Laboratory work 2

Basics of the NumPy library

Goal: Gaining knowledge and skills of using the NumPy library.

2. Tasks:

Notes: a) Perform all tasks using the NumPy library **without** using loops to access to the elements of the matrices, as well as the functions `map(), filter(), reduce()`.

b) In notebook cells the problem statement should be specified. To create formulas LaTex should be used.

c) Checking should be performed using `numpy.allclose()`

1. Perform a set of exercises (see tab 2.1).

2. Solve the system of linear equations using Cramer's formulas and perform a test using:

   a) matrix multiplication;

   b) `numpy.linalg.solve()`.

3. Calculate the value of the matrix expression.

4. Solve the matrix equation using an inverse matrix and perform a test using:

   a) matrix multiplication;

   b) `numpy.linalg.solve()`.

5. Place the created notebook on GitHub

6. Prepare a report

Table 2.1. Assignments for the lab

Ind. task	Assignment 1	Assignments 2-5
1	6, 7, 9, 10, 11, 15, 17, 18, 20, 22	2) $\begin{cases} x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 - x_2 - x_3 - 2x_4 = -4 \\ 2x_1 - 3x_2 - x_3 - x_4 = -6 \\ x_1 + 2x_2 + 3x_3 - x_4 = -4 \end{cases}$
		3) $2(A+B)(2B-A),\ A = \begin{pmatrix} 2 & 3 & -1 \\ 4 & 5 & 2 \\ -1 & 0 & 7 \end{pmatrix},\ B = \begin{pmatrix} -1 & 0 & 5 \\ 0 & 1 & 3 \\ 2 & -2 & 4 \end{pmatrix}$

		4) $\begin{pmatrix} 2 & 3 & 1 \\ -1 & 2 & 4 \\ 5 & 3 & 0 \end{pmatrix} * X = \begin{pmatrix} 2 & 7 & 13 \\ -1 & 0 & 5 \\ 5 & 13 & 21 \end{pmatrix}$
2	2, 5, 11, 12, 13, 14, 16, 17, 20	2) $\begin{cases} x_1 + 2x_2 + 3x_3 - 2x_4 = 6 \\ x_1 - x_2 - 2x_3 - 3x_4 = 8 \\ 3x_1 + 2x_2 - x_3 + 2x_4 = 4 \\ 2x_1 - 3x_2 + 2x_3 + x_4 = -8 \end{cases}$
		3) $3A - (A - 2B)B,\ A = \begin{pmatrix} 4 & 5 & -2 \\ 3 & -1 & 0 \\ 4 & 2 & 7 \end{pmatrix},\ B = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \\ 5 & 7 & 3 \end{pmatrix}$
		4) $\begin{pmatrix} 2 & 3 & 1 \\ -1 & 2 & 4 \\ 5 & 3 & 0 \end{pmatrix} * X = \begin{pmatrix} 2 & 7 & 13 \\ -1 & 0 & 5 \\ 5 & 13 & 21 \end{pmatrix}$
3	1, 3, 6, 7, 8, 14, 16, 18, 19, 22	2) $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 5 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1 \\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5 \end{cases}$
		3) $2(A - B)(A^2 + B),\ A = \begin{pmatrix} 5 & 1 & 7 \\ -10 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix},\ B \begin{pmatrix} 2 & 4 & 1 \\ 3 & 1 & 0 \\ 7 & 2 & 1 \end{pmatrix}$
		4) $\begin{pmatrix} 4 & -2 & 0 \\ 1 & 1 & 2 \\ 3 & -2 & 0 \end{pmatrix} * X = \begin{pmatrix} 0 & -2 & 6 \\ 2 & 4 & 3 \\ 0 & -3 & 4 \end{pmatrix}$
4	2, 4, 8, 9, 11, 12, 13, 14, 18, 19	2) $\begin{cases} x_2 - 3x_3 + 4x_4 = -5 \\ x_1 - 2x_3 + 3x_4 = -4 \\ 3x_1 + 2x_2 - 5x_4 = 12 \\ 4x_1 + 3x_2 - 5x_3 = 5 \end{cases}$
		3) $\left(A^2 - B^2\right) * (A + B),\ A = \begin{pmatrix} 7 & 2 & 0 \\ -7 & -2 & 1 \\ 1 & 1 & 0 \end{pmatrix},\ B = \begin{pmatrix} 0 & 2 & 3 \\ 1 & 0 & -2 \\ 3 & 1 & 1 \end{pmatrix}$
		4) $X * \begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 0 \\ 4 & -3 & 0 \end{pmatrix} = \begin{pmatrix} 22 & -14 & 3 \\ 6 & -7 & 0 \\ 11 & 3 & 15 \end{pmatrix}$

