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Cross-chain Settlement and Exchange in Cryptocurrency

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Abstract

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Glossary

Crypto currency - A currency backed not by centralized authorities but by mathematical evidence and clever mechanisms

Bitcoin - The very first and most mature cryptocurrency on earth

satoshi - The smallest fraction of a bitcoin. 100.000.000 satoshis = 1฿

Satoshi Nakamoto - The pseudonym used by the creator of bitcoin

Proof-of-work - A system where a someone can prove mathematically that work was put into doing something.

Mining - In this context refers to looking for a valid hash in a proof-of-work system. Most often by checking random numbers in the nonce field until a valid hash is found

Proof-of-work - afwaf

Proof-of-work - afwaf

Proof-of-work - afwaf

Proof-of-work - afwaf

Proof-of-work - afwaf

Atomic (Adjective) - Something that only has two outcomes. Either completed fully or no changes to the state at all. An atomic task can not be half completed

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Chapter 1

Introduction

Although blockchain and other types of distributed ledgers are still in their infancy a growing number of people and companies are starting to see that they hold great promise, especially when it comes to financial technology, where things like trust and security is held to be very important.

1.1 The origins of bitcoin

The most well known, developed and researched blockchain technology is know as Bitcoin. The mysterious nature of bitcoins creator makes hard to pinpoint how and when the idea was first thought up and when the development started, the most exact way and most well known would be to pinpoint at: 2009-01-03 18:15:05[1], which is the timestamp on the very first block in the bitcoin blockchain, however the white paper (**Bitcoin: A Peer-to-Peer Electronic Cash System**)[11] specifying the technical details circulated on cryptographic mailing lists as early as 31 October 2008, and the domain name `bitcoin.org` was registered 18 August 2008.[8]

1.1.1 Satoshi Nakamoto

The author name given in the white paper is **Satoshi Nakamoto**, this name is believed to be a pseudonym. Satoshi remained in the bitcoin community for a couple of years. Regularly posting on the forum `bitcointalk.org` and keeping up with conversations in the mailing list. Those who have been interested in finding out Satoshi's real identity have analyzed his active time and language used. The findings were that Satoshi was most active during Western European day time, and he also used a lot of Anglo-colloquialisms such as "bloody hard" [4], and "flat" instead of apartment, so a popular theory is that Satoshi lived in Britain at least during this time.[8]

April 23, 2011 was the last time anyone ever heard from Satoshi Nakamoto, in a mail to a fellow developer Mike Hearn he said "I've moved on to other things. It's in good hands with Gavin and everyone.".[6] Speculations on who Satoshi really is still going strong even to this day, but Nakamoto's true identity is so far unknown.[8][9]

1.2 Basics on Bitcoin

Most people, even the layman with no blockchain experience, have at least heard of bitcoin. But the exact details of how it works is not common knowledge. Described in a single sentence: bitcoin is a currency where a communal ledger that is shared between the whole world. The ledger holds the information on who owns what in terms of money or other

assets. The regular monetary system we are used to has a centralized authority, for example a central bank, who decides how much money is in circulation, who can transact with who etc. The monetary system presented by bitcoin has no centralized authority, instead it relies on decentralized, trust-less verification.

These usually are the main concerns people have with distributed ledgers:

- How is spending someone else's money prevented?
- How is spending the same money in different places in the world prevented?
- How is the consensus on the order of transactions reached?
- How do I interact with it?
- What type of transactions can you make?

These questions will be given answers in the sections below.

1.2.1 Digital signatures

Bitcoin would not be possible at all without the underlying cryptographic mathematics. When it comes to proving ownership in bitcoin ECDSA (Elliptic curve digital signature algorithm)[12] is used. Elliptic curve cryptography will not be covered in depth here but the basic idea is that you have a private and public key. The public key can be shared with anyone without danger, the public key can be derived from the private key, but not the other way around, this is something called trap-door mathematics.[12][7] This means that it is easy to go one way via equations. But going back is nearly impossible.

