CS 406 Discrete Mathematics 2

Exercises - Probability 1

1 . For each of the following experiments, define the sample spaces S mathematically and give the size of S. You can use roster notation with dots or set builder notation to define S.

(a) A coin is flipped 5 times.

(b) A 20-sided die is tossed 3 times.

(c) A player chooses 2 colored meeples from a bag $B = \{m_1, m_2, \ldots, m_7\}$ of 7 differently colored meeples.

(d) A player chooses a 5 card hand from a 32-card game C_{32}.

2 . Define the sample spaces S and the size of S for the following two experiments involving random graphs.

(a) A random ordered pair of distinct edges is chosen from a free tree $T = (V, E)$ with $|V| = n$.

(b) A random simple graph $R = \{V, E\}$ with $V = \{1, 2, \ldots, 7\}$ is chosen.

Start by finding complete set of (all possible) edges C, which consists of unordered pairs P.

3 . Understand the equations $n! \cdot (n+1) = (n+1)!$ and $(n-1)! \cdot n = n!$ by replacing n with concrete small numbers. Then, show that the following equation holds.

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$$

Notes: You can remember this formula from Pascal's triangle: If you sum up two neighboring binomial coeffients in row n you get the binomial coefficient in row $n+1$.

$$\begin{array}{ccccccccc} &&&&1&&&& \\ &&&1&&1&&& \\ &&1&&2&&1&& \\ &1&&3&&3&&1& \\ 1&&4&&6&&4&&1 \end{array} \qquad \begin{array}{ccccccccc} &&&&\binom{0}{0}&&&& \\ &&&\binom{1}{0}&&\binom{1}{1}&&& \\ &&\binom{2}{0}&&\binom{2}{1}&&\binom{2}{2}&& \\ &\binom{3}{0}&&\binom{3}{1}&&\binom{3}{2}&&\binom{3}{3}& \\ \binom{4}{0}&&\binom{4}{1}&&\binom{4}{2}&&\binom{4}{3}&&\binom{4}{4} \end{array}$$

By defining $\binom{n}{0} = 1$, $\binom{n}{n} = 1$, we can form all coefficients in Pascal's triangle using the above equation. Remember, that the coefficients in row n give the sizes of all subsets of a set of size k. Thus, all coefficients in row n sum up to 2^n, the size of the power set of size n and, equivalently, the number of binary strings of length n.

4 . Are the following functions p probability distributions over the respective sample space S? Explain your answer clearly using the definition.

(a) $S = \{a, b, c, d\}$ and $p(a) = 0.1$, $p(b) = 0.2$, $p(c) = 0.3$, and $p(d) = 0.4$.

(b) $S = \{a, b, c, d\}$ and $p(a) = -0.1$, $p(b) = 0.4$, $p(c) = 0.3$, and $p(d) = 0.4$.

(c) $S = \{1, \ldots, 4\}$ and $p(s) = \frac{1}{2^s}$.

(d) A fair coin is flipped repeatedly as long as it does not show heads. Once it shows heads, the number of flipps is the outcome of the experiment. The probabilities of the outcomes are: for one flipp $\frac{1}{2}$, for two flipps $\frac{1}{4}$, for three flipps $\frac{1}{8}$, and so forth.

Formally, $S = \{1, 2, 3, \ldots\} = \mathbb{N}^+$ and $p(s) = \frac{1}{2^s}$ (e.g., $p(\text{TTTH}) = \frac{1}{16}$).