Pit-Stop Revision Handwritten Notes

Feel free to pass this around!

Stats + Mech 2

① Regression, Correlation and Hypothesis Testing

Regression lines are used to model a linear relationships.

* If $y = ax^n$ then $\log y = \log a + n \log x$
  $y = c + m x$

* If $y = kb^x$ then $\log y = \log k + x \log b$
  $y = c + x m$

Product Moment Correlation Coefficient (PMCC), a measure of correlation between -1 – 1.

- $r = 1$ → perfect +ve correlation
- $r = -1$ → perfect -ve correlation

Calculate PMCC on calc:

1. Menu / Setup.
2. "6 : Statistics"
3. " 2 : $y = a + bx$"
4. Input data
5. "OPTN" button
6. "4 : Regression Calc"
7. Read "r = ..." value.

Hypothesis test for zero correlation.

r = PMCC for sample. ρ = PMCC of whole population

One-tailed test:

$H_0 : \rho = 0$, $H_1 : \rho > 0$

or

$H_0 : \rho = 0$, $H_1 : \rho < 0$

Two-tailed test:

$H_0 : \rho = 0$, $H_1 : \rho \neq 0$

2 Conditional Probability

$P(B|A)$ = Probability that B occurs, given that A has already happened.

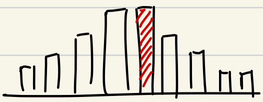
$P(B|A')$ = Probability that B occurs, given that A has <u>NOT</u> happened.

For independent events: $P(A|B) = P(A|B') = P(A)$
and
$P(B|A) = P(B|A') = P(B)$

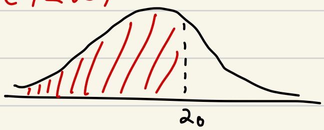
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \rightarrow P(B \cap A) = P(B|A) \times P(A)$$

3 Normal Distribution

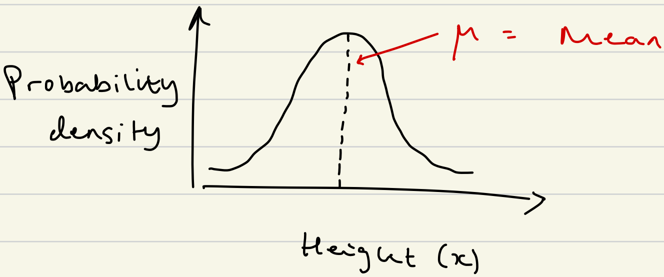


X is a discrete random variable



Y is a continuous random variable.

Continuous random variable has a continuous probability distribution. Shown as a curve - normal distribution.



Normal distribution:

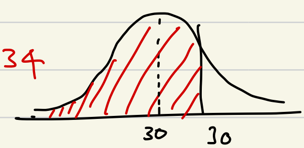
- μ = mean
- σ^2 = variance
- Is symmetrical. Mean = mode = median
- Bell-shaped. Asymptotes on each end.
- Points of inflection $\mu + \sigma$, $\mu - \sigma$
- Area under curve = 1.

Random variable notation:

* $X \sim N(\mu, \sigma^2)$

* Can use Normal CD on calc.

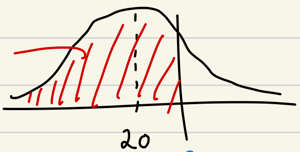
$X \sim N(30, 4^2)$ $\quad P(X < 33) = 0.7734$



Inverse Normal : $P(X < a) = p$, can use the inverse normal on the calc.

$X \sim N(20, 3^2)$

$P(X > a) = 0.4$ so $P(X < a) = 0.6$



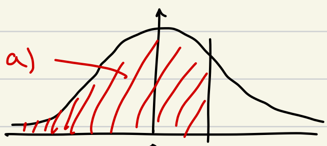
$a = 20.76$

Standard Normal:
(Z)
$Z \sim N(\mu, \sigma^2)$

$\mu = 0$
$\sigma = 1$

Use code: $Z = \frac{X - \mu}{\sigma}$

results in z-values



(You can find p and z values in the data booklet)

$$Z = \frac{X - \mu}{\sigma}$$ ← Rearrange this to find μ or σ.

