MATH 0401 - DISCRETE MATH STRUCTURES

Exam 1 Review Topics

Check full list of terms in the book's Table of Contents/Index

Chapter 1

Proposition, compound proposition
Connectives – conjunction, disjunction, implication, hypothesis, conclusion, biconditional, negation, symbols, truth values, truth tables
Contrapositive, converse, inverse
Tautology, contradiction, logically equivalent, De Morgan's Laws, other laws of propositional logic, dual
Quantifiers, predicates, domain, interpretation
Predicate propositions, scope, bound variable, free variable, main connective
Order of quantifiers

Chapter 2

Proof methods – exhaustion, direct proof, contrapositive, contradiction, proof by cases
Proving biconditional (if and only if)

Format of Test

20 points: Short Answer questions on concepts, terminology, notation
80 points: Problems

Date of Test

Monday October 16

Practice Exam 1 – Chapters 1, 2

Show all working where appropriate – partial credit will be awarded.

Part A

1. Write down the contrapositive of $p \rightarrow q$ **[1 point]**
2. Write down the converse of $p \rightarrow q$ **[1]**
3. In the statement $p \rightarrow q$, what is the hypothesis? **[1]**
4. If $p \rightarrow \neg q$ is false what can we conclude about q? **[1]**
5. What do we use to disprove a statement? **[1]**
6. Explain proof by exhaustion. **[2]**
7. Write down the dual of $(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$. **[2]**
8. Write down one of De Morgan's Laws. **[2]**

9a. What is the logic symbol for biconditional? **[1]**

b. Simplify $\neg(\neg p)$ **[1]**

10. Identify the scope of the quantifiers in the following expression and indicate any free variables (if any). **[3]**

$$\exists x\,(A(x) \wedge B(z) \wedge \forall y\, C(y))$$

scope of $\exists x$:
scope of $\forall y$:
free variables:

11. What is a tautology? **[2]**
12. Write down the associative rule for $\vee$ **[2]**

Part B

1. Let p be "Paul is a student", let q be "Quincy is a student" and let r be "Rachel is a student". Write each of the following symbolically: **[8]**

a. If Paul is a student, then so are Quincy and Rachel.

b. Paul is a student and Quincy and Rachel are not students.

Using the same values for p, q, and r, write a sentence for the statement $(p \vee q) \rightarrow r$

2. Write the truth table for:

a. $p \vee \neg q$ **[3]**

b. $(p \rightarrow q) \wedge \neg r$ **[4]**

3. Negate the statement: "All students like either Math or English." **[2]**

4. Let the domain be the integers. Determine the truth value of the following statements (no reasons necessary): **[5]**

a. $\exists x\ (x > 10)$

b. $\exists x\ ((x \leq 0) \wedge \forall y\ (x \leq y^2))$

c. $\forall x\ \forall y\ ((x = 0) \rightarrow (xy = 0))$

d. $\forall x\ \forall y\ ((xy = 0) \rightarrow (x = 0) \wedge (y = 0))$

e. $\forall x\ \exists y\ (x < y)$

5. Decide if the following is a tautology. Give reasons. **[6]**

$$p \wedge (p \vee q) \rightarrow p \vee q$$

6. Suppose the domain is everything in the world, T(x) is "x is tired", D(x) is "x is a doctor" and N(x) is "x is a nurse", O(x, y) is “x is older than y”. Write the following statements symbolically.

a. All doctors and nurses are tired. **[3]**

b. There are doctors who are not tired. **[3]**

c. Nurses are older than doctors. **[3]**

d. Exactly one doctor is tired. **[3]**

e. At least one doctor is older than all other doctors. **[3]**

f. Not every nurse is tired. **[3]**

7a. Find an interpretation in which the wff $\forall x\ \forall y\ (A(x) \wedge A(y) \rightarrow B(x,y))$ is true. **Use the integers as the domain. [3]**

b. Find an interpretation in which the wff $\forall x\ \forall y\ (A(x) \wedge A(y) \rightarrow B(x,y))$ is false. **Use the integers as the domain. [3]**

8. Use **proof by contraposition** to prove that if the product of two integers is not divisible by 7, then neither integer is divisible by 7. **[7]**

9. Use **proof by contradiction** to prove that if n is even then 7n + 9 is odd. **[7]**

11. Prove or disprove the following statement: **[7]**

The sum of the squares of two odd integers is divisible by 4

12. Prove or disprove the following: **[7]**

If two odd integers are multiplied then the answer is odd.