* the content of some module `Foo`, e.g. in remarks or examples, but none of the symbols in our current module actually *depend* on the content of `Foo`, we can use `\usemodule` instead – like `\importmodule`, this will make the module content available, but will *not* export it to other modules.

```
\symdef{GeometricSeries}[name=geometric-series]{\comp{S}}
```

\symdef Next, we introduce a new *symbol* with name `geometric-series` and assign it the semantic macro `\geometricSeries`. `\symdef` also immediately assigns this symbol a *notation*, namely `S`.

\comp The macro `\comp` marks the `S` in the notation as a *notational component*, as opposed to e.g. arguments to `\geometricSeries`. It is the notational components that get highlighted and associated with the corresponding symbol (i.e. in this case `geometricSeries`). Since `\geometricSeries` takes no arguments, we can wrap the whole notation in a `\comp`.

```

\begin{sdefinition}[for=geometricSeries]
...
\end{sdefinition}
\begin{sassertion}[name=geometricSeriesConverges,type=theorem]
...
\end{sassertion}

```

What follows are two \LaTeX -statements (e.g. definitions, theorems, examples, proofs, ...). These are semantically marked-up variants of the usual environments, which take additional optional arguments (e.g. `for=`, `type=`, `name=`). Since many \LaTeX templates predefine environments like `definition` or `theorem` with different syntax, we use `sdefinition`, `sassertion`, `sexample` etc. instead. You can customize these environments to e.g. simply wrap around some predefined `theorem`-environment. That way, we can still use `sassertion` to provide semantic information, while being fully compatible with (and using the document presentation of) predefined environments.

In our case, the `stexthm`-package patches e.g. `\begin{sassertion}[type=theorem]` to use a `theorem`-environment defined (as usual) using the `amsthm` package.

```
... is the \symname{?series}
```

`\symname`

The `\symname`-command prints the name of a symbol, highlights it (based on customizable settings) and associates the text printed with the corresponding symbol.

Note that the argument of `\symref` can be a local or imported symbol (here the `series` symbol is imported from the `series` module). \LaTeX tries to determine the full symbol URI from the argument. If there are name clashes in or with the imported symbols, the name of the exporting module can be prepended to the symbol name before the `?` character.

If you hover over the word `series` in the pdf output, you should see a tooltip showing the full URI of the symbol used.

`\symref`

The `\symname`-command is a special case of the more general `\symref`-command, which allows customizing the precise text associated with a symbol. `\symref` takes two arguments the first is the symbol name, and the second a variant verbalization of the symbol, e.g. an inflection variant, a different language or a synonym. In our example `\symname{?series}` abbreviates `\symref{?series}{series}`.

```
The \definame{geometricSeries} ...
```

`\definame`
`\definiendum`

The `sdefinition`-environment provides two additional macros, `\definame` and `\definiendum` which behave similarly to `\symname` and `\symref`, but explicitly mark the symbols as *being defined* in this environment, to allow for special highlighting.

```

\[\defeq{\geometricSeries}{\definiens{
  \infinitesum{\svar{n}}{1}{
    \realdivide[frac]{1}{
      \realpower{2}{\svar{n}}
    }
  }}
\].\]

```

The next snippet – set in a math environment – uses several semantic macros imported from (or recursively via) `series` and `realarithmetics`, such as `\defeq`, `\infinitesum`, etc. In math mode, using a semantic macro inserts its (default) definition. A semantic

macro can have several notations – in that case, we can explicitly choose a specific notation by providing its identifier as an optional argument; e.g. `\realdivide[frac]{a}{b}` will use the explicit notation named `frac` of the semantic macro `\realdivide`, which yields $\frac{a}{b}$ instead of a/b .

`\svar` The `\svar{n}` command marks up the `n` as a variable with name `n` and notation `n`.

`\definiens` The `sdefinition`-environment additionally provides the `\definiens`-command, which allows for explicitly marking up its argument as the *definiens* of the symbol currently being defined.

2.2.1 OMDoc/xhtml Conversion

So, if we run `pdflatex` on our document, then \TeX yields pretty colors and tooltips¹. But \TeX becomes a lot more powerful if we additionally convert our document to `xhtml` while preserving all the \TeX markup in the result.

TODO VSCode Plugin

Using `Ru\TeX` [], we can convert the document to `xhtml` using the command `rustex -i /path/to/file.tex -o /path/to/outfile.xhtml`. Investigating the resulting file, we notice additional semantic information resulting from our usage of semantic macros, `\symref` etc. Below is the (abbreviated) snippet inside our `\definiens` block:

```
<mrow resource="" property="stex:definiens">
  <mrow resource="...?series?infinitesum" property="stex:OMBIND">
    <munderover displaystyle="true">
      <mo resource="...?series?infinitesum" property="stex:comp">\Sigma</mo>
      <mrow>
        <mrow resource="1" property="stex:arg">
          <mi resource="var://n" property="stex:OMV">n</mi>
        </mrow>
        <mo resource="...?series?infinitesum" property="stex:comp">=</mo>
        <mi resource="2" property="stex:arg">1</mi>
      </mrow>
      <mi resource="...?series?infinitesum" property="stex:comp">\infty</mi>
    </munderover>
    <mrow resource="3" property="stex:arg">
      <mfrac resource="...?realarith?division#frac#" property="stex:OMA">
        <mi resource="1" property="stex:arg">1</mi>
        <mrow resource="2" property="stex:arg">
          <msup resource="...realarith?exponentiation" property="stex:OMA">
            <mi resource="1" property="stex:arg">2</mi>
            <mrow resource="2" property="stex:arg">
              <mi resource="var://n" property="stex:OMV">n</mi>
            </mrow>
          </msup>
        </mrow>
      </mfrac>
    </mrow>
  </mrow>
```

¹...and hyperlinks for symbols, and indices, and allows reusing document fragments modularly, and...

...containing all the semantic information. The MMT system can extract from this the following OPENMATH snippet:

```
<OMBIND>
  <OMID name="...?series?infinitesum"/>
  <OMV name="n"/>
  <OMLIT name="1"/>
  <OMA>
    <OMS name="...?realarith?division"/>
    <OMLIT name="1"/>
    <OMA>
      <OMS name="...realarith?exponentiation"/>
      <OMLIT name="2"/>
      <OMV name="n"/>
    </OMA>
  </OMA>
</OMBIND>
```

...giving us the full semantics of the snippet, allowing for a plurality of knowledge management services – in particular when serving the **xhtml**.

Remark 2.2.2:

Note that the **html** when opened in a browser will look slightly different than the **pdf** when it comes to highlighting semantic content – that is because naturally **html** allows for much more powerful features than **pdf** does. Consequently, the **html** is intended to be served by a system like MMT, which can pick up on the semantic information and offer much more powerful highlighting, linking and similar features, and being customizable by *readers* rather than being prescribed by an author.

Additionally, not all browsers (most notably Chrome) support MATHML natively, and might require additional external JavaScript libraries such as MathJax to render mathematical formulas properly.

Chapter 3

Creating sTeX Content

We can use sTeX by simply including the package with `\usepackage{stex}`, or – primarily for individual fragments to be included in other documents – by using the sTeX document class with `\documentclass{stex}` which combines the `standalone` document class with the `stex` package.

Both the `stex` package and document class offer the following options:

lang (*<language>**) Languages to load with the `babel` package.

mathhub (*<directory>*) MathHub folder to search for repositories – this is not necessary if the `MATHHUB` system variable is set.

sms (*<boolean>*) use *persisted* mode (not yet implemented).

image (*<boolean>*) passed on to `tikzinput`.

debug (*<log-prefix>**) Logs debugging information with the given prefixes to the terminal, or all if `all` is given. Largely irrelevant for the majority of users.

3.1 How Knowledge is Organized in sTeX

sTeX content is organized on multiple levels:

1. sTeX **archives** (see [section 3.2](#)) contain individual `.tex`-files.
2. These may contain sTeX **modules**, introduced via `\begin{smodule}{ModuleName}`.
3. Modules contain sTeX **symbol declarations**, introduced via `\symdecl{symbolname}`, `\symdef{symbolname}` and some other constructions. Most symbols have a *notation* that can be used via a *semantic macro* `\symbolname` generated by symbol declarations.
4. sTeX **expressions** finally are built up from usages of semantic macros.

\hookrightarrow

\rightarrow

\rightsquigarrow

- sTeX archives are simultaneously MMT archives, and the same directory structure is consequently used.

- sTeX modules correspond to OMDoc/MMT *theories*. `\importmodules`

- (and similar constructions) induce MMT `includes` and other *theory morphisms*, thus giving rise to a *theory graph* in the OMDoc sense [RabKoh:WSMSML13].
- Symbol declarations induce OMDoc/MMT *constants*, with optional (formal) *type* and *definiens* components.
 - Finally, \S T E X expressions are converted to OMDoc/MMT terms, which use the abstract syntax (and XML encoding) of OPENMATH [BusCapCar:2oms04].

