

Question

A spin-half particle is in a linear superposition $0.5|\uparrow\rangle + \sqrt{3}/2|\downarrow\rangle$ of its spin-up and spin-down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of σ_z , then what is the expectation value up to one decimal place, of the operator $10\sigma_z + 5\sigma_x$? Here, symbols have their usual meanings
A) -1.4
B) -0.7
C) 1.65
D) 0.85

Ground Truth Answer: B)

Ours Longest: 2766 Tokens

NRP: 2294 Tokens

Okay, so I have this quantum mechanics problem here, and I need to find the expectation value of the operator $10\sigma_z + 5\sigma_x$ for a given spin- $\frac{1}{2}$ particle state. Hmm, let me try to break this down step by step.
First, the state of the particle is given as a linear combination of the spin-up and spin-down states: $0.5|\uparrow\rangle + (\sqrt{3}/2)|\downarrow\rangle$. I remember that $|\uparrow\rangle$ and $|\downarrow\rangle$ are eigenstates of the Pauli matrix σ_z . So, σ_z acting on $|\uparrow\rangle$ gives +1 times $|\uparrow\rangle$, and σ_z acting on $|\downarrow\rangle$ gives -1 times $|\downarrow\rangle$, right?
Now, the operator we're dealing with is $10\sigma_z + 5\sigma_x$. I need to find the expectation value of this operator, which is $\langle\psi| (10\sigma_z + 5\sigma_x) |\psi\rangle$. Since expectation value is linear, I can separate this into $10\langle\psi|\sigma_z|\psi\rangle + 5\langle\psi|\sigma_x|\psi\rangle$. That should make things easier.
Let me handle each term separately. Starting with the first term, $10\langle\psi|\sigma_z|\psi\rangle$. I know that σ_z is diagonal in the σ_z basis, which is the same as the $|\uparrow\rangle$ and $|\downarrow\rangle$ basis. So, the expectation value of σ_z is just the weighted average of the eigenvalues, with the weights given by the square of the amplitudes in the state $|\psi\rangle$.
The state $|\psi\rangle$ is $0.5|\uparrow\rangle + (\sqrt{3}/2)|\downarrow\rangle$. So, the coefficient for $|\uparrow\rangle$ is 0.5, and for $|\downarrow\rangle$ is $\sqrt{3}/2$. The square of these gives the probabilities. Therefore, the expectation value of σ_z is $(0.5)^2*(+1) + (\sqrt{3}/2)^2*(-1)$. Calculating that:
 $0.25*(1) + (3/4)*(-1) = 0.25 - 0.75 = -0.5$. So, 10 times that would be $10*(-0.5) = -5$.
Okay, that's the first part. Now, the second term is $5\langle\psi|\sigma_x|\psi\rangle$. Hmm, σ_x is the Pauli X matrix, which is not diagonal in the σ_z basis. I remember that σ_x has eigenstates $|\pm x\rangle$ and $|\mp x\rangle$, which are superpositions of $|\uparrow\rangle$ and $|\downarrow\rangle$. Maybe I need to express $|\psi\rangle$ in the σ_x basis to compute this expectation value, or perhaps there's another way.
Alternatively, I recall that σ_x can be written in matrix form as $[[0,1],[1,0]]$. So, if I can write $|\psi\rangle$ as a column vector, then σ_x acting on $|\psi\rangle$ can be computed, and then the inner product $\langle\psi|\sigma_x|\psi\rangle$ can be found.
Let's represent $|\psi\rangle$ as a vector. Since $|\uparrow\rangle$ is $[1; 0]$ and $|\downarrow\rangle$ is $[0; 1]$, then $|\psi\rangle$ is $0.5*[1; 0] + (\sqrt{3}/2)*[0; 1] = [0.5; \sqrt{3}/2]$. So, the vector is $[0.5, \sqrt{3}/2]^T$.
Now, σ_x is $[[0,1],[1,0]]$. So, $\sigma_x|\psi\rangle$ is $[0.5*1 + (\sqrt{3}/2)*0, 0.5*0 + (\sqrt{3}/2)*1]^T$, which is $[0.5, \sqrt{3}/2]^T$. Wait, that's the same as $|\psi\rangle$? Hm, so $\sigma_x|\psi\rangle = |\psi\rangle$?
[omitted]
Since a and b are real in this case (the coefficients are real), this becomes $2ab$.
So, for our state, $a = 0.5$, $b = \sqrt{3}/2$. Therefore, $\langle\sigma_x\rangle = 2*(0.5)*(\sqrt{3}/2) = (\sqrt{3}/2) \approx 0.866$.
Therefore, $5*\langle\sigma_x\rangle = 5*(\sqrt{3}/2) \approx 5*0.866 \approx 4.33$.
So, adding the two terms together: $-5 + 4.33 \approx -0.67$. So, approximately -0.7 when rounded to one decimal place.
Ah, okay, so that makes sense now. My initial mistake was probably in the way I computed $\sigma_x|\psi\rangle$ and then the inner product. But using the formula for expectation value in terms of coefficients, it's much simpler. So, the expectation value of σ_x is $2ab$, which is $2*(0.5)*(\sqrt{3}/2) = \sqrt{3}/2 \approx 0.866$. Therefore, 5 times that is approximately 4.33, and 10 times the expectation value of σ_z is -5. So, adding them gives approximately -0.67, which rounds to -0.7. So, the answer should be option B) -0.7.