5	2, 3, 4, 5, 7, 9, 10, 14, 15, 21	2) $\begin{cases} x_1+3x_2+5x_3+7x_4=12 \\ 3x_1+5x_2+7x_3+x_4=0 \\ 5x_1+7x_2+x_3+3x_4=4 \\ 7x_1+x_2+3x_3+5x_4=16 \end{cases}$
		3) $\left(A-B^2\right)\left(2A+B\right),\ A=\begin{pmatrix} 5 & 2 & 0 \\ 10 & 4 & 1 \\ 7 & 3 & 2 \end{pmatrix},\ B=\begin{pmatrix} 3 & 6 & -1 \\ -1 & -2 & 0 \\ 2 & 1 & 3 \end{pmatrix}$
		4) $\begin{pmatrix} 2 & 3 & 1 \\ 4 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix} * X=\begin{pmatrix} 9 & 8 & 7 \\ 2 & 7 & 3 \\ 4 & 3 & 5 \end{pmatrix}$
6	1, 3, 4, 6, 10, 12, 15, 17, 18, 22	2) $\begin{cases} x_1+5x_2+3x_3-4x_4=20 \\ 3x_1+x_2-2x_3=9 \\ 5x_1-7x_2+10x_4=-9 \\ 3x_2-5x_3=1 \end{cases}$
		3) $\left(A-B^2\right)\cdot\left(2A+B\right),\ A=\begin{pmatrix} 5 & 2 & 0 \\ 10 & 4 & 1 \\ 7 & 3 & 2 \end{pmatrix},B=\begin{pmatrix} 3 & 6 & -1 \\ -1 & -2 & 0 \\ 2 & 1 & 3 \end{pmatrix}$
		4) $\begin{pmatrix} 2 & 3 & 1 \\ 4 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix} * X=\begin{pmatrix} 9 & 8 & 7 \\ 2 & 7 & 3 \\ 4 & 3 & 5 \end{pmatrix}$
7	3, 4, 5, 6, 7, 8, 9, 13, 17, 19	2) $\begin{cases} 2x_1+x_2-5x_3+x_4=8 \\ x_1-3x_2-6x_4=9 \\ 2x_2-x_3+2x_4=-5 \\ x_1+4x_2-7x_3+6x_4=0 \end{cases}$
		3) $2\left(A-0{,}5B\right)+AB\ \ A=\begin{pmatrix} 5 & 3 & -1 \\ 2 & -2 & 0 \\ 3 & -1 & 2 \end{pmatrix}\ B=\begin{pmatrix} 1 & 4 & 16 \\ -3 & -2 & 0 \\ 5 & 7 & 2 \end{pmatrix}$
		4) $\begin{pmatrix} 4 & 2 & 1 \\ 3 & -2 & 0 \\ 0 & -1 & 2 \end{pmatrix} * X=\begin{pmatrix} 2 & 0 & 2 \\ 5 & -7 & -2 \\ 1 & 0 & -1 \end{pmatrix}$
8	3, 4, 5, 6, 7, 10, 12, 16, 17, 21	2) $\begin{cases} 2x_1-x_2+3x_3+2x_4=4 \\ 3x_1+3x_2+3x_3+2x_4=6 \\ 3x_1-x_2-x_3+2x_4=6 \\ 3x_1-x_2+3x_3-x_4=6 \end{cases}$

