DISCRETE MATHEMATICAL STRUCTURES

Exam 2 Practice

1. What is the symbol for intersection? **(1)**
2. List elements of the set $\mathbf{N} \cap \mathbf{Z}$? **(2)**
3. If $A \subseteq B$ and $B \subseteq A$ then what can we conclude about A and B? **(2)**
4. For any set A what is $A \cup U$? **(2)**
5. Fill in the blanks: **[3]**
   A function maps each element of the domain to _____ one element in the target.
   A function is onto if every element of the _____ is used.
   A function is one-one if every element of the domain maps to a _____ element in the target.
6. Write down one of De Morgan's Laws for sets. **(2)**
7. What three properties does a partially ordered set have? **(3)**
8. What is meant by an identity function on a set A? **(2)**
9. Draw a Venn Diagram to represent $A \cup \overline{B}$ for two sets A, B subsets of the universal set U. **(3)**

10. Suppose U = {a, b, c, d, e, f, g, h} A = {a, c, g, h}
    B = {a, d, e, h} C = {b, c}
    D = {b, c, d}

    Find **(13)**

    a. $\overline{(A \cup C)} \cup D$
    b. $\bar{C} \cap D$
    c. $C - D$
    d. $\overline{A} - \overline{B}$
    e. C x D

11. Consider the following sets: U = {all students}
    A = {computer science majors}
    B = {physics majors}
    C = {science majors}
    D = {female students}

    Describe the following sets in terms of A, B, C, D and set operators: **(6)**

    a. set of female science majors
    b. set of male students who are not computer science majors
    c. set of all students who are female physics majors or female science majors.

12. Decide whether the following statements are True or False (no reasons necessary) **(4)**

    a. $((A \subseteq B) \wedge (B \subseteq C)) \rightarrow (A \subseteq C)$
    b. A x B = B x A
    c. $A \cup \bar{A} = U$
    d. $A - B = \overline{A} \cap B$

13. For each of the following binary relations, R on $\mathbf{N}$ decide which of the ordered pairs belong to R: **(4)**
   a. $x \, R \, y \leftrightarrow x + y = 3$; (1, 1), (2, 1), (3, 3)
   b. $x \, R \, y \leftrightarrow x$ is prime ; (2, 6), (4, 7), (7, 4)

14. R = {(1,1), (2,2), (3,3), (4,4), (5,5), (1,5), (2,3), (3,2), (2,4), (3,4), (4,2), (4,3), (5,1)} is an equivalence relation. **(6)**
   a. What is [2]?
   b. What is [3]?
   c. What is [5]?

15. Test the following binary relations for being reflexive, antireflexive, symmetric, antisymmetric and transitive. **(10)**
   a. S = people living in the United States.
   $x \, R \, y \leftrightarrow x$ is the sister of y
   b. S = $\mathbf{Z}$
   $x \, R \, y \leftrightarrow x - y$ is divisible by 7

16 For each of the following bijections f:$\mathbf{R} \rightarrow \mathbf{R}$ find the inverse of f: **(4)**
   a. $f(x) = 3x$
   b. $f(x) = (x + 4)/3$

17. Draw Hasse diagrams for the following partial orderings. Name any minimal and maximal elements. **(12)**
   a. S = {Ø, {a}, {b}, {d}, {a, b}, {a, c} {b, d}, {a, b, c}, {b, c, d}, {a, c, d}}
   A R B ↔ A is a subset of B
   b. S = {1, 2, 5, 10, 50, 100}
   x R y ↔ x divides y

18. Decide whether the following are functions or not. If they are functions test them for being one-to-one and onto. Give reasons. **(15)**
   a. f:{1, 2, 3} → {5, 6, 7} where f = {(1, 7), (2, 7), (3, 6)}
   b. g:$\mathbf{N} \rightarrow \mathbf{N}$ where $g(x) = x^2$
   c. S = {men over 18 in U.S.}, T = $\mathbf{N}$
   k:S → T where k(x) is "the height of x to the nearest inch"

19. Suppose S = {1, 2, 3, 4} and T = {5, 6, 7}. Give a counterexample for each of the following statements **(6)**
   a. All functions f:S → T are onto.
   b. All functions f:T → S are not one-one.
   c. All functions f:S → S are one-one