# The STEX3 Manual \*

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#### Abstract

 $\mbox{\sc ST}_{EX}$  is a collection of EaTeX packages that allow to markup documents semantically without leaving the document format.

Running 'pdflatex' over sTeX-annotated documents formats them into normal-looking PDF. But sTeX also comes with a conversion pipeline into semantically annotated HTML5, which can host semantic added-value services that make the documents active (i.e. interactive and user-adaptive) and essentially turning LATEX into a document format for (mathematical) knowledge management (MKM). STEX augments LATEX with

- semantic macros that denote and distinguish between mathematical concepts, operators, etc. independent of their notational presentation,
- a powerful module system that allows for authoring and importing individual fragments containing document text and/or semantic macros, independent of

   and without hard coding – directory paths relative to the current document,
- a mechanism for exporting STEX documents to (modular) XHTML, preserving all the semantic information for semantically informed knowledge management services

This is the user manual for the STEX package and associated software. It is primarily directed at end-users who want to use STEX to author semantically enriched documents. For the full documentation, see the STEX documentation.

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Boxes like this one contain implementation details that are mostly relevant for more advanced use cases, might be useful to know when debugging, or might be good to know to better understand how something works. They can easily be skipped on a first read.



 $\begin{array}{c} \overset{\longleftarrow}{M} \xrightarrow{\longrightarrow} \\ -M \xrightarrow{\longrightarrow} \\ \times T \xrightarrow{\longrightarrow} \end{array} \\ \text{Boxes like this one explain how some STEX concept relates to the MMT/OMDoc system, philosophy or language; see [MMT; Koh06] for introductions. } \\$ 

# Chapter 1

# What is STEX?

Formal systems for mathematics (such as interactive theorem provers) have the potential to significantly increase both the accessibility of published knowledge, as well as the confidence in its veracity, by rendering the precise semantics of statements machine actionable. This allows for a plurality of added-value services, from semantic search up to verification and automated theorem proving. Unfortunately, their usefulness is hidden behind severe barriers to accessibility; primarily related to their surface languages reminiscent of programming languages and very unlike informal standards of presentation.

STEX minimizes this gap between informal and formal mathematics by integrating formal methods into established and widespread authoring workflows, primarily LATEX, via non-intrusive semantic annotations of arbitrary informal document fragments. That way formal knowledge management services become available for informal documents, accessible via an IDE for authors and via generated *active* documents for readers, while remaining fully compatible with existing authoring workflows and publishing systems.

Additionally, an extensible library of reusable document fragments is being developed, that serve as reference targets for global disambiguation, intermediaries for content exchange between systems and other services.

Every component of the system is designed modularly and extensibly, and thus lay the groundwork for a potential full integration of interactive theorem proving systems into established informal document authoring workflows.

The general STEX workflow combines functionalities provided by several pieces of software:

- The STEX package collection to use semantic annotations in IATEX documents,
- RusTeX [] to convert tex sources to (semantically enriched) xhtml,
- The MMT system [MMT], that extracts semantic information from the thus generated xhtml and provides semantically informed added value services.

# Chapter 2

# Quickstart

# 2.1 Setup

There are two ways of using STEX: as a

- 1. way of writing LATEX more modularly (object-oriented Math) for creating PDF documents or
- 2. foundation for authoring active documents in HTML5 instrumented with knowledge management services.

Both are legitimate and useful. The first requires a significantly smaller tool-chain, so we describe it first. The second requires a much more substantial (and experimental) toolchain of knowledge management systems. Both workflows profit from an integrated development environment (IDE), which (also) automates setup as far as possible (see subsection 2.1.4).

## 2.1.1 Minimal Setup for the PDF-only Workflow

In the best of all worlds, there is no setup, as you already have a new version of TEXLive on your system as a LATEX enthusiast. If not now is the time to install it; see [TL]. You can usually update TEXLive via a package manager or the TEXLive manager tlmgr.

Alternatively, you can install STEX from CTAN, the Comprehensive TEX Archive Network; see [ST] for details.

# 2.1.2 GIT-based Setup for the STFX Development Version

If you want use the latest and greatest STEX packages, you can that have not even been released to CTAN, then you can directly clone them from the STEX development repository [sTeX] by the following command-line instructions:

```
cd <stexdir>
git clone https://github.com/slatex/sTeX.git
```

and keep it updated by pulling updates via git pull in the cloned STEX directory. Then update your TEXINPUTS environment variable, e.g. by placing the following line in your .bashrc:

3

BNP:1

 $<sup>^1\</sup>mathrm{New}$   $\mathrm{Part}\colon$  MK: reorganized, we do not need the full MKM tool chain

# 2.1.3 STEX Archives (Manual Setup)

Writing semantically annotated STEX becomes much easier, if we can use well-designed libraries of already annotated content. STEX provides such libraries as STEX archives—i.e. GIT repositories at https://gl.mathhub.info—most prominently the SMGLoM libraries at https://gl.mathhub.info/smglom.

To do so, we set up a **local MathHub** by creating a MathHub directory <mhdir>. Every STEX archive as an **archive path** <apath> and a name <archive>. We can clone the STEX archive by the following command-line instructions:

```
cd <mhdir>/<apath>
git clone https://gl.mathhub.info/smglom/<archive>.git
```

Note that STEX archives often depend on other archives, thus you should be prepared to clone these as well – e.g. if pdflatex reports missing files. To make sure that STEX too knows where to find its archives, we need to set a global system variable MATHHUB, that points to your local MathHub-directory (see section 3.2).

export MATHHUB="<mhdir>''

# 2.1.4 The STEX IDE

We are currently working on an STEX IDE as an STEX plugin for VScode; see [SIa]. It will feature a setup procedure that automates the setup described above (and below). For additional functionality see the (now obsolete) plugin for STEX 1 [SLS; SIb].

# 2.1.5 Manual Setup for Active Documents and Knowledge Management Services

Foregoing on the STEX IDE, we will need several additional (on top of the minimal setup above) pieces of software; namely:

• The Mmt System available here<sup>2</sup>. We recommend following the setup routine documented here.

Following the setup routine (Step 3) will entail designating a MathHub-directory on your local file system, where the MMT system will look for STEX/MMT content archives.

• STEX Archives If we only care about LATEX and generating pdfs, we do not technically need MMT at all; however, we still need the MATHHUB system variable to be set. Furthermore, MMT can make downloading content archives we might want to use significantly easier, since it makes sure that all dependencies of (often highly interrelated) STEX archives are cloned as well.

Once set up, we can run mmt in a shell and download an archive along with all of its dependencies like this: lmh install <name-of-repository>, or a whole group of archives; for example, lmh install smglom will download all smglom archives.

•  $R_{US}T_{EX}$  The MMT system will also set up  $R_{US}T_{EX}$  for you, which is used to generate (semantically annotated) xhtml from tex sources. In lieu of using MMT, you can also download and use  $R_{US}T_{EX}$  directly here.

ENP:1

EdN:2

<sup>&</sup>lt;sup>2</sup>EdNote: For now, we require the sTeX-branch, requiring manually compiling the MMT sources

# 2.2 A First STEX Document

Having set everything up, we can write a first STEX document. As an example, we will use the smglom/calculus and smglom/arithmetics archives, which should be present in the designated MathHub-folder, and write a small fragment defining the *geometric series*:

TODO: use some sTeX-archive instead of smglom, use a convergence-notion that includes the limit, mark-up the theorem properly

```
\documentclass{article}
  \usepackage{stex,xcolor,stexthm}
4 \begin{document}
 5 \begin{smodule}{GeometricSeries}
       \importmodule[smglom/calculus]{series}
      \importmodule[smglom/arithmetics]{realarith}
 7
 8
9
      \symdef{geometricSeries}[name=geometric-series]{\comp{S}}}
10
      \begin{sdefinition} [for=geometricSeries]
11
          The \definame{geometricSeries} is the \symname{?series}
13
          \[\defeq{\geometricSeries}{\definiens{
14
              \displaystyle \inf \{ \sup \{ svar\{n\} \} \} \} 
15
                  \realdivide[frac]{1}{
                      \realpower{2}{\svar{n}}
17
              }}
18
          }}.\]
19
      \end{sdefinition}
20
      \begin{sassertion} [name=geometricSeriesConverges, type=theorem]
21
      The \symname{geometricSeries} \symname{converges} towards $1$.
      \end{sassertion}
24 \end{smodule}
25 \end{document}
```

Compiling this document with pdflatex should yield the output

**Definition 0.1.** The **geometric series** is the series

$$S := \sum_{n=1}^{\infty} \frac{1}{2^n}.$$

**Theorem 0.2.** The geometric series converges towards 1.

Move your cursor over the various highlighted parts of the document – depending on your pdf viewer, this should yield some interesting (but possibly for now cryptic) information.

#### Remark 2.2.1:

Note that all of the highlighting, tooltips, coloring and the environment headers come from stexthm – by default, the amount of additional packages loaded is kept to a minimum and all the presentations can be customized, see <a href="https://chapter.com/chap

Let's investigate this document in detail to understand the respective parts of the STEX markup infrastructure:

```
\begin{smodule}{GeometricSeries}
...
\end{smodule}
```

smodule

First, we open a new *module* called GeometricSeries. The main purpose of the smodule environment is to group the contents and associate it with a *globally unique* identifier (URI), which is computed from the name GeometricSeries and the document context.

(Depending on your pdf viewer), the URI should pop up in a tooltip if you hover over the word **geometric series**.

```
\importmodule[smglom/calculus]{series}
\importmodule[smglom/arithmetics]{realarith}
```

\importmodule

Next, we *import* two modules – series from the STEX archive smglom/calculus, and realarith from the STEX archive smglom/arithmetics. If we investigate these archives, we find the files series.en.tex and realarith.en.tex (respectively) in their respective source-folders, which contain the statements \begin{smodule}{series} and \begin{smodule}{frealarith} (respectively).

The \importmodule-statements make all STEX symbols and associated semantic macros (e.g. \infinitesum, \realdivide, \realpower) in the imported module available to the current module GeometricSeries. The module GeometricSeries "exports" all of these symbols to all modules imports it via an \importmodule (GeometricSeries) instruction. Additionally it exports the local symbol \geometricSeries.

\usemodule

If we only want to *use* the content of some module Foo, e.g. in remarks or examples, but none of the symbols in our current module actually *depend* on the content of Foo, we can use \usemodule instead – like \importmodule, this will make the module content available, but will *not* export it to other modules.

```
\symdef{GeometricSeries} [name=geometric-series] {\comp{S}}
```

\symdef

Next, we introduce a new symbol with name geometric-series and assign it the semantic macro \geometricSeries. \symdef also immediately assigns this symbol a notation, namely S.

