Pit-Stop Revision Handwritten Notes

Feel free to pass this around!

PURE 1

(Not a substitute of your own work yeah... questions are still the best to progress in maths)

① Rules of indices

$a^m \times a^n = a^{m+n}$ $\qquad$ $a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{m \times n}$ $\qquad$ $a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ $\qquad$ $a^0 = 1$

$a^{-n} = \frac{1}{a^n}$ $\qquad$ $(ab)^n = a^n b^n$

(Practise these with question. Will be useful in differentiation and integration)

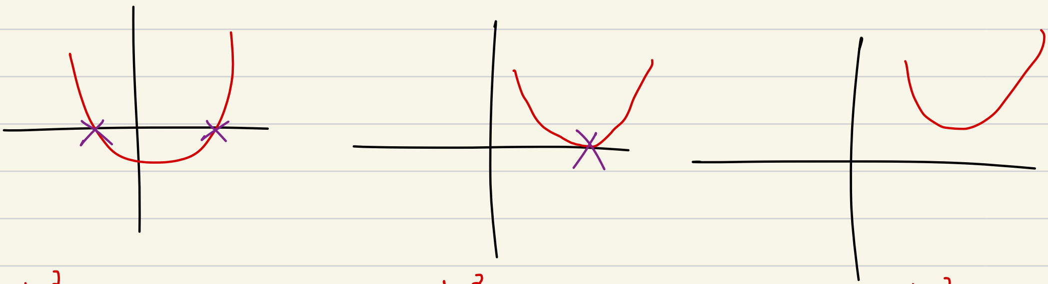
① Rationalising Surds:

$$\frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} \qquad \frac{1}{b+\sqrt{a}} \times \frac{b-\sqrt{a}}{b-\sqrt{a}}$$

② Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

② Discriminant: $b^2 - 4ac$



$b^2 - 4ac > 0$ — 2 roots

$b^2 - 4ac = 0$ — 1 root

$b^2 - 4ac < 0$ — 0 roots

② Completing the square: $a(x+p)^2 + q$ form

$$x^2 + bx + c = 0 \longrightarrow \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

$$ax^2 + bx + c = 0 \longrightarrow a\left[\left(x + \frac{b/a}{2}\right)^2 - \left(\frac{b/a}{2}\right)^2\right] + c$$

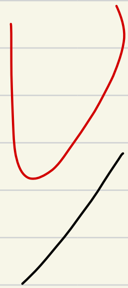
OR (I prefer this)

$$a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{2a}\right)$$

③ Simultaneous Equations:

$b^2 - 4ac > 0$
2 solutions

$b^2 - 4ac = 0$
1 solution



$b^2 - 4ac = 0$
0 solutions

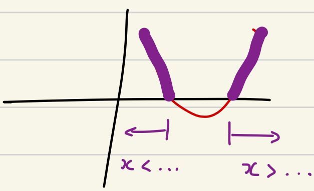
③ Inequalities:

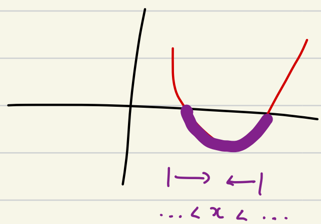
$ax^2 + bx + c > 0$

above axis

$ax^2 + bx + c < 0$

below axis





$f(x) > g(x)$

f(x) above g(x) for .... values of x.

$f(x) < g(x)$

f(x) below g(x) for .... values of x.

above f(x)

$y > f(x)$

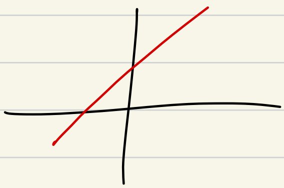
below f(x)

$y < f(x)$

④ Graphs:

