Stat 331 Project

2022-12-05

R Markdown

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When you click the **Knit** button a document will be generated that includes both content as well as the output of any embedded R code chunks within the document. You can embed an R code chunk like this:

```
library(glmnet)
## Loading required package: Matrix
## Loaded glmnet 4.1-6
set.seed(1000)
exp <- load("/Users/pivaldhingra/Downloads/exposome_NA.RData")</pre>
exposome <- exposomeNA[order(exposomeNA$ID),]</pre>
covariates <- covariatesNA[order(covariatesNA$ID),]</pre>
phenotype <- phenotypeNA[order(phenotypeNA$ID),]</pre>
## read in data
a <- phenotype[5]
b <- covariates[12]</pre>
c <- exposome[,86:105]</pre>
expm <- data.frame(a,b,c)</pre>
expm <- expm[sample(nrow(expm)),]</pre>
## bounds for model selection
MO <- lm(hs_correct_raven ~ 1,data=expm) # minimal model
Mfull <- lm(hs correct raven ~., data=expm) # full model
length(coef(Mfull))
## [1] 22
anyNA(coef(Mfull))
## [1] FALSE
df.penalty <- 2
## forward
system.time({
  Mfwd <- step(object = M0, # base model
               scope = list(lower = MO, upper = Mfull), # smallest and largest model
               trace = 1, # trace prints out information
                direction = "forward" )
```

```
## Start: AIC=4810.55
## hs_correct_raven ~ 1
##
##
                       Df Sum of Sq RSS
                                              ATC
## + hs_child_age_None
                       1
                            25099.7 28422 3995.4
## + hs cs c Log2
                            11300.4 42221 4506.4
                        1
## + hs hg c Log2
                             3283.3 50238 4730.8
                        1
                             2678.2 50844 4746.3
## + hs_cu_m_Log2
                        1
## + hs_as_c_Log2
                        1
                             2447.8 51074 4752.1
## + hs_cd_m_Log2
                        1
                           1619.3 51902 4772.9
## + hs_cs_m_Log2
                           1608.0 51914 4773.2
                        1
## + hs_co_c_Log2
                            1268.1 52254 4781.6
                        1
                           1231.0 52291 4782.5
## + hs_as_m_Log2
                        1
## + hs_pb_c_Log2
                           1171.9 52350 4784.0
## + hs_mo_c_Log2
                              927.7 52594 4790.0
                        1
## + hs_pb_m_Log2
                        1
                              738.1 52784 4794.6
## + hs_cu_c_Log2
                              723.3 52798 4795.0
                        1
## + hs_hg_m_Log2
                              472.4 53049 4801.1
                        1
## + hs_co_m_Log2
                              253.5 53268 4806.4
                        1
## + hs mn m Log2
                        1
                              207.8 53314 4807.5
## + hs_tl_mdich_None
                        1
                              110.2 53412 4809.9
## <none>
                                    53522 4810.5
                               16.3 53505 4812.2
## + hs_mn_c_Log2
                        1
## + hs tl cdich None
                                4.2 53518 4812.4
                        1
## + hs_cd_c_Log2
                        1
                                2.7 53519 4812.5
## + hs_mo_m_Log2
                        1
                                0.3 53521 4812.5
##
## Step: AIC=3995.45
## hs_correct_raven ~ hs_child_age_None
##
##
                      Df Sum of Sq
                                     RSS
                                             AIC
## + hs_cs_m_Log2
                       1
                            863.04 27559 3957.6
## + hs_pb_c_Log2
                            667.70 27754 3966.8
                            482.96 27939 3975.3
## + hs_co_m_Log2
                       1
## + hs_cu_c_Log2
                       1
                            454.41 27968 3976.6
## + hs_cd_m_Log2
                            347.56 28074 3981.6
                       1
## + hs as c Log2
                       1
                            307.82 28114 3983.4
## + hs_co_c_Log2
                            186.83 28235 3988.9
                       1
## + hs_cs_c_Log2
                       1
                            169.42 28253 3989.7
## + hs_hg_m_Log2
                            141.74 28280 3991.0
                       1
## + hs_mn_m_Log2
                             49.68 28372 3995.2
## <none>
                                   28422 3995.4
                             41.41 28381 3995.6
## + hs_hg_c_Log2
                       1
## + hs_cu_m_Log2
                       1
                             39.19 28383 3995.7
                             39.01 28383 3995.7
## + hs_mo_c_Log2
                       1
## + hs_tl_cdich_None
                             38.72 28383 3995.7
                       1
## + hs_pb_m_Log2
                       1
                             28.33 28394 3996.2
## + hs_tl_mdich_None
                             23.91 28398 3996.4
                       1
## + hs_cd_c_Log2
                       1
                             11.34 28411 3996.9
## + hs_mn_c_Log2
                       1
                             11.03 28411 3996.9
## + hs_mo_m_Log2
                              3.68 28418 3997.3
                       1
                              0.82 28421 3997.4
## + hs_as_m_Log2
##
## Step: AIC=3957.64
```

```
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2
##
##
                      Df Sum of Sq
                                      RSS
                                             AIC
                             470.09 27089 3937.4
## + hs_pb_c_Log2
                       1
## + hs_cu_c_Log2
                       1
                             340.56 27218 3943.6
## + hs cd m Log2
                            309.07 27250 3945.1
                       1
## + hs as c Log2
                             262.27 27297 3947.3
                       1
## + hs_co_c_Log2
                       1
                            177.15 27382 3951.3
## + hs_co_m_Log2
                       1
                             116.51 27442 3954.2
## + hs_mo_c_Log2
                       1
                             76.20 27483 3956.1
## + hs_as_m_Log2
                       1
                             43.12 27516 3957.6
## <none>
                                    27559 3957.6
## + hs_pb_m_Log2
                             42.64 27516 3957.6
                       1
## + hs_cd_c_Log2
                       1
                             42.36 27517 3957.7
## + hs_cu_m_Log2
                             41.56 27517 3957.7
                       1
## + hs_hg_c_Log2
                       1
                             37.15 27522 3957.9
## + hs_cs_c_Log2
                             30.26 27529 3958.2
                       1
## + hs tl cdich None
                             30.18 27529 3958.2
                       1
## + hs_tl_mdich_None
                             24.34 27535 3958.5
                       1
## + hs mo m Log2
                       1
                             22.59 27536 3958.6
## + hs_hg_m_Log2
                       1
                             19.24 27540 3958.7
## + hs_mn_c_Log2
                             16.09 27543 3958.9
                       1
## + hs_mn_m_Log2
                              1.11 27558 3959.6
                       1
##
## Step: AIC=3937.43
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2
##
##
                      Df Sum of Sq
                                      RSS
                                             AIC
## + hs_as_c_Log2
                           258.432 26830 3927.1
                       1
## + hs_cd_m_Log2
                           232.218 26857 3928.3
                       1
## + hs_cu_c_Log2
                       1
                           201.989 26887 3929.8
## + hs_co_c_Log2
                       1
                           113.730 26975 3934.0
## + hs_co_m_Log2
                       1
                            85.577 27003 3935.3
                             81.627 27007 3935.5
## + hs_mo_c_Log2
                       1
## + hs_as_m_Log2
                            73.397 27016 3935.9
## <none>
                                    27089 3937.4
## + hs hg c Log2
                            41.358 27048 3937.5
## + hs_cd_c_Log2
                            37.353 27052 3937.6
                       1
## + hs mo m Log2
                       1
                             34.640 27054 3937.8
## + hs_cs_c_Log2
                            32.425 27056 3937.9
                       1
## + hs tl cdich None
                             28.901 27060 3938.0
                       1
## + hs tl mdich None
                             20.124 27069 3938.5
                       1
## + hs cu m Log2
                       1
                            18.294 27071 3938.6
## + hs_hg_m_Log2
                            13.159 27076 3938.8
                       1
                             10.558 27078 3938.9
## + hs_mn_m_Log2
                       1
                             4.907 27084 3939.2
## + hs_mn_c_Log2
                       1
                             1.377 27088 3939.4
## + hs_pb_m_Log2
                       1
##
## Step: AIC=3927.05
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2
##
##
                      Df Sum of Sq
                                      RSS
                                             ATC
## + hs cd m Log2
                           273.163 26557 3915.8
```

```
## + hs cu c Log2
                           258.938 26572 3916.5
## + hs_hg_c_Log2
                           200.305 26630 3919.4
                       1
                           122.840 26708 3923.1
## + hs as m Log2
                           114.806 26716 3923.5
## + hs_co_c_Log2
                       1
## + hs_co_m_Log2
                       1
                           106.849 26724 3923.9
## + hs mo c Log2
                            99.889 26731 3924.2
                       1
## + hs mo m Log2
                            50.706 26780 3926.6
## <none>
                                    26830 3927.1
## + hs_cd_c_Log2
                            34.696 26796 3927.4
                       1
## + hs_cu_m_Log2
                            23.315 26807 3927.9
                       1
## + hs_tl_cdich_None
                       1
                            19.821 26811 3928.1
## + hs_mn_m_Log2
                            16.281 26814 3928.3
                       1
## + hs_tl_mdich_None
                       1
                            14.887 26816 3928.3
## + hs_mn_c_Log2
                             8.432 26822 3928.6
                       1
## + hs_cs_c_Log2
                             6.514 26824 3928.7
                       1
## + hs_pb_m_Log2
                       1
                              1.032 26830 3929.0
## + hs_hg_m_Log2
                             0.490 26830 3929.0
                       1
##
## Step: AIC=3915.84
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2
##
##
                      Df Sum of Sq
                                     RSS
                                             ATC
                           224.593 26333 3906.9
## + hs cu c Log2
                           195.040 26362 3908.3
## + hs_hg_c_Log2
                       1
## + hs_mo_c_Log2
                       1
                           109.006 26448 3912.5
## + hs_as_m_Log2
                           105.065 26452 3912.7
                       1
                            90.429 26467 3913.4
## + hs_co_c_Log2
                       1
                            88.029 26469 3913.6
## + hs_co_m_Log2
                       1
## <none>
                                    26557 3915.8
## + hs_cd_c_Log2
                       1
                            38.065 26519 3916.0
## + hs_mo_m_Log2
                       1
                            33.249 26524 3916.2
## + hs_mn_m_Log2
                            27.454 26530 3916.5
## + hs_tl_mdich_None
                            20.114 26537 3916.9
                       1
## + hs tl cdich None
                       1
                            19.122 26538 3916.9
## + hs_mn_c_Log2
                            10.257 26547 3917.3
                       1
## + hs cs c Log2
                            7.226 26550 3917.5
## + hs_cu_m_Log2
                             6.620 26551 3917.5
                       1
## + hs_hg_m_Log2
                             4.273 26553 3917.6
                             2.924 26554 3917.7
## + hs_pb_m_Log2
##
## Step: AIC=3906.88
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2
##
##
                      Df Sum of Sq
                                      RSS
                                             AIC
## + hs_hg_c_Log2
                       1
                           164.441 26168 3900.8
## + hs_as_m_Log2
                            96.109 26237 3904.2
                       1
## + hs_mo_c_Log2
                       1
                            93.284 26239 3904.3
## + hs_co_m_Log2
                       1
                            81.516 26251 3904.9
## + hs_co_c_Log2
                            76.894 26256 3905.1
                       1
## + hs_cd_c_Log2
                            47.679 26285 3906.5
## <none>
                                    26333 3906.9
                            28.442 26304 3907.5
## + hs mo m Log2
```

```
## + hs mn m Log2
                            25.932 26307 3907.6
## + hs_cs_c_Log2
                            23.971 26309 3907.7
                       1
                            19.482 26313 3907.9
## + hs tl mdich None
## + hs_tl_cdich_None 1
                            12.508 26320 3908.3
## + hs_hg_m_Log2
                       1
                             9.016 26324 3908.4
## + hs_pb_m_Log2
                             3.668 26329 3908.7
                       1
## + hs mn c Log2
                             3.241 26330 3908.7
                       1
                             1.574 26331 3908.8
## + hs_cu_m_Log2
                       1
##
## Step: AIC=3900.79
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2
##
                      Df Sum of Sq
##
                                      RSS
                                             AIC
                           100.055 26068 3897.8
## + hs_mo_c_Log2
                       1
## + hs_co_c_Log2
                       1
                            76.176 26092 3899.0
                            64.243 26104 3899.6
## + hs_as_m_Log2
                       1
## + hs co m Log2
                            52.783 26116 3900.2
                            51.690 26117 3900.2
## + hs_cd_c_Log2
                       1