\comp

The macro \comp marks the S in the notation as a notational component, as opposed to e.g. arguments to \geometricSeries. It is the notational components that get highlighted and associated with the corresponding symbol (i.e. in this case geometricSeries). Since \geometricSeries takes no arguments, we can wrap the whole notation in a \comp.

```
\begin{sdefinition} [for=geometricSeries]
...
\end{sdefinition}
\begin{sassertion} [name=geometricSeriesConverges, type=theorem]
...
\end{sassertion}
```

What follows are two STEX-statements (e.g. definitions, theorems, examples, proofs, ...). These are semantically marked-up variants of the usual environments, which take additional optional arguments (e.g. for=, type=, name=). Since many LATEX templates predefine environments like definition or theorem with different syntax, we use sdefinition, sassertion, sexample etc. instead. You can customize these environments to e.g. simply wrap around some predefined theorem-environment. That way, we can still use sassertion to provide semantic information, while being fully compatible with (and using the document presentation of) predefined environments.

In our case, the stexthm-package patches e.g. \begin{sassertion}[type=theorem] to use a theorem-environment defined (as usual) using the amsthm package.

```
... is the \symname{?series}
```

 $\sqrt{symname}$ 

The \symname-command prints the name of a symbol, highlights it (based on customizable settings) and associates the text printed with the corresponding symbol.

Note that the argument of \symref can be a local or imported symbol (here the series symbol is imported from the series module). STEX tries to determine the full symbol URI from the argument. If there are name clashes in or with the imported symbols, the name of the exporting module can be prepended to the symbol name before the ? character.

If you hover over the word series in the pdf output, you should see a tooltip showing the full URI of the symbol used.

\symref

The \symname-command is a special case of the more general \symref-command, which allows customizing the precise text associated with a symbol. \symref takes two arguments the first ist the symbol name, and the second a variant verbalization of the symbol, e.g. an inflection variant, a different language or a synonym. In our example \symname{?series} abbreviates \symref{?series}.

The \definame{geometricSeries} ...

\definame \definiendum

The sdefinition-environment provides two additional macros, \definame and \definiendum which behave similarly to \symname and \symref, but explicitly mark the symbols as being defined in this environment, to allow for special highlighting.

```
\[\defeq{\geometricSeries}{\definiens{
   \infinitesum{\svar{n}}{1}{
      \realdivide[frac]{1}{
      \realpower{2}{\svar{n}}
   }
}}.\]
```

The next snippet – set in a math environment – uses several semantic macros imported from (or recursively via) series and realarithmetics, such as \defeq, \infinitesum, etc. In math mode, using a semantic macro inserts its (default) definition. A semantic

macro can have several notations – in that case, we can explicitly choose a specific notation by providing its identifier as an optional argument; e.g.  $\realdivide[frac]{a}{b}$  will use the explicit notation named frac of the semantic macro  $\realdivide$ , which yields  $\frac{a}{b}$  instead of a/b.

\svar

The  $\sqrt{n}$  command marks up the n as a variable with name n and notation n.

\definiens

The **sdefinition**-environment additionally provides the \definiens-command, which allows for explicitly marking up its argument as the *definiens* of the symbol currently being defined.

# 2.2.1 OMDoc/xhtml Conversion

So, if we run pdflatex on our document, then STEX yields pretty colors and tooltips<sup>1</sup>. But STEX becomes a lot more powerful if we additionally convert our document to xhtml while preserving all the STEX markup in the result.

### TODO VSCode Plugin

Using  $R_{US}T_{EX}$  [], we can convert the document to xhtml using the command rustex -i /path/to/file.tex -o /path/to/outfile.xhtml. Investigating the resulting file, we notice additional semantic information resulting from our usage of semantic macros, \symmetric Elow is the (abbreviated) snippet inside our \definiens block:

```
<mrow resource="" property="stex:definiens">
  <mrow resource="...?series?infinitesum" property="stex:OMBIND">
  <munderover displaystyle="true">
   <mo resource="...?series?infinitesum" property="stex:comp">∑</mo>
    <mrow resource="1" property="stex:arg">
     <mi resource="var://n" property="stex:OMV">n</mi>
    </mrow>
    <mo resource="...?series?infinitesum" property="stex:comp">=</mo>
    <mi resource="2" property="stex:arg">1</mi>
   <mi resource="...?series?infinitesum" property="stex:comp">\infty/mi>
  </munderover>
  <mrow resource="3" property="stex:arg">
   <mfrac resource="...?realarith?division#frac#" property="stex:OMA">
    <mi resource="1" property="stex:arg">1</mi>
<mrow resource="2" property="stex:arg">
  <msup resource="...realarith?exponentiation" property="stex:OMA">
      <mi resource="1" property="stex:arg">2</mi>
<mrow resource="2" property="stex:arg"></mi>
       <mi resource="var://n" property="stex:OMV">n</mi>
      </mrow>
     </msup>
    </mrow>
   </mfrac>
  </mrow>
 </mrow>
</mrow>
```

<sup>1...</sup>and hyperlinks for symbols, and indices, and allows reusing document fragments modularly, and...

...containing all the semantic information. The MMT system can extract from this the following OPENMATH snippet:

```
<OMBIND>
  <OMID name="...?series?infinitesum"/>
  <OMV name="n"/>
  <OMLIT name="1"/>
  <OMA>
      <OMS name="...?realarith?division"/>
      <OMLIT name="1"/>
      <OMA>
        <OMS name="...realarith?exponentiation"/>
        <OMLIT name="2"/>
        <OMLIT name="2"/>
        <OMV name="n"/>
      </OMA>
    </OMA>
  </OMBIND>
```

...giving us the full semantics of the snippet, allowing for a plurality of knowledge management services – in particular when serving the xhtml.

#### **Remark 2.2.2:**

Note that the html when opened in a browser will look slightly different than the pdf when it comes to highlighting semantic content – that is because naturally html allows for much more powerful features than pdf does. Consequently, the html is intended to be served by a system like MMT, which can pick up on the semantic information and offer much more powerful highlighting, linking and similar features, and being customizable by readers rather than being prescribed by an author.

Additionally, not all browsers (most notably Chrome) support MATHML natively, and might require additional external JavaScript libraries such as MathJax to render mathematical formulas properly.

# Chapter 3

# Creating STeX Content

We can use STEX by simply including the package with \usepackage{stex}, or - primarily for individual fragments to be included in other documents - by using the STEX document class with \documentclass{stex} which combines the standalone document class with the stex package.

Both the stex package and document class offer the following options:

lang  $(\langle language \rangle *)$  Languages to load with the babel package.

mathhub ( $\langle directory \rangle$ ) MathHub folder to search for repositories – this is not necessary if the MATHHUB system variable is set.

sms  $(\langle boolean \rangle)$  use persisted mode (not yet implemented).

image  $(\langle boolean \rangle)$  passed on to tikzinput.

**debug**  $(\langle log\text{-}prefix\rangle*)$  Logs debugging information with the given prefixes to the terminal, or all if all is given. Largely irrelevant for the majority of users.

# 3.1 How Knowledge is Organized in STEX

STFX content is organized on multiple levels:

- 1. STEX archives (see section 3.2) contain individual .tex-files.
- 2. These may contain STEX modules, introduced via \begin{smodule}{ModuleName}.
- 3. Modules contain STEX symbol declarations, introduced via \symdecl{symbolname}, \symdef{symbolname} and some other constructions. Most symbols have a notation that can be used via a semantic macro \symbolname generated by symbol declarations.
- 4. STeX expressions finally are built up from usages of semantic macros.



- STEX archives are simultaneously MMT archives, and the same directory structure is consequently used.
- STEX modules correspond to OMDoc/MMT theories. \importmodules (and



similar constructions) induce MMT includes and other theory morphisms, thus giving rise to a theory graph in the OMDOC sense [RK13].

- Symbol declarations induce OMDoc/MMT constants, with optional (formal) type and definiens components.
- Finally, STEX expressions are converted to OMDoc/MMT terms, which use the abstract syntax (and XML encoding) of OPENMATH [Bus+04].

# 3.2 STEX Archives

# 3.2.1 The Local MathHub-Directory

\userodule, \importmodule, \inputref etc. allow for including content modularly without having to specify absolute paths, which would differ between users and machines. Instead, STEX uses archives that determine the global namespaces for symbols and statements and make it possible for STEX to find content referenced via such URIs.

All STEX archives need to exist in the local MathHub-directory. STEX knows where this folder is via one of four means:

- 1. If the STEX package is loaded with the option mathhub=/path/to/mathhub, then STEX will consider /path/to/mathhub as the local MathHub-directory.
- 2. If the mathhub package option is *not* set, but the macro \mathhub exists when the \STEX-package is loaded, then this macro is assumed to point to the local MathHub-directory; i.e. \def\mathhub{/path/to/mathhub}\usepackage{stex} will set the MathHub-directory as path/to/mathhub.
- 3. Otherwise, STEX will attempt to retrieve the system variable MATHHUB, assuming it will point to the local MathHub-directory. Since this variant needs setting up only once and is machine-specific (rather than defined in tex code), it is compatible with collaborating and sharing tex content, and hence recommended.
- 4. Finally, if all else fails, STEX will look for a file ~/.stex/mathhub.path. If this file exists, STEX will assume that it contains the path to the local MathHub-directory. This method is recommended on systems where it is difficult to set environment variables.

## 3.2.2 The Structure of STeX Archives

An STEX archive group/name is stored in the directory /path/to/mathhub/group/name; e.g. assuming your local MathHub-directory is set as /user/foo/MathHub, then in order for the smglom/calculus-archive to be found by the STEX system, it needs to be in /user/foo/MathHub/smglom/calculus.

Each such archive needs two subdirectories:

- /source this is where all your tex files go.
- $\bullet\,$  /META-INF a directory containing a single file MANIFEST.MF, the content of which we will consider shortly

An additional lib-directory is optional, and is where STEX will look for files included via \\libinput.

Additionally a *group* of archives group/name may have an additional archive group/meta-inf. If this meta-inf-archive has a /lib-subdirectory, it too will be searched by \libinput from all tex files in any archive in the group/\*-group.

