Laboratory work 3

Optimization in Python

Goal: Learning main Python features for solving optimization problems.

2. Tasks:

1. Modelling and solving linear programming problem:

   a. Choose assignment according to the number in the group journal.
   b. Describe the entered variables.
   c. Create a mathematical model of the problem.
   d. Find the solution using graphical representation of linear programming problem. The figure should contain lines with their equations, shaded half-plane that satisfy the original inequalities, the feasible region, the point of optimal solution, level curve.
   e. Solve the problem using `scipy.optimize.`
   f. Solve the problem using `pulp.LpProblem.`
   g. Compare solutions using `np.allclose()`.
   h. Compare the time to solve the problem using `scipy.optimize` and `pulp.LpProblem.`
   i. Explain the result.

2. Solving transportation problem.

   a. Describe the entered variables.

   b. Create a mathematical model of the problem.

   c. Solve the problem using `scipy.optimize.`

   d. Explain the result.

Assignments for task 1

1. The company has to produce at least 1100 details of type 1, 900 details of type 2 and 700 details of type 3. The cutting of one standard sheet using first cutting pattern gives 7 details of type 1, 5 – of type 2, 9 – of type 3 and 0.2 unit of offcuts. The second cutting pattern gives 3 details of type 1, 4 – of type 2, 2 of type 3 and 0.3 unit of offcuts. It should be found out how many sheets are needed to be cutting using each pattern to produce not less than the specified quantity of details and the total sum of offcuts will be minimal.

2. 4 types of raw material C_1, C_2, C_3 і C_4 are used to produce 2 types of goods P_1 і P_2. The total reserve and consumption rates for producing one unit of goods are given in the table. Income from selling the unit P_1 is 7 units and P_2 is 5 units. You should found out how many goods of each type the factory has to manufacture to maximize the total income.

Types of product / Types of raw material	P_1	P_2	Total reserves
C_1	2	8	19
C_2	2	1	13
C_3	0	3	15
C_4	3	0	18
Income	7	5	

3. To make the concrete it is necessary at least 300 units of cement and 850 units of granite chippings. The content of ingredients in two types of raw materials C_1 i C_2 that are offered to the company is as follows: C_1 – 12 and 24 units, C_2 – 25 and 13 units. How many raw materials of each type should be bought to minimize the total cost, if the unit of raw material C_1 and C_2 cost \$18 and \$13 respectively?

4. To save health and working ability the person have to obtain at least 12 units of protein, 10 units of fat, 16 units of carbohydrates, 10 units of water and 3 units of vitamins. The content of these nutrients in the two types of foods F_1 i F_2 and cost of food are presented in the table. You have to create a diet to provide the needed amount of all nutrients and to minimize the total cost of the meal.

	Protein	Fat	Carbohydrates	Water	Vitamins	Cost, \$
F_1	3	1	2	2	1	2
F_2	2	5	4	2	0.5	3

5. To make 1 item of some product it is necessary to have 2 items of detail D_1 and 1 item of detail D_2. These details can be obtained by two ways: the first way gives 6 items of details D_1 and 4 items of details D_2 from one unit of raw material. This way provides 0.06 units of waste. The second way provide 1 item of detail D_1, 2 items of detail D_2 and 0.05 units of waste. How many units of raw materials by each way should be processed to produce not less than 70 items of this product?

6. To feed the plants, it is necessary to apply: at least 24 units of the chemical substance B_1, 30 units of the chemical substance B_2 and 15 units of the chemical substance B_3 per hectare. All these substances are contained in combined additives U_1 and U_2 in certain amounts that is shown in the table. What amount of each fertilizer should be acquired to provide the necessary fertilizing but minimize the total costs of the fertilizers?

	B_1	B_2	B_3	Cost of an unit of fertilizer, $
C_1	3	4	1	3
C_2	2	5	3	5

7. It is necessary to build the reinforced concrete structures of two types P_1 and P_2, the costs of which are 25 and 15 units respectively. For this purpose three types of metal structures are used: K_1, K_2 and K_3. The suppliers can provide with 40, 65 and 80 units of each metal structure respectively. For building the construction P_1 it is necessary to use 2 units of K_1, 3 units of K_2 and 4 units of K_3, and for building the construction P_2 we need 1 unit of K_1, 2 units of K_2 and 2 units K_3. Make a plan for the release of reinforced concrete structures that would provide the company with the maximum income.

8. The monthly plan of the cutting shop is 1000 units of parts P_1 and 6000 units of parts P_2.Cutting 1 unit of material by 1st method allows us to obtain 10 units of parts P_1 and 90 units of P_2. 2-d method of cutting gives, respectively, 30 units of P_1 and 40 units P_2. Determine how many units of material to be cut by each of the methods to ensure the total minimum of waste, if 1st method of each unit of material gives 0.2 unit of waste and the 2nd method gives 0.31 unit of waste.