Remaining: None

NRP Ratio: 100%

ThinkPrune Longest: 8098 Tokens

NRP: 1535 Tokens

Okay, so I have this quantum mechanics problem here about a spin-half particle. Hmm, let's see. The particle is in a linear superposition given by $0.5|\uparrow\rangle + (\sqrt{3}/2)|\downarrow\rangle$. And I need to find the expectation value of the operator $10\sigma_z + 5\sigma_x$. The answer choices are A) -1.4, B) -0.7, C) 1.65, D) 0.85.
Alright, first, let me recall what the expectation value of an operator is. For a quantum state $|\psi\rangle$, the expectation value $\langle O \rangle$ is given by $\langle\psi| O |\psi\rangle$. So, I need to compute $\langle\psi| (10\sigma_z + 5\sigma_x) |\psi\rangle$.
Since expectation value is linear, I can separate this into $10\langle\psi|\sigma_z|\psi\rangle + 5\langle\psi|\sigma_x|\psi\rangle$. So, I need to find the expectation values of σ_z and σ_x separately and then combine them.
Let me write down the given state $|\psi\rangle$. It is $0.5|\uparrow\rangle + (\sqrt{3}/2)|\downarrow\rangle$. Let me note that $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of σ_z , which is the Pauli z-matrix. So, $\sigma_z|\uparrow\rangle = +|\uparrow\rangle$ and $\sigma_z|\downarrow\rangle = -|\downarrow\rangle$.
First, compute $\langle\sigma_z\rangle$. That is, $\langle\psi|\sigma_z|\psi\rangle$. Since σ_z is diagonal in the σ_z basis, which is the same as the basis in which $|\uparrow\rangle$ and $|\downarrow\rangle$ are defined, this should be straightforward.
[omitted...]
Looking at the options, B is -0.7, which would be the answer.
But wait, let me verify the calculations again because sometimes I might have made a mistake.
First, $\langle\sigma_z\rangle$: probabilities 0.25 and 0.75, so $0.25 - 0.75 = -0.5$. That seems right.
Then, for $\langle\sigma_x\rangle$: the expectation value.
Another way to compute $\langle\sigma_x\rangle$ is to note that $\sigma_x = |\rightarrow\rangle\langle\rightarrow| - |\leftarrow\rangle\langle\leftarrow|$, but maybe that's more complicated.
Alternatively, for a general state $|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$, $\langle\sigma_x\rangle = |a|^2 - |b|^2$? Wait, no, that can't be.
Wait, σ_x is $[[0,1],[1,0]]$, so when you compute $\langle\psi|\sigma_x|\psi\rangle$, it's $|a|^2*2*(\downarrow|\sigma_x|\uparrow) + |b|^2*2*(\uparrow|\sigma_x|\downarrow) + \text{cross terms}$.
Wait, let's compute it again step by step.
 $\langle\psi|\sigma_x|\psi\rangle = [0.5; \sqrt{3}/2]^T \cdot [[0,1],[1,0]] \cdot [0.5; \sqrt{3}/2] = [0.5, \sqrt{3}/2]$. So, in matrix form, it's a column vector $[0.5; \sqrt{3}/2]$.
 $\langle\psi|\sigma_x|\psi\rangle = [(\sqrt{3}/2)/2 ; 0.5]^T \cdot [0.5; \sqrt{3}/2]$.
Then, $\langle\psi|\sigma_x|\psi\rangle$ is $[0.5, (\sqrt{3}/2)]$ multiplied by $[(\sqrt{3}/2)/2 ; 0.5]^T$.
So, $0.5*(\sqrt{3}/2)^2 + (\sqrt{3}/2)^2 * 0.5 = (\sqrt{3})/4 + (\sqrt{3})/4 = \sqrt{3}/2 \approx 0.866$.
So, that seems correct.
Therefore, $5\langle\sigma_x\rangle = 5*(\sqrt{3}/2)/2 \approx 4.3301$, and $10\langle\sigma_z\rangle = -5$, so total expectation value is $-5 + 4.3301 \approx -0.6699$, which is approximately -0.67, so -0.7 when rounded to one decimal.
Therefore, the answer should be B) -0.7.

Remaining: 5990 Tokens

Wait, but let me think again. Is there another way to compute this? Maybe using the properties of spin states.
[omitted...]
So, in conclusion, $\langle\sigma_x\rangle$ is approximately 0.866, which is $\sqrt{3}/2$.
Therefore, $5\langle\sigma_x\rangle$ is $5*(\sqrt{3}/2) \approx 5*0.866 \approx 4.33$.
Then, $10\langle\sigma_z\rangle = -5$, as computed earlier.
Therefore, the total expectation value is $-5 + 4.33 \approx -0.67$, which is approximately -0.7 when rounded to one decimal place.
Therefore, the answer is B) -0.7.

NRP Ratio: 20%