We recommend the following additional directory structure in the source-folder of an ST<sub>E</sub>X archive:

- /source/mod/ individual STEX modules, containing symbol declarations, notations, and \begin{sparagraph} [type=symdoc,for=...] environments for "encyclopaedic" symbol documentations
- /source/def/ definitions
- /source/ex/ examples
- /source/thm/ theorems, lemmata and proofs; preferably proofs in separate files to allow for multiple proofs for the same statement
- /source/snip/ individual text snippets such as remarks, explanations etc.
- /source/frag/ individual document fragments, ideally only \inputrefing snippets, definitions, examples etc. in some desirable order
- /source/tikz/ tikz images, as individual .tex-files
- /source/pic/ image files.<sup>3</sup>

#### 3.2.3 MANIFEST.MF-Files

The MANIFEST.MF in the META-INF-directory consists of key-value-pairs, informing  $ST_EX$  (and associated software) of various properties of an archive. For example, the MANIFEST.MF of the smglom/calculus-archive looks like this:

```
id: smglom/calculus
```

source-base: http://mathhub.info/smglom/calculus
narration-base: http://mathhub.info/smglom/calculus

dependencies: smglom/arithmetics, smglom/sets, smglom/topology,

smglom/mv,smglom/linear-algebra,smglom/algebra

responsible: Michael.Kohlhase@FAU.de

title: Elementary Calculus

teaser: Terminology for the mathematical study of change.

description: desc.html

Many of these are in fact ignored by ST<sub>E</sub>X, but some are important:

id: The name of the archive, including its group (e.g. smglom/calculus),

source-base or

ns: The namespace from which all symbol and module URIs in this repository are formed, see (TODO),

EdN:3

<sup>&</sup>lt;sup>3</sup>Ednote: MK: bisher habe ich immer PIC subdirs, soll ich das ändern?

narration-base: The namespace from which all document URIs in this repository are formed, see (TODO),

url-base: The URL that is formed as a basis for external references, see (TODO),

dependencies: All archives that this archive depends on. STEX ignores this field, but MMT can pick up on them to resolve dependencies, e.g. for lmh install.

# 3.2.4 Using Files in STEX Archives Directly

Several macros provided by STEX allow for directly including files in repositories. These are:

\mhinput

\mhinput [Some/Archive] {some/file} directly inputs the file some/file in the source-folder of Some/Archive.

\inputref

\inputref[Some/Archive]{some/file} behaves like \mhinput, but wraps the input in a \begingroup ... \endgroup. When converting to xhtml, the file is not input at all, and instead an html-annotation is inserted that references the file, e.g. for lazy loading. In the majority of practical cases \inputref is likely to be preferred over \mhinput because it leads to less duplication in the generated xhtml.

\ifinput

Both \mhinput and \inputref set \iffinput to "true" during input. This allows for selectively including e.g. bibliographies only if the current file is not being currently included in a larger document.

\addmhbibresource

\addmhbibresource [Some/Archive] {some/file} searches for a file like \mhinput does, but calls \addbibresource to the result and looks for the file in the archive root directory directly, rather than the source directory. Typical invocations are

- \addmhbibresource{lib/refs.bib}, which specifies a bibliography in the lib folder in the local archive or
- \addmhbibresource[HW/meta-inf]{lib/refs.bib} in another.

\libinput

\libinput{some/file} searches for a file some/file in

- the lib-directory of the current archive, and
- the lib-directory of a meta-inf-archive in (any of) the archive groups containing the current archive

and include all found files in reverse order; e.g. \libinput{preamble} in a .tex-file in smglom/calculus will first input .../smglom/meta-inf/lib/preamble.tex and then ../smglom/calculus/lib/preamble.tex.

\libinput will throw an error if no candidate for some/file is found.

\libusepackage

\lambda libusepackage [package-options] {some/file} searches for a file some/file.sty in the same way that \libinput does, but will call

\usepackage[package-options]{path/to/some/file} instead of \input.

\libusepackage throws an error if not exactly one candidate for some/file is found.

#### Remark 3.2.1:

Then the preamble.tex files can take care of loading the generally required packages, setting presentation customizations etc. (per archive or archive group or both), and postamble.tex can e.g. print the bibliography, index etc.

\lambda libusepackage is particularly useful in preamble.tex when we want to use custom packages that are not part of TeXLive. In this case we commit the respective packages in one of the lib folders and use \libusepackage to load them.

# 3.3 Module, Symbol and Notation Declarations

#### 3.3.1 The smodule-Environment

smodule A new module is declared using the basic syntax

```
\begin{smodule} [options] {ModuleName}...\end{smodule}.
```

A module is required to declare any new formal content such as symbols or notations (but not variables, which may be introduced anywhere).

The smodule-environment takes several keyword arguments, all of which are optional:

```
title (\langle token \ list \rangle) to display in customizations.
```

type  $(\langle string \rangle *)$  for use in customizations.

deprecate  $(\langle module \rangle)$  if set, will throw a warning when loaded, urging to use  $\langle module \rangle$  instead.

id  $(\langle string \rangle)$  for cross-referencing.

ns  $(\langle URI \rangle)$  the namespace to use. Should not be used, unless you know precisely what you're doing. If not explicitly set, is computed using  $\text{stex_modules_current_namespace:}$ .

lang  $(\langle language \rangle)$  if not set, computed from the current file name (e.g. foo.en.tex).

sig (\language\rangle) if the current file is a translation of a file with the same base name but a different language suffix, setting sig=<lang> will preload the module from that language file. This helps ensuring that the (formal) content of both modules is (almost) identical across languages and avoids duplication.

```
creators (\langle string \rangle *) names of the creators. contributors (\langle string \rangle *) names of contributors. srccite (\langle string \rangle) a source citation for the content of this module.
```

```
\begin{tabular}{ll} & \begin{tabular}{ll}
```

By default, opening a module will produce no output whatsoever, e.g.:

# Example 1

```
Input:
```

```
1 \begin{smodule}[title={This is Some Module}]{SomeModule}
2 Hello World
3 \end{smodule}
```

#### Output:

Hello World

\stexpatchmodule

We can customize this behavior either for all modules or only for modules with a specific type using the command \stexpatchmodule[optional-type]{begin-code}{end-code}. Some optional parameters are then available in \smodule\*-macros, specifically \smoduletitle, \smoduletype and \smoduleid.

For example:

#### Example 2

```
Input:
```

```
1 \stexpatchmodule[display]
2 {\textbf{Module (\smoduletitle)}\par}
3 {\par\noindent\textbf{End of Module (\smoduletitle)}}
5 \begin{smodule}[type=display,title={Some New Module}]{SomeModule2}
6 Hello World
7 \end{smodule}
```

Output:

```
Module (Some New Module)

Hello World

End of Module (Some New Module)
```

.

## 3.3.2 Declaring New Symbols and Notations

Inside an smodule environment, we can declare new STFX symbols.

\symdecl

The most basic command for doing so is using \symdecl{symbolname}. This introduces a new symbol with name symbolname, arity 0 and semantic macro \symbolname.

The starred variant \symdecl\*{symbolname} will declare a symbol, but not introduce a semantic macro. If we don't want to supply a notation (for example to introduce concepts like "abelian", which is not something that has a notation), the starred variant is likely to be what we want.

Without a semantic macro or a notation, the only meaningful way to reference a symbol is via \symref,\symname etc.

```
Example 3
Input:

1 \symdecl*{foo}
2 Given a \symname{foo}, we can...

Output:

Given a foo, we can...
```

.

Obviously, most semantic macros should take actual *arguments*, implying that the symbol we introduce is an *operator* or *function*. We can let \symdecl know the *arity* (i.e. number of arguments) of a symbol like this:

```
Example 4
Input:

1 \symdecl{binarysymbol}[args=2]
2 \symref{binarysymbol}{this} is a symbol taking two arguments.

Output:

this is a symbol taking two arguments.
```

.

So far we have gained exactly  $\dots$  nothing by adding the arity information: we cannot do anything with the arguments in the text.

We will now see what we can gain with more machinery.

\notation

We probably want to supply a notation as well, in which case we can finally actually use the semantic macro in math mode. We can do so using the **\notation** command, like this:

```
this: Example 5
```

## Input:

```
1 \notation{binarysymbol}{\text{First: }#1\text{; Second: }#2} $\binarysymbol{a}{b}$
```

#### Output:

```
First: a; Second: b
```

.

\comp

For many semantic services e.g. semantic highlighting or **wikification** (linking uservisible notation components to the definition of the respective symbol they come from), we need to specify the notation components. Unfortunately, there is currently no way the STEX engine can infer this by itself, so we have to specify it manually in the notation specification. We can do so with the \comp command.

We can introduce a new notation highlight for \binarysymbol that fixes this flaw, which we can subsequently use with \binarysymbol[highlight]:

# Example 6

#### Input:

```
1 \notation{binarysymbol}[highlight]
2 {\comp{\text{First: }}#1\comp{\text{; Second: }}#2}
3 $\binarysymbol[highlight]{a}{b}$
```

#### Output:

```
First: a; Second: b
```



Ideally, \comp would not be necessary: Everything in a notation that is not an argument should be a notation component. Unfortunately, it is computationally expensive to determine where an argument begins and ends, and the argument markers #n may themselves be nested in other macro applications or TeX groups, making it ultimately almost impossible to determine them automatically while also remaining compatible with arbitrary highlighting customizations (such as tooltips, hyperlinks, colors) that users might employ, and that are ultimately invoked by \comp.

Note that it is required that

- 1. the argument markers #n never occur inside a \comp, and
- 2. no semantic arguments may ever occur inside a notation.

Both criteria are not just required for technical reasons, but conceptionally meaningful:

The underlying principle is that the arguments to a semantic macro represent arguments to the mathematical operation represented by a symbol. For example, a semantic macro  $\addition\{a\}\{b\}$  taking two arguments would represent the actual addition of (mathematical objects) a and b. It should therefore be impossible for a or b to be part of a notation component of  $\addition$ .



Similarly, a semantic macro can not conceptually be part of the notation of \addition, since a semantic macro represents a distinct mathematical concept with its own semantics, whereas notations are syntactic representations of the very symbol to which the notation belongs.

If you want an argument to a semantic macro to be a purely syntactic parameter, then you are likely somewhat confused with respect to the distinction between the precise syntax and semantics of the symbol you are trying to declare (which happens quite often even to experienced STEX users), and might want to give those another thought - quite likely, the macro you aim to implement does not actually represent a semantically meaningful mathematical concept, and you will want to use \def and similar native LATEX macro definitions rather than semantic macros.

\symdef

In the vast majority of cases where a symbol declaration should come with a semantic macro, we will want to supply a notation immediately. For that reason, the \symdef command combines the functionality of both \symdecl and \notation with the optional arguments of both:

```
Example 7
```

```
Input:
```

```
1 \symdef{newbinarysymbol}[h1,args=2]
2     {\comp{\text{1.: }}#1\comp{\text{; 2.: }}#2}
3 $\newbinarysymbol{a}{b}$
```

Output:

```
1.: a; 2.: b
```

We just declared a new symbol newbinarysymbol with args=2 and immediately provided it with a notation with identifier hl. Since hl is the *first* (and so far, only) notation supplied for newbinarysymbol, using \newbinarysymbol without optional argument defaults to this notation.

But one man's meat is another man's poison: it is very subjective what the "default notation" of an operator should be. Different communities have different practices. For instance, the complex unit is written as i in Mathematics and as j in electrical engineering.

So to allow modular specification and facilitate re-use of document fragments  $ST_EX$  allows to re-set notation defaults.