3.2 \S T E X Archives

3.2.1 The Local MathHub-Directory

`\usemodule`, `\importmodule`, `\inputref` etc. allow for including content modularly without having to specify absolute paths, which would differ between users and machines. Instead, \S T E X uses *archives* that determine the global namespaces for symbols and statements and make it possible for \S T E X to find content referenced via such URIs.

All \S T E X archives need to exist in the local **MathHub**-directory. \S T E X knows where this folder is via one of four means:

1. If the \S T E X package is loaded with the option `mathhub=/path/to/mathhub`, then \S T E X will consider `/path/to/mathhub` as the local **MathHub**-directory.
2. If the `mathhub` package option is *not* set, but the macro `\mathhub` exists when the \S T E X -package is loaded, then this macro is assumed to point to the local **MathHub**-directory; i.e. `\def\mathhub{/path/to/mathhub}\usepackage{stex}` will set the **MathHub**-directory as `path/to/mathhub`.
3. Otherwise, \S T E X will attempt to retrieve the system variable `MATHHUB`, assuming it will point to the local **MathHub**-directory. Since this variant needs setting up only *once* and is machine-specific (rather than defined in tex code), it is compatible with collaborating and sharing tex content, and hence recommended.
4. Finally, if all else fails, \S T E X will look for a file `~/stex/mathhub.path`. If this file exists, \S T E X will assume that it contains the path to the local **MathHub**-directory. This method is recommended on systems where it is difficult to set environment variables.

3.2.2 The Structure of \S T E X Archives

An \S T E X archive `group/name` is stored in the directory `/path/to/mathhub/group/name`; e.g. assuming your local **MathHub**-directory is set as `/user/foo/MathHub`, then in order for the `smglom/calculus`-archive to be found by the \S T E X system, it needs to be in `/user/foo/MathHub/smglom/calculus`.

Each such archive needs two subdirectories:

- `/source` – this is where all your tex files go.

- `/META-INF` – a directory containing a single file `MANIFEST.MF`, the content of which we will consider shortly

An additional `lib`-directory is optional, and is where $\text{\texttt{S}}\text{\texttt{T}}\text{\texttt{E}}\text{\texttt{X}}$ will look for files included via `\libinput`.

Additionally a *group* of archives `group/name` may have an additional archive `group/meta-inf`. If this `meta-inf`-archive has a `/lib`-subdirectory, it too will be searched by `\libinput` from all tex files in any archive in the `group/*`-group.

We recommend the following additional directory structure in the `source`-folder of an $\text{\texttt{S}}\text{\texttt{T}}\text{\texttt{E}}\text{\texttt{X}}$ archive:

- `/source/mod/` – individual $\text{\texttt{S}}\text{\texttt{T}}\text{\texttt{E}}\text{\texttt{X}}$ modules, containing symbol declarations, notations, and `\begin{spargraph}[type=symdoc,for=...]` environments for “encyclopaedic” symbol documentations
- `/source/def/` – definitions
- `/source/ex/` – examples
- `/source/thm/` – theorems, lemmata and proofs; preferably proofs in separate files to allow for multiple proofs for the same statement
- `/source/snip/` – individual text snippets such as remarks, explanations etc.
- `/source/frag/` – individual document fragments, ideally only `\inputrefing` snippets, definitions, examples etc. in some desirable order
- `/source/tikz/` – tikz images, as individual `.tex`-files
- `/source/pic/` – image files.³

3.2.3 MANIFEST.MF-Files

The `MANIFEST.MF` in the `META-INF`-directory consists of key-value-pairs, informing $\text{\texttt{S}}\text{\texttt{T}}\text{\texttt{E}}\text{\texttt{X}}$ (and associated software) of various properties of an archive. For example, the `MANIFEST.MF` of the `smglom/calculus`-archive looks like this:

```
id: smglom/calculus
source-base: http://mathhub.info/smglob/calculus
narration-base: http://mathhub.info/smglob/calculus
dependencies: smglom/arithmetic,smglom/sets,smglom/topology,
              smglom/mv,smglom/linear-algebra,smglom/algebra
responsible: Michael.Kohlhase@FAU.de
title: Elementary Calculus
teaser: Terminology for the mathematical study of change.
description: desc.html
```

Many of these are in fact ignored by $\text{\texttt{S}}\text{\texttt{T}}\text{\texttt{E}}\text{\texttt{X}}$, but some are important:

`id`: The name of the archive, including its group (e.g. `smglom/calculus`),

³EdNOTE: MK: bisher habe ich immer PIC subdirs, soll ich das ändern?

source-base or

ns: The namespace from which all symbol and module URIs in this repository are formed, see (TODO),

narration-base: The namespace from which all document URIs in this repository are formed, see (TODO),

url-base: The URL that is formed as a basis for *external references*, see (TODO),

dependencies: All archives that this archive depends on. $\text{\texttt{S\TeX}}$ ignores this field, but MMT can pick up on them to resolve dependencies, e.g. for `lmh install`.

3.2.4 Using Files in $\text{\texttt{S\TeX}}$ Archives Directly

Several macros provided by $\text{\texttt{S\TeX}}$ allow for directly including files in repositories. These are:

$\text{\texttt{\backslash mhinput}}$	$\text{\texttt{\backslash mhinput}}$ [Some/Archive]{some/file} directly inputs the file some/file in the source-folder of Some/Archive.
--------------------------------------	---

$\text{\texttt{\backslash inputref}}$	$\text{\texttt{\backslash inputref}}$ [Some/Archive]{some/file} behaves like $\text{\texttt{\backslash mhinput}}$, but wraps the input in a <code>\begingroup ... \endgroup</code> . When converting to <code>xhtml</code> , the file is not input at all, and instead an <code>html</code> -annotation is inserted that references the file, e.g. for lazy loading.
---------------------------------------	---

In the majority of practical cases $\text{\texttt{\backslash inputref}}$ is likely to be preferred over $\text{\texttt{\backslash mhinput}}$ because it leads to less duplication in the generated `xhtml`.

$\text{\texttt{\backslash ifinput}}$	Both $\text{\texttt{\backslash mhinput}}$ and $\text{\texttt{\backslash inputref}}$ set $\text{\texttt{\backslash ifinput}}$ to “true” during input. This allows for selectively including e.g. bibliographies only if the current file is not being currently included in a larger document.
--------------------------------------	---

$\text{\texttt{\backslash addmhbibresource}}$	$\text{\texttt{\backslash addmhbibresource}}$ [Some/Archive]{some/file} searches for a file like $\text{\texttt{\backslash mhinput}}$ does, but calls $\text{\texttt{\backslash addbibresource}}$ to the result and looks for the file in the archive root directory directly, rather than the <code>source</code> directory. Typical invocations are
---	---

- $\text{\texttt{\backslash addmhbibresource\{lib/refs.bib\}}}$, which specifies a bibliography in the `lib` folder in the local archive or
- $\text{\texttt{\backslash addmhbibresource[HW/meta-inf]\{lib/refs.bib\}}}$ in another.

`\libinput` `\libinput{some/file}` searches for a file `some/file` in

- the `lib`-directory of the current archive, and
- the `lib`-directory of a `meta-inf`-archive in (any of) the archive groups containing the current archive

and include all found files in reverse order; e.g. `\libinput{preamble}` in a `.tex`-file in `smglom/calculus` will *first* input `../smglom/meta-inf/lib/preamble.tex` and then `../smglom/calculus/lib/preamble.tex`.

`\libinput` will throw an error if *no* candidate for `some/file` is found.

`\libusepackage` `\libusepackage[package-options]{some/file}` searches for a file `some/file.sty` in the same way that `\libinput` does, but will call `\usepackage[package-options]{path/to/some/file}` instead of `\input`.

`\libusepackage` throws an error if not *exactly one* candidate for `some/file` is found.

Remark 3.2.1:

A good practice is to have individual \TeX fragments follow basically this document frame:

```
1 \documentclass{stex}
2 \libinput{preamble}
3 \begin{document}
4   ...
5   \ifinputref \else \libinput{postamble} \fi
6 \end{document}
```

Then the `preamble.tex` files can take care of loading the generally required packages, setting presentation customizations etc. (per archive or archive group or both), and `postamble.tex` can e.g. print the bibliography, index etc.

`\libusepackage` is particularly useful in `preamble.tex` when we want to use custom packages that are not part of \TeX Live. In this case we commit the respective packages in one of the `lib` folders and use `\libusepackage` to load them.

3.3 Module, Symbol and Notation Declarations

3.3.1 The `smodule`-Environment

`smodule` A new module is declared using the basic syntax

`\begin{smodule}[options]{ModuleName}...\end{smodule}`.

A module is required to declare any new formal content such as symbols or notations (but not variables, which may be introduced anywhere).

The `smodule`-environment takes several optional arguments, all of which are optional:

`title` (*token list*) to display in customizations.