If n is large and p is close to 0.5, $X \sim B(n, p)$ can be approximated by $N(\mu, \sigma^2)$:
* $\mu = np$
* $\sigma = \sqrt{np(1-p)}$

Apply continuity correction when applying normal approximation to binomial distribution.

For a sample of size 'n' taken from $X \sim N(\mu, \sigma^2)$ the sample mean: $\bar{X}$ is normally distributed using $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$.

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n}) \longrightarrow Z = \frac{\bar{X} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} \quad \text{with } Z \sim N(0, 1)$$

This can be used to find critical region and value. Use inverse normal.

④ Moments

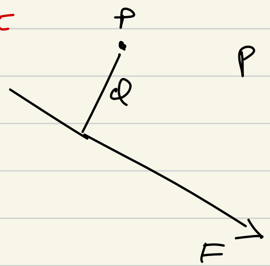
* Using rigid + uniform rods

Moment (Nm) = Turning Force.

- Clockwise
- Anti clockwise

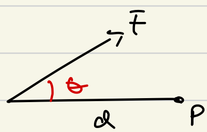
= Force (N) × distance (m)

Anticlockwise moment



$P = |F| \times d$

Clockwise moment

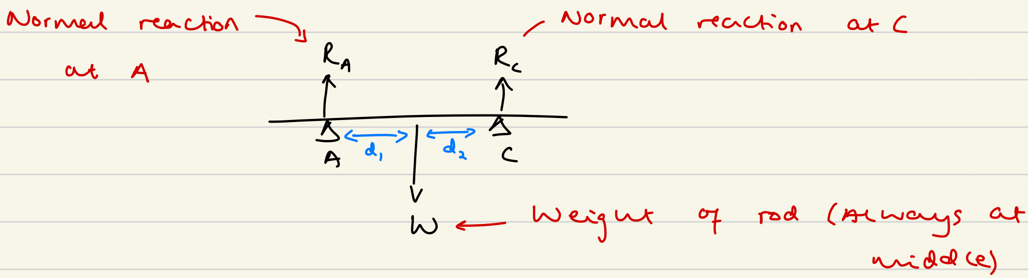


$P = |F| \times d \sin\theta$

Sum of moments = Resultant moment

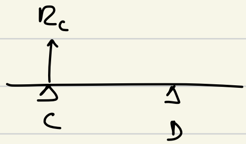
When a rigid body is in equilibrium, all resultant forces and moments = 0

↳ Clockwise = Anticlockwise



* If a rod is non-uniform, the centre of mass may not be in the middle of the rod.

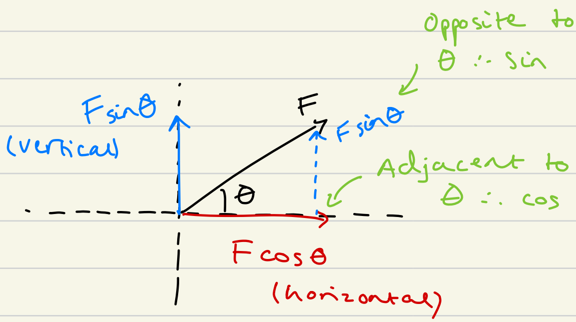
* When a rigid body is tilting, the reaction at the other end is 0.



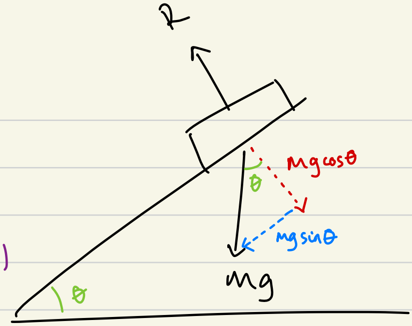
- Tilting at C means $R_D = 0$.

⑤ Forces and Friction

- Resolving vertically and horizontally



Incline plane :



R(↑) : . . .

(Resolve along normal reaction)

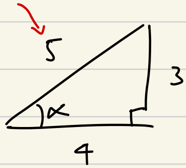
R(↙) : . . .