\setnotation

The first notation provided will stay the default notation unless explicitly changed — this is enabled by the \setnotation command: \setnotation{symbolname}{notation-id} sets the default notation of \symbolname to notation-id, i.e. henceforth, \symbolname behaves like \symbolname[notation-id] from now on.

Often, a default notation is set right after the corresponding notation is introduced – the starred version \notation\* for that reason introduces a new notation and immediately sets it to be the new default notation. So expressed differently, the first \notation for a symbol behaves exactly like \notation\*, and \notation\*{foo}[bar]{...} behaves exactly like \notation{foo}{bar}.

#### **Operator Notations**

Once we have a semantic macro with arguments, such as *\newbinarysymbol*, the semantic macro represents the *application* of the symbol to a list of arguments. What if we want to refer to the operator *itself*, though?

We can do so by supplying the \notation (or \symdef) with an operator notation, indicated with the optional argument op=. We can then invoke the operator notation using \symbolname! [notation-identifier]. Since operator notations never take arguments, we do not need to use \comp in it, the whole notation is wrapped in a \comp automatically:

## 3.3.3 Argument Modes

directly.

The notations so far used <code>simple</code> arguments which we call <code>mode-i</code> arguments. Declaring a new symbol with <code>\symdecl{foo}[args=3]</code> is equivalent to writing <code>\symdecl{foo}[args=iii]</code>, indicating that the semantic macro takes three mode-i arguments. However, there are three more argument modes which we will investigate now, namely mode-b, mode-a and mode-B arguments.

#### Mode-b Arguments

A mode-b argument represents a variable that is bound by the symbol in its application, making the symbol a binding operator. Typical examples of binding operators are e.g. sums  $\sum$ , products  $\prod$ , integrals  $\int$ , quantifiers like  $\forall$  and  $\exists$ , that  $\lambda$ -operator, etc.

```
—M→ Mode-b arguments behave exactly like mode-i arguments within TeX, but appli—M→ cations of binding operators, i.e. symbols with mode-b arguments, are translated ~T → to OMBIND-terms in OMDoc/MMT, rather than OMA.
```

For example, we can implement a summation operator binding an index variable and taking lower and upper index bounds and the expression to sum over like this:

where the variable x is now bound by the \summation-symbol in the expression.

### Mode-a Arguments

Mode-a arguments represent a *flexary argument sequence*, i.e. a sequence of arguments of arbitrary length. Formally, operators that take arbitrarily many arguments don't "exist", but in informal mathematics, they are ubiquitous. Mode-a arguments allow us to write e.g. \addition{a,b,c,d,e} rather than having to write something like \addition{a}{\addition{b}{\addition{b}}}!

\notation (and consequently \symdef, too) take one additional argument for each mode-a argument that indicates how to "accumulate" a comma-separated sequence of arguments. This is best demonstrated on an example.

Let's say we want an operator representing quantification over an ascending chain of elements in some set, i.e.  $\ascendingchain{S}{a,b,c,d,e}{t}$  should yield  $\forall a < sb < sc < sd < se$ . The "base"-notation for this operator is simply

 ${\comp{\forall} #2\comp{.\,}#3}, where #2 represents the full notation fragment $accumulated from {a,b,c,d,e}.}$ 

The additional argument to \notation (or \symdef) takes the same arguments as the base notation and two additional arguments ##1 and ##2 representing successive pairs in the mode-a argument, and accumulates them into #2, i.e. to produce  $a <_S b <_S c <_S d <_S e$ , we do {##1 \comp{<}\_{#1} ##2}:

```
Example 10 Input:
```

```
1 \symdef{ascendingchain}[args=iai]
2 {\comp{\forall} #2\comp{.\,}#3}
3 {##1 \comp{<}_{#1} ##2}
4
5 Tadaa: $\ascendingchain{S}{a,b,c,d,e}{t}$</pre>
```

Output:

```
Tadaa: \forall a <_S b <_S c <_S d <_S e.t
```

If this seems overkill, keep in mind that you will rarely need the single-hash arguments #1,#2 etc. in the a-notation-argument. For a much more representative and simpler example, we can introduce flexary addition via:

#### Example 11

```
Input:

1 \symdef{addition}[args=a]{#1}{##1 \comp{+} ##2}
2 3 Tadaa: $\addition{a,b,c,d,e}$

Output:
```

Tadaa: a+b+c+d+e

**The assoc-key** We mentioned earlier that "formally", flexary arguments don't really "exist". Indeed, formally, addition is usually defined as a binary operation, quantifiers bind a single variable etc.

Consequently, we can tell STEX (or, rather, MMT/OMDoc) how to "resolve" flexary arguments by providing \symdecl or \symdef with an optional assoc-argument, as in \symdecl{addition}[args=a,assoc=bin]. The possible values for the assoc-key are:

bin: A binary, associative argument, e.g. as in \addition

binl: A binary, left-associative argument, e.g.  $a^{b^{c^d}}$ , which stands for  $((a^b)^c)^d$ 

binr: A binary, right-associative argument, e.g. as in  $A \to B \to C \to D$ , which stands for  $A \to (B \to (C \to D))$ 

**pre**: Successively prefixed, e.g. as in  $\forall x, y, z. P$ , which stands for  $\forall x. \forall y. \forall z. P$ 

conj: Conjunctive, e.g. as in a=b=c=d or  $a,b,c,d\in A$ , which stand for  $a=d\wedge b=d\wedge c=d$  and  $a\in A\wedge b\in A\wedge c\in A\wedge d\in A$ , respectively

pwconj: Pairwise conjunctive, e.g. as in  $a \neq b \neq c \neq d$ , which stands for  $a \neq b \land a \neq c \land a \neq d \land b \neq c \land b \neq d \land c \neq d$ 

As before, at the PDF level, this annotation is invisible (and without effect), but at the level of the generated OMDoc/MMT this leads to more semantical expressions.

#### Mode-B Arguments

Finally, mode-B arguments simply combine the functionality of both a and b - i.e. they represent an arbitrarily long sequence of variables to be bound, e.g. for implementing quantifiers:

#### Example 12

```
Input:

1 \symdef{quantforall}[args=Bi]
2 {\comp{\forall}#1\comp{.}#2}
3 {##1\comp,##2}
4
5 $\quantforall{\svar{x},\svar{y},\svar{z}}{P}$
```

#### Output:

```
\forall x,y,z.P
```

# 3.3.4 Type and Definiens Components

\symdecl and \symdef take two more optional arguments. TEX largely ignores them (except for special situations we will talk about later), but MMT can pick up on them for additional services. These are the type and def keys, which expect expressions in math-mode (ideally using semantic macros, of course!)

```
The type and def keys correspond to the type and definiens components of 

M > OMDoc/Mmt constants.

M > Correspondingly, the name "type" should be taken with a grain of salt, since 

OMDoc/Mmt being foundation-independent – does not a priori implement a fixed typing system.
```

The type-key allows us to provide additional information (given the necessary STEX symbols), e.g. for addition on natural numbers:

# Example 13

Input:

```
1 \symdef{Nat}[type=\set]{\comp{\mathbb N}}
2 \symdef{addition}[
3     type=\funtype{\Nat,\Nat}{\Nat},
4     op=+,
5     args=a
6 ]{#1}{##1 \comp+ ##2}
7
8 \symname{addition} is an operation $\funtype{\Nat,\Nat}{\Nat}$
```

## Output:

```
addition is an operation \mathbb{N}{\times}\mathbb{N}{\to}\mathbb{N}
```

.

The def-key allows for declaring symbols as abbreviations:

## Example 14

Input:

```
1 \symdef{successor}[
2    type=\funtype{\Nat}{\Nat},
3    def=\fun{\svar{x}}{\addition{\svar{x},1}},
4    op=\mathtt{succ},
5    args=1
6 ]{\comp{\mathtt{succ(}#1\comp{)}}}
7
8 The \symname{successor} operation $\funtype{\Nat}{\Nat}$
9 is defined as $\fun{\svar{x}}{\addition{\svar{x},1}}$
```

Output:

```
The successor operation \mathbb{N} \to \mathbb{N} is defined as x \mapsto x+1
```

# 3.3.5 Precedences and Automated Bracketing

Having done  $\addition$ , the obvious next thing to implement is  $\mbox{\it multiplication}$ . This is straight-forward in theory:

# Example 15

```
Input:
```

```
1 \symdef{multiplication}[
2  type=\funtype{\Nat,\Nat}{\Nat},
3  op=\cdot,
4  args=a
5 ]{#1}{##1 \comp\cdot ##2}
6
7 \symname{multiplication} is an operation $\funtype{\Nat,\Nat}{\Nat}$
```

#### Output:

```
multiplication is an operation \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}
```

However, if we *combine* \addition and \multiplication, we notice a problem:

## Example 16

Input:

```
1 \addition{a,\multiplication{b,\addition{c,\multiplication{d,e}}}} \\
```

Output:

```
a+b\cdot c+d\cdot e
```

.

We all know that  $\cdot$  binds stronger than +, so the output  $a+b\cdot c+d\cdot e$  does not actually reflect the term we wrote. We can of course insert parentheses manually

```
Example 17 Input:  1 \addition{a, \multiplication{b, (\addition{c, \multiplication{d,e}})}}  Output:  a+b\cdot(c+d\cdot e)
```

but we can also do better by supplying *precedences* and have ST<sub>E</sub>X insert parentheses automatically.

For that purpose, \notation (and hence \symdef) take an optional argument prec=<opprec>;<argprec1>x...x<argprec n>.

We will investigate the precise meaning of <opprec> and the <argprec>s shortly – in the vast majority of cases, it is perfectly sufficient to think of prec= taking a single number and having that be *the* precedence of the notation, where lower precedences (somewhat counterintuitively) bind stronger than higher precedences. So fixing our notations for \addition and \multiplication, we get:

#### Example 18

Input:

```
1 \notation{multiplication}[
2    op=\cdot,
3    prec=50
4]{#1}{##1 \comp\cdot ##2}
5 \notation{addition}[
6    op=+,
7    prec=100
8]{#1}{##1 \comp+ ##2}
9
10 $\addition{a,\multiplication{b,\addition{c,\multiplication{d,e}}}}$
```

Output:

```
a + b \cdot (c + d \cdot e)
```

.

Note that the precise numbers used for precedences are pretty arbitrary - what matters is which precedences are higher than which other precedences when used in conjunction.

\infprec \neginfprec

It is occasionally useful to have "infinitely" high or low precedences to enforce or forbid automated bracketing entirely – for those purposes, \infprec and \neginfprec exist (which are implemented as the maximal and minimal integer values accordingly).

More precisely, each notation takes

- 1. One operator precedence and
- 2. one argument precedence for each argument.

By default, all precedences are 0, unless the symbol takes no argument, in which case the operator precedence is \neginfprec (negative infinity). If we only provide a single number, this is taken as both the operator precedence and all argument precedences.