`type` ($\langle string \rangle^*$) for use in customizations.
`deprecate` ($\langle module \rangle$) if set, will throw a warning when loaded, urging to use $\langle module \rangle$ instead.
`id` ($\langle string \rangle$) for cross-referencing.
`ns` ($\langle URI \rangle$) the namespace to use. *Should not be used, unless you know precisely what you're doing.* If not explicitly set, is computed using `\stex_modules_current_namespace:`.
`lang` ($\langle language \rangle$) if not set, computed from the current file name (e.g. `foo.en.tex`).
`sig` ($\langle language \rangle$) if the current file is a translation of a file with the same base name but a different language suffix, setting `sig=<lang>` will preload the module from that language file. This helps ensuring that the (formal) content of both modules is (almost) identical across languages and avoids duplication.
`creators` ($\langle string \rangle^*$) names of the creators.
`contributors` ($\langle string \rangle^*$) names of contributors.
`srccite` ($\langle string \rangle$) a source citation for the content of this module.

\hookrightarrow An $\text{\texttt{sTeX}}$ module corresponds to an MMT/OMDOC *theory*. As such it
 \hookrightarrow gets assigned a module URI (*universal resource identifier*) of the form
 \hookrightarrow `<namespace>?<module-name>`.

By default, opening a module will produce no output whatsoever, e.g.:

Example 1

Input:

```

1 \begin{smodule}[title={This is Some Module}]{SomeModule}
2   Hello World
3 \end{smodule}

```

Output:

Hello World

`\stexpatchmodule`

We can customize this behavior either for all modules or only for modules with a specific type using the command `\stexpatchmodule[optional-type]{begin-code}{end-code}`. Some optional parameters are then available in `\smodule*`-macros, specifically `\smodulename`, `\smoduletype` and `\smoduleid`.

For example:

Example 2

Input:

```

1 \stexpatchmodule[display]
2 {\textbf{Module (\smoduletitle))}\par}
3 {\par\noindent\textbf{End of Module (\smoduletitle))}}
4
5 \begin{smodule}[type=display,title={Some New Module}]{SomeModule2}
6   Hello World
7 \end{smodule}

```

Output:

```

Module (Some New Module)
  Hello World
End of Module (Some New Module)

```

3.3.2 Declaring New Symbols and Notations

Inside an `smodule` environment, we can declare new \TeX symbols.

`\symdecl`

The most basic command for doing so is using `\symdecl{symbolname}`. This introduces a new symbol with name `symbolname`, arity 0 and semantic macro `\symbolname`.

The starred variant `\symdecl*{symbolname}` will declare a symbol, but not introduce a semantic macro. If we don't want to supply a notation (for example to introduce concepts like “abelian”, which is not something that has a notation), the starred variant is likely to be what we want.

\hookrightarrow `\symdecl` introduces a new OMDoc/MMT constant in the current module (=OMDoc/MMT theory). Correspondingly, they get assigned the URI `<module-URI>?<constant-name>`.

Without a semantic macro or a notation, the only meaningful way to reference a symbol is via `\symref`, `\symname` etc.

Example 3

Input:

```

1 \symdecl*{foo}
2 Given a \symname{foo}, we can...

```

Output:

```

Given a foo, we can...

```

Obviously, most semantic macros should take actual *arguments*, implying that the symbol we introduce is an *operator* or *function*. We can let `\symdecl` know the *arity* (i.e. number of arguments) of a symbol like this:

Example 4

Input:

```

1 \symdecl{binarysymbol}[args=2]
2 \symref{binarysymbol}{this} is a symbol taking two arguments.

```

Output:

this is a symbol taking two arguments.

\notation

In that case, we probably want to supply a notation as well, in which case we can finally actually use the semantic macro in math mode. We can do so using the `\notation` command, like this:

Example 5

Input:

```

1 \notation{binarysymbol}{\text{First: }#1\text{; Second: }#2}
2 $\binarysymbol{a}{b}$

```

Output:

First: a ; Second: b

\hookrightarrow Applications of semantic macros, such as `\binarysymbol{a}{b}` are translated to
 \rightarrow MMT/OMDOC as OMA-terms with head `<OMS name="...?binarysymbol"/>`.
 \rightsquigarrow Semantic macros with no arguments correspond to OMS directly.

\comp

Unfortunately, we have no highlighting whatsoever now. That is because we need to tell \TeX explicitly which parts of the notation are *notation components* which *should* be highlighted. We can do so with the `\comp` command.

We can introduce a new notation `highlight` for `\binarysymbol` that fixes this flaw, which we can subsequently use with `\binarysymbol[highlight]`:

Example 6

Input:

```

1 \notation{binarysymbol}[highlight]
2 {\comp{\text{First: }#1\comp{\text{; Second: }#2}
3 $\binarysymbol[highlight]{a}{b}$

```

Output:

First: a ; Second: b



Ideally, `\comp` would not be necessary: Everything in a notation that is *not* an argument should be a notation component. Unfortunately, it is computationally expensive to determine where an argument begins and ends, and the argument markers `#n` may themselves be nested in other macro applications or \TeX groups, making it ultimately almost impossible to determine them automatically while also remaining compatible with arbitrary highlighting customizations (such as tooltips, hyperlinks, colors) that users might employ, and that are ultimately invoked by `\comp`.



Note that it is required that

1. the argument markers `#n` never occur inside a `\comp`, and
2. no semantic arguments may ever occur inside a notation.

Both criteria are not just required for technical reasons, but conceptionally meaningful:

The underlying principle is that the arguments to a semantic macro represent *arguments to the mathematical operation* represented by a symbol. For example, a semantic macro `\addition{a}{b}` taking two arguments would represent *the actual addition of (mathematical objects) a and b*. It should therefore be impossible for *a* or *b* to be part of a notation component of `\addition`.

Similarly, a semantic macro can not conceptually be part of the notation of `\addition`, since a semantic macro represents a *distinct mathematical concept* with *its own semantics*, whereas notations are syntactic representations of the very symbol to which the notation belongs.

If you want an argument to a semantic macro to be a purely syntactic parameter, then you are likely somewhat confused with respect to the distinction between the precise *syntax* and *semantics* of the symbol you are trying to declare (which happens quite often even to experienced \TeX users), and might want to give those another thought - quite likely, the macro you aim to implement does not actually represent a semantically meaningful mathematical concept, and you will want to use `\def` and similar native \LaTeX macro definitions rather than semantic macros.

`\symdef`

In the vast majority of cases where a symbol declaration should come with a semantic macro, we will want to supply a notation immediately. For that reason, the `\symdef` command combines the functionality of both `\symdecl` and `\notation` with the optional arguments of both:

Example 7

Input:

```
1 \symdef{newbinarysymbol}[h1,args=2]
2   {\comp{\text{1.: }}#1\comp{\text{; 2.: }}#2}
3 $\newbinarysymbol{a}{b}$
```

Output:

1.: *a*; 2.: *b*

We just declared a new symbol `newbinarysymbol` with `args=2` and immediately provided it with a notation with identifier `h1`. Since `h1` is the *first* (and so far, only) notation supplied for `newbinarysymbol`, using `\newbinarysymbol` without optional argument defaults to this notation.

\setnotation

The first notation provided will stay the default notation unless explicitly changed – this is enabled by the `\setnotation` command: `\setnotation{symbolname}{notation-id}` sets the default notation of `\symbolname` to `notation-id`, i.e. henceforth, `\symbolname` behaves like `\symbolname[notation-id]` from now on.

Often, a default notation is set right after the corresponding notation is introduced – the starred version `\notation*` for that reason introduces a new notation and immediately sets it to be the new default notation. So expressed differently, the *first* `\notation` for a symbol behaves exactly like `\notation*`, and `\notation*{foo}[bar]{...}` behaves exactly like `\notation{foo}[bar]{...}\setnotation{foo}{bar}`.

Operator Notations

Once we have a semantic macro with arguments, such as `\newbinarysymbol`, the semantic macro represents the *application* of the symbol to a list of arguments. What if we want to refer to the operator *itself*, though?

We can do so by supplying the `\notation` (or `\symdef`) with an *operator notation*, indicated with the optional argument `op=`. We can then invoke the operator notation using `\symbolname![notation-identifier]`. Since operator notations never take arguments, we do not need to use `\comp` in it, the whole notation is wrapped in a `\comp` automatically:

Example 8

Input:

```
1 \notation{newbinarysymbol}[ab,
2   op={\text{a:}\cdot\text{; b:}\cdot}]
3   {\comp{\text{a:}}#1\comp{\text{; b:}}#2}
4 \symname{newbinarysymbol} is also occasionally written
5 $\newbinarysymbol![ab]$
```

Output:

`newbinarysymbol` is also occasionally written `a: · ; b: ·`

\hookrightarrow `\symbolname!` is translated to OMDoc/MMT as `<OMS name="...?symbolname"/>` directly.
 \hookrightarrow `\T` directly.