Linear

$y = x$

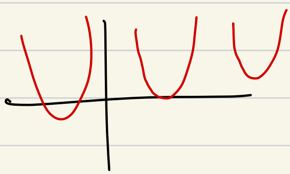


1 solution

Quadratic

$y = x^2$

$y = (x \quad)(x \quad)$

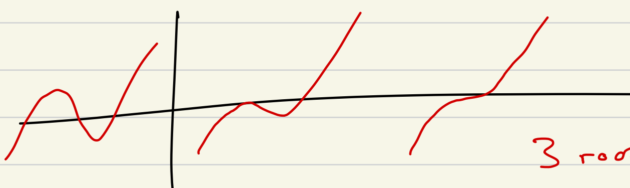


2 roots,
1 root,
0 roots

Cubic

$y = x^3$

$y = (x \quad)(x \quad)(x \quad)$



3 roots,
2 roots,
1 root

Quartic

$y = x^4$

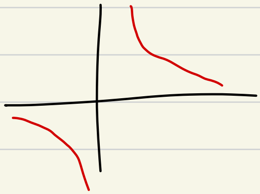
$y = (x \quad)(x \quad)(x \quad)(x \quad)$

4 roots, 3 roots, 2 roots,
1 root

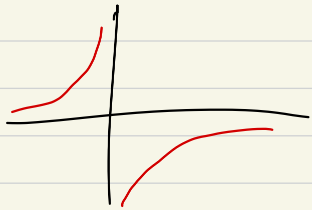
Reciprocal

The bigger this no. the further out the graph

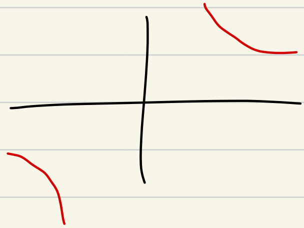
$y = \frac{1}{x}$



$y = -\frac{1}{x}$

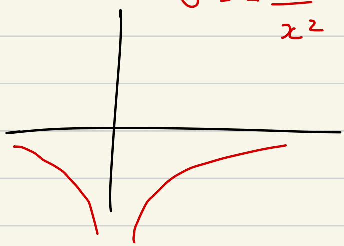


$y = \frac{10}{x}$



$y = \frac{1}{x^2}$

$y = -\frac{1}{x^2}$



Asymptotes at $x=0$, $y=0$

A line the graph tends towards but never touches.

$y = f(x) + a$

Add "a" to all y-values only

$y = f(x+a)$

Subtract "a" from all x-values only.

$y = af(x)$

Multiply all y-values by "a".

$y = f(ax)$

Divide all x-values by "a"

⑤ Straight Line Graphs:

$y = mx + c$ $\xrightarrow{\text{can also be}}$ $ax + by + c = 0$

Gradient

y-intercept

Gradient Equation:

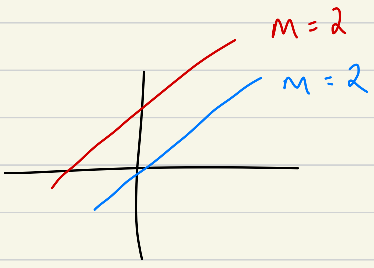
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Straight line equation:

$$y - y_1 = m(x - x_1)$$

Parallel lines : SAME GRADIENT

E.g



Perpendicular Lines = Inverse reciprocal

$m_1 \times m_2 = -1$

E.g.

Length of a line : $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

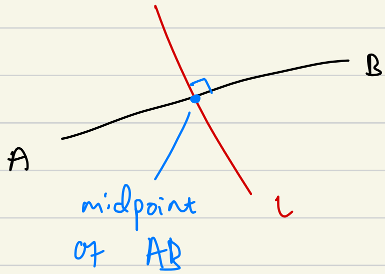
6 Circles

Equation of a circle : $(x-a)^2 + (y-b)^2 = r^2$

centre = (a, b)
radius = r

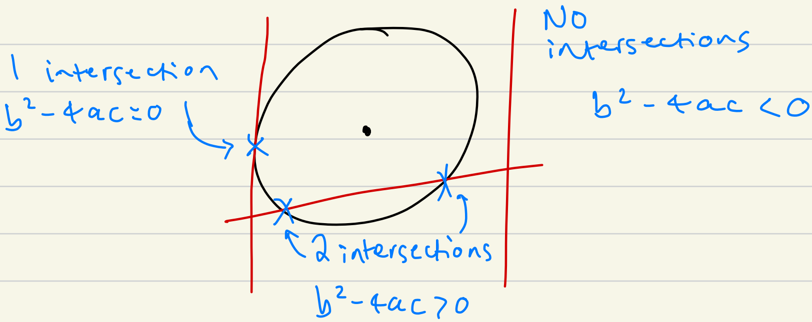
Midpoint = $\left(\dfrac{x_1 + x_2}{2} , \dfrac{y_1 + y_2}{2} \right)$

Perpendicular bisector: Perpendicular AND goes through midpoint

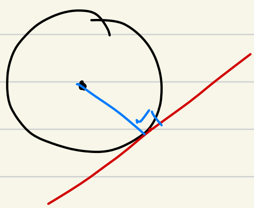


Make use of -ve reciprocal rule!