STEX decides whether to insert parentheses by comparing operator precedences to a downward precedence  $p_d$  with initial value \infprec. When encountering a semantic macro, STEX takes the operator precedence  $p_{op}$  of the notation used and checks whether  $p_{op} > p_d$ . If so, STEX insert parentheses.

When SIEX steps into an argument of a semantic macro, it sets  $p_d$  to the respective argument precedence of the notation used.

In the example above:

- 1. STEX starts out with  $p_d = \$
- 2. STeX encounters \addition with  $p_{op} = 100$ . Since  $100 \not>$ \infprec, it inserts no parentheses.
- 3. Next, STeX encounters the two arguments for \addition. Both have no specifically provided argument precedence, so STeX uses  $p_d = p_{op} = 100$  for both and recurses.
- 4. Next, STEX encounters \multiplication{b,...}, whose notation has  $p_{op} = 50$
- 5. We compare to the current downward precedence  $p_d$  set by \addition, arriving at  $p_{op} = 50 > 100 = p_d$ , so SIEX again inserts no parentheses.
- 6. Since the notation of \multiplication has no explicitly set argument precedences, STEX uses the operator precedence for all arguments of \multiplication, hence sets  $p_d = p_{op} = 50$  and recurses.
- 7. Next, STEX encounters the inner \addition{c,...} whose notation has  $p_{op} = 100$ .
- 8. We compare to the current downward precedence  $p_d$  set by \multiplication, arriving at  $p_{op} = 100 > 50 = p_d$  which finally prompts STEX to insert parentheses, and we proceed as before.

#### 3.3.6 Variables

All symbol and notation declarations require a module with which they are associated, hence the commands \symdecl, \notation, \symdef etc. are disabled outside of smodule-environments.

Variables are different – variables are allowed everywhere, are not exported when the current module (if one exists) is imported (via \importmodule or \usemodule) and (also unlike symbol declarations) "disappear" at the end of the current TeX group.

\svar

So far, we have always used variables using  $\operatorname{n}$ , which marks-up n as a variable with name n. More generally,  $\operatorname{svar}[foo]$  (<texcode>) marks-up the arbitrary <texcode> as representing a variable with name foo.

Of course, this makes it difficult to reuse variables, or introduce "functional" variables with arities > 0, or provide them with a type or definiens.

\vardef

For that, we can use the \vardef command. Its syntax is largely the same as that of \symdef, but unlike symbols, variables have only one notation (TODO: so far?), hence there is only \vardef and no \vardecl.

# Example 19 Input:

```
1 \vardef{varf}[
2     name=f,
3     type=\funtype{\Nat}{\Nat},
4     op=f,
5     args=1,
6     prec=0;\neginfprec
7 ]{\comp{f}#1}
8 \vardef{varn} [name=n,type=\Nat]{\comp{n}}
9 \vardef{varx} [name=x,type=\Nat]{\comp{x}}
10
11 Given a function $\varf!:\funtype{\Nat}{\Nat}$,
12 by $\addition{\varf!,\varn}$ we mean the function
13 $\fun{\varx}{\varf}\addition{\varx,\varn}}$
```

Output:

```
Given a function f: \mathbb{N} \to \mathbb{N}, by f+n we mean the function x \mapsto f(x+n)
```

(of course, "lifting" addition in the way described in the previous example is an operation that deserves its own symbol rather than abusing  $\addition$ , but... well.)

TODO: bind=forall/exists

## 3.3.7 Variable Sequences

Variable sequences occur quite frequently in informal mathematics, hence they deserve special support. Variable sequences behave like variables in that they disappear at the end of the current TEX group and are not exported from modules, but their declaration is quite different.

\varseq

A variable sequence is introduced via the command \warseq, which takes the usual optional arguments name and type. It then takes a starting index, an end index and a notation for the individual elements of the sequence parametric in an index. Note that both the starting as well as the ending index may be variables.

This is best shown by example:

Example 20 Input:

```
1 \vardef{varn}[name=n,type=\Nat]{\comp{n}}
2 \varseq{seqa}[name=a,type=\Nat]{1}{\varn}{\comp{a}_{#1}}
3
4 The $i$th index of $\seqa!$ is $\seqa{i}$.
```

Output:

```
The ith index of a_1, \ldots, a_n is a_i.
```

.

Note that the syntax \seqa! now automatically generates a presentation based on the starting and ending index.

TODO: more notations for invoking sequences.

Notably, variable sequences are nicely compatible with a-type arguments, so we can do the following:

# Example 21

Input:

```
1 $\addition{\seqa}$
```

Output:

```
a_1 + \ldots + a_n
```

.

Sequences can be multidimensional using the args-key, in which case the notation's arity increases and starting and ending indices have to be provided as a comma-separated list:

#### Example 22

```
Input:
```

```
1 \vardef{varm}[name=m,type=\Nat]{\comp{m}}
2 \varseq{seqa}[
3     name=a,
4     args=2,
5     type=\Nat,
6 ]{1,1}{\varm,\varm}{\comp{a}_{#1}^{#2}}
7
8 $\seqa!$ and $\addition{\seqa}$
```

Output:

```
a_1^1, \ldots, a_n^m and a_1^1 + \ldots + a_n^m
```

'We can also explicitly provide a "middle" segment to be used, like such:

# Example 23

Input:

```
1 \varseq{seqa}[
2    name=a,
3    type=\Nat,
4    args=2,
5    mid={\comp{a}_{\varn}^1,\comp{a}_1^2,\ellipses,\comp{a}_{1}^{\varn}}
6]{1,1}{\varn,\varm}{\comp{a}_{\alpha}^{\#1}^{\#2}}
7
8 $\seqa!$ and $\addition{\seqa}$
```

Output:

```
a_1^1, \dots, a_n^1, a_1^2, \dots, a_1^m, \dots, a_n^m \text{ and } a_1^1 + \dots + a_n^1 + a_1^2 + \dots + a_n^m + \dots + a_n^m
```

## 3.4 Module Inheritance and Structures

The STEX features for modular document management are inherited from the OM-Doc/MMT model that organizes knowledge into a graph, where the nodes are theories (called modules in STEX) and the edges are truth-preserving mappings (called theory morphismes in MMT). We have already seen modules/theories above.

Before we get into theory morphisms in STEX we will see a very simple application of modules: managing multilinguality modularly.

# 3.4.1 Multilinguality and Translations

If we load the STeX document class or package with the option lang=<lang>, STeX will load the appropriate babel language for you – e.g. lang=de will load the babel language ngerman. Additionally, it makes STeX aware of the current document being set in (in this example) german. This matters for reasons other than mere babel-purposes, though:

Every module is assigned a language. If no STEX package option is set that allows for inferring a language, STEX will check whether the current file name ends in e.g. .en.tex (or .de.tex or .fr.tex, or...) and set the language accordingly. Alternatively, a language can be explicitly assigned via \begin{smodule}[lang=<language>]{Foo}.

```
Technically, each smodule-environment induces two OMDoc/MMT theories: \begin{smodule}[lang=<lang>]{Foo} generates a theory some/namespace?Foo that only contains the "formal" part of the module – i.e. exactly the content—M→ that is exported when using \importmodule.

TAMAGINATION Additionally, MMT generates a language theory some/namespace/Foo?<lang> that includes some/namespace?Foo and contains all the other document content – variable declarations, includes for each \usenbodule, etc.
```

Notably, the language suffix in a filename is ignored for \usemodule, \importmodule and in generating/computing URIs for modules. This however allows for providing translations for modules between languages without needing to duplicate content:

If a module Foo exists in e.g. english in a file Foo.en.tex, we can provide a file Foo.de.tex right next to it, and write \begin{smodule}[sig=en]{Foo}. The sig-key

then signifies, that the "signature" of the module is contained in the *english* version of the module, which is immediately imported from there, just like \importmodule would.

Additionally to translating the informal content of a module file to different languages, it also allows for customizing notations between languages. For example, the least common multiple of two numbers is often denoted as lcm(a,b) in english, but is called kleinstes gemeinsames Vielfaches in german and consequently denoted as kgV(a,b) there.

We can therefore imagine a german version of an lcm-module looking something like this:

```
1 \begin{smodule}[sig=en]{lcm}
2 \notation*{lcm}[de]{\comp{\mathtt{kgV}}(#1,#2)}
3
4 Das \symref{lcm}{kleinste gemeinsame Vielfache}
5 $\lcm{a,b}$ von zwei Zahlen $a,b$ ist...
6 \end{smodule}
```

If we now do \importmodule{lcm} (or \usemodule{lcm}) within a german document, it will also load the content of the german translation, including the de-notation for \lcm.

#### 3.4.2 Simple Inheritance and Namespaces

\importmodule \usemodule

\importmodule[Some/Archive] {path?ModuleName} is only allowed within an smodule-environment and makes the symbols declared therein available. Additionally the content of ModuleName will be exported if the current module is imported somewhere else via \importmodule.

\userbound usemodule behaves the same way, but without exporting the content of the used module.

It is worth going into some detail how exactly \importmodule and \usemodule resolve their arguments to find the desired module – which is closely related to the namespace generated for a module, that is used to generate its URI.

Ideally, STeX would use arbitrary URIs for modules, with no forced relationships between the *logical* namespace of a module and the *physical* location of the file declaring the module – like MMT does things.

Unfortunately, TEX only provides very restricted access to the file system, so we are forced to generate namespaces systematically in such a way that they reflect the physical location of the associated files, so that SIEX can resolve them accordingly. Largely, users need not concern themselves with namespaces at all, but for completenesses sake, we describe how they are constructed:



- If  $\begin{smodule}{foo} occurs in a file /path/to/file/Foo[.<math>\langle lang \rangle$ ].tex which does not belong to an archive, the namespace is file://path/to/file.
- If the same statement occurs in a file /path/to/file/bar[. \(\lang\rang\rang\right)\]. tex, the namespace is file://path/to/file/bar.

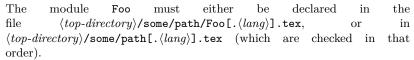
In other words: outside of archives, the namespace corresponds to the file URI with the filename dropped iff it is equal to the module name, and ignoring the (optional) language suffix.



If the current file is in an archive, the procedure is the same except that the initial segment of the file path up to the archive's **source**-folder is replaced by the archive's namespace URI.

Conversely, here is how namespaces/URIs and file paths are computed in import statements, examplary \importmodule:

- \importmodule{Foo} outside of an archive refers to module Foo in the current namespace. Consequently, Foo must have been declared earlier in the same document or, if not, in a file Foo[.\langle].tex in the same directory.
- The same statement within an archive refers to either the module Foo declared earlier in the same document, or otherwise to the module Foo in the archive's top-level namespace. In the latter case, is has to be declared in a file Foo[. $\langle lang \rangle$ ].tex directly in the archive's source-folder.
- Similarly, in \importmodule{some/path?Foo} the path some/path refers to either the sub-directory and relative namespace path of the current directory and namespace outside of an archive, or relative to the current archive's top-level namespace and source-folder, respectively.



