CS 406 Discrete Mathematics 2

Homework - Edge Relations, Adjacency and Weight Matrices

1 . Given is the following directed graph. Form E^2, E^3, E^4, E^5, and E^+.

$G = (V, E) \quad V = \{a, b, c, d, e\} \quad E = \{(a, c), (b, c), (c, d), (d, e), (e, c)\}$

2 . Let $G = (V, E)$ be a directed graph with 5 vertices $V = \{a, b, c, d, e\}$. The composed edge set E^5 is $\{(a, d), (a, e), (b, d), (b, e), (d, d), (d, e), (e, d), (e, e)\}$. Write down A^5.

$A^5 =$

3 . Calculate the scalar products using boolean algebra.

$$\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \qquad \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} =$$

4 . Calculate the scalar products using min-plus algebra.

$$\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \qquad \begin{bmatrix} 2 & \infty & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ \infty \\ 4 \end{bmatrix} =$$

5 . Matrix multiplication can be used to describe geometric transformations. The following 2×2 matrix $R(\alpha)$ rotates points in the plane α degrees around the origin.

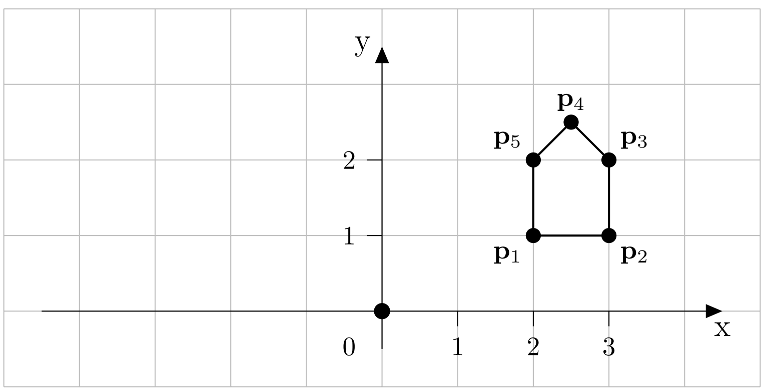
$$R(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

You transform a point by forming the product of the rotation matrix times and the column vector containing the x and y coordinate of the point. For instance,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = R(30°) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

rotates some point $\mathbf{p} = (x, y)$ 30 degrees (counter-clockwise) around the origin, which results in the transformed point $\mathbf{p}' = (x', y')$.

Illustration of a polygonal house:



(a) Calculate the entries in $R(100°)$ numerically using your calculator. Round the entries of $R(100°)$ to two places after the point.

$R(100°) =$

(b) Rotate all points $\{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_5\}$ of the house 100 degrees and write down the vectors $\{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_5\}$. Use your calculator to do the calculations numerically.

(c) Draw the transformed points into the illustration and convince yourself that you rotated the house around the origin.

6 . Fill in the neutral elements for the following matrix multiplications.

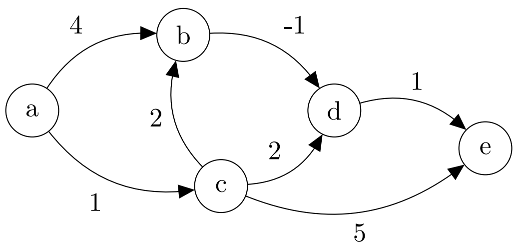
(a) using normal algebra

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(b) using min-plus algebra

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

7 . Given is the following directed graph G.



$W =$

(a) Write down the weight matrix W.

(b) Use min-plus algebra to calculate W^2, W^3, and W^4. Note that the last two rows will never change.

$W^2 =$

$W^3 =$

$W^4 =$

(c) Which matrix entry contains the length of the shortest path from a to e?