		3) $(A-B)A+3B,\ A=\begin{pmatrix}3&2&-5\\4&2&0\\1&1&2\end{pmatrix}, B=\begin{pmatrix}-1&2&4\\0&3&2\\-1&-3&4\end{pmatrix}$
		4) $X*\begin{pmatrix}1&4&2\\2&1&-2\\0&1&-1\end{pmatrix}=\begin{pmatrix}4&6&-2\\4&10&1\\2&4&-5\end{pmatrix}$
9	1, 2, 4, 7, 8, 12, 13, 15, 16, 20	2) $\begin{cases}x_1+2x_2+x_3+x_4=8\\2x_1+x_2+x_3+x_4=5\\x_1-x_2+2x_3+x_4=-1\\x_1+x_2-x_3+3x_4=10\end{cases}$
		3) $2A-\left(A^2+B\right)B,\ A=\begin{pmatrix}1&4&2\\2&1&-2\\0&1&-1\end{pmatrix},\ B=\begin{pmatrix}4&6&-2\\4&10&1\\2&4&-5\end{pmatrix}$
		4) $\begin{pmatrix}3&2&-5\\4&2&0\\1&1&2\end{pmatrix}*X=\begin{pmatrix}-1&2&4\\0&3&2\\-1&-3&4\end{pmatrix}$
10	1, 2, 3, 4, 6, 8, 11, 20, 21, 22	1) $\begin{cases}4x_1+x_2-x_4=-9\\x_1-3x_2+4x_3=7\\3x_2-2x_3+4x_4=12\\x_1+2x_3+4x_4=12\end{cases}$
		3) $3\left(A^2-B^2\right)-2AB\ \ A=\begin{pmatrix}4&2&1\\3&-2&0\\0&1&2\end{pmatrix}B=\begin{pmatrix}2&0&2\\5&-7&-2\\1&0&-1\end{pmatrix}$
		4) $X*\begin{pmatrix}5&3&-1\\-2&0&4\\3&5&-1\end{pmatrix}=\begin{pmatrix}1&4&16\\-3&-2&0\\5&7&-2\end{pmatrix}$
11	1, 5, 9, 10, 12, 15,16, 21, 22	2) $\begin{cases}2x_1-x_2+x_3-x_4=1\\2x_1-x_2-3x_4=2\\3x_1-x_3+x_4=-3\\2x_1+2x_2-2x_3+5x_4=-6\end{cases}$
		3) $(2A-B)(3A+B)-2AB, A=\begin{pmatrix}1&0&3\\-2&0&1\\-1&3&1\end{pmatrix},\ B=\begin{pmatrix}7&5&2\\0&1&2\\-3&-1&-1\end{pmatrix}$

		4) $X * \begin{pmatrix} 5 & -1 & 3 \\ 0 & 2 & -1 \\ -2 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 7 & -2 \\ 1 & 1 & -2 \\ 0 & 1 & 3 \end{pmatrix}$
12	1, 2, 13, 14, 17, 18, 19, 20, 21, 22	2) $\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_2 + 2x_3 - x_4 = 2 \\ x_1 - x_2 - x_4 = -1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$
		3) $A\left(A^2 - B\right) - 2\left(B + A\right)B,\ A = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 2 & 4 \\ 5 & 3 & 0 \end{pmatrix} B = \begin{pmatrix} 2 & 7 & 13 \\ -1 & 0 & 5 \\ 5 & 13 & 21 \end{pmatrix}$
		4) $\begin{pmatrix} 4 & 5 & -2 \\ 3 & -1 & 0 \\ 4 & 2 & 7 \end{pmatrix} * X = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \\ 5 & 7 & 3 \end{pmatrix}$
13	1, 5, 7, 8, 11, 13, 16, 17, 20, 22	2) $\begin{cases} 5x_1 + x_2 - x_4 = -9 \\ 3x_1 - 3x_2 + x_3 + 4x_4 = -7 \\ 3x_1 - 2x_3 + x_4 = -16 \\ x_1 - 4x_2 + x_4 = 0 \end{cases}$
		3) $\left(A + B\right)A - B\left(2A + 3B\right),\ A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & 5 \\ 1 & 4 & -1 \end{pmatrix} B = \begin{pmatrix} 4 & 11 & 3 \\ 1 & 6 & 1 \\ 2 & 2 & 16 \end{pmatrix}$
		4) $\begin{pmatrix} 1 & 1 & -1 \\ 4 & -3 & 1 \\ 0 & 2 & 1 \end{pmatrix} * X = \begin{pmatrix} 7 & 0 & -5 \\ 4 & 11 & 2 \\ 1 & 3 & 1 \end{pmatrix}$
14	6, 9, 10, 11, 12, 15, 18, 19, 20, 21	2) $\begin{cases} 2x_1 + x_3 + 4x_4 = 9 \\ x_1 + 2x_2 - x_3 + x_4 = 8 \\ 2x_1 + x_2 + x_3 + x_4 = 5 \\ x_1 - x_2 + 2x_3 + x_4 = -1 \end{cases}$
		3) $A\left(2A + B\right) - B\left(A - B\right),\ A = \begin{pmatrix} 2 & 3 & 1 \\ 4 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix}, B = \begin{pmatrix} 9 & 8 & 7 \\ 2 & 7 & 3 \\ 4 & 3 & 5 \end{pmatrix}$
		4) $\begin{pmatrix} 5 & 3 & -1 \\ 2 & 0 & 4 \\ 3 & 5 & -1 \end{pmatrix} * X = \begin{pmatrix} 1 & 4 & 16 \\ -3 & -2 & 0 \\ 5 & 7 & 2 \end{pmatrix}$