9. Using certain equipment, the enterprise can produce 400 products of type P_1, or 100 products of type P_2 per shift, the costs of each product are $18 and $54 per unit. Make a production plan that would ensure maximum profit, if the painting shop can process no more than 300 products of any type per shift.

10. For the production of two types of concrete mixtures B_1 and B_2, raw materials are used, which are mined in three quarries C_1, C_2 and C_3, the capacities of which are 34, 42 and 60 units, respectively. Manufacturing 1 unit of B_1 requires 3 units of raw material C_1, 1 unit of raw material C_2 and 4 units of raw material C_3. Manufacturing 1 unit of B_2, requires 1, 4 and 5 units of C_1, C_2 and C_3 respectively, Draw up a plan for the production of concrete mixes, which provides the maximum profit, if 1 unit of B_1 and B_2 costs $3 and $5 respectively.

11. A farmer mixes two brands *P* and *Q* of cattle feed. Brand *P*, costing $250 per bag, contains 3 units of nutritional element *A*, 2.5 units of element *B* and 2 units of element *C*. Brand *Q* costing $200 per bag contains 1.5 units of nutritional element *A*, 11.25 units of element *B*, and 3 units of element *C*. The minimum requirements of nutrients *A*, *B* and *C* are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag. What is the minimum cost of the mixture per bag?

12. A dietician wishes to mix together two kinds of food *X* and *Y* in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin *B* and 8 units of vitamin *C*. The vitamin content of one kg of food is given below:

Food	Vitamin *A*	Vitamin *B*	Vitamin *C*
X	1	2	3
Y	2	2	1

One *kg* of food *X* costs \$16 and one *kg* of food *Y* costs \$20. Find the amount of food of each type that should be in the diet to ensure the least cost of the mixture which will produce the required diet?

Assignments for the task 2

Transportation problem: There are 5 destinations (in columns) where the products/goods are to be delivered from different 4 sources (in rows). a_i is the supply form each source; b_i is the demand of each destination; c_{ij} is the cost when the product is delivered from i^{th} source to j^{th} destination. Find a transportation plan that will provide all customers with the necessary goods at the lowest cost.

	1						2						3					
i/j	**1**	**2**	**3**	**4**	**5**	a_i	**1**	**2**	**3**	**4**	**5**	a_i	**1**	**2**	**3**	**4**	**5**	a_i
1	12	8	7	10	9	**85**	12	5	7	10	9	**85**	10	8	7	12	9	**85**
2	4	5	11	3	14	**110**	4	5	11	3	10	**110**	4	6	5	3	14	**110**
3	15	10	6	5	9	**65**	15	12	6	5	9	**65**	15	10	6	5	7	**65**
4	16	8	6	4	5	**80**	16	8	6	4	5	**80**	11	3	6	4	5	**80**
b_i	**90**	**70**	**70**	**60**	**50**	***340***	**90**	**70**	**70**	**60**	**50**	***340***	**90**	**70**	**70**	**60**	**50**	***340***
	4						5						6					
i/j	**1**	**2**	**3**	**4**	**5**	a_i	**1**	**2**	**3**	**4**	**5**	a_i	**1**	**2**	**3**	**4**	**5**	a_i
1	12	6	7	5	9	**85**	17	8	3	9	5	**85**	14	8	7	15	9	**85**
2	4	5	11	3	16	**110**	4	5	4	3	18	**110**	4	5	11	8	14	**110**
3	15	9	6	5	9	**65**	11	10	6	5	9	**65**	15	12	6	5	9	**65**
4	12	10	6	4	5	**80**	7	8	9	4	5	**80**	10	8	6	4	5	**80**
b_i	**90**	**70**	**70**	**60**	**50**	***340***	**90**	**70**	**70**	**60**	**50**	***340***	**90**	**70**	**70**	**60**	**50**	***340***
	7						8						9					
i/j	**1**	**2**	**3**	**4**	**5**	a_i	**1**	**2**	**3**	**4**	**5**	a_i	**1**	**2**	**3**	**4**	**5**	a_i
1	9	5	7	10	9	**90**	11	3	7	10	9	**85**	19	4	7	10	9	**85**
2	8	15	11	3	12	**70**	4	5	11	3	7	**110**	4	5	11	3	14	**110**
3	10	14	6	11	9	**65**	15	10	6	5	8	**65**	15	10	6	5	9	**65**
4	16	5	7	4	8	**75**	6	8	6	4	5	**80**	12	8	9	8	5	**80**
b_i	**70**	**90**	**50**	**60**	**30**	***300***	**90**	**70**	**70**	**60**	**50**	***340***	**90**	**70**	**70**	**60**	**50**	***340***
	10						11						12					
i/j	**1**	**2**	**3**	**4**	**5**	a_i	**1**	**2**	**3**	**4**	**5**	a_i	**1**	**2**	**3**	**4**	**5**	a_i
1	16	4	7	10	15	**85**	5	17	7	10	9	**85**	12	8	7	10	9	**85**
2	4	5	11	3	10	**110**	4	5	11	3	14	**110**	4	5	11	3	14	**110**
3	15	12	6	5	9	**65**	12	10	6	5	9	**65**	15	10	6	5	9	**65**
4	10	8	12	4	5	**80**	11	8	6	4	16	**80**	16	8	6	4	5	**80**
b_i	**90**	**70**	**70**	**60**	**50**	***340***	**90**	**70**	**70**	**60**	**50**	***340***	**80**	**80**	**70**	**60**	**50**	***340***