## + hs_cs_c_Log2
                       1
                            44.149 26124 3900.6
## <none>
                                    26168 3900.8
## + hs hg m Log2
                            36.625 26132 3901.0
## + hs_mn_m_Log2
                            28.229 26140 3901.4
                       1
## + hs mo m Log2
                            21.923 26146 3901.7
                       1
## + hs tl mdich None
                      - 1
                            16.781 26152 3902.0
## + hs_tl_cdich_None
                      1
                            10.282 26158 3902.3
## + hs_pb_m_Log2
                             4.478 26164 3902.6
                       1
                             3.710 26165 3902.6
## + hs_cu_m_Log2
                       1
## + hs_mn_c_Log2
                             3.637 26165 3902.6
                       1
##
## Step: AIC=3897.84
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_mo_c_Log2
##
##
                      Df Sum of Sq
                                      RSS
                                             ATC
## + hs co c Log2
                            62.830 26005 3896.7
## + hs_as_m_Log2
                            58.313 26010 3897.0
                       1
## + hs_co_m_Log2
                       1
                            57.816 26010 3897.0
## + hs_cd_c_Log2
                            49.182 26019 3897.4
                       1
                            45.934 26022 3897.6
## + hs cs c Log2
## <none>
                                    26068 3897.8
                            33.626 26035 3898.2
## + hs hg m Log2
                       1
## + hs_mo_m_Log2
                       1
                            21.491 26047 3898.8
## + hs_mn_m_Log2
                       1
                            21.449 26047 3898.8
                            17.247 26051 3899.0
## + hs_tl_mdich_None
                       1
## + hs_tl_cdich_None
                       1
                             7.117 26061 3899.5
## + hs_cu_m_Log2
                             3.783 26064 3899.7
                       1
## + hs_pb_m_Log2
                       1
                             3.312 26065 3899.7
## + hs_mn_c_Log2
                       1
                             2.012 26066 3899.7
##
## Step: AIC=3896.73
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
```

```
##
       hs_mo_c_Log2 + hs_co_c_Log2
##
                      Df Sum of Sq
##
                                      RSS
                                             AIC
                             59.369 25946 3895.8
## + hs_as_m_Log2
                       1
## + hs co m Log2
                       1
                             51.368 25954 3896.2
## + hs cs c Log2
                             51.165 25954 3896.2
                       1
## + hs cd c Log2
                             41.108 25964 3896.7
## <none>
                                    26005 3896.7
## + hs_hg_m_Log2
                             34.529 25971 3897.0
                       1
## + hs_mn_m_Log2
                             21.204 25984 3897.7
                       1
## + hs_mo_m_Log2
                       1
                             19.298 25986 3897.8
                             17.194 25988 3897.9
## + hs_tl_mdich_None
                       1
## + hs_tl_cdich_None
                       1
                             6.059 25999 3898.4
## + hs_pb_m_Log2
                             4.441 26001 3898.5
                       1
## + hs_cu_m_Log2
                             3.337 26002 3898.6
                       1
## + hs_mn_c_Log2
                       1
                              0.378 26005 3898.7
##
## Step: AIC=3895.78
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
       hs as c Log2 + hs cd m Log2 + hs cu c Log2 + hs hg c Log2 +
##
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2
##
##
                      Df Sum of Sq
                                             AIC
                                      RSS
                             64.946 25881 3894.5
## + hs hg m Log2
                             59.485 25886 3894.8
## + hs_cs_c_Log2
                       1
## + hs_co_m_Log2
                       1
                             52.787 25893 3895.1
## <none>
                                    25946 3895.8
## + hs_cd_c_Log2
                             39.467 25907 3895.8
                       1
## + hs_tl_mdich_None
                            16.578 25930 3897.0
                       1
## + hs_mn_m_Log2
                       1
                            14.660 25931 3897.0
## + hs_mo_m_Log2
                       1
                             11.823 25934 3897.2
## + hs_tl_cdich_None
                       1
                             6.921 25939 3897.4
                             3.766 25942 3897.6
## + hs_cu_m_Log2
                       1
                             2.444 25944 3897.7
## + hs_pb_m_Log2
                       1
## + hs mn c Log2
                       1
                             0.368 25946 3897.8
##
## Step: AIC=3894.54
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2 + hs_hg_m_Log2
##
##
                      Df Sum of Sq
                                      RSS
                                             AIC
                             53.586 25828 3893.9
## + hs_co_m_Log2
                       1
                             48.774 25832 3894.1
## + hs_cs_c_Log2
                       1
                                    25881 3894.5
## <none>
                             39.184 25842 3894.6
## + hs_cd_c_Log2
                       1
## + hs_mn_m_Log2
                       1
                             18.486 25863 3895.6
## + hs_tl_mdich_None
                             17.628 25864 3895.7
                       1
## + hs_mo_m_Log2
                       1
                             14.426 25867 3895.8
## + hs_tl_cdich_None
                       1
                             5.596 25876 3896.3
## + hs_cu_m_Log2
                             1.571 25880 3896.5
                       1
                             1.387 25880 3896.5
## + hs_pb_m_Log2
                       1
## + hs_mn_c_Log2
                       1
                             0.020 25881 3896.5
##
```

```
## Step: AIC=3893.87
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
       hs as c Log2 + hs cd m Log2 + hs cu c Log2 + hs hg c Log2 +
##
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2 + hs_hg_m_Log2 +
##
       hs_co_m_Log2
##
                      Df Sum of Sq
                                     RSS
##
                            50.741 25777 3893.3
## + hs cs c Log2
                       1
## + hs_cd_c_Log2
                            43.570 25784 3893.7
## <none>
                                    25828 3893.9
## + hs_mn_m_Log2
                       1
                            34.699 25793 3894.1
## + hs_tl_mdich_None
                            15.244 25812 3895.1
                       1
## + hs_mo_m_Log2
                       1
                            12.836 25815 3895.2
                             3.754 25824 3895.7
## + hs_cu_m_Log2
                       1
## + hs_tl_cdich_None
                             3.672 25824 3895.7
                       1
## + hs_pb_m_Log2
                       1
                             0.472 25827 3895.8
## + hs_mn_c_Log2
                             0.023 25828 3895.9
                       1
##
## Step: AIC=3893.33
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2 + hs_hg_m_Log2 +
##
       hs_co_m_Log2 + hs_cs_c_Log2
##
##
                      Df Sum of Sq
                                     RSS
                                             ATC
## + hs_cd_c_Log2
                       1
                            46.538 25730 3893.0
                            46.078 25731 3893.0
## + hs_mn_m_Log2
                       1
                                    25777 3893.3
## <none>
## + hs_mo_m_Log2
                            14.874 25762 3894.6
                       1
## + hs_tl_mdich_None
                            12.847 25764 3894.7
                       1
## + hs_tl_cdich_None
                       1
                             4.652 25772 3895.1
## + hs_cu_m_Log2
                       1
                             2.776 25774 3895.2
## + hs_pb_m_Log2
                       1
                             1.041 25776 3895.3
                             0.622 25776 3895.3
## + hs_mn_c_Log2
                       1
##
## Step: AIC=3892.99
## hs correct raven ~ hs child age None + hs cs m Log2 + hs pb c Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2 + hs_hg_m_Log2 +
##
       hs_co_m_Log2 + hs_cs_c_Log2 + hs_cd_c_Log2
##
##
                      Df Sum of Sq
                                     RSS
                                             ATC
## + hs_mn_m_Log2
                            40.878 25689 3892.9
## <none>
                                    25730 3893.0
                            12.770 25717 3894.4
## + hs_mo_m_Log2
                       1
                            10.444 25720 3894.5
## + hs_tl_mdich_None
                       1
## + hs_tl_cdich_None
                       1
                             6.254 25724 3894.7
## + hs_cu_m_Log2
                       1
                             2.904 25727 3894.8
## + hs_pb_m_Log2
                       1
                             0.888 25729 3894.9
## + hs_mn_c_Log2
                       1
                             0.156 25730 3895.0
##
## Step: AIC=3892.94
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
```

```
##
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2 + hs_hg_m_Log2 +
##
       hs_co_m_Log2 + hs_cs_c_Log2 + hs_cd_c_Log2 + hs_mn_m_Log2
##
##
                                             AIC
                      Df Sum of Sq
                                     RSS
## <none>
                                    25689 3892.9
                           13.7055 25676 3894.3
## + hs mo m Log2
                       1
## + hs tl mdich None
                           11.3186 25678 3894.4
                      1
## + hs_tl_cdich_None
                            5.9937 25683 3894.6
                       1
## + hs cu m Log2
                       1
                            2.7974 25686 3894.8
                            1.7825 25688 3894.9
## + hs_mn_c_Log2
                       1
## + hs_pb_m_Log2
                            0.1704 25689 3894.9
##
      user system elapsed
##
             0.008
                     0.124
     0.115
## backward
system.time({
  Mback <- step(object = Mfull, # base model</pre>
                scope = list(lower = M0, upper = Mfull),
                direction = "backward", trace = 1)
})
## Start: AIC=3903.35
## hs_correct_raven ~ hs_child_age_None + hs_as_c_Log2 + hs_as_m_Log2 +
       hs cd c Log2 + hs cd m Log2 + hs co c Log2 + hs co m Log2 +
##
       hs_cs_c_Log2 + hs_cs_m_Log2 + hs_cu_c_Log2 + hs_cu_m_Log2 +
##
       hs_hg_c_Log2 + hs_hg_m_Log2 + hs_mn_c_Log2 + hs_mn_m_Log2 +
##
       hs_mo_c_Log2 + hs_mo_m_Log2 + hs_pb_c_Log2 + hs_pb_m_Log2 +
##
       hs_tl_cdich_None + hs_tl_mdich_None
##
                       Df Sum of Sq
                                      RSS
                                0.3 25658 3901.4
## - hs_pb_m_Log2
                        1
## - hs_cu_m_Log2
                                1.4 25659 3901.4
                        1
## - hs_mn_c_Log2
                        1
                                1.5 25659 3901.4
## - hs_tl_cdich_None
                                4.6 25662 3901.6
                        1
## - hs_tl_mdich_None
                        1
                                9.1 25667 3901.8
## - hs_mo_m_Log2
                               10.8 25669 3901.9
                        1
## - hs_cd_c_Log2
                        1
                               37.5 25695 3903.2
## <none>
                                     25658 3903.4
## - hs_mn_m_Log2
                        1
                               42.7 25700 3903.5
## - hs_co_c_Log2
                               50.1 25708 3903.9
                        1
## - hs hg m Log2
                               55.7 25713 3904.2
                        1
## - hs_cs_c_Log2
                               65.9 25724 3904.7
                        1
## - hs_mo_c_Log2
                        1
                               70.5 25728 3904.9
## - hs_co_m_Log2
                        1
                               73.8 25732 3905.1
## - hs_as_m_Log2
                               75.7 25733 3905.2
                        1
## - hs_hg_c_Log2
                              162.3 25820 3909.5
                        1
## - hs_cu_c_Log2
                              189.7 25847 3910.9
                        1
## - hs_cd_m_Log2
                        1
                              213.7 25871 3912.1
## - hs_pb_c_Log2
                        1
                              243.8 25902 3913.6
## - hs_cs_m_Log2
                              297.4 25955 3916.2
## - hs_as_c_Log2
                              476.2 26134 3925.1
                        1
## - hs_child_age_None 1
                           11576.6 37234 4382.1
##
## Step: AIC=3901.37
```

```
## hs_correct_raven ~ hs_child_age_None + hs_as_c_Log2 + hs_as_m_Log2 +
##
       hs_cd_c_Log2 + hs_cd_m_Log2 + hs_co_c_Log2 + hs_co_m_Log2 +
##
       hs cs c Log2 + hs cs m Log2 + hs cu c Log2 + hs cu m Log2 +
##
       hs_hg_c_Log2 + hs_hg_m_Log2 + hs_mn_c_Log2 + hs_mn_m_Log2 +
##
       hs_mo_c_Log2 + hs_mo_m_Log2 + hs_pb_c_Log2 + hs_tl_cdich_None +
##
       hs tl mdich None
##
##
                                       RSS
                                              AIC
                       Df Sum of Sq
## - hs_cu_m_Log2
                        1
                                1.4 25659 3899.4
                                1.5 25660 3899.4
## - hs_mn_c_Log2
                        1
## - hs_tl_cdich_None
                                4.6 25663 3899.6
                        1
## - hs_tl_mdich_None
                                9.0 25667 3899.8
                        1
## - hs_mo_m_Log2
                        1
                               10.8 25669 3899.9
                               37.5 25696 3901.3
## - hs_cd_c_Log2
                        1
## <none>
                                     25658 3901.4
## - hs_mn_m_Log2
                        1
                               43.6 25702 3901.6
## - hs_co_c_Log2
                               49.9 25708 3901.9
                        1
## - hs hg m Log2
                               56.3 25714 3902.2
## - hs_cs_c_Log2
                               65.7 25724 3902.7
                        1
## - hs mo c Log2
                        1
