COSC 312

Homework Assignment 7

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1. $A = \{0^n \# 0^{2n} \# 0^{3n} | n \geq 0\}$.

   Assume that A is context-free and let p be the pumping length of A. Choose $s = \{0^p \# 0^{2p} \# 0^{3p} | n \geq 0\} \in$ A so that $s > p$. By the Pumping Lemma, s can be partitioned into 5 pieces $s = uvxyz$ such that for all $i \geq 0$, $uv^ixy^iz \in$ A. We can analyze the following cases:

   Case 1. If *vxy* is the left sequence of zeroes. If $i = 0$, the number of zeroes in the left sequence would decrease while the zeros in the next two sequences remain the same. That would mean the word isn't in the language.
   Case 2. If *vx* is the left sequence of zeroes and y includes the first #. If $i = 2$, we'll be left with $w' = uv^2xy^2z$ which has too many #s, and is not in the language.
   Case 3. If *v* is the left sequence of zeroes, *x* includes the first #, and *y* includes the middle sequence of zeroes. If $i = 0$, the zeroes in the first two sequences will decrease but not the zeroes in the third. That word would not be in the language.
   Case 4. If *vxy* is the middle or right sequence of zeroes, we'll have the same problems as in Case 1.
   Case 5. If *x* or *y* includes the far right #, we'll have the same problems as Case 2 or 3.

   Based on these contradictions, A is not a context free language.

2. $C = \left\{ w \middle| \begin{array}{c} \text{the number of } 1s \text{ equals the number of } 2s \\ \text{and the number of } 3s \text{ equals the number of } 4s \end{array} \right\}$ and $\Sigma = \{1, 2, 3, 4\}$

   Assume that C is context-free and let p be the pumping length of C. Choose $s = 1^p 3^p 2^p 4^p \in$ C so that $|s| > p$. By the Pumping Lemma, s can be partitioned into 5 pieces $s = uvxyz$ such that for all $i \geq 0$, $uv^ixy^iz \in$ C. We can analyze the following cases:

   Case 1. If *vxy* contains a 1, then uv^2xy^2z will not be in C because it won't have the same number of 1s and 2s.
   Case 2. If *vxy* contains a 2, then uv^2xy^2z will not be in C for the same reason as Case 1- it won't have the same number of 1s and 2s.
   Case 3. If *vxy* contains a 3, then uv^2xy^2z will not be in C for the same reason as Cases 1 and 2- it won't have the same number of 3s and 4s.
   Case 4. If *vxy* contains a 3, then uv^2xy^2z will not be in C for the same reason as Cases 1, 2, and 3- it won't have the same number of 3s and 4s.
   Case 5. If *vxy* contains a 4, then uv^2xy^2z will not be in C for the same reason as Cases 1, 2, 3, and 4- it won't have the same number of 3s and 4s.

   Based on these contradictions, C is not a context free language.

3. $B = \{\text{w\#t} \mid w\ is\ a\ substring\ of\ t, where\ w, t \in \{\text{a, b}\} *\}$. Assume that B is context-free and let p be the pumping length of C. Choose $s = a^p b^p \# a^p b^p\ \in$ B so that $|s| > p$. By the Pumping Lemma, s can be partitioned into 5 pieces $s = uvxyz$ such that for all $i \geq 0$, $uv^i xy^i z \in$ C. We can analyze the following cases:

Case 1. Because # separates sections, it can neither go into *v* or *y*.
Case 2. If vxy is contained on the right side, then the right side gets smaller while w stays the same. This means this word isn't in the language.
Case 3. If x straddles #, then vxy will have some bs on the left and some as on the right. In this case, we'd have more bs in w than bs on the right side, which is not in the language.
Case 4. If v or y are fully on the left, then w will be bigger than t.
Case 5. If v or y is fully on the right, then t has fewer as or bs than w.

Based on these contradictions, C is not a context free language.