- Similarly, \importmodule[Some/Archive]{some/path?Foo} is resolved like the previous cases, but relative to the archive Some/Archive in the mathhub-directory.
- Finally, \importmodule{full://uri?Foo} naturally refers to the module Foo in the namespace full://uri. Since the file this module is declared in can not be determined directly from the URI, the module must be in memory already, e.g. by being referenced earlier in the same document. Since this is less compatible with a modular development, using full URIs directly is strongly discouraged, unless the module is delared in the current file directly.

\STEXexport

\importmodule and \usemodule import all symbols, notations, semantic macros and (recursively) \importmodules. If you want to additionally export e.g. convenience macros and other code from a module, you can use the command \STEXexport{<code>} in your module. Then <code> is executed (both immediately and) every time the current module is opened via \importmodule or \usemodule.



Note, that **\newcommand** defines macros *globally* and throws an error if the macro already exists, potentially leading to low-level LATEX errors if we put a **\newcommand** in an **\STEXexport** and the **<code>** is executed more than once in a document – which can happen easily.

A safer alternative is to use macro definition principles, that are safe to use even if the macro being defined already exists, and ideally are local to the current  $T_EX$ 

## 3.4.3 The mathstructure Environment

A common occurrence in mathematics is bundling several interrelated "declarations" together into *structures*. For example:

- A monoid is a structure  $\langle M, \circ, e \rangle$  with  $\circ : M \times M \to M$  and  $e \in M$  such that...
- A topological space is a structure  $\langle X, \mathcal{T} \rangle$  where X is a set and  $\mathcal{T}$  is a topology on X
- A partial order is a structure  $\langle S, \leq \rangle$  where  $\leq$  is a binary relation on S such that...

This phenomenon is important and common enough to warrant special support, in particular because it requires being able to *instantiate* such structures (or, ratherer, structure *signatures*) in order to talk about (concrete or variable) *particular* monoids, topological spaces, partial orders etc.

mathstructure

The mathstructure environment allows us to do exactly that. It behaves exactly like the smodule environment, but is itself only allowed inside an smodule environment, and allows for instantiation later on.

How this works is again best demonstrated by example:

#### Example 24

Input:

```
1 \begin{mathstructure} {monoid}
2 \symdef {universe} [type=\set] {\comp{U}}
3 \symdef {op}[
4 \args=2,
5 \type=\funtype{\universe,\universe} {\universe},
6 \op=\circ
7 ]{#1 \comp{\circ} #2}
8 \symdef {unit} [type=\universe] {\comp{e}}
9 \end{mathstructure}
10
11 A \symname{monoid} is...
```

Output:

```
A monoid is...
```

Note that the \symname{monoid} is appropriately highlighted and (depending on your pdf viewer) shows a URI on hovering – implying that the mathstructure environment has generated a *symbol* monoid for us. It has not generated a semantic macro though, since we can not use the monoid-symbol *directly*. Instead, we can instantiate it, for example for integers:

```
Example 25 Input:
```

```
1 \symdef{Int}[type=\set]{\comp{\mathbb Z}}
2 \symdef{addition}[
3     type=\funtype{\Int,\Int}{\Int},
4     args=2,
5     op=+
6 ]{##1 \comp{+} ##2}
7 \symdef{zero}[type=\Int]{\comp{0}}
8
9 $\mathstruct{\Int,\addition!,\zero}$ is a \symname{monoid}.
```

#### Output:

```
\langle \mathbb{Z}, +, 0 \rangle is a monoid.
```

So far, we have not actually instantiated monoid, but now that we have all the symbols to do so, we can:

#### Example 26

```
Input:
```

```
1 \instantiate{intmonoid}{
2    universe = Int ,
3    op = addition ,
4    unit = zero
5 }{monoid}{\mathbb{Z}_{+,0}}
6
7    $\intmonoid{\universe}$, $\intmonoid{\unit}$ and $\intmonoid{\op}{a}{b}$.
8
9    Also: $\intmonoid!$
```

#### Output:

```
\mathbb{Z}, 0 and a+b.
Also: \mathbb{Z}_{+,0}
```

\instantiate

So summarizing: \instantiate takes four arguments: The (macro-)name of the instance, a key-value pair assigning declarations in the corresponding mathstructure to symbols currently in scope, the name of the mathstructure to instantiate, and lastly a notation for the instance itself.

It then generates a semantic macro that takes as argument the name of a declaration in the instantiated mathstructure and resolves it to the corresponding instance of that particular declaration.

```
\instantiate and mathstructure make use of the Theories-as-Types paradigm:

\[ \bigcup_M \rightarrow \text{mathstructure} \{ \cap does in fact simply create a nested theory with name \]

\[ -\bigcup_M \rightarrow \text{rame} \rightarrow \text{structure} \]

\[ -\bigcup_N \rightarrow \text{ame} \rightarrow \text{structure} \]

\[ -\bigcup_N \rightarrow \text{ame} \rightarrow \text{structure} \]

\[ \alpha \text{T} \rightarrow \text{adependent record type with manifest fields}, \text{ the fields of which are generated from (and correspond to) the constants in \( \cap \text{name} \rightarrow \text{structure}. \)
```

```
\begin{tabular}{ll} $\longleftarrow M$ & \label{table_interpolate} $$ \mbox{`instantiate appropriately generates a constant whose definiens is a record term $$ $-M$ & of type Mod(\name>-structure), with the fields assigned appropriately based on $$ $\sim T$ & the key-value-list. \end{tabular}
```

Notably, \instantiate throws an error if not every declaration in the instantiated mathstructure is being assigned.

You might consequently ask what the usefulness of mathstructure even is.

#### \varinstantiate

The answer is that we can also instantiate a mathstructure with a *variable*. The syntax of \varianstantiate is equivalent to that of \instantiate, but all of the key-value-pairs are optional, and if not explicitly assigned (to a symbol *or* a variable declared with \vardef) inherit their notation from the one in the mathstructure environment.

This allows us to do things like:

```
Input:

1  \varinstantiate{varM}{}{monoid}{M}
2
3 A \symname{monoid} is a structure
4 $\varM!:=\mathstruct{\varM{universe},\varM{op}!,\varM{unit}}$
5 such that
6 $\varM{op}!:\funtype{\varM{universe}},\varM{universe}}$
7 and
```

9 \varinstantiate{varMb}{universe = Int}{monoid}{M\_2}

11 \noindent Let  $\$  \varMb!:=\mathstruct{\varMb{universe},\varMb{op}!,\varMb{unit}}\$ 12 a \symname{monoid} on \$\Int\$...

#### Output:

Example 27

```
A monoid is a structure M := \langle U, \circ, e \rangle such that \circ : U \times U \to U and...
Let M_2 := \langle \mathbb{Z}, \circ, e \rangle a monoid on \mathbb{Z}...
```

We will return to this example later, when we also know how to handle the *axioms* of a monoid.

#### 3.4.4 The copymodule Environment

# TODO: explain Given modules: Example 28 Input:

```
1 \begin{smodule}{magma}
2 \symdef{universe}{\comp{\mathcal U}}
3 \symdef{operation}[args=2,op=\circ]{#1 \comp\circ #2}
4 \end{smodule}
5 \begin{smodule}{monoid}
6 \importmodule{magma}
7 \symdef{unit}{\comp e}
8 \end{smodule}
9 \begin{smodule}{group}
10 \importmodule{monoid}
11 \symdef{inverse}[args=1]{{#1}^{\comp{-1}}}
12 \end{smodule}

Output:
```

We can form a module for *rings* by "cloning" an instance of <code>group</code> (for addition) and <code>monoid</code> (for multiplication), respectively, and "glueing them together" to ensure they share the same universe:

#### Example 29

```
Input:
   \begin{smodule}{ring}
 2
      \begin{copymodule}{group}{addition}
3
          \renamedecl[name=universe] {universe} {runiverse}
          \renamedecl[name=plus]{operation}{rplus}
 4
 5
          \renamedecl[name=zero]{unit}{rzero}
 6
          \renamedecl[name=uminus]{inverse}{ruminus}
 7
      \end{copymodule}
      \notation*{rplus}[plus,op=+,prec=60]{#1 \comp+ #2}
9
          \notation*{rzero}[zero]{\comp0}
10
          \notation*{ruminus}[uminus,op=-]{\comp- #1}
11
          \begin{copymodule}{monoid}{multiplication}
12
          \assign{universe}{\runiverse}
13
          \renamedecl[name=times] {operation}{rtimes}
14
          \renamedecl[name=one] {unit}{rone}
15
      \end{copymodule}
16
      \notation*{rtimes}[cdot,op=\cdot,prec=50]{#1 \comp\cdot #2}
17
          \notation*{rone}[one]{\comp1}
18
          Test: $\rtimes a{\rplus c{\rtimes de}}$
19 \end{smodule}
```

Output:

```
Test: a \cdot (c + d \cdot e)
```

TODO: explain donotclone

#### 3.4.5 The interpret module Environment

TODO: explain

# Input: 1 \begin{smodule}{int} 2 \symdef{Integers}{\comp{\mathbb Z}} 3 \symdef{plus}[args=2,op=+]{#1 \comp+ #2} 4 \symdef{zero}{\comp0} 5 \symdef{uminus}[args=1,op=-]{\comp-#1} 6 7 \begin{interpretmodule}{group}{intisgroup} 8 \assign{universe}{\Integers} 9 \assign{operation}{\plus!} 10 \assign{unit}{\zero} 11 \assign{inverse}{\uminus!} 12 \end{interpretmodule} 13 \end{smodule} Output:

# 3.5 Primitive Symbols (The STEX Metatheory)

TODO: metatheory documentation

# Using STEX Symbols

Given a symbol declaration \symdecl{symbolname}, we obtain a semantic macro \symbolname. We can use this semantic macro in math mode to use its notation(s), and we can use \symbolname! in math mode to use its operator notation(s). What else can we do?

#### 4.1 \symmet and its variants

\symref \symname

We have already seen \symname and \symref, the latter being the more general.

\symref{<symbolname>}{<code>} marks-up <code> as referencing <symbolname>. Since quite often, the <code> should be (a variant of) the name of the symbol anyway, we also have \symname{<symbolname>}.

Note that \symname uses the *name* of a symbol, not its macroname. More precisely, \symname will insert the name of the symbol with "-" replaced by spaces. If a symbol does not have an explicit name = given, the two are equal – but for \symname it often makes sense to make the two explicitly distinct. For example:

#### Example 31

```
Input:

1 \symdef{Nat}[
2    name=natural-number,
3    type=\set
4 ]{\comp{\mathbb{N}}}
5
6 A \symname{Nat} is...
```

Output:

A natural number is...

\symname takes two additional optional arguments, pre= and post= that get prepended or appended respectively to the symbol name.