                               70.8 25729 3902.9
## - hs_co_m_Log2
                        1
                               75.1 25733 3903.1
## - hs as m Log2
                               76.9 25735 3903.2
                        1
## - hs_hg_c_Log2
                              162.0 25820 3907.5
                        1
## - hs cu c Log2
                              189.5 25848 3908.9
                        1
                              216.2 25874 3910.2
## - hs_cd_m_Log2
                        1
## - hs_pb_c_Log2
                        1
                              251.2 25909 3911.9
## - hs_cs_m_Log2
                              300.5 25959 3914.4
                        1
                              475.9 26134 3923.1
## - hs_as_c_Log2
                        1
## - hs_child_age_None
                            11586.3 37244 4380.4
                        1
##
## Step: AIC=3899.44
## hs_correct_raven ~ hs_child_age_None + hs_as_c_Log2 + hs_as_m_Log2 +
##
       hs_cd_c_Log2 + hs_cd_m_Log2 + hs_co_c_Log2 + hs_co_m_Log2 +
##
       hs_cs_c_Log2 + hs_cs_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_hg_m_Log2 + hs_mn_c_Log2 + hs_mn_m_Log2 + hs_mo_c_Log2 +
##
       hs_mo_m_Log2 + hs_pb_c_Log2 + hs_tl_cdich_None + hs_tl_mdich_None
##
##
                       Df Sum of Sq
                                      RSS
## - hs_mn_c_Log2
                        1
                                1.6 25661 3897.5
## - hs_tl_cdich_None
                                5.2 25665 3897.7
                        1
## - hs tl mdich None
                                9.0 25668 3897.9
                        1
## - hs mo m Log2
                        1
                               11.3 25671 3898.0
## - hs cd c Log2
                        1
                                37.4 25697 3899.3
## <none>
                                     25659 3899.4
                               43.8 25703 3899.6
## - hs_mn_m_Log2
                        1
                               50.1 25710 3900.0
## - hs_co_c_Log2
                        1
## - hs_hg_m_Log2
                        1
                               58.1 25718 3900.4
## - hs_cs_c_Log2
                               66.8 25726 3900.8
                        1
## - hs_mo_c_Log2
                        1
                               70.5 25730 3901.0
## - hs_co_m_Log2
                        1
                               73.9 25733 3901.2
## - hs_as_m_Log2
                               76.9 25736 3901.3
                        1
## - hs_hg_c_Log2
                        1
                              161.5 25821 3905.5
## - hs cu c Log2
                              194.2 25854 3907.2
                        1
## - hs cd m Log2
                              225.5 25885 3908.7
                        1
```

```
## - hs_pb_c_Log2
                              254.9 25914 3910.2
                        1
## - hs_cs_m_Log2
                              300.0 25960 3912.4
                        1
## - hs as c Log2
                              474.5 26134 3921.1
## - hs_child_age_None 1
                            11934.0 37593 4390.5
## Step: AIC=3897.52
## hs correct raven ~ hs child age None + hs as c Log2 + hs as m Log2 +
       hs_cd_c_Log2 + hs_cd_m_Log2 + hs_co_c_Log2 + hs_co_m_Log2 +
##
       hs_cs_c_Log2 + hs_cs_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_hg_m_Log2 + hs_mn_m_Log2 + hs_mo_c_Log2 + hs_mo_m_Log2 +
##
       hs_pb_c_Log2 + hs_tl_cdich_None + hs_tl_mdich_None
##
##
                       Df Sum of Sq
                                       RSS
                                              AIC
## - hs_tl_cdich_None
                        1
                                5.0 25666 3895.8
## - hs_tl_mdich_None
                                9.1 25670 3896.0
                        1
## - hs_mo_m_Log2
                        1
                               11.5 25673 3896.1
                               38.6 25700 3897.5
## - hs_cd_c_Log2
                        1
## <none>
                                     25661 3897.5
## - hs_mn_m_Log2
                               42.3 25703 3897.6
                        1
## - hs co c Log2
                        1
                               52.4 25714 3898.2
## - hs_hg_m_Log2
                        1
                               60.5 25722 3898.6
## - hs cs c Log2
                               65.4 25726 3898.8
                        1
## - hs_mo_c_Log2
                               71.6 25733 3899.1
                        1
## - hs co m Log2
                               72.7 25734 3899.2
                        1
## - hs_as_m_Log2
                        1
                               77.6 25739 3899.4
## - hs_hg_c_Log2
                        1
                              161.6 25823 3903.6
## - hs_cu_c_Log2
                              198.6 25860 3905.5
                        1
## - hs_cd_m_Log2
                        1
                              224.5 25886 3906.8
## - hs_pb_c_Log2
                              257.3 25918 3908.4
                        1
## - hs_cs_m_Log2
                              299.2 25960 3910.5
                        1
## - hs_as_c_Log2
                        1
                              473.3 26134 3919.1
## - hs_child_age_None 1
                            11934.5 37596 4388.6
##
## Step: AIC=3895.77
## hs_correct_raven ~ hs_child_age_None + hs_as_c_Log2 + hs_as_m_Log2 +
       hs_cd_c_Log2 + hs_cd_m_Log2 + hs_co_c_Log2 + hs_co_m_Log2 +
##
       hs cs c Log2 + hs cs m Log2 + hs cu c Log2 + hs hg c Log2 +
##
       hs_hg_m_Log2 + hs_mn_m_Log2 + hs_mo_c_Log2 + hs_mo_m_Log2 +
##
       hs_pb_c_Log2 + hs_tl_mdich_None
##
##
                       Df Sum of Sq
                                      RSS
## - hs_tl_mdich_None
                                9.5 25676 3894.3
                        1
                               11.9 25678 3894.4
## - hs mo m Log2
                        1
## - hs_cd_c_Log2
                               37.2 25703 3895.6
                        1
                                     25666 3895.8
## <none>
                               42.6 25709 3895.9
## - hs_mn_m_Log2
                        1
## - hs_co_c_Log2
                        1
                               53.3 25719 3896.5
## - hs_hg_m_Log2
                        1
                               62.0 25728 3896.9
## - hs_cs_c_Log2
                        1
                               64.2 25730 3897.0
## - hs_mo_c_Log2
                        1
                               73.9 25740 3897.5
## - hs_co_m_Log2
                               75.2 25741 3897.6
                        1
## - hs_as_m_Log2
                        1
                               76.9 25743 3897.6
## - hs_hg_c_Log2
                              163.0 25829 3901.9
                        1
## - hs cu c Log2
                        1
                              201.7 25868 3903.9
```

```
## - hs cd m Log2
                               224.6 25891 3905.0
                        1
## - hs_pb_c_Log2
                               256.5 25923 3906.6
                        1
                               298.5 25965 3908.7
## - hs cs m Log2
                               482.7 26149 3917.8
## - hs_as_c_Log2
## - hs child age None
                       1
                            11932.1 37598 4386.7
##
## Step: AIC=3894.25
## hs_correct_raven ~ hs_child_age_None + hs_as_c_Log2 + hs_as_m_Log2 +
       hs_cd_c_Log2 + hs_cd_m_Log2 + hs_co_c_Log2 + hs_co_m_Log2 +
##
       hs_cs_c_Log2 + hs_cs_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_hg_m_Log2 + hs_mn_m_Log2 + hs_mo_c_Log2 + hs_mo_m_Log2 +
##
       hs_pb_c_Log2
##
##
                       Df Sum of Sq
                                       RSS
                                              AIC
                                13.7 25689 3892.9
## - hs_mo_m_Log2
                        1
## - hs_cd_c_Log2
                        1
                                39.2 25715 3894.2
                                     25676 3894.3
## <none>
## - hs mn m Log2
                        1
                                41.8 25717 3894.4
                               53.0 25729 3894.9
## - hs_co_c_Log2
                        1
## - hs hg m Log2
                        1
                               61.1 25737 3895.3
## - hs_cs_c_Log2
                        1
                               66.7 25742 3895.6
## - hs mo c Log2
                               73.7 25749 3896.0
                        1
## - hs_as_m_Log2
                               76.9 25753 3896.1
                        1
                               77.3 25753 3896.1
## - hs co m Log2
                        1
## - hs_hg_c_Log2
                        1
                               164.4 25840 3900.5
## - hs_cu_c_Log2
                        1
                               202.5 25878 3902.4
## - hs_cd_m_Log2
                               220.6 25896 3903.3
                        1
                               259.0 25935 3905.2
## - hs_pb_c_Log2
                        1
## - hs_cs_m_Log2
                               296.4 25972 3907.1
                        1
## - hs_as_c_Log2
                               489.1 26165 3916.6
                        1
## - hs_child_age_None 1
                            11941.2 37617 4385.3
##
## Step: AIC=3892.94
## hs_correct_raven ~ hs_child_age_None + hs_as_c_Log2 + hs_as_m_Log2 +
##
       hs_cd_c_Log2 + hs_cd_m_Log2 + hs_co_c_Log2 + hs_co_m_Log2 +
##
       hs_cs_c_Log2 + hs_cs_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_hg_m_Log2 + hs_mn_m_Log2 + hs_mo_c_Log2 + hs_pb_c_Log2
##
##
                       Df Sum of Sq
                                       RSS
                                              AIC
## <none>
                                     25689 3892.9
## - hs_mn_m_Log2
                                40.9 25730 3893.0
                        1
## - hs cd c Log2
                               41.3 25731 3893.0
                        1
## - hs co c Log2
                        1
                               54.4 25744 3893.7
## - hs_hg_m_Log2
                               58.9 25748 3893.9
                        1
                               64.5 25754 3894.2
## - hs_cs_c_Log2
                        1
                               73.7 25763 3894.6
## - hs_mo_c_Log2
                        1
## - hs_co_m_Log2
                        1
                               79.1 25768 3894.9
## - hs_as_m_Log2
                        1
                               85.5 25775 3895.2
## - hs_hg_c_Log2
                        1
                               165.9 25855 3899.3
## - hs_cu_c_Log2
                        1
                               204.3 25894 3901.2
## - hs_cd_m_Log2
                               229.2 25919 3902.4
                        1
## - hs_pb_c_Log2
                        1
                              253.1 25942 3903.6
## - hs_cs_m_Log2
                              291.4 25981 3905.5
                        1
## - hs as c Log2
                        1
                              485.2 26175 3915.1
```

```
## - hs_child_age_None 1
                            12009.2 37699 4386.1
      user system elapsed
     0.099
            0.005
                    0.105
## stepwise (both directions)
system.time({
  Mstep <- step(object = M0,</pre>
                scope = list(lower = MO, upper = Mfull),
                direction = "both", trace = 1)
})
## Start: AIC=4810.55
## hs_correct_raven ~ 1
##
                       Df Sum of Sq RSS
                                             AIC
                           25099.7 28422 3995.4
## + hs_child_age_None 1
## + hs_cs_c_Log2
                           11300.4 42221 4506.4
                        1
## + hs_hg_c_Log2
                          3283.3 50238 4730.8
                        1
## + hs_cu_m_Log2
                        1
                             2678.2 50844 4746.3
## + hs_as_c_Log2
                             2447.8 51074 4752.1
                        1
## + hs_cd_m_Log2
                        1 1619.3 51902 4772.9
## + hs_cs_m_Log2
                          1608.0 51914 4773.2
                        1
                           1268.1 52254 4781.6
## + hs co c Log2
                        1
## + hs_as_m_Log2
                        1 1231.0 52291 4782.5
## + hs pb c Log2
                        1 1171.9 52350 4784.0
## + hs_mo_c_Log2
                             927.7 52594 4790.0
                        1
## + hs_pb_m_Log2
                        1
                              738.1 52784 4794.6
## + hs_cu_c_Log2
                             723.3 52798 4795.0
                        1
## + hs_hg_m_Log2
                             472.4 53049 4801.1
                        1
                              253.5 53268 4806.4
## + hs_co_m_Log2
                        1
## + hs_mn_m_Log2
                              207.8 53314 4807.5
                        1
## + hs_tl_mdich_None
                        1
                              110.2 53412 4809.9
## <none>
                                    53522 4810.5
## + hs_mn_c_Log2
                              16.3 53505 4812.2
                        1
## + hs_tl_cdich_None
                              4.2 53518 4812.4
                        1
## + hs_cd_c_Log2
                        1
                                2.7 53519 4812.5
## + hs_mo_m_Log2
                               0.3 53521 4812.5
                        1
##
## Step: AIC=3995.45
## hs_correct_raven ~ hs_child_age_None
##
##
                       Df Sum of Sq
                                    RSS
                                             AIC
## + hs_cs_m_Log2
                        1
                              863.0 27559 3957.6
## + hs_pb_c_Log2
                              667.7 27754 3966.8
                        1
## + hs co m Log2
                              483.0 27939 3975.3
                        1
                              454.4 27968 3976.6
## + hs cu c Log2
                        1
## + hs_cd_m_Log2
                        1
                              347.6 28074 3981.6
## + hs_as_c_Log2
                        1
                              307.8 28114 3983.4
## + hs_co_c_Log2
                             186.8 28235 3988.9
                        1
                           169.4 28253 3989.7
## + hs_cs_c_Log2
                        1
## + hs_hg_m_Log2
                            141.7 28280 3991.0
                        1
## + hs_mn_m_Log2
                             49.7 28372 3995.2
                        1
## <none>
                                    28422 3995.4
```