3.3.3 Argument Types

The notations so far used *simple* arguments which we call *i-type* arguments. Declaring a new symbol with `\symdecl{foo}[args=3]` is equivalent to writing `\symdecl{foo}[args=iii]`, indicating that the semantic macro takes three i-type arguments. However, there are three more argument types which we will investigate now, namely **b-type**, **a-type** and **B-type** arguments.

b-Type Arguments

A **b-type** argument represents a *variable* that is *bound* by the symbol in its application, making the symbol a *binding operator*. Typical examples of binding operators are e.g. sums \sum , products \prod , integrals \int , quantifiers like \forall and \exists , that λ -operator, etc.

\hookrightarrow **b-type** arguments behave exactly like **i-type** arguments within TeX, but applications of binding operators, i.e. symbols with **b-type** arguments, are translated to \rightsquigarrow **OMBIND**-terms in OMDoc/MMT, rather than **OMA**.

For example, we can implement a summation operator binding an index variable and taking lower and upper index bounds and the expression to sum over like this:

Example 9

Input:

```
1 \symdef{summation}[args=biii]
2 {\mathop{\comp{\sum}}_{\#1\comp{=}\#2}^{\#3}\#4}
3 $\summation{\svar{x}}{1}{\svar{n}}{\svar{x}}^2$
```

Output:

$$\sum_{x=1}^n x^2$$

where the variable x is now *bound* by the `\summation`-symbol in the expression.

a-Type Arguments

a-type arguments represent a *flexary argument sequence*, i.e. a sequence of arguments of arbitrary length. Formally, operators that take arbitrarily many arguments don't “exist”, but in informal mathematics, they are ubiquitous. **a-type** arguments allow us to write e.g. `\addition{a,b,c,d,e}` rather than having to write something like `\addition{a}{\addition{b}{\addition{c}{\addition{d}{e}}}}`!

`\notation` (and consequently `\symdef`, too) take one additional argument for each **a-type** argument that indicates how to “accumulate” a comma-separated sequence of arguments. This is best demonstrated on an example.

Let's say we want an operator representing quantification over an ascending chain of elements in some set, i.e. `\ascendingchain{S}{a,b,c,d,e}{t}` should yield $\forall a <_{sb} b <_{sc} c <_{sd} d <_{se} e. t$. The “base”-notation for this operator is simply `\comp{forall} #2 \comp{. ,} #3`, where `#2` represents the full notation fragment *accumulated* from `{a,b,c,d,e}`.

The *additional* argument to `\notation` (or `\symdef`) takes the same arguments as the base notation and two *additional* arguments `##1` and `##2` representing successive pairs in the *a*-type argument, and accumulates them into `#2`, i.e. to produce $a <_S b <_S c <_S d <_S e$, we do `{##1 \comp{<}_#1} ##2`:

Example 10

Input:

```
1 \symdef{ascendingchain}[args=iai]
2 {\comp{\forall} #2\comp{.},#3}
3 {##1 \comp{<}_#1} ##2}
4
5 Tadaa: $\ascendingchain{S}{a,b,c,d,e}{t}$
```

Output:

Tadaa: $\forall a <_S b <_S c <_S d <_S e. t$

If this seems overkill, keep in mind that you will rarely need the single-hash arguments `#1`, `#2` etc. in the *a*-notation-argument. For a much more representative and simpler example, we can introduce flexary addition via:

Example 11

Input:

```
1 \symdef{addition}[args=a]{#1}{##1 \comp{+} ##2}
2
3 Tadaa: $\addition{a,b,c,d,e}$
```

Output:

Tadaa: $a+b+c+d+e$

The `assoc-key` We mentioned earlier that “formally”, flexary arguments don’t really “exist”. Indeed, formally, addition is usually defined as a binary operation, quantifiers bind a single variable etc.

Consequently, we can tell `TeX` (or, rather, `MMT/OMDOC`) how to “resolve” flexary arguments by providing `\symdecl` or `\symdef` with an optional `assoc`-argument, as in `\symdecl{addition}[args=a,assoc=bin]`. The possible values for the `assoc`-key are:

- `bin`: A binary, associative argument, e.g. as in `\addition`
- `binl`: A binary, left-associative argument, e.g. $a^{b^{c^d}}$, which stands for $((a^b)^c)^d$
- `binr`: A binary, right-associative argument, e.g. as in $A \rightarrow B \rightarrow C \rightarrow D$, which stands for $A \rightarrow (B \rightarrow (C \rightarrow D))$
- `pre`: Successively prefixed, e.g. as in $\forall x, y, z. P$, which stands for $\forall x. \forall y. \forall z. P$
- `conj`: Conjunctive, e.g. as in $a = b = c = d$ or $a, b, c, d \in A$, which stand for $a = d \wedge b = d \wedge c = d$ and $a \in A \wedge b \in A \wedge c \in A \wedge d \in A$, respectively

pwconj: Pairwise conjunctive, e.g. as in $a \neq b \neq c \neq d$, which stands for $a \neq b \wedge a \neq c \wedge a \neq d \wedge b \neq c \wedge b \neq d \wedge c \neq d$

B-Type Arguments

Finally, B-type arguments simply combine the functionality of both **a** and **b** - i.e. they represent an arbitrarily long sequence of variables to be bound, e.g. for implementing quantifiers:

Example 12

Input:

```
1 \symdef{quantforall}[args=Bi]
2 {\comp{\forall}#1\comp{.}#2}
3 {##1\comp,##2}
4
5 $\quantforall{\svar{x},\svar{y},\svar{z}}{P}$
```

Output:

$\forall x,y,z.P$

3.3.4 Type and Definiens Components

`\symdecl` and `\symdef` take two more optional arguments. \TeX largely ignores them (except for special situations we will talk about later), but MMT can pick up on them for additional services. These are the **type** and **def** keys, which expect expressions in math-mode (ideally using semantic macros, of course!)

The **type** and **def** keys correspond to the **type** and **definiens** components of

- \hookrightarrow OMDoc/MMT constants.
- \rightarrow Correspondingly, the name “type” should be taken with a grain of salt, since
- \rightsquigarrow OMDoc/MMT- being foundation-independent – does not a priori implement a fixed typing system.

The **type**-key allows us to provide additional information (given the necessary \TeX symbols), e.g. for addition on natural numbers:

Example 13

Input:

```
1 \symdef{Nat}[type=\set]{\comp{\mathbb N}}
2 \symdef{addition}[
3   type=\funtype{\Nat,\Nat}{\Nat},
4   op=+,
5   args=a
6 ]{##1}{##1 \comp+ ##2}
7
8 \symname{addition} is an operation $\funtype{\Nat,\Nat}{\Nat}$
```

Output:

addition is an operation $\mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$

The `def`-key allows for declaring symbols as abbreviations:

Example 14

Input:

```
1 \symdef{successor}[
2   type=\funtype{\Nat}{\Nat},
3   def=\fun{\svar{x}}{\addition{\svar{x},1}},
4   op=\mathtt{succ},
5   args=1
6 ]{\comp{\mathtt{succ}{}#1\comp{}}}
7
8 The \symname{successor} operation $\funtype{\Nat}{\Nat}$
9 is defined as $\fun{\svar{x}}{\addition{\svar{x},1}}$
```

Output:

The `successor` operation $\mathbb{N} \rightarrow \mathbb{N}$ is defined as $x \mapsto x+1$

3.3.5 Precedences and Automated Bracketing

Having done `\addition`, the obvious next thing to implement is `\multiplication`. This is in theory straight-forward:

Example 15

Input:

```
1 \symdef{multiplication}[
2   type=\funtype{\Nat,\Nat}{\Nat},
3   op=\cdot,
4   args=a
5 ]{#1}{##1 \comp{\cdot} ##2}
6
7 \symname{multiplication} is an operation $\funtype{\Nat,\Nat}{\Nat}$
```

Output:

`multiplication` is an operation $\mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$

However, if we *combine* `\addition` and `\multiplication`, we notice a problem:

Example 16

Input:

```
1 $\addition{a,\multiplication{b,\addition{c,\multiplication{d,e}}}}$
```

Output:

$$a+b\cdot c+d\cdot e$$

We all know that \cdot binds stronger than $+$, so the output $a+b\cdot c+d\cdot e$ does not actually reflect the term we wrote. We can of course insert parentheses manually

Example 17

Input:

```
1 $\addition{a,\multiplication{b,(\addition{c,\multiplication{d,e}})}}$
```

Output:

$$a+b\cdot(c+d\cdot e)$$

but we can also do better by supplying *precedences* and have \TeX insert parentheses automatically.

For that purpose, `\notation` (and hence `\symdef`) take an optional argument `prec=<opprec>;<argprec1>x...x<argprec n>`.

We will investigate the precise meaning of `<opprec>` and the `<argprec>`s shortly – in the vast majority of cases, it is perfectly sufficient to think of `prec=` taking a single number and having that be *the* precedence of the notation, where lower precedences (somewhat counterintuitively) bind stronger than higher precedences. So fixing our notations for `\addition` and `\multiplication`, we get:

Example 18

Input:

```
1 \notation{multiplication}[
2   op=\cdot,
3   prec=50
4 ]{#1}{##1 \comp\cdot ##2}
5 \notation{addition}[
6   op=+,
7   prec=100
8 ]{#1}{##1 \comp+ ##2}
9
10 $\addition{a,\multiplication{b,\addition{c,\multiplication{d,e}}}}$
```

Output:

$$a+b\cdot(c+d\cdot e)$$

Note that the precise numbers used for precedences are pretty arbitrary – what matters is which precedences are higher than which other precedences when used in conjunction.