\Symname

Additionally, \Symname behaves exactly like \symname, but will capitalize the first letter of the name:

#### Example 32

Input:

1 \Symname[post=s]{Nat} are...

#### Output:

Natural numbers are...

This is as good a place as any other to explain how STEX resolves a string symbolname to an actual symbol.

If \symbolname is a semantic macro, then STEX has no trouble resolving symbolname to the full URI of the symbol that is being invoked.

However, especially in \symname (or if a symbol was introduced using \symdecl\* without generating a semantic macro), we might prefer to use the *name* of a symbol directly for readability — e.g. we would want to write A \symname{natural-number} is... rather than A \symname{Nat} is... SIEX attempts to handle this case thusly:



If string does *not* correspond to a semantic macro \string and does *not* contain a ?, then STEX checks all symbols currently in scope until it finds one, whose name is string. If string is of the form pre?name, STEX first looks through all modules currently in scope, whose full URI ends with pre, and then looks for a symbol with name name in those. This allows for disambiguating more precisely, e.g. by saying \symname{Integers?addition} or \symname{RealNumbers?addition} in the case where several additions are in scope.

#### 4.2 Marking Up Text and On-the-Fly Notations

We can also use semantic macros outside of text mode though, which allows us to annotate arbitrary text fragments.

Let us assume again, that we have  $\symdef{addition}[args=2]{\#1 \comp+ \#2}$ . Then we can do

#### Example 33

Input:

1 \addition{\comp{The sum of} \arg{\$\svar{n}\$} \comp{ and }\arg{\$\svar{m}\$}} 2 is...

#### Output:

The sum of n and m is...

 $\therefore$  which marks up the text fragment as representing an *application* of the addition-symbol to two argument n and m.

\arg

In text mode, every semantic macro takes exactly one argument, namely the text-fragment to be annotated. The \arg command is only valid within the argument to a semantic macro and marks up the *individual arguments* for the symbol.

We can also use semantic macros in text mode to invoke an operator itself instead of its application, with the usual syntax using !:

```
Example 34
Input:

1 \addition! {Addition} is...

Output:

Addition is...
```

In deed, \symbolname! {<code>} is exactly equivalent to \symref {symbolname} {<code>} (the latter is in fact implemented in terms of the former).

\arg also allows us to switch the order of arguments around and "hide" arguments: For example, \arg[3]{<code>} signifies that <code> represents the *third* argument to the current operator, and \arg\*[i]{<code>} signifies that <code> represents the *i*th argument, but it should not produce any output (it is exported in the xhtml however, so that MMT and other systems can pick up on it)

## Example 35

```
Input:

1 \addition{\comp{adding}
2 \arg[2]{$\svar{k}$}
3 \arg*{$\addition{\svar{n}}{\svar{m}}$}} yields...

Output:

adding k yields...
```

Note that since the second  $\arg$  has no explicit argument number, it automatically represents the first not-yet-given argument – i.e. in this case the first one.

The same syntax can be used in math mode, too, which allows us to spontaneously introduce new notations on the fly. We can activate it using the starred variants of semantic macros:

#### Example 36

```
Input:
```

```
1 Given $\addition{\svar{n}}{\svar{m}}$, then
2 $\addition*{
3  \arg*{\addition{\svar{n}}}{\svar{m}}}
4  \comp{+}
5  \arg{\svar{k}}
6 }$ yields...
```

#### Output:

```
Given n+m, then +k yields...
```

## 4.3 Referencing Symbols and Statements

TODO: references documentation

# STEX Statements

#### 5.1 Definitions, Theorems, Examples, Paragraphs

As mentioned earlier, we can semantically mark-up *statements* such as definitions, theorems, lemmata, examples, etc.

The corresponding environments for that are:

- sdefinition for definitions,
- sassertion for assertions, i.e. propositions that are declared to be *true*, such as theorems, lemmata, axioms,
- sexample for examples, and
- sparagraph for other semantic paragraphs, such as comments, remarks, conjectures, etc.

The *presentation* of these environments can be customized to use e.g. predefined theorem-environments, see chapter 6 for details.

All of these environments take optional arguments in the form of key=value-pairs. Common to all of them are the keys id= (for cross-referencing, see section 4.3), type= for customization (see chapter 6) and additional information (e.g. definition principles, "difficulty" etc), title=, and for=.

The for= key expects a comma-separated list of existing symbols, allowing for e.g. things like

#### Example 37

```
Input:

1 \begin{sexample}[
2   id=additionandmultiplication.ex,
3   for={addition,multiplication},
4   type={trivial,boring},
5   title={An Example}
6]
7   $\addition{2,3}$ is $5$, $\multiplication{2,3}$ is $6$.
8 \end{sexample}
```

Output:

**Example 5.1.1** (An Example). 2+3 is 5,  $2\cdot 3$  is 6.

\definiendum \definame \definiens \Definame

sdefinition (and sparagraph with type=symdoc) introduce three new macros: definiendum behaves like symref (and definame/Definame like symname/Symname, respectively), but highlights the referenced symbol as being defined in the current definition.

\definiens [<optional symbolname>] {<code>} marks up <code> as being the explicit definiens of <optional symbolname> (in case for= has multiple symbols).

The special type=symdoc for sparagraph is intended to be used for "informal definitions", or encyclopedia-style descriptions for symbols.

The MMT-system can use those (in lieu of an actual sdefinition in scope) to present to users, e.g. when hovering over symbols.

All four environments also take an optional parameter name = – if this one is given a value, the environment will generate a *symbol* by that name (but with no semantic macro). Not only does this allow for \symref et al, it allows us to resume our earlier example for monoids much more nicely:

#### Example 38

```
Input:
    \begin{mathstructure} { monoid}
 2
       \symdef{universe}[type=\set]{\comp{U}}}
 3
       \symdef{op}[
 4
           args=2,
 5
           type=\funtype{\universe, \universe}{\universe},
 6
           op=\circ
 7
       ]{#1 \comp{\circ} #2}
 8 9
       \symdef{unit}[type=\universe]{\comp{e}}
10
       \begin{sparagraph} [type=symdoc,for=monoid]
           A \definame{monoid} is a structure
11
12
           $\mathstruct{\universe,\op!,\unit}$
13
           where $\op!:\funtype{\universe}{\universe}$ and
14
           $\inset{\unit}{\universe}$ such that
15
\frac{16}{17}
           \begin{sassertion} [name=associative,
               type=axiom,
18
               title=Associativity]
19
               $\op!$ is associative
20
           \end{sassertion}
\frac{21}{22}
           \begin{sassertion} [name=isunit,
               type=axiom,
23
               title=Unit]
24
               \displaystyle {\displaystyle \{ \op{\svar}(x)}{\svar}(x)} 
25
               for all $\inset{\svar{x}}{\universe}$
26
           \end{sassertion}
27
       \end{sparagraph}
28 \end{mathstructure}
30 An example for a \symname{monoid} is...
```

#### Output:

```
A monoid is a structure \langle U, \circ, e \rangle where \circ : U \rightarrow U and e \in U such that 
Axiom 5.1.2 (Associativity). \circ is associative 
Axiom 5.1.3 (Unit). x \circ e = x for all x \in U 
An example for a monoid is...
```

Now the mathstructure monoid contains two additional symbols, namely the axioms for associativity and that e is a unit. Note that both symbols do not represent the mere propositions that e.g.  $\circ$  is associative, but the assertion that it is actually true that  $\circ$  is associative.

If we now want to instantiate monoid (unless with a variable, of course), we also need to assign associative and neutral to analogous assertions. So the earlier example

```
1 \instantiate{intmonoid}{
2    universe = Int ,
3    op = addition ,
4    unit = zero
5 }{monoid}{\mathbb{Z}_{+,0}}
```

...will not work anymore. We now need to give assertions that addition is associative and that zero is a unit with respect to addition.<sup>2</sup>

#### 5.2 Proofs

#### 5.2.1 Introduction & Running Example

The stex-proof package supplies macros and environment that allow to annotate the structure of mathematical proofs in STEX document. This structure can be used by MKM systems for added-value services, either directly from the STEX sources, or after translation.

We will go over the general intuition by way of a running example:

```
1 \begin{sproof} [id=simple-proof]
     {We prove that \sum_{i=1}^n{2i-1}=n^{2} by induction over n}
    \begin{spfcases}{For the induction we have to consider the following cases:}
     \begin{spfcase}{$n=1$}
5
      \begin{spfstep}[type=inline] then we compute $1=1^2$\end{spfstep}
6
     \end{spfcase}
7
     \begin{spfcase}{$n=2$}
8
        \begin{sproofcomment}[type=inline]
9
         This case is not really necessary, but we do it for the
10
         fun of it (and to get more intuition).
11
        \end{sproofcomment}
12
        \begin{spfstep}[type=inline] We compute $1+3=2^{2}=4$.\end{spfstep}
13
     \end{spfcase}
     \begin{spfcase}{$n>1$}
14
15
        \begin{spfstep}[type=assumption,id=ind-hyp]
```

 $<sup>^2</sup>$ Of course, STeX can not check that the assertions are the "correct" ones – but if the assertions (both in monoid as well as those for addition and zero) are properly marked up, MMT can. TODO: should

```
Now, we assume that the assertion is true for a certain $k\geq 1$,
16
         i.e. \sum_{i=1}^k{(2i-1)}=k^{2}.
17
18
        \end{spfstep}
19
        \begin{sproofcomment}
20
          We have to show that we can derive the assertion for $n=k+1$ from
21
          this assumption, i.e. \sum_{i=1}^{k+1}{(2i-1)}=(k+1)^{2}.
22
        \end{sproofcomment}
23
        \begin{spfstep}
24
         We obtain \sum_{i=1}^{k+1}{2i-1}=\sum_{i=1}^k{2i-1}+2(k+1)-1
25
          \begin{justification} [method=arith:split-sum]
26
           by splitting the sum.
27
          \end{justification}
28
        \end{spfstep}
        \begin{spfstep}
29
30
          Thus we have \sum_{i=1}^{k+1}{(2i-1)}=k^2+2k+1
31
          \begin{justification} [method=fertilize]
32
           by inductive hypothesis.
33
          \end{justification}
34
        \end{spfstep}
35
        \begin{spfstep}[type=conclusion]
36
         We can \begin{justification}[method=simplify]simplify\end{justification}
37
          the right-hand side to ${k+1}^2$, which proves the assertion.
38
        \end{spfstep}
39
     \end{spfcase}
40
      \begin{spfstep}[type=conclusion]
        We have considered all the cases, so we have proven the assertion.
41
42
      \end{spfstep}
43
    \end{spfcases}
44 \end{sproof}
```

This yields the following result:

```
Proof: We prove that \sum_{i=1}^{n} 2i - 1 = n^2 by induction over n

1. For the induction we have to consider the following cases:

1.1. n = 1: then we compute 1 = 1^2

1.2. n = 2: This case is not really necessary, but we do it for the fun of it (and to get more intuition). We compute 1 + 3 = 2^2 = 4

1.3. n > 1:

1.3.1. Now, we assume that the assertion is true for a certain k \ge 1, i.e. \sum_{i=1}^{k} (2i-1) = k^2.

1.3.2. We have to show that we can derive the assertion for n = k+1 from this assumption, i.e. \sum_{i=1}^{k+1} (2i-1) = (k+1)^2.

1.3.3. We obtain \sum_{i=1}^{k+1} (2i-1) = \sum_{i=1}^{k} (2i-1) + 2(k+1) - 1 by splitting the sum 1.3.4. Thus we have \sum_{i=1}^{k+1} (2i-1) = k^2 + 2k + 1 by inductive hypothesis.

1.3.5. We can simplify the right-hand side to (k+1)^2, which proves the assertion.
```

1.4. We have considered all the cases, so we have proven the assertion.

5.2.2 Proofs and Proof steps

sproof

The sproof environment is the main container for proofs. It takes an optional KeyVal argument that allows to specify the id (identifier) and for (for which assertion is this a proof) keys. The regular argument of the proof environment contains an introductory comment, that may be used to announce the proof style. The proof environment contains a sequence of spstep, spfcomment, and spfcases environments that are used to markup the proof steps.