41.4 28381 3995.6

+ hs_hg_c_Log2

1

```
## + hs cu m Log2
                               39.2 28383 3995.7
                        1
                               39.0 28383 3995.7
## + hs_mo_c_Log2
                        1
## + hs tl cdich None
                               38.7 28383 3995.7
## + hs_pb_m_Log2
                               28.3 28394 3996.2
                        1
## + hs_tl_mdich_None
                        1
                               23.9 28398 3996.4
## + hs_cd_c_Log2
                               11.3 28411 3996.9
                        1
## + hs mn c Log2
                               11.0 28411 3996.9
                        1
                                3.7 28418 3997.3
## + hs_mo_m_Log2
                        1
## + hs as m Log2
                        1
                                0.8 28421 3997.4
## - hs_child_age_None 1
                            25099.7 53522 4810.5
## Step: AIC=3957.64
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2
##
##
                       Df Sum of Sq
                                      RSS
                                              ATC:
## + hs_pb_c_Log2
                        1
                              470.1 27089 3937.4
                              340.6 27218 3943.6
## + hs_cu_c_Log2
                        1
## + hs_cd_m_Log2
                              309.1 27250 3945.1
## + hs_as_c_Log2
                              262.3 27297 3947.3
                        1
## + hs co c Log2
                        1
                              177.1 27382 3951.3
## + hs_co_m_Log2
                        1
                              116.5 27442 3954.2
## + hs_mo_c_Log2
                              76.2 27483 3956.1
                        1
## + hs_as_m_Log2
                               43.1 27516 3957.6
                        1
## <none>
                                     27559 3957.6
## + hs_pb_m_Log2
                        1
                               42.6 27516 3957.6
## + hs_cd_c_Log2
                        1
                               42.4 27517 3957.7
## + hs_cu_m_Log2
                               41.6 27517 3957.7
                        1
                               37.2 27522 3957.9
## + hs_hg_c_Log2
                        1
## + hs_cs_c_Log2
                               30.3 27529 3958.2
                        1
## + hs_tl_cdich_None
                               30.2 27529 3958.2
                        1
## + hs_tl_mdich_None
                        1
                               24.3 27535 3958.5
## + hs_mo_m_Log2
                        1
                               22.6 27536 3958.6
## + hs_hg_m_Log2
                               19.2 27540 3958.7
                               16.1 27543 3958.9
## + hs_mn_c_Log2
                        1
## + hs mn m Log2
                                1.1 27558 3959.6
                        1
                              863.0 28422 3995.4
## - hs_cs_m_Log2
                        1
## - hs_child_age_None
                            24354.7 51914 4773.2
## Step: AIC=3937.43
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2
##
##
                       Df Sum of Sq RSS
                                              ATC
                              258.4 26830 3927.1
## + hs_as_c_Log2
                        1
## + hs_cd_m_Log2
                              232.2 26857 3928.3
                        1
                              202.0 26887 3929.8
## + hs_cu_c_Log2
                        1
## + hs_co_c_Log2
                              113.7 26975 3934.0
                        1
## + hs_co_m_Log2
                        1
                               85.6 27003 3935.3
## + hs_mo_c_Log2
                        1
                               81.6 27007 3935.5
## + hs_as_m_Log2
                               73.4 27016 3935.9
                        1
## <none>
                                     27089 3937.4
                               41.4 27048 3937.5
## + hs_hg_c_Log2
                        1
## + hs_cd_c_Log2
                        1
                               37.4 27052 3937.6
## + hs_mo_m_Log2
                        1
                               34.6 27054 3937.8
## + hs cs c Log2
                        1
                               32.4 27056 3937.9
```

```
## + hs tl cdich None
                                28.9 27060 3938.0
                               20.1 27069 3938.5
## + hs_tl_mdich_None
                        1
## + hs cu m Log2
                        1
                               18.3 27071 3938.6
## + hs_hg_m_Log2
                               13.2 27076 3938.8
                        1
## + hs_mn_m_Log2
                        1
                               10.6 27078 3938.9
## + hs mn c Log2
                                 4.9 27084 3939.2
                        1
## + hs_pb_m_Log2
                                 1.4 27088 3939.4
                        1
## - hs_pb_c_Log2
                        1
                               470.1 27559 3957.6
## - hs cs m Log2
                        1
                               665.4 27754 3966.8
## - hs_child_age_None 1
                            24016.3 51105 4754.9
## Step: AIC=3927.05
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2
##
##
                       Df Sum of Sq
                                       RSS
                                              AIC
                               273.2 26557 3915.8
## + hs_cd_m_Log2
                        1
## + hs_cu_c_Log2
                               258.9 26572 3916.5
                        1
                               200.3 26630 3919.4
## + hs_hg_c_Log2
                        1
## + hs as m Log2
                        1
                               122.8 26708 3923.1
## + hs_co_c_Log2
                        1
                              114.8 26716 3923.5
## + hs co m Log2
                              106.8 26724 3923.9
                        1
## + hs_mo_c_Log2
                               99.9 26731 3924.2
                        1
## + hs mo m Log2
                               50.7 26780 3926.6
                        1
## <none>
                                     26830 3927.1
## + hs_cd_c_Log2
                        1
                                34.7 26796 3927.4
## + hs_cu_m_Log2
                               23.3 26807 3927.9
                        1
## + hs_tl_cdich_None
                        1
                               19.8 26811 3928.1
## + hs_mn_m_Log2
                               16.3 26814 3928.3
                        1
## + hs_tl_mdich_None
                               14.9 26816 3928.3
                        1
## + hs_mn_c_Log2
                        1
                                8.4 26822 3928.6
## + hs_cs_c_Log2
                        1
                                 6.5 26824 3928.7
## + hs_pb_m_Log2
                                 1.0 26829 3929.0
                                 0.5 26830 3929.0
## + hs_hg_m_Log2
                        1
## - hs_as_c_Log2
                               258.4 27089 3937.4
                        1
                               466.3 27297 3947.3
## - hs_pb_c_Log2
                        1
## - hs cs m Log2
                               627.1 27458 3954.9
## - hs_child_age_None 1
                            22086.1 48917 4700.4
##
## Step: AIC=3915.84
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2
##
##
                       Df Sum of Sq
                                       RSS
                                              AIC
                               224.6 26333 3906.9
## + hs_cu_c_Log2
                        1
## + hs_hg_c_Log2
                               195.0 26362 3908.3
                        1
## + hs_mo_c_Log2
                        1
                               109.0 26448 3912.5
## + hs_as_m_Log2
                        1
                               105.1 26452 3912.7
## + hs_co_c_Log2
                               90.4 26467 3913.4
                        1
## + hs_co_m_Log2
                        1
                               88.0 26469 3913.6
## <none>
                                     26557 3915.8
## + hs_cd_c_Log2
                        1
                               38.1 26519 3916.0
## + hs_mo_m_Log2
                        1
                               33.2 26524 3916.2
## + hs mn m Log2
                        1
                               27.5 26530 3916.5
```

```
## + hs tl mdich None
                                20.1 26537 3916.9
                               19.1 26538 3916.9
## + hs_tl_cdich_None
                        1
## + hs mn c Log2
                        1
                               10.3 26547 3917.3
                                7.2 26550 3917.5
## + hs_cs_c_Log2
                        1
## + hs_cu_m_Log2
                        1
                                 6.6 26551 3917.5
                                 4.3 26553 3917.6
## + hs hg m Log2
                        1
                                 2.9 26554 3917.7
## + hs pb m Log2
                        1
                               273.2 26830 3927.1
## - hs_cd_m_Log2
                        1
## - hs_as_c_Log2
                        1
                               299.4 26857 3928.3
## - hs_pb_c_Log2
                        1
                               383.5 26941 3932.3
## - hs_cs_m_Log2
                               607.9 27165 3943.1
                        1
                            20938.6 47496 4664.3
## - hs_child_age_None
                        1
## Step: AIC=3906.88
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2
##
##
                       Df Sum of Sq
                                       RSS
                               164.4 26168 3900.8
## + hs_hg_c_Log2
                        1
## + hs as m Log2
                        1
                                96.1 26237 3904.2
## + hs_mo_c_Log2
                        1
                               93.3 26239 3904.3
## + hs co m Log2
                               81.5 26251 3904.9
                        1
## + hs_co_c_Log2
                               76.9 26256 3905.1
                        1
                               47.7 26285 3906.5
## + hs cd c Log2
                        1
## <none>
                                     26333 3906.9
## + hs_mo_m_Log2
                        1
                               28.4 26304 3907.5
## + hs_mn_m_Log2
                               25.9 26307 3907.6
                        1
                               24.0 26309 3907.7
## + hs_cs_c_Log2
                        1
## + hs_tl_mdich_None
                               19.5 26313 3907.9
                        1
## + hs_tl_cdich_None
                               12.5 26320 3908.3
                        1
## + hs_hg_m_Log2
                        1
                                 9.0 26324 3908.4
## + hs_pb_m_Log2
                        1
                                 3.7 26329 3908.7
## + hs_mn_c_Log2
                                 3.2 26329 3908.7
                                 1.6 26331 3908.8
## + hs_cu_m_Log2
                        1
## - hs_cu_c_Log2
                               224.6 26557 3915.8
                        1
                               238.8 26572 3916.5
## - hs_cd_m_Log2
                        1
## - hs pb c Log2
                              259.7 26592 3917.5
## - hs_as_c_Log2
                               352.9 26686 3922.1
                        1
## - hs_cs_m_Log2
                               551.4 26884 3931.6
                        1
## - hs_child_age_None 1
                            20781.2 47114 4655.9
## Step: AIC=3900.79
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2
##
##
                       Df Sum of Sq
                                       RSS
                                              AIC
## + hs_mo_c_Log2
                        1
                               100.1 26068 3897.8
                                76.2 26092 3899.0
## + hs_co_c_Log2
                        1
## + hs_as_m_Log2
                        1
                                64.2 26104 3899.6
## + hs_co_m_Log2
                        1
                               52.8 26116 3900.2
                               51.7 26117 3900.2
## + hs_cd_c_Log2
                        1
## + hs_cs_c_Log2
                               44.1 26124 3900.6