`\infprec`
`\neginfprec`

It is occasionally useful to have “infinitely” high or low precedences to enforce or forbid automated bracketing entirely – for those purposes, `\infprec` and `\neginfprec` exist (which are implemented as the maximal and minimal integer values accordingly).

More precisely, each notation takes

1. One *operator precedence* and
2. one *argument precedence* for each argument.

By default, all precedences are 0, unless the symbol takes no argument, in which case the operator precedence is `\neginfprec` (negative infinity). If we only provide a single number, this is taken as both the operator precedence and all argument precedences.

$\text{\S}\text{\TeX}$ decides whether to insert parentheses by comparing operator precedences to a *downward precedence* p_d with initial value `\infprec`. When encountering a semantic macro, $\text{\S}\text{\TeX}$ takes the operator precedence p_{op} of the notation used and checks whether $p_{op} > p_d$. If so, $\text{\S}\text{\TeX}$ insert parentheses.

When $\text{\S}\text{\TeX}$ steps into an argument of a semantic macro, it sets p_d to the respective argument precedence of the notation used.

In the example above:



1. $\text{\S}\text{\TeX}$ starts out with $p_d = \text{\code{\infprec}}$.
2. $\text{\S}\text{\TeX}$ encounters `\addition` with $p_{op} = 100$. Since $100 \not> \text{\code{\infprec}}$, it inserts no parentheses.
3. Next, $\text{\S}\text{\TeX}$ encounters the two arguments for `\addition`. Both have no specifically provided argument precedence, so $\text{\S}\text{\TeX}$ uses $p_d = p_{op} = 100$ for both and recurses.
4. Next, $\text{\S}\text{\TeX}$ encounters `\multiplication{b,...}`, whose notation has $p_{op} = 50$.
5. We compare to the current downward precedence p_d set by `\addition`, arriving at $p_{op} = 50 \not> 100 = p_d$, so $\text{\S}\text{\TeX}$ again inserts no parentheses.
6. Since the notation of `\multiplication` has no explicitly set argument precedences, $\text{\S}\text{\TeX}$ uses the operator precedence for all arguments of `\multiplication`, hence sets $p_d = p_{op} = 50$ and recurses.
7. Next, $\text{\S}\text{\TeX}$ encounters the inner `\addition{c,...}` whose notation has $p_{op} = 100$.
8. We compare to the current downward precedence p_d set by `\multiplication`, arriving at $p_{op} = 100 > 50 = p_d$ – which finally prompts $\text{\S}\text{\TeX}$ to insert parentheses, and we proceed as before.

3.3.6 Variables

All symbol and notation declarations require a module with which they are associated, hence the commands `\symdecl`, `\notation`, `\symdef` etc. are disabled outside of `smodule`-environments.

Variables are different – variables are allowed everywhere, are not exported when the current module (if one exists) is imported (via `\importmodule` or `\usemodule`) and (also unlike symbol declarations) “disappear” at the end of the current \TeX group.

`\svar`

So far, we have always used variables using `\svar{n}`, which marks-up n as a variable with name `n`. More generally, `\svar[foo]{<texcode>}` marks-up the arbitrary `<texcode>` as representing a variable with name `foo`.

Of course, this makes it difficult to reuse variables, or introduce “functional” variables with arities > 0 , or provide them with a type or definiens.

`\vardef`

For that, we can use the `\vardef` command. Its syntax is largely the same as that of `\symdef`, but unlike symbols, variables have only one notation (**TODO: so far?**), hence there is only `\vardef` and no `\vardecl`.

Example 19

Input:

```

1 \vardef{varf}[
2   name=f,
3   type=\funtype{\Nat}{\Nat},
4   op=f,
5   args=1,
6   prec=0;\neginfprec
7 ]{\comp{f}#1}
8 \vardef{varn}[name=n,type=\Nat]{\comp{n}}
9 \vardef{varx}[name=x,type=\Nat]{\comp{x}}
10
11 Given a function  $\mathit{varf}!:\mathit{funtype}\{\mathit{Nat}\}\{\mathit{Nat}\}$ ,
12 by  $\mathit{addition}\{\mathit{varf}!,\mathit{varn}\}$  we mean the function
13  $\mathit{fun}\{\mathit{varx}\}\{\mathit{varf}\{\mathit{addition}\{\mathit{varx},\mathit{varn}\}\}\}$ 

```

Output:

Given a function $f : \mathbb{N} \rightarrow \mathbb{N}$, by $f+n$ we mean the function $x \mapsto f(x+n)$

(of course, “lifting” addition in the way described in the previous example is an operation that deserves its own symbol rather than abusing `\addition`, but... well.)

TODO: bind=forall/exists

3.3.7 Variable Sequences

Variable *sequences* occur quite frequently in informal mathematics, hence they deserve special support. Variable sequences behave like variables in that they disappear at the end of the current \TeX group and are not exported from modules, but their declaration is quite different.

`\varseq`

A variable sequence is introduced via the command `\varseq`, which takes the usual optional arguments `name` and `type`. It then takes a starting index, an end index and a *notation* for the individual elements of the sequence parametric in an index.

This is best shown by example:

Example 20

Input:

```

1 \vardef{varn}[name=n,type=\Nat]{\comp{n}}
2 \varseq{seqa}[name=a,type=\Nat]{1}{\varn}{\comp{a}_{#1}}
3
4 The  $i$ th index of  $\mathit{seqa}!$  is  $\mathit{seqa}_i$ .

```

Output:

The i th index of a_1, \dots, a_n is a_i .

Note that the syntax `\seqa!` now automatically generates a presentation based on the starting and ending index.

TODO: more notations for invoking sequences.

Notably, variable sequences are nicely compatible with **a**-type arguments, so we can do the following:

Example 21

Input:

```
1 \addition{\seqa}
```

Output:

$$a_1 + \dots + a_n$$

Sequences can be *multidimensional* using the `args`-key, in which case the notation's arity increases and starting and ending indices have to be provided as a comma-separated list:

Example 22

Input:

```
1 \vardef{var}{name=m,type=\Nat}{\comp{m}}
2 \varseq{seqa}[
3   name=a,
4   args=2,
5   type=\Nat,
6 ]{1,1}{\varn,\var}{\comp{a}_{#1}^{\#2}}
7
8 \seqa! and \addition{\seqa}
```

Output:

$$a_1^1, \dots, a_n^m \text{ and } a_1^1 + \dots + a_n^m$$

We can also explicitly provide a “middle” segment to be used, like such:

Example 23

Input:

```
1 \varseq{seqa}[
2   name=a,
3   type=\Nat,
4   args=2,
5   mid={\comp{a}_{\varn}^1,\comp{a}_{1^2},\ellipses,\comp{a}_{1^{\varn}}}
6 ]{1,1}{\varn,\var}{\comp{a}_{#1}^{\#2}}
7
8 \seqa! and \addition{\seqa}
```

Output:

$$a_1^1, \dots, a_n^1, a_1^2, \dots, a_n^m, \dots, a_1^m, \dots, a_n^m \text{ and } a_1^1 + \dots + a_n^1 + a_1^2 + \dots + a_n^2 + \dots + a_1^m + \dots + a_n^m$$

3.4 Module Inheritance and Structures

3.4.1 Multilinguality and Translations

If we load the \LaTeX document class or package with the option `lang=<lang>`, \LaTeX will load the appropriate `babel` language for you – e.g. `lang=de` will load the `babel` language `ngerman`. Additionally, it makes \LaTeX aware of the current document being set in (in this example) *german*. This matters for reasons other than mere `babel`-purposes, though:

Every *module* is assigned a language. If no \LaTeX package option is set that allows for inferring a language, \LaTeX will check whether the current file name ends in e.g. `.en.tex` (or `.de.tex` or `.fr.tex`, or...) and set the language accordingly. Alternatively, a language can be explicitly assigned via `\begin{smodule}[lang=<language>]{Foo}`.

Technically, each `smodule`-environment induces *two* OMDoc/MMT theories:
 \LaTeX \rightarrow `\begin{smodule}[lang=<lang>]{Foo}` generates a theory `some/namespace?Foo` that only contains the “formal” part of the module – i.e. exactly the content that is exported when using `\importmodule`.
 \LaTeX \rightarrow Additionally, MMT generates a *language theory* `some/namespace/Foo?<lang>` that includes `some/namespace?Foo` and contains all the other document content – variable declarations, includes for each `\usemodule`, etc.

Notably, the language suffix in a `lfile` is ignored for `\usemodule`, `\importmodule` and in generating/computing URIs for modules. This however allows for providing *translations* for modules between languages without needing to duplicate content:

If a module `Foo` exists in e.g. *english* in a file `Foo.en.tex`, we can provide a file `Foo.de.tex` right next to it, and write `\begin{smodule}[sig=en]{Foo}`. The `sig`-key then signifies, that the “signature” of the module is contained in the *english* version of the module, which is immediately imported from there, just like `\importmodule` would.