(Resolve down plane)

* Use $F = ma$ in a question like this

* If $\tan\alpha = \frac{3}{4}$

$(\sqrt{3^2+4^2})$



* Maximum value of friction:

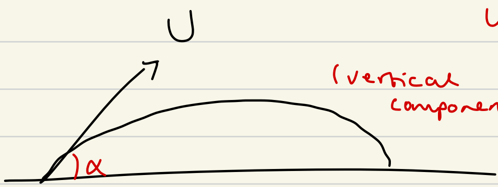
$$F_{MAX} = \mu R$$

Normal reaction.

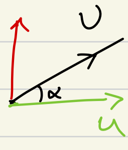
Coefficient of friction

⑥ Projectiles (Using suvat a lot here)

Projectile = Something being thrown.



(vertical component)



(horizontal component)

$u_y = U\sin\alpha$

$u_x = U\cos\alpha$

* A projectile reaches its greatest height when the vertical component of velocity is 0.

* On horizontal plane, vertical displacement = 0

- Time of flight = $\frac{2U\sin\alpha}{g}$

- Time to reach greatest height: $\frac{U\sin\alpha}{g}$

- Range on horizontal plane: $\frac{U^2\sin 2\alpha}{g}$

- Equation of trajectory: $y = x\tan\alpha - gx^2\frac{(1+\tan^2\alpha)}{2U^2}$

vertical height (y)   horizontal distance (x)

⑦ Application of Forces

Static equilibrium = At rest and resultant is 0.

* $F = ma$ with $a = 0$ $\therefore$ $F = 0$

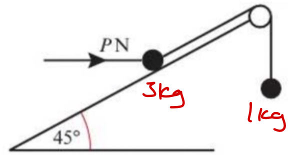
(Make sure to practise resolving vertically and horizontally, shows up a lot).

Force diagrams: Identify and show all the forces acting on an object.

No friction

tension

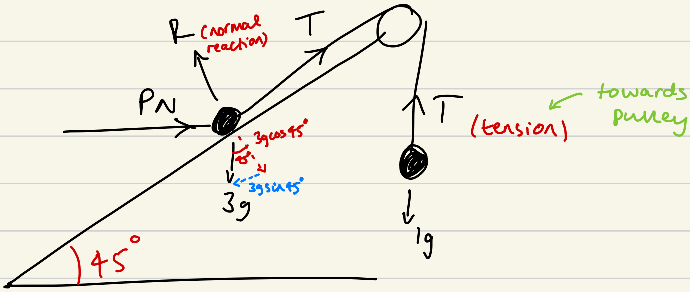
A mass of 3 kg rests on the surface of a smooth plane which is inclined at an angle of 45° to the horizontal. The mass is attached to a cable which passes up the plane along the line of greatest slope and then passes over a smooth pulley at the top of the plane. The cable carries a mass of 1 kg freely suspended at the other end. The masses are modelled as particles, and the cable as a light inextensible string. There is a force of P N acting horizontally on the 3 kg mass and the system is in equilibrium.



1. $R(\uparrow)$

2. $R(\nearrow)$

3. $R(\swarrow)$



$\star$ $F_{MAX} = \mu R$. (Limited equilibrium)

$\star$ $F \leq \mu R$. Opposite direction to motion.

⑧ Further Kinematics

Starting position vector $= r_0$ Constant velocity $= v$.

$$r = r_0 + vt$$

displacement from initial position

$$v = u + at$$
$$r = ut + \frac{1}{2}at^2$$

constant acc.

displacement

$v = \frac{ds}{dt} \longrightarrow s = \int v \, dt$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \longrightarrow v = \int a \, dt$

Variable acceleration = Model as function against time.

$\underline{r} = x\underline{i} + y\underline{j} \longrightarrow \underline{v} = \frac{dr}{dt} = \dot{r} + \dot{x}\underline{i} + \dot{y}\underline{j}$

differentiate with respect to time $\longrightarrow \underline{a} = \frac{dv}{dt} = \frac{d^2r}{dt^2} = \ddot{r} = \ddot{x}\underline{i} + \ddot{y}\underline{j}$

$\dot{r} = \frac{dr}{dt} \qquad \ddot{r} = \frac{d^2r}{dt^2}$

$* \; r = \int v \, dt$