## <none>
                                     26168 3900.8
## + hs hg m Log2
                        1
                               36.6 26132 3901.0
```

```
## + hs mn m Log2
                               28.2 26140 3901.4
## + hs_mo_m_Log2
                               21.9 26146 3901.7
                        1
## + hs tl mdich None
                               16.8 26152 3902.0
## + hs_tl_cdich_None
                               10.3 26158 3902.3
                        1
## + hs_pb_m_Log2
                        1
                                4.5 26164 3902.6
## + hs cu m Log2
                                3.7 26165 3902.6
                        1
## + hs mn c Log2
                                3.6 26165 3902.6
                        1
## - hs_hg_c_Log2
                              164.4 26333 3906.9
                        1
## - hs_cu_c_Log2
                        1
                              194.0 26362 3908.3
## - hs_cd_m_Log2
                        1
                              236.5 26405 3910.4
## - hs_pb_c_Log2
                              272.6 26441 3912.2
                        1
## - hs_as_c_Log2
                              498.6 26667 3923.2
                        1
## - hs_cs_m_Log2
                              533.7 26702 3924.9
                        1
## - hs_child_age_None
                            19432.1 45600 4615.8
                       1
## Step: AIC=3897.84
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_mo_c_Log2
##
##
                       Df Sum of Sq
                                       RSS
                                              AIC
## + hs_co_c_Log2
                               62.8 26005 3896.7
                        1
## + hs_as_m_Log2
                               58.3 26010 3897.0
                        1
## + hs co m Log2
                               57.8 26010 3897.0
                        1
## + hs_cd_c_Log2
                        1
                               49.2 26019 3897.4
## + hs_cs_c_Log2
                        1
                               45.9 26022 3897.6
## <none>
                                     26068 3897.8
                               33.6 26035 3898.2
## + hs_hg_m_Log2
                        1
## + hs_mo_m_Log2
                               21.5 26047 3898.8
                        1
## + hs_mn_m_Log2
                        1
                               21.4 26047 3898.8
## + hs_tl_mdich_None
                        1
                               17.2 26051 3899.0
## + hs_tl_cdich_None
                        1
                                7.1 26061 3899.5
## + hs_cu_m_Log2
                        1
                                3.8 26064 3899.7
                                3.3 26065 3899.7
## + hs_pb_m_Log2
                        1
## + hs mn c Log2
                        1
                                2.0 26066 3899.7
## - hs_mo_c_Log2
                              100.1 26168 3900.8
                        1
## - hs hg c Log2
                              171.2 26239 3904.3
## - hs_cu_c_Log2
                              178.4 26247 3904.6
                        1
## - hs_cd_m_Log2
                              245.6 26314 3908.0
                        1
## - hs_pb_c_Log2
                              279.8 26348 3909.6
                        1
## - hs as c Log2
                              524.2 26592 3921.5
                        1
## - hs cs m Log2
                              567.9 26636 3923.7
                        1
                            18593.2 44661 4590.9
## - hs_child_age_None 1
##
## Step: AIC=3896.73
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_mo_c_Log2 + hs_co_c_Log2
##
##
                       Df Sum of Sq
                                       RSS
                                              AIC
                               59.4 25946 3895.8
## + hs_as_m_Log2
                        1
## + hs_co_m_Log2
                        1
                               51.4 25954 3896.2
## + hs_cs_c_Log2
                               51.2 25954 3896.2
                        1
## + hs cd c Log2
                        1
                               41.1 25964 3896.7
```

```
26005 3896.7
## <none>
## + hs_hg_m_Log2
                               34.5 25971 3897.0
                        1
## + hs mn m Log2
                               21.2 25984 3897.7
## + hs_mo_m_Log2
                               19.3 25986 3897.8
                        1
## - hs co c Log2
                        1
                               62.8 26068 3897.8
## + hs tl mdich None
                               17.2 25988 3897.9
                        1
## + hs tl cdich None
                                6.1 25999 3898.4
                        1
## + hs_pb_m_Log2
                                4.4 26001 3898.5
                        1
## + hs cu m Log2
                        1
                                3.3 26002 3898.6
## + hs_mn_c_Log2
                        1
                                0.4 26005 3898.7
## - hs_mo_c_Log2
                        1
                               86.7 26092 3899.0
                              168.6 26174 3903.1
## - hs_cu_c_Log2
                        1
## - hs_hg_c_Log2
                        1
                              170.1 26175 3903.1
                              226.4 26232 3905.9
## - hs_cd_m_Log2
                        1
## - hs_pb_c_Log2
                              249.1 26254 3907.0
                        1
## - hs_as_c_Log2
                        1
                              519.2 26525 3920.2
## - hs_cs_m_Log2
                              570.0 26575 3922.7
                        1
## - hs_child_age_None 1
                            18196.5 44202 4579.6
##
## Step: AIC=3895.78
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2
##
##
##
                       Df Sum of Sq
                                       RSS
                                              AIC
## + hs_hg_m_Log2
                        1
                               64.9 25881 3894.5
## + hs_cs_c_Log2
                               59.5 25887 3894.8
                        1
                               52.8 25893 3895.1
## + hs_co_m_Log2
                        1
## <none>
                                     25946 3895.8
                               39.5 25907 3895.8
## + hs_cd_c_Log2
                        1
## - hs_as_m_Log2
                        1
                               59.4 26005 3896.7
## + hs_tl_mdich_None
                        1
                               16.6 25929 3897.0
## - hs_co_c_Log2
                        1
                               63.9 26010 3897.0
                               14.7 25931 3897.0
## + hs_mn_m_Log2
                        1
## + hs mo m Log2
                        1
                               11.8 25934 3897.2
## + hs_tl_cdich_None
                                6.9 25939 3897.4
                        1
## + hs cu m Log2
                        1
                                3.8 25942 3897.6
## + hs_pb_m_Log2
                                2.4 25944 3897.7
                        1
## + hs_mn_c_Log2
                        1
                                0.4 25946 3897.8
## - hs_mo_c_Log2
                               81.1 26027 3897.8
                        1
## - hs_hg_c_Log2
                              138.4 26084 3900.6
                        1
## - hs cu c Log2
                              165.2 26111 3902.0
                        1
                              214.3 26160 3904.4
## - hs cd m Log2
                        1
## - hs_pb_c_Log2
                              270.3 26216 3907.2
                        1
                              541.7 26488 3920.5
## - hs_as_c_Log2
                        1
                              625.8 26572 3924.5
## - hs_cs_m_Log2
                        1
## - hs_child_age_None 1
                            18178.9 44125 4579.3
##
## Step: AIC=3894.54
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2 + hs_hg_m_Log2
##
##
                       Df Sum of Sq
                                       RSS
                                              AIC
```

```
## + hs co m Log2
                        1
                               53.6 25827 3893.9
## + hs_cs_c_Log2
                               48.8 25832 3894.1
                        1
## <none>
                                     25881 3894.5
                               39.2 25842 3894.6
## + hs_cd_c_Log2
                        1
## + hs mn m Log2
                        1
                               18.5 25863 3895.6
## + hs tl mdich None
                               17.6 25863 3895.7
                        1
## - hs hg m Log2
                               64.9 25946 3895.8
                        1
## - hs_co_c_Log2
                               65.5 25947 3895.8
                        1
## + hs mo m Log2
                        1
                               14.4 25867 3895.8
## + hs_tl_cdich_None
                        1
                               5.6 25875 3896.3
## - hs_mo_c_Log2
                        1
                               75.7 25957 3896.3
## + hs_cu_m_Log2
                                1.6 25880 3896.5
                        1
## + hs_pb_m_Log2
                        1
                                1.4 25880 3896.5
## + hs_mn_c_Log2
                                0.0 25881 3896.5
## - hs_as_m_Log2
                               89.8 25971 3897.0
                        1
## - hs_hg_c_Log2
                        1
                              171.7 26053 3901.1
## - hs_cu_c_Log2
                              173.9 26055 3901.2
                        1
## - hs cd m Log2
                              229.3 26110 3903.9
                        1
## - hs_pb_c_Log2
                              262.6 26144 3905.6
                        1
## - hs cs m Log2
                        1
                              515.0 26396 3918.0
## - hs_as_c_Log2
                        1
                              519.1 26400 3918.2
## - hs_child_age_None 1
                            18219.4 44100 4580.6
##
## Step: AIC=3893.87
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2 + hs_hg_m_Log2 +
##
       hs_co_m_Log2
##
                       Df Sum of Sq
                                       RSS
                                              AIC
## + hs_cs_c_Log2
                        1
                               50.7 25777 3893.3
## + hs_cd_c_Log2
                        1
                               43.6 25784 3893.7
## <none>
                                     25827 3893.9
## + hs_mn_m_Log2
                               34.7 25793 3894.1
                        1
## - hs co m Log2
                               53.6 25881 3894.5
                        1
## - hs_co_c_Log2
                               58.8 25886 3894.8
                        1
## + hs tl mdich None
                               15.2 25812 3895.1
## - hs_hg_m_Log2
                               65.7 25893 3895.1
                        1
## + hs mo m Log2
                               12.8 25815 3895.2
## + hs_cu_m_Log2
                               3.8 25824 3895.7
                        1
## + hs tl cdich None
                                3.7 25824 3895.7
                        1
## + hs_pb_m_Log2
                                0.5 25827 3895.8
                        1
                                0.0 25827 3895.9
## + hs mn c Log2
                        1
## - hs_mo_c_Log2
                               80.4 25908 3895.9
                        1
## - hs_as_m_Log2
                               91.7 25919 3896.4
                        1
                              143.1 25971 3899.0
## - hs_hg_c_Log2
                        1
## - hs_cu_c_Log2
                        1
                              171.7 25999 3900.4
## - hs_cd_m_Log2
                              217.4 26045 3902.7
                        1
## - hs_pb_c_Log2
                              248.5 26076 3904.2
                        1
## - hs_cs_m_Log2
                        1
                              330.5 26158 3908.3
## - hs_as_c_Log2
                              519.3 26347 3917.6
                        1
## - hs_child_age_None
                            17805.6 43633 4568.8
##
## Step: AIC=3893.33
```