Additionally to translating the informal content of a module file to different languages, it also allows for customizing notations between languages. For example, the *least common multiple* of two numbers is often denoted as `lcm(a,b)` in *english*, but is called *kleinstes gemeinsames Vielfaches* in *german* and consequently denoted as `kgV(a,b)` there.

We can therefore imagine a *german* version of an `lcm`-module looking something like this:

```
1 \begin{smodule}[sig=en]{lcm}
2   \notation*{lcm}[de]{\comp{\mathtt{kgV}}}{\#1,\#2}
3
4   Das \symref{lcm}{kleinste gemeinsame Vielfache}
5   $\lcm{a,b}$ von zwei Zahlen $a,b$ ist...
6 \end{smodule}
```

If we now do `\importmodule{lcm}` (or `\usemodule{lcm}`) within a *german* document, it will also load the content of the *german* translation, including the `de`-notation for `\lcm`.

3.4.2 Simple Inheritance and Namespaces

`\importmodule`
`\usemodule`

`\importmodule`[Some/Archive]{path?ModuleName} is only allowed within an `smodule`-environment and makes the symbols declared therein available. Additionally the content of ModuleName will be exported if the current module is imported somewhere else via `\importmodule`.

`\usemodule` behaves the same way, but without exporting the content of the used module.

It is worth going into some detail how exactly `\importmodule` and `\usemodule` resolve their arguments to find the desired module – which is closely related to the *namespace* generated for a module, that is used to generate its URI.



Ideally, \TeX would use arbitrary URIs for modules, with no forced relationships between the *logical* namespace of a module and the *physical* location of the file declaring the module – like MMT does things.

Unfortunately, \TeX only provides very restricted access to the file system, so we are forced to generate namespaces systematically in such a way that they reflect the physical location of the associated files, so that \TeX can resolve them accordingly. Largely, users need not concern themselves with namespaces at all, but for completeness sake, we describe how they are constructed:

- If `\begin{smodule}{Foo}` occurs in a file `/path/to/file/Foo[.<lang>].tex` which does not belong to an archive, the namespace is `file://path/to/file`.
- If the same statement occurs in a file `/path/to/file/bar[.<lang>].tex`, the namespace is `file://path/to/file/bar`.

In other words: outside of archives, the namespace corresponds to the file URI with the filename dropped iff it is equal to the module name, and ignoring the (optional) language suffix.

If the current file is in an archive, the procedure is the same except that the initial segment of the file path up to the archive's `source`-folder is replaced by the archive's namespace URI.



Conversely, here is how namespaces/URIs and file paths are computed in import statements, exemplary `\importmodule`:

- `\importmodule{Foo}` outside of an archive refers to module `Foo` in the current namespace. Consequently, `Foo` must have been declared earlier in the same document or, if not, in a file `Foo[.<lang>].tex` in the same directory.
- The same statement *within* an archive refers to either the module `Foo` declared earlier in the same document, or otherwise to the module `Foo` in the archive's top-level namespace. In the latter case, it has to be declared in a file `Foo[.<lang>].tex` directly in the archive's `source`-folder.
- Similarly, in `\importmodule{some/path?Foo}` the path `some/path` refers to either the sub-directory and relative namespace path of the current directory and namespace outside of an archive, or relative to the current archive's top-level namespace and `source`-folder, respectively.

The module `Foo` must either be declared in the



file $\langle\text{top-directory}\rangle/\text{some/path}/\text{Foo}[\langle\text{lang}\rangle].\text{tex}$, or in $\langle\text{top-directory}\rangle/\text{some/path}[\langle\text{lang}\rangle].\text{tex}$ (which are checked in that order).

- Similarly, `\importmodule[Some/Archive]{some/path?Foo}` is resolved like the previous cases, but relative to the archive `Some/Archive` in the mathhub-directory.
- Finally, `\importmodule{full://uri?Foo}` naturally refers to the module `Foo` in the namespace `full://uri`. Since the file this module is declared in can not be determined directly from the URI, the module must be in memory already, e.g. by being referenced earlier in the same document. Since this is less compatible with a modular development, using full URIs directly is strongly discouraged, unless the module is declared in the current file directly.

`\STEXexport`

`\importmodule` and `\usemodule` import all symbols, notations, semantic macros and (recursively) `\importmodules`. If you want to additionally export e.g. convenience macros and other code from a module, you can use the command `\STEXexport{<code>}` in your module. Then `<code>` is executed (both immediately and) every time the current module is opened via `\importmodule` or `\usemodule`.



Note, that `\newcommand` defines macros *globally* and throws an error if the macro already exists, potentially leading to low-level L^AT_EX errors if we put a `\newcommand` in an `\STEXexport` and the `<code>` is executed more than once in a document – which can happen easily.

A safer alternative is to use macro definition principles, that are safe to use even if the macro being defined already exists, and ideally are local to the current T_EX group, such as `\def` or `\let`.

3.4.3 The `mathstructure` Environment

A common occurrence in mathematics is bundling several interrelated “declarations” together into *structures*. For example:

- A *monoid* is a structure $\langle M, \circ, e \rangle$ with $\circ : M \times M \rightarrow M$ and $e \in M$ such that...
- A *topological space* is a structure $\langle X, \mathcal{T} \rangle$ where X is a set and \mathcal{T} is a topology on X
- A *partial order* is a structure $\langle S, \leq \rangle$ where \leq is a binary relation on S such that...

This phenomenon is important and common enough to warrant special support, in particular because it requires being able to *instantiate* such structures (or, ratherer, structure *signatures*) in order to talk about (concrete or variable) *particular* monoids, topological spaces, partial orders etc.

`mathstructure` The `mathstructure` environment allows us to do exactly that. It behaves exactly like the `smodule` environment, but is itself only allowed inside an `smodule` environment, and allows for instantiation later on.

How this works is again best demonstrated by example:

Example 24

Input:

```

1 \begin{mathstructure}{monoid}
2   \symdef{universe}[type=\set]{\comp{U}}
3   \symdef{op}[
4     args=2,
5     type=\funtype{\universe,\universe}{\universe},
6     op=\circ
7   ]{##1 \comp{\circ} ##2}
8   \symdef{unit}[type=\universe]{\comp{e}}
9 \end{mathstructure}
10
11 A \symname{monoid} is...
```

Output:

A `monoid` is...

Note that the `\symname{monoid}` is appropriately highlighted and (depending on your pdf viewer) shows a URI on hovering – implying that the `mathstructure` environment has generated a *symbol* `monoid` for us. It has not generated a semantic macro though, since we can not use the `monoid`-symbol *directly*. Instead, we can instantiate it, for example for integers:

Example 25

Input:

```

1 \symdef{Int}[type=\set]{\comp{\mathbb Z}}
2 \symdef{addition}[
3   type=\funtype{\Int,\Int}{\Int},
4   args=2,
5   op=+
6 ]{##1 \comp{+} ##2}
7 \symdef{zero}[type=\Int]{\comp{0}}
8
9 $\mathstruct{\Int,\addition!,\zero}$ is a \symname{monoid}.
```

Output:

$\langle \mathbb{Z}, +, 0 \rangle$ is a `monoid`.

So far, we have not actually instantiated `monoid`, but now that we have all the symbols to do so, we can:

Example 26

Input:

```

1 \instantiate{intmonoid}{
2   universe = Int ,
3   op = addition ,
4   unit = zero
5 }{monoid}{\mathbb{Z}_{+,0}}
6
7 $\intmonoid{universe}$, $\intmonoid{unit}$ and $\intmonoid{op}{a}{b}$.
8
9 Also: $\intmonoid!$

```

Output:

\mathbb{Z} , 0 and $a+b$.
Also: $\mathbb{Z}_{+,0}$

\instantiate

So summarizing: `\instantiate` takes four arguments: The (macro-)name of the instance, a key-value pair assigning declarations in the corresponding `mathstructure` to symbols currently in scope, the name of the `mathstructure` to instantiate, and lastly a notation for the instance itself.

It then generates a semantic macro that takes as argument the name of a declaration in the instantiated `mathstructure` and resolves it to the corresponding instance of that particular declaration.

`\instantiate` and `mathstructure` make use of the *Theories-as-Types* paradigm: `mathstructure{<name>}` does in fact simply create a nested theory with name `<name>-structure`. The *constant* `<name>` is defined as `Mod(<name>-structure)` – a *dependent record type with manifest fields*, the fields of which are generated from (and correspond to) the constants in `<name>-structure`.
 $\hookrightarrow M$ `\instantiate` appropriately generates a constant whose definiens is a record term of type `Mod(<name>-structure)`, with the fields assigned appropriately based on the key-value-list.

Notably, `\instantiate` throws an error if not *every* declaration in the instantiated `mathstructure` is being assigned.

You might consequently ask what the usefulness of `mathstructure` even is.