15	1, 3, 6, 8, 12	2) $\begin{cases} 2x_1 - 6x_2 + 2x_3 + 2x_4 = 12 \\ x_1 + 3x_2 + 5x_3 + 7x_4 = 12 \\ 3x_1 + 5x_2 + 7x_3 + x_4 = 0 \\ 5x_1 + 7x_2 + x_3 + 3x_4 = 4 \end{cases}$
		3) $3(A+B)(AB-2A)$, $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 0 \\ 4 & -3 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 22 & -14 & 3 \\ 6 & -7 & 0 \\ 11 & 3 & 15 \end{pmatrix}$
		4) $X * \begin{pmatrix} 2 & 3 & -1 \\ 4 & 5 & 2 \\ -1 & 0 & 7 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 5 \\ 2 & 1 & 3 \\ 0 & -2 & 4 \end{pmatrix}$

Assignment 1

Note: In all examples `Out []` are provided just to specify the outputs. In fact, there are shown the results of the `print()` function.

Excercise 1. Enter the number *n*. Output an array of size *n*×*n*, in which the diagonals are numbers from 1 to *n*, and other elements are 0

```
In  [2]   task_1()
          5
Out [2]   [[1 0 0 0 0]
           [0 2 0 0 0]
           [0 0 3 0 0]
           [0 0 0 4 0]
           [0 0 0 0 5]]
```

Excercise 2. Enter the number *n*. Arrange 1 and 0 in a checkerboard pattern, starting with 0 in a matrix of size *n*×*n*, using slicing.

```
In  [2]   task_2()
          5
Out [2]   [[0 1 0 1 0]
           [1 0 1 0 1]
           [0 1 0 1 0]
           [1 0 1 0 1]
           [0 1 0 1 0]]
```

Excercise 3. Enter 4 numbers *n*, *m*, *r*, *c*. Output an array of size *n*×*m*, in which in each *r* -th row and in each *c*-th column are 0, and other elements are equal to 1.

```
In  [2]   task_3()
          6 7 2 3
Out [2]   [[0 0 0 0 0 0 0]
           [0 1 1 0 1 1 0]
           [0 0 0 0 0 0 0]
           [0 1 1 0 1 1 0]
           [0 0 0 0 0 0 0]
```

```
          [0 1 1 0 1 1 0]]
```

Excercise 4. Enter the numbers *n* and *m*. Output an array of size *n*×*m*, in which the first line (line with zero index) are numbers from 0 to *m*-1, and all other elements of the matrix are equal to 0.

```
In  [2]   task_4()
          3 4
Out [2]   [[0 1 2]
           [0 0 0]
           [0 0 0]
           [0 0 0]]
```

Excercise 5. Enter the number *n*. Output an array of size *n*×*n*, in which the rows with the even indices are equal to 1, and in others are equal to 0.

```
In  [2]   task_5()
          5
Out [2]   [[1 1 1 1 1]
           [0 0 0 0 0]
           [1 1 1 1 1]
           [0 0 0 0 0]
           [1 1 1 1 1]]
```

Excercise 6. An array is entered from the keyboard. Replace all non-zero elements with -1.

```
In  [2]   task_6()
          3 4 0 9 7 0 6 0 4 0 3
Out [2]   [-1 -1  0 -1 -1  0 -1  0 -1  0 -1]
```

Excercise 7. An array is entered from the keyboard. Replace all zeros with -1.

```
In  [2]   task_7()
           3 4 0 6 5 0 3 0 4
Out [2]   [3  4 -1  6  5 -1  3 -1  4]
```

Excercise 8. An array is entered from the keyboard. Count the number of zero and non-zero elements in it.

```
In  [2]   task_8()
           3 4 0 9 8 2 4 0 8 4 0
Out [2]   Zeros: 3
          Non_zeros: 8
```

Excercise 9. Enter the number *n*. Create an array of values from *n* to 0.

```
In  [2]   task_9()
           10
Out [2]   [10  9  8  7  6  5  4  3  2  1  0]
```

Excercise 10. Enter the numbers *n* and *m*. Create an array of random values of size *n*×*n* and calculate the minimum, maximum value, mean and standard deviation,

rounded to 3 decimal places

```
In  [2]   task_10()

          4 5
Out [2]   minimum: 0.038
          maksimum: 0.946
          mean: 0.593
          deviation: 0.302
```