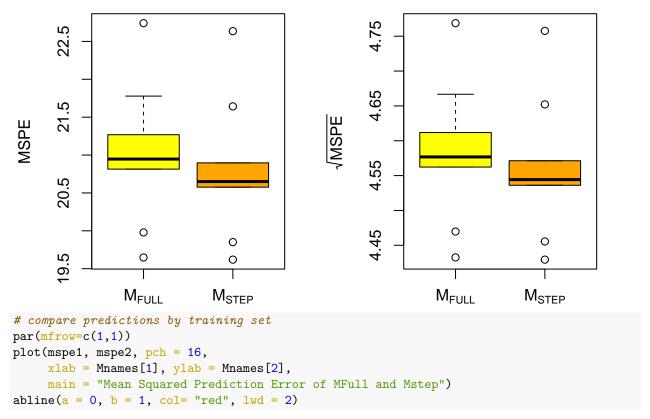
```
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2 + hs_hg_m_Log2 +
##
       hs_co_m_Log2 + hs_cs_c_Log2
##
##
                       Df Sum of Sq
                                      RSS
                                              AIC
                               46.5 25730 3893.0
## + hs cd c Log2
                        1
                               46.1 25731 3893.0
## + hs mn m Log2
                        1
## <none>
                                     25777 3893.3
## - hs_cs_c_Log2
                        1
                               50.7 25827 3893.9
## - hs_hg_m_Log2
                               54.8 25832 3894.1
                        1
## - hs_co_m_Log2
                               55.6 25832 3894.1
                        1
## - hs_co_c_Log2
                               63.7 25840 3894.5
                        1
                              14.9 25762 3894.6
## + hs_mo_m_Log2
                        1
## + hs_tl_mdich_None
                               12.8 25764 3894.7
                        1
## + hs_tl_cdich_None
                        1
                                4.7 25772 3895.1
## + hs_cu_m_Log2
                                2.8 25774 3895.2
                        1
## + hs pb m Log2
                        1
                               1.0 25776 3895.3
                                0.6 25776 3895.3
## + hs_mn_c_Log2
                        1
## - hs mo c Log2
                        1
                               81.8 25859 3895.4
## - hs_as_m_Log2
                        1
                               97.8 25875 3896.2
## - hs hg c Log2
                              158.7 25935 3899.3
                        1
                              193.2 25970 3901.0
## - hs_cu_c_Log2
                        1
## - hs cd m Log2
                              213.9 25991 3902.0
                        1
                              245.6 26022 3903.6
## - hs_pb_c_Log2
                        1
## - hs_cs_m_Log2
                        1
                              262.0 26039 3904.4
## - hs_as_c_Log2
                              478.3 26255 3915.1
                        1
                           12022.4 37799 4385.5
## - hs_child_age_None
                       1
##
## Step: AIC=3892.99
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
##
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
       hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2 + hs_hg_m_Log2 +
       hs_co_m_Log2 + hs_cs_c_Log2 + hs_cd_c_Log2
##
##
##
                                      RSS
                       Df Sum of Sq
                                              ATC
## + hs mn m Log2
                               40.9 25689 3892.9
## <none>
                                     25730 3893.0
## - hs cd c Log2
                               46.5 25777 3893.3
                        1
## - hs_cs_c_Log2
                               53.7 25784 3893.7
                        1
                               54.2 25784 3893.7
## - hs hg m Log2
                        1
## - hs co c Log2
                               54.9 25785 3893.7
                        1
## - hs co m Log2
                        1
                               60.2 25790 3894.0
## + hs_mo_m_Log2
                        1
                               12.8 25717 3894.4
                               10.4 25720 3894.5
## + hs_tl_mdich_None
                        1
## + hs_tl_cdich_None
                                6.3 25724 3894.7
                        1
## + hs_cu_m_Log2
                        1
                                2.9 25727 3894.8
## + hs_pb_m_Log2
                        1
                                0.9 25729 3894.9
## + hs_mn_c_Log2
                        1
                                0.2 25730 3895.0
## - hs_mo_c_Log2
                        1
                               80.7 25811 3895.0
## - hs_as_m_Log2
                               95.7 25826 3895.8
                        1
## - hs_hg_c_Log2
                        1
                              162.1 25892 3899.1
## - hs_cu_c_Log2
                              203.1 25933 3901.1
                        1
## - hs cd m Log2
                        1
                              217.2 25947 3901.8
```

```
## - hs_pb_c_Log2
                              240.1 25970 3903.0
                       1
                             274.5 26005 3904.7
## - hs_cs_m_Log2
                       1
## - hs as c Log2
                             477.6 26208 3914.7
## - hs_child_age_None 1 12032.8 37763 4386.3
##
## Step: AIC=3892.94
## hs_correct_raven ~ hs_child_age_None + hs_cs_m_Log2 + hs_pb_c_Log2 +
       hs_as_c_Log2 + hs_cd_m_Log2 + hs_cu_c_Log2 + hs_hg_c_Log2 +
##
      hs_mo_c_Log2 + hs_co_c_Log2 + hs_as_m_Log2 + hs_hg_m_Log2 +
##
      hs_co_m_Log2 + hs_cs_c_Log2 + hs_cd_c_Log2 + hs_mn_m_Log2
##
##
                                     RSS
                       Df Sum of Sq
                                             AIC
## <none>
                                    25689 3892.9
## - hs_mn_m_Log2
                               40.9 25730 3893.0
## - hs_cd_c_Log2
                               41.3 25731 3893.0
                        1
## - hs_co_c_Log2
                        1
                               54.4 25744 3893.7
## - hs_hg_m_Log2
                              58.9 25748 3893.9
                       1
## - hs cs c Log2
                              64.5 25754 3894.2
                       1
## + hs_mo_m_Log2
                              13.7 25676 3894.3
                       1
                             11.3 25678 3894.4
## + hs tl mdich None 1
## + hs_tl_cdich_None
                       1
                              6.0 25683 3894.6
## - hs_mo_c_Log2
                             73.7 25763 3894.6
                       1
## + hs_cu_m_Log2
                              2.8 25687 3894.8
                       1
                              1.8 25688 3894.9
## + hs mn c Log2
                       1
## - hs_co_m_Log2
                       1
                              79.1 25768 3894.9
## + hs_pb_m_Log2
                       1
                              0.2 25689 3894.9
## - hs_as_m_Log2
                             85.5 25775 3895.2
                       1
                             165.9 25855 3899.3
## - hs_hg_c_Log2
                       1
## - hs_cu_c_Log2
                             204.3 25894 3901.2
                       1
## - hs_cd_m_Log2
                            229.2 25919 3902.4
                       1
## - hs_pb_c_Log2
                        1
                              253.1 25942 3903.6
## - hs_cs_m_Log2
                       1
                              291.4 25981 3905.5
## - hs_as_c_Log2
                              485.2 26175 3915.1
## - hs_child_age_None 1
                          12009.2 37699 4386.1
##
      user system elapsed
##
     0.156
           0.008 0.168
## compare the three different models
beta.fwd <- coef(Mfwd)
beta.back <- coef(Mback)</pre>
beta.step <- coef(Mstep)</pre>
c(fwd = length(beta.fwd), back = length(beta.back),
step = length(beta.step)) # number of coefficients in each
##
  fwd back step
##
    16
         16
              16
# check if models are nested
names(beta.fwd)[!names(beta.fwd) %in% names(beta.back)]
## character(0)
names(beta.back)[!names(beta.back) %in% names(beta.fwd)]
## character(0)
```