3. The content of the report

1. Cover page of the report.
2. Topic and goal of the lab.
3. Progress of the work:
    a. Problem statement;
    b. Table for more convenient representation of problem statement;
    c. Explanation of problem variables;
    d. Mathematical models for problems 1 and 2;
    e. Graphical representation for problem 1;
    f. Listings of input cells;
    g. Responses in output cells;
    h. Explanation of the solutions.
4. Link to the created Jupyter Notebook on GitHub, rendered by nbviewer.
5. Conclusions.

Appendix 1.

Example of a graphical solution

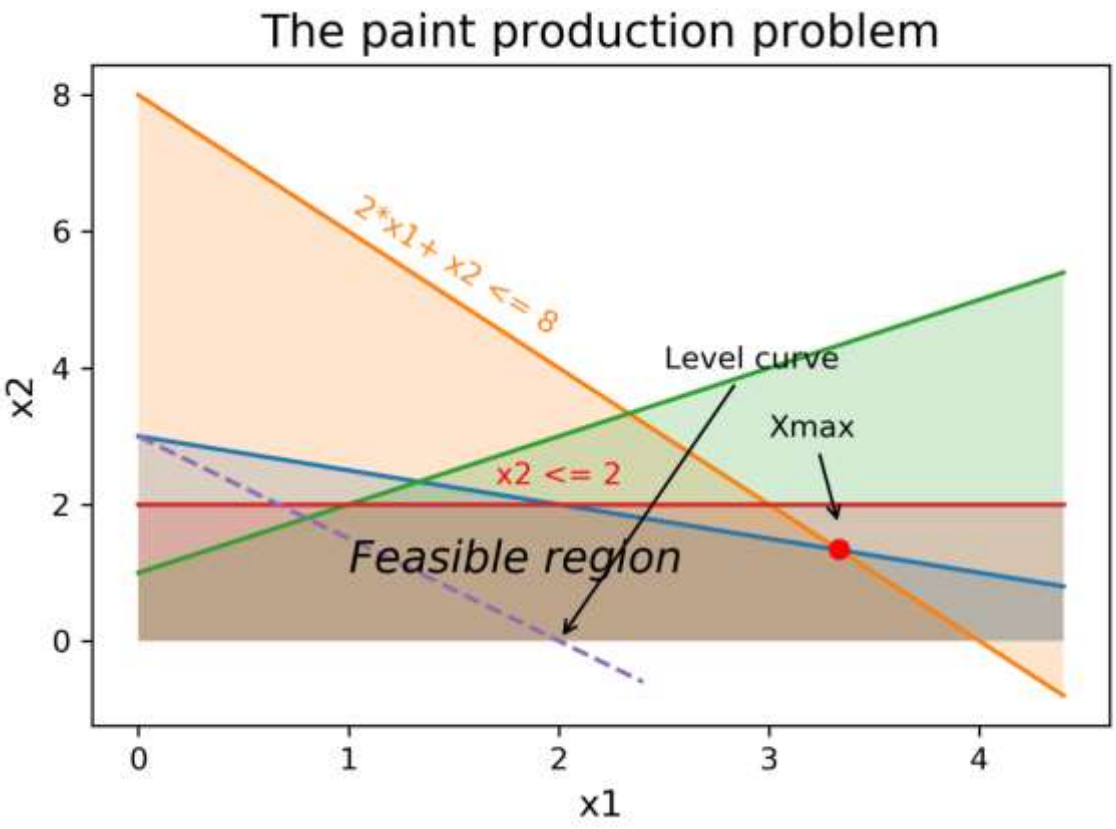

Fig. 1. Approximate view of the graphical solution

Appendix 2.

An example of explanation the problem solution.

```
     con: array([], dtype=float64)
     fun: -12.66666666663677
 message: 'Optimization terminated successfully.'
     nit: 5
   slack: array([9.946e-12, 1.994e-11, 3.000e+00, 6.667e-01])
  status: 0
 success: True
       x: array([3.333, 1.333])
Time :
0.0140008926391601 56
```

Fig. 2. A solution obtained with scipy.optimize()

Explanation: In order to get the maximum profit, that is equal to 12.667, we need to produce the first type of paints in the amount of 3.333, and the second type paints in the amount of 1.3333. At the same time, raw materials of the first and second types are in short supply, that is, they are fully used.