Excercise 11. Create an array of 1 of size *n*x*n* and add to it a "border" formed from 0

```
In  [2]   task_11()
          4
Out [2]   [[0 0 0 0 0 0]
           [0 1 1 1 1 0]
           [0 1 1 1 1 0]
           [0 1 1 1 1 0]
           [0 1 1 1 1 0]
           [0 0 0 0 0 0]]
```

Excercise 12. Create an array of 1 of size *n*x*n* and create an inside "border" formed from 0.

```
In  [2]   task_12()
          4
Out [2]   [[0. 0. 0. 0.]
           [0. 1. 1. 0.]
           [0. 1. 1. 0.]
           [0. 0. 0. 0.]]
```

Excercise 13. Determine the index of 70-th element in a vector, if it is converted into a matrix of size 4x5x6.

```
Out [2]   In the matrix of 4x5x6 the 70-th element index is (2, 1, 3)
```

Excercise 14. Arrange 1 and 0 on the 8x8 field in a checkerboard pattern using the repeat function (*).

```
Out [2]   [[0 1 0 1 0 1 0 1]
           [1 0 1 0 1 0 1 0]
           [0 1 0 1 0 1 0 1]
           [1 0 1 0 1 0 1 0]
           [0 1 0 1 0 1 0 1]
           [1 0 1 0 1 0 1 0]
           [0 1 0 1 0 1 0 1]
           [1 0 1 0 1 0 1 0]]
```

Excercise 15. Arrange 1 and 0 on the 8x8 field in a checkerboard pattern using the `numpy tile()`.

```
Out [2]   [[0 1 0 1 0 1 0 1]
           [1 0 1 0 1 0 1 0]
           [0 1 0 1 0 1 0 1]
           [1 0 1 0 1 0 1 0]
```

```
          [0 1 0 1 0 1 0 1]
          [1 0 1 0 1 0 1 0]
          [0 1 0 1 0 1 0 1]
          [1 0 1 0 1 0 1 0]]
```

Excercise 16. Fill the even columns of the matrix of *n*x*n* size with 1, and the odd ones with 0.

```
In  [2]   task_16()
          5
Out [2]   [[0 1 0 1 0]
           [0 1 0 1 0]
           [0 1 0 1 0]
           [0 1 0 1 0] [0 1 0 1 0]]
```

Excercise 17. Fill the vector with values from 0 to *n*. Replace all values greater than *n*/2 and less than 3*n*/4 with zeros.

```
In  [2]   task_17()
          10
Out [2]   [ 0  1  2  0  0  0  0  0  8  9 10]
```

Excercise 18. Fill the vector with values from 0 to *n*. Replace signs for all values less than n/2 and greater than 3n/4 to opposite ones.

```
In  [2]   task_18()
          10
Out [2]   [  0  -1   2   3   4   5   6   7   8  -9 -10]
```

Excercise 19. Generate a vector of *n* random numbers in the range from 0 to 99. Determine the number of unique elements in the sequence.

```
In  [2]   task_19()
          30
Out [2]   In vector [ 5 77 15 21  8 44 58 77  6 49 35 35 25  3 51 65
73 81 67 78 85 68 40 77 58  1 10  9 59 13] there are 26 unique numbers
```

Excercise 20. Generate a vector of *n* elements that are evenly spaced on the interval (0,1) i.e. both ends of the interval are not included. Output values up to 3 decimal places.

```
In  [2]   task_20()
          10
Out [2]  [0.091 0.182 0.273 0.364 0.455 0.545 0.636 0.727 0.818 0.909]
```

Excercise 21. Generate a vector of *n* random elements, lying in the interval (0,1). Replace the maximum element with 0.

```
In  [2]   task_21()
          5
Out [2]  Initial vector [0.593 0.071 0.85  0.646 0.827]
         Modified vector [0.593 0.071 0.    0.646 0.827]
```

Excercise 22. Enter the numbers *n* and *m*. Generate a matrix *n*x*n* of random integer elements from 0 to *m*. Change the signs of all maximum elements to the opposite.

```
In  [2]   task_22()
           4 10
Out [2]
Initial vector
 [[9 6 0 9]
 [4 3 7 9]
 [4 1 0 8]
 [5 5 4 8]]
Modified vector
 [[-9  6  0 -9]
 [ 4  3  7 -9]
 [ 4  1  0  8]
 [ 5  5  4  8]]
```

3. The content of the report

1. Cover page of the report.
2. Topic and goal of the lab.
3. Progress of the work with the listings of input cells and responses in output cells.
4. Screen shots with inputs and outputs.
5. Link to the created Jupyter Notebook on GitHub, rendered by nbviewer.
6. Conclusions.