Straight line intersecting circle:

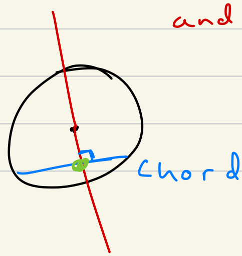


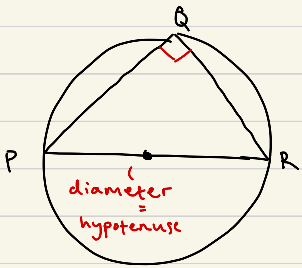
Tangent: Touches circle perp. to radius

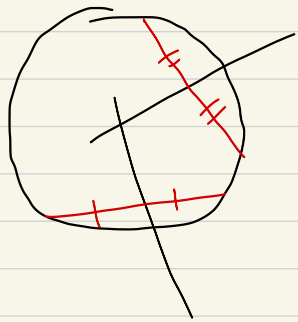


Perp. bisector of chord:

Perp. to chord and goes through at midpoint







Centre of circle
=
Coordinate of intersection
of perp. bisectors

⑦ Algebraic Fractions

* Proof by exhaustion = Break the statement into smaller bits and prove each one seperately.

* Proof by counter-example = Give one example where something is NOT true. Try to disprove.

* Proof by deduction: Start from known factors and use logical steps to reach a conclusion.

⑧ Binomial

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + b^n$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

9 Trig Ratios

Cosine Rule : $a^2 = b^2 + c^2 - 2bc \cos A$ (Length)

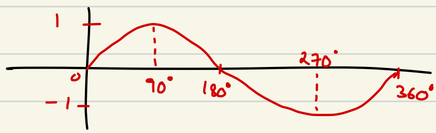
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad \text{(Angle)}$$

Sine Rule : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (Length)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{(Angle)}$$

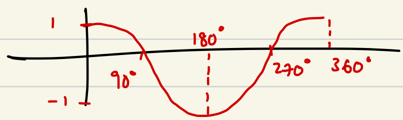
Area of Triangle : $A = \frac{1}{2} ab \sin C$

$y = \sin\theta$

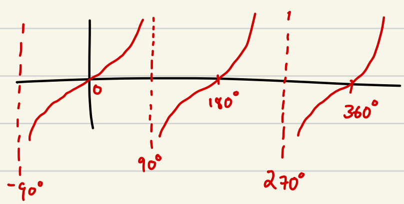


(Make sure you know all the intersections and asymptotes)

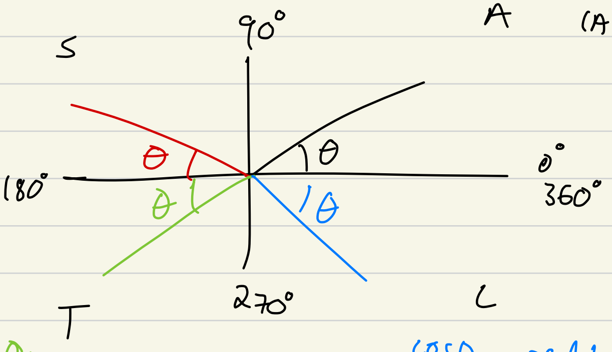
$y = \cos\theta$



$y = \tan\theta$



⑩ Trig Identities



(ALL +ve)

sinθ reflection (only sin +ve)

tanθ reflection (only tan +ve)

cosθ reflection (only cos +ve)

$$\sin^2\theta + \cos^2\theta \equiv 1 \qquad \frac{\sin\theta}{\cos\theta} \equiv \tan\theta$$

⑪ Vectors

A vector is parallel if it's a multiple.

$$2\underline{a} + \underline{b} = 6\underline{a} + 3\underline{b}$$

$$= 3(2\underline{a} + \underline{b})$$

can be written as λ.

