COSC 312

Homework Assignment 4

Submission Format: docx, tex or pdf to Canvas

Points: 10 per proof; total = 40

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1. Prove that C = {w|w has an equal number of 0s and 1s} is not regular. Consider the string s=0^p1^p.

Assume that C is regular and let p be the pumping length of C.

Choose $s = 0^p1^p \in C$ so that $|0^p1^p| > p$. According to the Pumping Lemma, s can be partitioned into $s = xyz$, such that for all $i \geq 0$, there is $xy^iz \in C$. By condition 3 of the Pumping Lemma, we should have $|xy| \leq p$. Therefore, y must contain 0s and $xy^iz \neq C$ and $xz \neq C$. This means that there would be an unequal number of 0s in the string as 1s. Hence, s cannot be pumped and due to this contradiction, we can conclude that C is not regular.

2. Prove that F = {ww|w is a string from {0,1}*} is not regular. Consider the string s=0^p10^p1. Note that {0,1}* is any string (including the empty string) containing any number of 0s and 1s (in any order).

Assume that F is regular and let p be the pumping length of F.

Choose $s = 0^p10^p1 \in F$ so that $|0^p10^p1| > p$. According to the Pumping Lemma, s can be partitioned into $s = xyz$, such that for all $i \geq 0$, there is $xy^iz \in F$. By condition 3 of the Pumping Lemma, we should have $|xy| \leq p$. Therefore, y must contain 0s and $xy^iz = xy^2z \neq F$. This means that the first part of s (substring w) is not the same as the 2nd part of s. Hence, s cannot be pumped due to condition 1 of the Pumping Lemma, and due to this contradiction, we can conclude that F is not regular.

3. Prove that A = {www|w is a string from {a,b}*} is not regular. Consider the string $s=a^pba^pba^pb$.

Assume that A is regular and let p be the pumping length of A.

Choose $s = a^p ba^p ba^p b \in A$ so that $|a^p ba^p ba^p b| > p$. According to the Pumping Lemma, s can be partitioned into $s = xyz$, such that for all $i \geq 0$, there is $xy^iz \in A$. By condition 3 of the Pumping Lemma, we should have $|xy| \leq p$. Therefore, y must only contain as and $xy^iz = A$. This means that there would be more as at the start of the string than any other part. Hence, s cannot be pumped and due to this contradiction, we can conclude that A is not regular.

4. Prove that L= $\{0^n1^m0^n | m,n \geq 0\}$ is not regular. Consider the string $s=0^p10^p$.

Assume that L is regular and let p be the pumping length of L.

Choose $s = 0^p10^p \in L$ so that $|0^p10^p| > p$. According to the Pumping Lemma, s can be partitioned into $s = xyz$, such that for all $i \geq 0$, there is $xy^iz \in L$. By condition 3 of the Pumping Lemma, we should have $|xy| \leq p$. Therefore, y must only contain *0*s and $xy^iz = L$. This means that there would be more *0*s in the first part of the string than any subsequent part. Hence, s cannot be pumped and due to this contradiction, we can conclude that L is not regular.