\varinstantiate

The answer is that we can also instantiate a `mathstructure` with a *variable*. The syntax of `\varinstantiate` is equivalent to that of `\instantiate`, but all of the key-value-pairs are optional, and if not explicitly assigned (to a symbol *or* a variable declared with `\vardef`) inherit their notation from the one in the `mathstructure` environment.

This allows us to do things like:

Example 27

Input:

```

1 \varinstantiate{varM}{\monoid}{M}
2
3 A \symname{monoid} is a structure
4 $\varM!:=\mathstrut{\varM{universe},\varM{op}!,\varM{unit}}{\varM{universe}}$
5 such that
6 $\varM{op}!:\mathstrut{\varM{universe},\varM{universe}}{\varM{universe}}$
7 and...
8
9 \varinstantiate{varMb}{universe = Int}{monoid}{M_2}
10
11 \noindent Let $\varMb!:=\mathstrut{\varMb{universe},\varMb{op}!,\varMb{unit}}{\varMb{universe}}$
12 a \symname{monoid} on $\mathbb{Z}$...

```

Output:

A **monoid** is a structure $M := \langle U, \circ, e \rangle$ such that $\circ : U \times U \rightarrow U$ and...
 Let $M_2 := \langle \mathbb{Z}, \circ, e \rangle$ a **monoid** on \mathbb{Z} ...

We will return to this example later, when we also know how to handle the *axioms* of a monoid.

3.4.4 The copymodule Environment

TODO: explain

Given modules:

Example 28

Input:

```

1 \begin{smodule}{magma}
2   \symdef{universe}{\comp{\mathcal U}}
3   \symdef{operation}[args=2,op=\circ]{\#1 \comp \circ \#2}
4 \end{smodule}
5 \begin{smodule}{monoid}
6   \importmodule{magma}
7   \symdef{unit}{\comp e}
8 \end{smodule}
9 \begin{smodule}{group}
10  \importmodule{monoid}
11  \symdef{inverse}[args=1]{\#1^{\comp{-1}}}
12 \end{smodule}

```

Output:

We can form a module for *rings* by “cloning” an instance of **group** (for addition) and **monoid** (for multiplication), respectively, and “glueing them together” to ensure they share the same universe:

Example 29

Input:

```

1 \begin{smodule}{ring}
2   \begin{copymodule}{group}{addition}
3     \renamedecl[name=universe]{universe}{runiverse}
4     \renamedecl[name=plus]{operation}{rplus}
5     \renamedecl[name=zero]{unit}{rzero}
6     \renamedecl[name=uminus]{inverse}{ruminus}
7   \end{copymodule}
8   \notation*{rplus}[plus,op=+,prec=60]{#1 \comp+ #2}
9   \notation*{rzero}[zero]{\comp0}
10  \notation*{ruminus}[uminus,op=-]{\comp- #1}
11  \begin{copymodule}{monoid}{multiplication}
12    \assign{universe}{\runiverse}
13    \renamedecl[name=times]{operation}{rtimes}
14    \renamedecl[name=one]{unit}{rone}
15  \end{copymodule}
16  \notation*{rtimes}[cdot,op=\cdot,prec=50]{#1 \comp\cdot #2}
17  \notation*{rone}[one]{\comp1}
18  Test: $\rtimes a\{rplus c\{rtimes de\}}$
19 \end{smodule}

```

Output:

Test: $a \cdot (c + d \cdot e)$

TODO: explain donotclone

3.4.5 The interpretmodule Environment

TODO: explain

Example 30

Input:

```

1 \begin{smodule}{int}
2   \symdef{Integers}{\comp{\mathbb Z}}
3   \symdef{plus}[args=2,op=+]{#1 \comp+ #2}
4   \symdef{zero}{\comp0}
5   \symdef{uminus}[args=1,op=-]{\comp-#1}
6
7   \begin{interpretmodule}{group}{intisgroup}
8     \assign{universe}{\Integers}
9     \assign{operation}{\plus!}
10    \assign{unit}{\zero}
11    \assign{inverse}{\uminus!}
12  \end{interpretmodule}
13 \end{smodule}

```

Output:

3.5 Primitive Symbols (The \TeX Metatheory)

TODO: metatheory documentation

Chapter 4

Using \TeX Symbols

Given a symbol declaration `\symdecl{symbolname}`, we obtain a semantic macro `\symbolname`. We can use this semantic macro in math mode to use its notation(s), and we can use `\symbolname!` in math mode to use its operator notation(s). What else can we do?

4.1 `\symref` and its variants

`\symref`
`\symname`

We have already seen `\symname` and `\symref`, the latter being the more general.

`\symref{<symbolname>}{<code>}` marks-up `<code>` as referencing `<symbolname>`. Since quite often, the `<code>` should be (a variant of) the name of the symbol anyway, we also have `\symname{<symbolname>}`.

Note that `\symname` uses the *name* of a symbol, not its macroname. More precisely, `\symname` will insert the name of the symbol with “-” replaced by spaces. If a symbol does not have an explicit `name=` given, the two are equal – but for `\symname` it often makes sense to make the two explicitly distinct. For example:

Example 31

Input:

```
1 \symdef{Nat}[
2   name=natural-number,
3   type=\set
4 ]{\comp{\mathbb{N}}}
5
6 A \symname{Nat} is...
```

Output:

A natural number is...

`\symname` takes two additional optional arguments, `pre=` and `post=` that get prepended or appended respectively to the symbol name.

`\Symname`

Additionally, `\Symname` behaves exactly like `\symname`, but will capitalize the first letter of the name:

Example 32

Input:

```
1 \Symname[post=s]{Nat} are...
```

Output:

```
Natural numbers are...
```



This is as good a place as any other to explain how \TeX resolves a string `symbolname` to an actual symbol.

If `\symbolname` is a semantic macro, then \TeX has no trouble resolving `symbolname` to the full URI of the symbol that is being invoked.

However, especially in `\symname` (or if a symbol was introduced using `\symdecl*` without generating a semantic macro), we might prefer to use the *name* of a symbol directly for readability – e.g. we would want to write `A \symname{natural-number} is...` rather than `A \symname{Nat} is...`. \TeX attempts to handle this case thusly:

If `string` does *not* correspond to a semantic macro `\string`, then \TeX checks all symbols currently in scope until it finds one, whose full URI ends with `string`. This allows for disambiguating more precisely, e.g. by saying `\symname{Integers?addition}` or `\symname{RealNumbers?addition}` in the case where several `additions` are in scope.

However, this also means that if we have symbols `foo` and e.g. `miraculous-foo`, then \TeX might resolve `\symname{foo}` to `miraculous-foo` if it finds this symbol first. It is therefore a good idea to prefix symbol names with a `?`, thus ensuring that \TeX will find the symbol `...?foo` rather than `...?miraculous-foo`.

4.2 Marking Up Text and On-the-Fly Notations

We can also use semantic macros outside of text mode though, which allows us to annotate arbitrary text fragments.

Let us assume again, that we have `\symdef{addition}[args=2]{#1 \comp+ #2}`. Then we can do

Example 33

Input:

```
1 \addition{\comp{The sum of} \arg{\$svar{n}} \comp{ and } \arg{\$svar{m}}}  
2 is...
```

Output:

```
The sum of  $n$  and  $m$  is...
```


...which marks up the text fragment as representing an *application* of the **addition**-symbol to two argument n and m .

\hookrightarrow M \rightarrow As expected, the above example is translated to OMDOC/MMT as an
 \rightarrow M \rightarrow OMA with `<OMS name="...?addition"/>` as head and `<OMV name="n"/>` and
 \rightsquigarrow T \rightsquigarrow `<OMV name="m"/>` as arguments.

\arg

In text mode, every semantic macro takes exactly one argument, namely the text-fragment to be annotated. The `\arg` command is only valid within the argument to a semantic macro and marks up the *individual arguments* for the symbol.

We can also use semantic macros in text mode to invoke an operator itself instead of its application, with the usual syntax using `!`:

Example 34

Input:

```
1 \addition!{Addition} is...
```

Output:

Addition is...

In deed, `\symbolname!{<code>}` is exactly equivalent to `\symref{symbolname}{<code>}` (the latter is in fact implemented in terms of the former).

`\arg` also allows us to switch the order of arguments around and “hide” arguments: For example, `\arg[3]{<code>}` signifies that `<code>` represents the *third* argument to the current operator, and `\arg*[i]{<code>}` signifies that `<code>` represents the *i*th argument, but it should not produce any output (it is exported in the `xhtml` however, so that MMT and other systems can pick up on it)

Example 35

Input:

```
1 \addition{\comp{adding}
2 \arg[2]{\svar{k}}
3 \arg*{\svar{n}}{\svar{m}} yields...
```

Output:

adding k yields...

Note that since the second `\arg` has no explicit argument number, it automatically represents the first not-yet-given argument – i.e. in this case the first one.