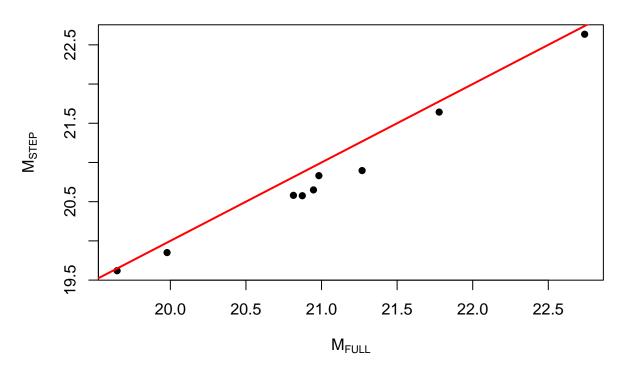
```
## AIC
n <- nrow(expm)</pre>
ll_fwd \leftarrow -n/2 * (1 + log(sum(resid(Mfwd)^2)/n) + log(2*pi))
aic_fwd <- -2*11_fwd + 2*(n - Mfwd$df + 1) # total number of parameters includes sigma
aic_fwd - AIC(Mfwd)
## [1] 68.24673
aic_step <- AIC(Mstep)</pre>
aic_back <- AIC(Mback)</pre>
aic_all <- round(c(aic_fwd,aic_step,aic_back),2)</pre>
names(aic_all) <- c("FWD", "Step", "Back")</pre>
aic_all
##
       FWD
               Step
                        Back
## 7626.89 7558.64 7558.64
bic_fwd \leftarrow -2*ll_fwd + log(n)*(n - Mfwd$df + 1) # total number of parameters includes sigma
bic_fwd - BIC(Mfwd)
## [1] 120.0868
bic_step <- BIC(Mstep)</pre>
bic_back <- BIC(Mback)</pre>
bic_all <- round(c(bic_fwd,bic_step,bic_back),2)</pre>
names(bic_all) <- c("FWD", "Step", "Back")</pre>
bic_all
       FWD
               Step
                        Back
## 7766.50 7646.41 7646.41
## Random subsets cross-validation
# compare Mfull to Mstep
M1 <- Mfull
M2 <- Mstep
Mnames <- expression(M[FULL], M[STEP])</pre>
# number of cross-validation replications
nreps <- 1e3
ntot <- nrow(expm) # total number of observations</pre>
ntrain <- 800 # for fitting MLE's</pre>
ntest <- ntot-ntrain # for out-of-sample prediction</pre>
# storage space
mspe1 <- rep(NA, nreps) # mspe for M1
mspe2 <- rep(NA, nreps) # mspe for M2
system.time({
  for(ii in 1:nreps) {
    if(ii%%100 == 0) message("ii = ", ii)
    train.ind <- sample(ntot, ntrain) # training observations</pre>
   # long-form cross-validation
```

```
M1.cv <- update(M1, subset = train.ind)</pre>
    M2.cv <- update(M2, subset = train.ind)</pre>
    # cross-validation residuals
    M1.res <- expm$hs_correct_raven[-train.ind] - # test observations
      predict(M1.cv, newdata = expm[-train.ind,]) # prediction with training data
    M2.res <- expm$hs_correct_rave[-train.ind] -predict(M2.cv, newdata = expm[-train.ind,])
    # mspe for each model
    mspe1[ii] <- mean(M1.res^2)</pre>
    mspe2[ii] <- mean(M2.res^2)</pre>
 }
})
## ii = 100
## ii = 200
## ii = 300
## ii = 400
## ii = 500
## ii = 600
## ii = 700
## ii = 800
## ii = 900
## ii = 1000
##
     user system elapsed
##
     4.752
            0.135 4.998
# compare
par(mfrow = c(1,2))
cex <- 1
boxplot(x = list(mspe1, mspe2), names = Mnames,
        main = "MSPE using Random subsets cross-validation",
        #ylab = expression(sqrt(bar(SSE)[CV])),
        ylab = expression(MSPE),
        col = c("yellow", "orange"),
        cex = cex, cex.lab = cex, cex.axis = cex, cex.main = cex)
boxplot(x = list(sqrt(mspe1), sqrt(mspe2)), names = Mnames,
        main = "Root MSPE using Random subsets cross-validation",
        ylab = expression(sqrt(MSPE)),
        ## ylab = expression(SSE[CV]),
        col = c("yellow", "orange"),
        cex = cex, cex.lab = cex, cex.axis = cex, cex.main = cex)
```