\spfidea

The \spfidea macro allows to give a one-paragraph description of the proof idea.

\spfsketch

For one-line proof sketches, we use the \spfsketch macro, which takes the same optional argument as sproof and another one: a natural language text that sketches the proof.

spfstep

Regular proof steps are marked up with the step environment, which takes an optional KeyVal argument for annotations. A proof step usually contains a local assertion (the text of the step) together with some kind of evidence that this can be derived from already established assertions.

#### 5.2.3 Justifications

justification

This evidence is marked up with the justification environment in the stex-proofs package. This environment totally invisible to the formatted result; it wraps the text in the proof step that corresponds to the evidence. The environment takes an optional KeyVal argument, which can have the method key, whose value is the name of a proof method (this will only need to mean something to the application that consumes the semantic annotations). Furthermore, the justification can contain "premises" (specifications to assertions that were used justify the step) and "arguments" (other information taken into account by the proof method).

\premise

The \premise macro allows to mark up part of the text as reference to an assertion that is used in the argumentation. In the example in Figure ?? we have used the \premise macro to identify the inductive hypothesis.

\justarg

The \justarg macro is very similar to \premise with the difference that it is used to mark up arguments to the proof method. Therefore the content of the first argument is interpreted as a mathematical object rather than as an identifier as in the case of \premise. In our example, we specified that the simplification should take place on the right hand side of the equation. Other examples include proof methods that instantiate. Here we would indicate the substituted object in a \justarg macro.

Note that both \premise and \justarg can be used with an empty second argument to mark up premises and arguments that are not explicitly mentioned in the text.

#### 5.2.4 Proof Structure

subproof

spfsketch

The spfcases environment is used to mark up a subproof. This environment takes an optional KeyVal argument for semantic annotations and a second argument that allows to specify an introductory comment (just like in the proof environment). The method key can be used to give the name of the proof method executed to make this subproof.

spfcases

The spfcases environment is used to mark up a proof by cases. Technically it is a variant of the subproof where the method is by-cases. Its contents are spfcase environments that mark up the cases one by one.

spfcase

The content of a spfcases environment are a sequence of case proofs marked up in the pfcase environment, which takes an optional KeyVal argument for semantic annotations. The second argument is used to specify the the description of the case under consideration. The content of a spfcase environment is the same as that of a sproof, i.e. spfsteps, spfcomments, and spfcases environments.

#### \spfcasesketch

\spfcasesketch is a variant of the spfcase environment that takes the same arguments, but instead of the spfsteps in the body uses a third argument for a proof sketch.

spfcomment

The spfcomment environment is much like a step, only that it does not have an object-level assertion of its own. Rather than asserting some fact that is relevant for the proof, it is used to explain where the proof is going, what we are attempting to to, or what we have achieved so far. As such, it cannot be the target of a \premise.

#### 5.2.5 Proof End Markers

\sproofend

Traditionally, the end of a mathematical proof is marked with a little box at the end of the last line of the proof (if there is space and on the end of the next line if there isn't), like so:

The stex-proofs package provides the \sproofend macro for this.

 $\space{2mbol}$ 

If a different symbol for the proof end is to be used (e.g. q.e.d), then this can be obtained by specifying it using the  $\sproofEndSymbol$  configuration macro (e.g. by specifying  $\sproofEndSymbol{q.e.d}$ ).

Some of the proof structuring macros above will insert proof end symbols for subproofs, in most cases, this is desirable to make the proof structure explicit, but sometimes this wastes space (especially, if a proof ends in a case analysis which will supply its own proof end marker). To suppress it locally, just set proofend={} in them or use use \sProofEndSymbol{}.

#### 5.2.6 Configuration of the Presentation

Finally, we provide configuration hooks in Figure ?? for the keywords in proofs. These are mainly intended for package authors building on statements, e.g. for multi-language support.

#### \pstlabelstyle

 ${\tt method}$ 

The proof step labels can be customized via the \pstlabelstyle macro: \pstlabelstyle{ $\langle style \rangle$ } sets the style; see Figure ?? for an overview of styles. Package writers can add additional styles by adding a macro \pst@make@label@ $\langle style \rangle$  that takes two arguments: a comma-separated list of ordinals that make up the prefix and the current ordinal. Note that comma-separated lists can be conveniently iterated over by the LATEX \@for...:=...\do{...} macro; see Figure ?? for examples.

# Highlighting and Presentation Customizations

The environments starting with s (i.e. smodule, sassertion, sexample, sdefinition, sparagraph and sproof) by default produce no additional output whatsoever (except for the environment content of course). Instead, the document that uses them (whether directly or e.g. via inputref) can decide how these environments are supposed to look like.

The stexthm defines some default customizations that can be used, but of course many existing LATEX templates come with their own definition, theorem and similar environments that authors are supposed (or even required) to use. Their concrete syntax however is usually not compatible with all the additional arguments that STEX allows for semantic information.

Therefore we introduced the separate environments **sdefinition** etc. instead of using **definition** directly, and allow authors to specify how these environments should be styled via the commands **stexpatch\***.

\stexpatchmodule \stexpatchdefinition \stexpatchassertion \stexpatchexample \stexpatchparagraph \stexpatchproof All of these commands take one optional and two proper arguments, i.e. \stexpatch\*[<type>]{<begin-code>}{end-code}.

After SIEX reads and processes the optional arguments for these environments, (some of) their values are stored in the macros \s\*<field> (i.e. sexampleid, \sassertionname, etc.). It then checks for all the values <type> in the type=-list, whether an \stexpatch\*[<type>] for the current environment has been called. If it finds one, it uses that patches <begin-code> and <end-code> to mark up the current environment. If no patch for (any of) the type(s) is found, it checks whether and \stexpatch\* was called without optional argument.

For example, if we want to use a predefined theorem environment for sassertions with type=theorem, we can do

#### 1 \stexpatchassertion[theorem] {\begin{theorem}} {\end{theorem}}

...or, rather, since e.g. theorem-environments defined using amsthm take an optional title as argument, we can do:

- 1 \stexpatchassertion[theorem]
- 2 {\ifx\sassertiontitle\@empty
- 3 \begin{theorem}

```
4 \else
5 \begin{theorem}[\sassertiontitle]
6 \fi}
7 {\end{theorem}}
```

Or, if we want all sdefinitions to use a predefined definition-environment, we can do

```
1 \stexpatchdefinition
2 {\ifx\sdefinitiontitle\@empty
3 \begin{definition}
4 \else
5 \begin{definition}[\sdefinitiontitle]
6 \fi}
7 {\end{definition}}
```

\compemph
\varemph
\symrefemph
\defemph

Apart from the environments, we can control how STEX highlights variables, notation components, \symrefs and \definiendums, respectively.

To do so, we simply redefine these four macros. For example, to highlight notation components (i.e. everything in a \comp) in blue, as in this document, we can do \def\compemph#1{\textcolor{blue}{#1}}. By default, \compemph et al do nothing.

\compemph@uri \varemph@uri \symrefemph@uri \defemph@uri

For each of the four macros, there exists an additional macro that takes the full URI of the relevant symbol currently being highlighted as a second argument. That allows us to e.g. use pdf tooltips and links. For example, this document uses

```
1 \protected\def\symrefemph@uri#1#2{
2 \pdftooltip{
3 \srefsymuri{#2}{\symrefemph{#1}}}
4 }{
5 URI:~\detokenize{#2}
6 }
7 }
```

By default, \compemph@uri is simply defined as \compemph{#1} (analogously for the other three commands).

# **Additional Packages**

TODO: tikzinput documentation

#### 7.1 Modular Document Structuring

TODO: document-structure documentation

#### 7.2 Slides and Course Notes

TODO: notesslides documentation

#### 7.3 Homework, Problems and Exams

TODO: problem documentation
TODO: hwexam documentation

#### References

- [] sLaTeX/RusTeX. URL: https://github.com/sLaTeX/RusTeX (visited on 04/22/2022).
- [Bus+04] Stephen Buswell et al. *The Open Math Standard, Version 2.0.* Tech. rep. The OpenMath Society, 2004. URL: http://www.openmath.org/standard/om20.
- [Koh06] Michael Kohlhase. OMDoc An open markup format for mathematical documents [Version 1.2]. LNAI 4180. Springer Verlag, Aug. 2006. URL: http://omdoc.org/pubs/omdoc1.2.pdf.
- [MMT] MMT Language and System for the Uniform Representation of Knowledge. Project web site. URL: https://uniformal.github.io/ (visited on 01/15/2019).
- [RK13] Florian Rabe and Michael Kohlhase. "A Scalable Module System". In: Information & Computation 0.230 (2013), pp. 1–54. URL: https://kwarc.info/frabe/Research/mmt.pdf.

- [SIa] sLaTeX/sTeX-IDE. URL: https://github.com/slatex/sTeX-IDE (visited on 04/22/2022).
- [SIb] sLaTeX/stexls-vscode-plugin. URL: https://github.com/slatex/stexls-vscode-plugin (visited on 04/22/2022).
- [SLS] sLaTeX/stexls. URL: https://github.com/slatex/stexls (visited on 04/22/2022).
- [ST] sTeX An Infrastructure for Semantic Preloading of LaTeX Documents. URL: https://ctan.org/pkg/stex (visited on 04/22/2022).
- [sTeX] sTeX: A semantic Extension of TeX/LaTeX. URL: https://github.com/sLaTeX/sTeX (visited on 05/11/2020).
- [TL] TeX Live. URL: http://www.tug.org/texlive/ (visited on 12/11/2012).