Multiply vectors:

$$\lambda \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \lambda p \\ \lambda q \end{pmatrix}$$

Adding vectors: $\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p + r \\ q + s \end{pmatrix}$

$i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad$ $j = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad$ $\begin{pmatrix} p \\ q \end{pmatrix} = pi + qj$

(x - axis) $\quad$ (y - axis)

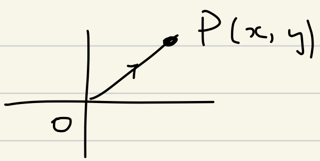
For $\underline{a} = xi + yj$ magnitude of a:

$$|\underline{a}| = \sqrt{x^2 + y^2}$$

Unit vector of $\underline{a}$:

$$\frac{\underline{a}}{|\underline{a}|}$$

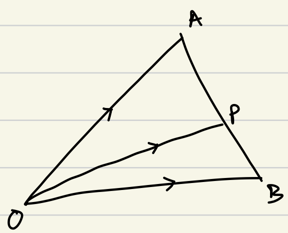
If the point P has coords (x, y):

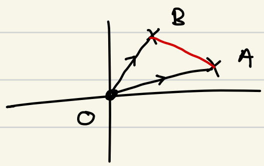


$\overrightarrow{OP} = xi + yj$

↳ Position vector

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$





P divides AB $\longrightarrow$ AP : PB

$\lambda : \mu$

$$\overrightarrow{OP} = \overrightarrow{OA} + \frac{\lambda}{\lambda + \mu}(\overrightarrow{AB})$$

(" $\overrightarrow{OA}$ + fraction along $\overrightarrow{AB}$ ")

how I like to think about it

If $\underline{a}$ and $\underline{b}$ are two non-parallel vectors:

$$p\underline{a} + q\underline{b} = r\underline{a} + s\underline{b}$$

$$p = r \qquad q = s$$

(12) Differentiation

Differential gives gradient of graph.

Notation: $f'(x)$ and $\frac{dy}{dx}$

Increasing function: $f'(x) \geq 0$ for interval $[a, b]$

Decreasing function: $f'(x) \leq 0$ for interval $[a, b]$

Second Derivative: $f''(x)$ and $\frac{d^2y}{dx^2}$

↓

Differentiate, then differentiate again.

Stationary point = $f'(x) = 0$ or $\frac{dy}{dx} = 0$

Local maximum $f''(x) < 0$

point of inflection $f''(x) = 0$

local minimum $f''(x) > 0$

13 Integration

Notation : $\int x^n \, dx$

Integrate

with respect to x.

$$\int_1^2 3x^2 \, dx = \left[x^3 \right]_1^2$$

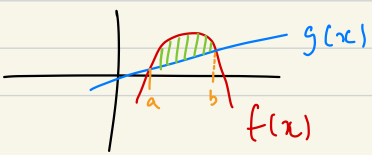
Definite Integral

$$= \left[(2)^3 \right] - \left[(1)^3 \right]$$

$$= 8 - 1$$

$$= 7$$

Integration = Area under curve



Area $= \int_a^b f(x) - g(x) \, dx$

(14) Exponentials and Logs

If $f(x) = e^x \quad f'(x) = e^x$

$f(x) = e^{kx} \quad f'(x) = ke^{kx}$

$\log_a n = x \longrightarrow a^x = n$

Log Laws:

$\log_a x + \log_a y = \log_a xy$

$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$

$\log_a (x^k) = k \log_a x$

$\log_a \left(\frac{1}{x}\right) = \log_a (x^{-1}) = -\log_a x$

$\log_a a = 1 \qquad \log_a 1 = 0$

* $\ln x$ is inverse of e^x * $\longrightarrow e^{\ln x} = x$

When $f(x) = g(x) \longrightarrow \log_a f(x) = \log_a g(x)$

If $y = ax^n \longrightarrow \log y = n \log x + \log a$

$Y = mX + C$

$Y = \log y \quad m = n \quad X = \log x$

$C = \log a$

If $y = ab^x \longrightarrow \log y = x \log b + \log a$

$Y = mX + C$

$Y = \log y \quad M = \log b \quad C = \log a$