DISCRETE MATHEMATICAL STRUCTURES

Sections 1.6 – 1.10 Homework

1. **Write a predicate expression that contains one free variable and one bound variable.**

2. **Write a predicate expression to represent the phrase "A(x) is sometimes false".**

3. **Write a predicate expression to represent the phrase "For all x and y both A(x, y) and A(y, x) are false".**

4. **What is the truth value of each of the expressions below in the interpretation where the domain is the set of integers, V(x) means "x is even", L(x) means "x < 100" and G(x) means "x > 90", C(x, y) means "x < y". Write intermediate statements for all cases.**

   $\exists x \neg L(x)$

   $\forall x (\neg V(x) \vee G(x))$

   $\forall x (\neg G(x) \rightarrow L(x))$

   $\exists x (L(x) \wedge G(x))$

   $\exists y \forall x (G(y) \wedge C(x,y))$

   $\forall x \exists y (G(x) \rightarrow C(x,y))$

   $\exists x \exists y (\neg C(x,y) \wedge \neg L(x) \wedge \neg G(y))$

   $\forall x \forall y (\neg L(x) \wedge \neg G(y) \rightarrow C(y,x))$

5. **Domain = towns and cities in Pennsylvania. A(x,y) means "x has a larger population than y", B(x) means "x is closer than 100 miles to Pittsburgh", j is Johnstown and p is Philadelphia. Which of the following are true? (No reasons necessary)**

   $\forall x\, \forall y\, \forall z\, (A(x,y) \wedge A(y,z) \rightarrow A(x,z))$ $A(j,p)$

   $\exists x\, (A(x, p) \wedge B(x))$ $\forall x\, (B(x) \rightarrow \neg(x = p))$

6. **Negate the following expressions and English statements:**

   $\forall x\, (\neg A(x) \vee B(x))$ $\exists x\, \exists y\, (A(x) \rightarrow B(y))$

   Everybody is either hungry or tired

   No students are tired

   Some people are hungry

Do the following questions on a separate sheet and staple them to this sheet.

7. **In this question, the set of integers is the domain. For each expression below give one interpretation in which it is true and one in which it is false:**

   $\forall x\, (A(x) \oplus B(x))$ $\exists x\, \exists y\, (A(x) \wedge B(y) \wedge C(x, y))$

   $\forall x\, \forall y\, (A(x, y) \wedge A(y, x))$ $(\exists x)(A(x) \wedge (\forall y)(B(x, y) \rightarrow C(y)))$

8. **Domain is all people in the world. F(x) means "x is my friend", Y(x) means "x is young", S(x) means "x likes soccer". Write intermediate statements and predicate expressions for the following:**

   Some of my friends are young

   All my friends like soccer

   Not all my friends are young

   Every young soccer fan is my friend

   Only soccer fans are my friends

9. **Domain is the set of professional athletes. F(x) means "x is a football player", B(x) means "x is a basketball player", R(x, y) means "x runs faster than y". Write intermediate statements and predicate expressions for the following:**

   All basketball players run faster than all football players

   Some basketball players run faster than all football players

   Exactly one professional athlete is both a basketball player and a football player

   Only basketball players run faster than football players

10. **Do Additional Exercises 1.10.3 and 1.10.4. Write intermediate statements and predicate expressions for each part.**