MSPE using Random subsets cross-valiot MSPE using Random subsets cross-valion



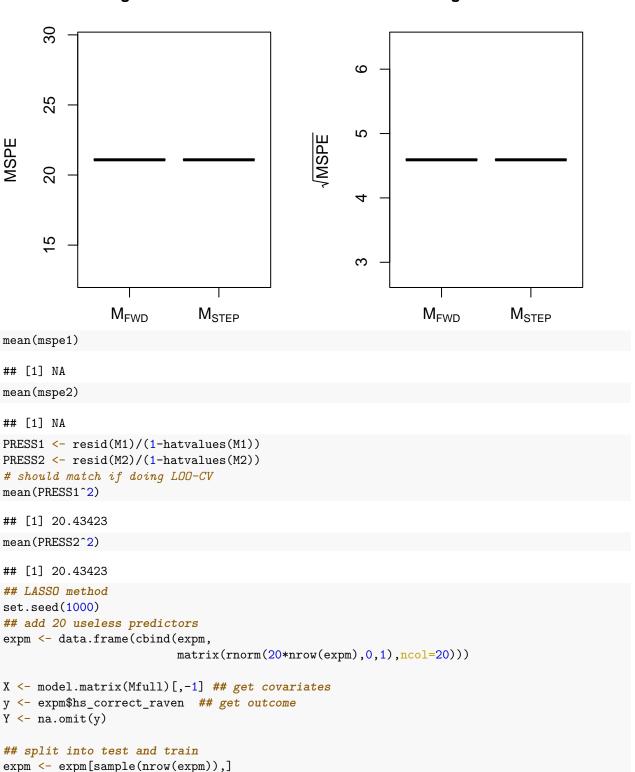
Mean Squared Prediction Error of MFull and Mstep



```
## K-fold cross-validation
# compare Mfwd to Mstep
set.seed(1000)
M1 <- Mfwd
M2 <- Mstep
Mnames <- expression(M[FWD], M[STEP])</pre>
expm \leftarrow expm[-1300,]
ntot <- nrow(expm) # total number of observations</pre>
# number of cross-validation replications
Kfolds <- 5
expm <- expm[sample(ntot),] # permute rows</pre>
expm$index <- rep(1:Kfolds,each=ntot/Kfolds)</pre>
# storage space
mspe1 <- rep(NA, Kfolds) # mspe for M1</pre>
mspe2 <- rep(NA, Kfolds) # mspe for M2
system.time({
  for(ii in 1:Kfolds) {
    train.ind <- which(expm$index!=ii) # training observations</pre>
    # using R functions
    M1.cv <- update(M1, subset = train.ind)</pre>
    M2.cv <- update(M2, subset = train.ind)</pre>
    # cross-validation residuals
    M1.res <- expm$hs_correct_raven[-train.ind] - # test observations
      predict(M1.cv, newdata = expm[-train.ind,]) # prediction with training data
    M2.res <- expm$hs_correct_raven[-train.ind] -predict(M2.cv, newdata = expm[-train.ind,])
    # mspe for each model
    mspe1[ii] <- mean(M1.res^2)</pre>
    mspe2[ii] <- mean(M2.res^2)</pre>
 }
})
##
      user system elapsed
##
     0.021
            0.000 0.021
# compare
par(mfrow = c(1,2))
cex <- 1
boxplot(x = list(mspe1, mspe2), names = Mnames,
        main = " MSPE using K-fold cross-validation",
        #ylab = expression(sqrt(bar(SSE)[CV])),
        ylab = expression(MSPE),
        col = c("yellow", "orange"),
        cex = cex, cex.lab = cex, cex.axis = cex, cex.main = cex)
boxplot(x = list(sqrt(mspe1), sqrt(mspe2)), names = Mnames,
        main = "Root MSPE using K-fold cross-validation",
        ylab = expression(sqrt(MSPE)),
```

```
## ylab = expression(SSE[CV]),
col = c("yellow", "orange"),
cex = cex, cex.lab = cex, cex.axis = cex, cex.main = cex)
```

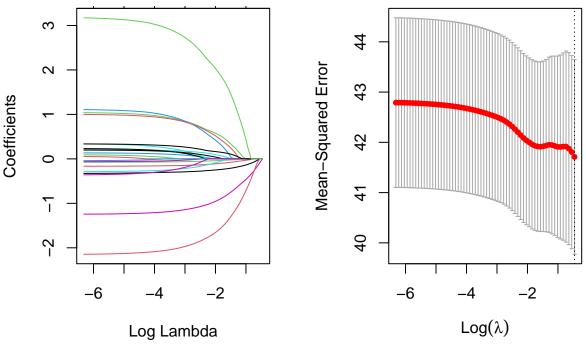
MSPE using K-fold cross-validation Root MSPE using K-fold cross-validation



```
ntrain <- 500
train_id <- 1:ntrain
X_train <- X[train_id,]
X_test <- X[-train_id,]
Y_train <- Y[train_id]
Y_test <- Y[-train_id]

## fit models
M_lasso <- glmnet(x=X_train,y=Y_train,alpha = 1)
## plot paths
plot(M_lasso,xvar = "lambda",label=TRUE, main= "plotting paths according to LASSO method")
## fit with crossval
cvfit_lasso <- cv.glmnet(x=X_train,y=Y_train,alpha = 1)
## plot MSPEs by lambda
plot(cvfit_lasso, main="plotting Mean Sqaured Prediction Error by lambda for LASSO method")</pre>
```

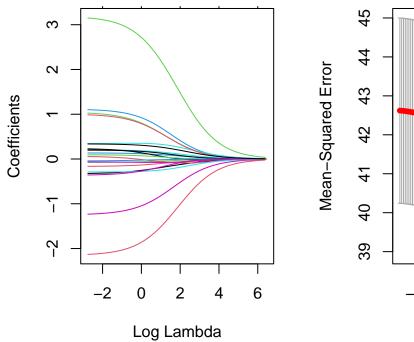
otting paths according to LASSO magaured Prediction Error by lambda



estimated betas for minimum lambda
coef(cvfit_lasso, s = "lambda.min")## alternatively could use "lambda.1se"

```
## hs_cs_m_Log2
## hs_cu_c_Log2
## hs_cu_m_Log2
## hs_hg_c_Log2
## hs_hg_m_Log2
## hs_mn_c_Log2
## hs_mn_m_Log2
## hs_mo_c_Log2
## hs_mo_m_Log2
## hs_pb_c_Log2
## hs_pb_m_Log2
## hs_tl_cdich_NoneUndetected
## hs_tl_mdich_NoneUndetected
## predictions
pred_lasso <- predict(cvfit_lasso,newx=X_test, s="lambda.min")</pre>
## MSPE in test set
MSPE_lasso <- mean((pred_lasso-Y_test)^2)</pre>
## Warning in pred_lasso - Y_test: longer object length is not a multiple of
## shorter object length
## RIDGE
## fit models
M_ridge <- glmnet(x=X_train,y=Y_train,alpha = 0)</pre>
## plot paths
plot(M_ridge,xvar = "lambda",label=TRUE, main= "plotting paths according to RIDGE method")
## fit with crossval
cvfit_ridge <- cv.glmnet(x=X_train,y=Y_train,alpha = 0)</pre>
## plot MSPEs by lambda
plot(cvfit_ridge, main ="plotting Mean Squured Prediction Error by lambda for RIDGE method")
```

otting paths, according to RIDGE naquired Prediction Error, by Jambda



```
coef(cvfit_ridge, s = "lambda.min")## alternatively could use "lambda.1se"
## 22 x 1 sparse Matrix of class "dgCMatrix"
## (Intercept)
                              20.402658745
## hs_child_age_None
                              -0.081897174
## hs_as_c_Log2
                              -0.039663496
## hs as m Log2
                              -0.018929286
## hs_cd_c_Log2
                               0.042914075
## hs_cd_m_Log2
                               0.072899580
## hs_co_c_Log2
                              -0.320351836
## hs_co_m_Log2
                              -0.016381096
## hs_cs_c_Log2
                              -0.083620480
## hs_cs_m_Log2
                               0.194306520
## hs_cu_c_Log2
                               0.234928523
## hs_cu_m_Log2
                               0.188430322
## hs_hg_c_Log2
                              -0.049447118
## hs_hg_m_Log2
                               0.043888199
## hs_mn_c_Log2
                               0.186373982
## hs_mn_m_Log2
                              0.007028066
## hs mo c Log2
                              -0.003814262
## hs_mo_m_Log2
                              -0.126631143
## hs_pb_c_Log2
                              -0.039207153
## hs_pb_m_Log2
                               0.134761994
## hs_tl_cdich_NoneUndetected -0.646247409
## hs_tl_mdich_NoneUndetected 0.961720045
## predictions
pred_ridge <- predict(cvfit_ridge,newx=X_test, s="lambda.min")</pre>
## MSPE in test set
MSPE_ridge <- mean((pred_ridge-Y_test)^2)</pre>
## Warning in pred_ridge - Y_test: longer object length is not a multiple of
## shorter object length
## compare prediction error for lasso and ridge
MSPE_lasso
## [1] 41.57002
MSPE_ridge
## [1] 41.7584
```

estimated betas for minimum lambda