The same syntax can be used in math mode, too, which allows us to spontaneously introduce new notations on the fly. We can activate it using the starred variants of semantic macros:

Example 36

Input:

```
1 Given $\addition{\svar{n}}{\svar{m}}$, then
2 $\addition*{
3   \arg*{\addition{\svar{n}}{\svar{m}}}
4   \comp{+}
5   \arg{\svar{k}}
6 }$ yields...
```

Output:

Given $n+m$, then $+k$ yields...

4.3 Referencing Symbols and Statements

TODO: references documentation

Chapter 5

sTEX Statements

5.1 Definitions, Theorems, Examples, Paragraphs

As mentioned earlier, we can semantically mark-up *statements* such as definitions, theorems, lemmata, examples, etc.

The corresponding environments for that are:

- `sdefinition` for definitions,
- `sassertion` for assertions, i.e. propositions that are declared to be *true*, such as theorems, lemmata, axioms,
- `sexample` for examples, and
- `sparagraph` for other semantic paragraphs, such as comments, remarks, conjectures, etc.

The *presentation* of these environments can be customized to use e.g. predefined theorem-environments, see [chapter 6](#) for details.

All of these environments take optional arguments in the form of `key=value`-pairs. Common to all of them are the keys `id=` (for cross-referencing, see [section 4.3](#)), `type=` for customization (see [chapter 6](#)) and additional information (e.g. definition principles, “difficulty” etc), `title=`, and `for=`.

The `for=` key expects a comma-separated list of existing symbols, allowing for e.g. things like

Example 37

Input:

```
1 \begin{sexample}[
2   id=additionandmultiplication.ex,
3   for={addition,multiplication},
4   type={trivial,boring},
5   title={An Example}
6 ]
7   $\addition{2,3}$ is $5$, $\multiplication{2,3}$ is $6$.
8 \end{sexample}
```

Output:

Example 5.1.1 (An Example). $2+3$ is 5, $2\cdot 3$ is 6.

`\definiendum`
`\definame`
`\definiens`
`\Definame`

`sdefinition` (and `sparagraph` with `type=symdoc`) introduce three new macros: `definiendum` behaves like `symref` (and `definame/Definame` like `symname/Symname`, respectively), but highlights the referenced symbol as *being defined* in the current definition.

`\definiens`[<optional symbolname>]{<code>} marks up <code> as being the explicit *definiens* of <optional symbolname> (in case `for=` has multiple symbols).

- \hookrightarrow The special `type=symdoc` for `sparagraph` is intended to be used for “informal definitions”, or encyclopedia-style descriptions for symbols.
- \hookrightarrow The MMT-system can use those (in lieu of an actual `sdefinition` in scope) to present to users, e.g. when hovering over symbols.

All four environments also take an optional parameter `name=` – if this one is given a value, the environment will generate a *symbol* by that name (but with no semantic macro). Not only does this allow for `\symref` et al, it allows us to resume our earlier example for monoids much more nicely:

Example 38

Input:

```

1 \begin{mathstructure}{monoid}
2   \symdef{universe}[type=\set]{\comp{U}}
3   \symdef{op}[
4     args=2,
5     type=\funtype{\universe,\universe}{\universe},
6     op=\circ
7   ]{\#1 \comp{\circ} \#2}
8   \symdef{unit}[type=\universe]{\comp{e}}
9
10  \begin{sparagraph}[type=symdoc,for=monoid]
11    A \definame{monoid} is a structure
12    $\mathstruct{\universe,\op!,\unit}$
13    where $\op!: \funtype{\universe}{\universe}$ and
14    $\inset{\unit}{\universe}$ such that
15
16    \begin{sassertion}[name=associative,
17      type=axiom,
18      title=Associativity]
19      $\op!$ is associative
20    \end{sassertion}
21    \begin{sassertion}[name=isunit,
22      type=axiom,
23      title=Unit]
24      $\equal{\op{\svar{x}}{\unit}}{\svar{x}}$
25      for all $\inset{\svar{x}}{\universe}$
26    \end{sassertion}
27  \end{sparagraph}
28 \end{mathstructure}
29
30 An example for a \symname{monoid} is...
```

Output:

A **monoid** is a structure $\langle U, \circ, e \rangle$ where $\circ : U \rightarrow U$ and $e \in U$ such that

Axiom 5.1.2 (Associativity). \circ is associative

Axiom 5.1.3 (Unit). $x \circ e = x$ for all $x \in U$

An example for a **monoid** is...

Now the **mathstructure monoid** contains two additional symbols, namely the axioms for associativity and that e is a unit. Note that both symbols do not represent the mere *propositions* that e.g. \circ is associative, but *the assertion that it is actually true* that \circ is associative.

If we now want to instantiate **monoid** (unless with a variable, of course), we also need to assign **associative** and **neutral** to analogous assertions. So the earlier example

```
1 \instantiate{intmonoid}{  
2   universe = Int ,  
3   op = addition ,  
4   unit = zero  
5 }{monoid}{\mathbb{Z}_{+,0}}
```

...will not work anymore. We now need to give assertions that **addition** is associative and that **zero** is a unit with respect to addition.²

5.2 Proofs

TODO

²Of course, $\text{\texttt{S}\text{\textsf{T}\text{\textsf{E}\text{\textsf{X}}}}$ can not check that the assertions are the “correct” ones – but if the assertions (both in **monoid** as well as those for addition and zero) are properly marked up, MMT can. **TODO: should**

Chapter 6

Highlighting and Presentation Customizations

The environments starting with `s` (i.e. `smodule`, `sassertion`, `sexample`, `sdefinition`, `sparagraph` and `sproof`) by default produce no additional output whatsoever (except for the environment content of course). Instead, the document that uses them (whether directly or e.g. via `inputref`) can decide how these environments are supposed to look like.

The `stexthm` defines some default customizations that can be used, but of course many existing L^AT_EX templates come with their own `definition`, `theorem` and similar environments that authors are supposed (or even required) to use. Their concrete syntax however is usually not compatible with all the additional arguments that `gTEX` allows for semantic information.

Therefore we introduced the separate environments `sdefinition` etc. instead of using `definition` directly, and allow authors to specify how these environments should be styled via the commands `stexpatch*`.

```
\stexpatchmodule
\stexpatchdefinition
\stexpatchassertion
\stexpatchexample
\stexpatchparagraph
\stexpatchproof
```

All of these commands take one optional and two proper arguments, i.e.

```
\stexpatch*{<type>}{<begin-code>}{<end-code>}.
```

After `gTEX` reads and processes the optional arguments for these environments, (some of) their values are stored in the macros `\s*<field>` (i.e. `sexampleid`, `\sassertionname`, etc.). It then checks for all the values `<type>` in the `type=`-list, whether an `\stexpatch*{<type>}` for the current environment has been called. If it finds one, it uses that patches `<begin-code>` and `<end-code>` to mark up the current environment. If no patch for (any of) the type(s) is found, it checks whether and `\stexpatch*` was called without optional argument.

For example, if we want to use a predefined `theorem` environment for `sassertions` with `type=theorem`, we can do

```
1 \stexpatchassertion[theorem]{\begin{theorem}}{\end{theorem}}
```

...or, rather, since e.g. `theorem`-environments defined using `amsthm` take an optional title as argument, we can do:

```
1 \stexpatchassertion[theorem]
2   {\ifx\sassertiontitle\@empty
3     \begin{theorem}
```

```

4   \else
5     \begin{theorem}[\sassertiontitle]
6   \fi}
7   {\end{theorem}}

```

Or, if we want all **sdefinitions** to use a predefined **definition**-environment, we can do

```

1 \stexpatchdefinition
2   {\ifx\sdefinitiontitle\@empty
3     \begin{definition}
4   \else
5     \begin{definition}[\sdefinitiontitle]
6   \fi}
7   {\end{definition}}

```

`\compemph`
`\varemp`
`\symrefemph`
`\defemph`

Apart from the environments, we can control how \TeX highlights variables, notation components, `\symrefs` and `\definiendums`, respectively.

To do so, we simply redefine these four macros. For example, to highlight notation components (i.e. everything in a `\comp`) in blue, as in this document, we can do `\def\compemph#1{\textcolor{blue}{#1}}`. By default, `\compemph` et al do nothing.

`\compemph@uri`
`\varemp@uri`
`\symrefemph@uri`
`\defemph@uri`

For each of the four macros, there exists an additional macro that takes the full URI of the relevant symbol currently being highlighted as a second argument. That allows us to e.g. use pdf tooltips and links. For example, this document uses

```

1 \protected\def\symrefemph@uri#1#2{
2   \pdftooltip{
3     \srefsymuri{#2}{\symrefemph{#1}}
4   }{
5     URI:~\detokenize{#2}
6   }
7 }

```

By default, `\compemph@uri` is simply defined as `\compemph{#1}` (analogously for the other three commands).

Chapter 7

Additional Packages

TODO: tikzinput documentation

7.1 Modular Document Structuring

TODO: document-structure documentation

7.2 Slides and Course Notes

TODO: notesslides documentation

7.3 Homework, Problems and Exams

TODO: problem documentation

TODO: hwexam documentation