0-1 BFS

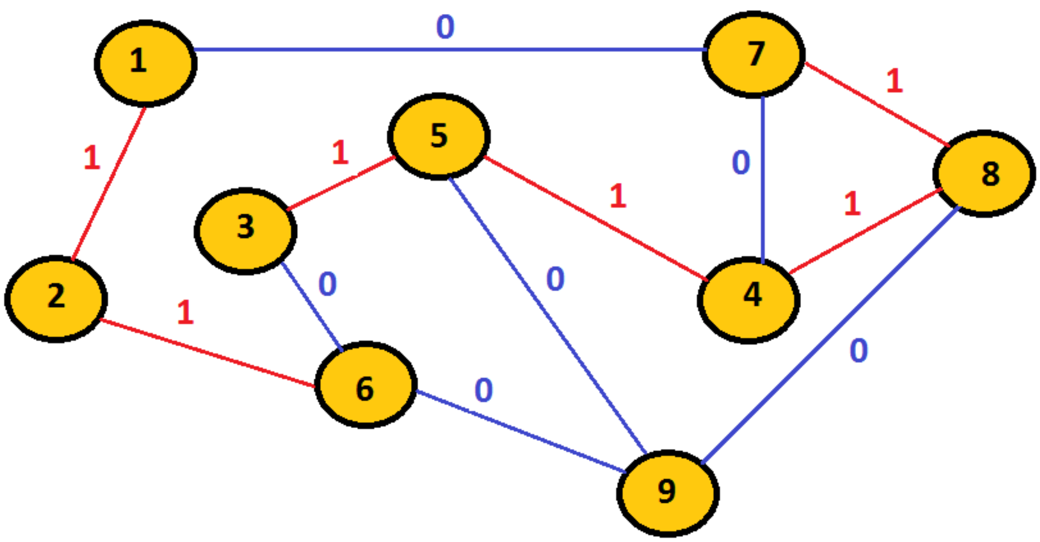
Because even Dijkstra isn't that fast always

Pre Requisites

Basics of Graph Theory , BFS , Shortest Path

Problem

You have a graph G with V vertices and E edges. The graph is a weighted graph but the weights have a contraint that they can only be 0 or 1. Write an efficient code to calculate shortest path from a given source.



P.S. : Try to come up with the most optimal solution you can implement before moving onto the next page.

Solution

Naive Solution-*Dijkstra's Algorithm*

This has a complexity of $O(ElogV)$ in its best implementation. You might try heuristics , but the worst case remains the same. At this point you maybe thinking about how you could optimise Dijkstra or why do I write such an efficient algorithm as the naive solution? Ok , so firstly the efficient solution isn't an optimisation of Dijkstra. Secondly , this is provided as the naive solution because almost everyone would code this up the first time they see such a question , assuming they know Dijkstra's algorithm.

Supposing Dijkstra's algorithm is your best code forward , I would like to present to you a very simple yet elegant trick to solve a question on this type of graph using Breadth First Search (BFS).

Before we dive into the more efficient algorithm, a lemma is required to get things crystal clear later on.

Lemma- "During the execution of BFS, the queue holding the vertices only contains elements from at max two successive levels of the BFS tree."

Explanation- The BFS tree is the tree built during the execution of BFS on any graph. This lemma is true since at every point in the execution of BFS , we only traverse to the adjacent vertices of a vertex and thus every vertex in the queue is at max one level away from all other vertices in the queue.

So let's get started with *0-1 BFS*

0-1 BFS

This is so named , since it works on graphs with edge weights 0 and 1. Let's take a point of execution of *BFS* when you are at an arbitrary vertex u having edges with weight 0 and 1. Similar to *Dijkstra* , we only put a vertex in the queue if it has been relaxed by a previous vertex (distance is reduced by travelling on this edge) and we also keep the queue sorted by distance from source at every point of time.

Now , when we are at u , we know one thing for sure : Travelling an edge *(u,v)* would make sure that v is either in the same level as u or at the next successive level. This is because the edge weights are 0 and 1. An edge weight of 0 would mean that they lie on the same level , whereas an edge weight of 1 means they lie on the level below. We also know that during *BFS* our queue holds vertices of two successive levels at max. So, when we are at vertex u , our queue contains elements of level $L[u]$ or $L[u] + 1$. And we also know that for an edge *(u,v)* , $L[v]$ is either $L[u]$ or $L[u] + 1$. Thus , if the vertex v is relaxed and has the same level , we can push it to the front of our queue and if it has the very next level , we can push it to the end of the queue. This helps us keep the queue sorted by level for the *BFS* to work properly.

But, using a normal queue data structure , we cannot insert and keep it sorted in $O(1)$. Using priority queue cost us $O(logN)$ to keep it sorted. The problem with the normal queue is the absence of methods which helps us to perform all of these functions :

1. Remove Top Element (To get vertex for BFS)
2. Insert At the beginning (To push a vertex with same level)
3. Insert At the end (To push a vertex on next level)

Fortunately, all of these operations are supported by a double ended queue (or deque in C++ STL).

Let's have a look at pseudocode for this trick :

```
for all v in vertices:
        dist[v] = inf
dist[source] = 0;
deque d
d.push_front(source)
while d.empty() == false:
        vertex=get front element and pop as in BFS
        for all edges e of form (vertex , u):
             if travelling e relaxes distance to u:
                  relax dist[u]
                  if e.weight = 1:
                       d.push_back(u)
                  else:
                       d.push_front(u)
```

As you can see , this is quite similar to BFS + Dijkstra. But the time complexity of this code is $O(E+V)$, which is linear and more efficient than Dijkstra. The analysis and proof of correctness is also same as that of BFS.

Before moving into solving problems from online judges , try these exercises to make sure you completely understand why and how 0-1 BFS works :

1. Can we apply the same trick if our edge weights can only be 0 and $x(x >= 0)$?
2. Can we apply the same trick if our edge weights are x and $x+1(x >= 0)$?
3. Can we apply the same trick if our edge weights are x and $y(x, y >= 0)$?

This trick is actually quite a simple trick, but not many people know this. Here are some problems you can try this hack at :

1. Spoj - KATHTHI

2. Topcoder SRM 436 Div-1 500

Div1 - 500 on topcoder are tough to crack. So congrats on being able to solve one of them using such a simple trick :).

Happy Coding!