The reason for it not being completely impossible is because any potential attacker could always just keep guessing private keys until the right one is found. However with a sufficiently large private key, let's say 256-bits, and a computer that could check one billion billion (10^{18}) private keys every second (If such a machine could exist at all) it would still take $\frac{(2^{256}/10^{18})}{(60 \cdot 60 \cdot 24 \cdot 365)} \approx 3.67 \cdot 10^{51}$ years to check all possible private keys.

The most common transaction made on the blockchain is one where you "pay to" a public key. Who ever holds the private key paired with that public key can then via a mathematical equation prove that they hold the private key without actually revealing the private key.[5]

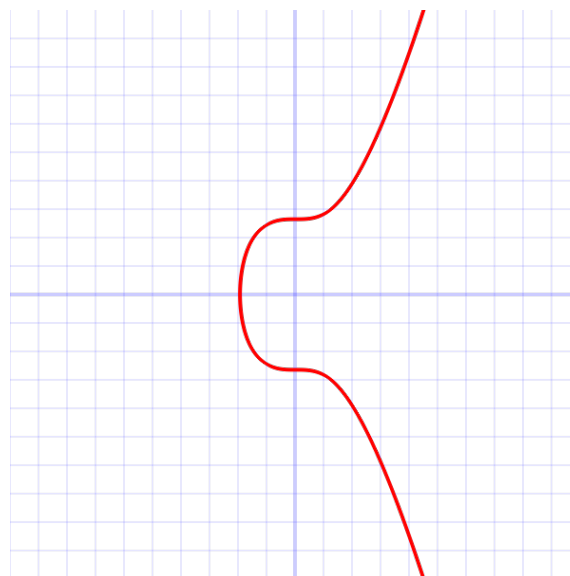


Figure 1.1: The **Secp256k1** plotted over real numbers. Note that the real curve is over a field, and thus looks more like a scattering of random points

The elliptic curve used can hold different parameters that defines it, certain elliptic curves are standardized and have their own names. The curve used in bitcoin is named **Secp256k1**.^{[13][7]}

The typical curve used in Elliptic curve cryptography is on the form $y^2 = x^3 + ax + b$. The **Secp256k1** is defined with $a = 0$ and $b = 7$, making the full **Secp256k1** equation: $y^2 = x^3 + 7$. Which is plotted in figure 1.1. For more in depth on elliptic curve cryptography see section 2.3

1.2.2 Blocks and the blockchain

A block is fairly simple to understand, it is simply a datatype or structure that holds information about it self, all the transactions that can fit and the previous block in the chain (more on that in a bit). Because every block holds information about the previous block you can follow all blocks backwards in time all the way back to the original, also called the genesis block.^[1] This is what is referred to as the blockchain.

A block could be added to the chain by anyone in the world. However it will only be accepted if it has sufficient proof of work, and this is the key to how consensus is reached in the network.^[7]

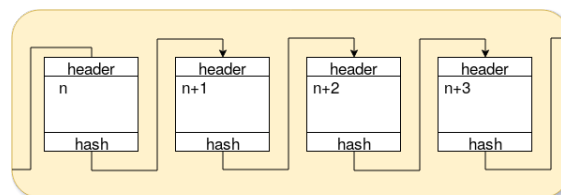


Figure 1.2: A basic overview of a blockchain

1.2.3 Proof of work

Proof of work is, just like the digital signatures, based in cryptographic mathematics. Before we go on you first need to understand what a hashing algorithm does, the hashing algorithm used in bitcoin is called **SHA256**. **SHA256** takes in data of any size and produces a sort of finger print of 256 bits.

For example the **SHA256** of the text "cool":

```
echo "cool" | openssl sha256
```

Produces:

```
27c16ce7e3861da034af1bb356d6a4f38cb84fa65d51fa62f69727143b4c6b60
```

The text produced is actually bytes represented in a hexadecimal number system, in fact the entire string can be considered to be a very large number. Just like with the digital signature there is no known viable way that can take a hash and find what the original data that produced it was.

There is a term in bitcoin called mining difficulty, or just difficulty. This is a large number, 256 bits to be exact. When you want to add a new block to the chain you have to do something called mining, this is a process where you change certain variables in the block until the hash of the block header (Think of this a number) is less than the target difficulty. The term hashpower refers to how many times the machine you are using to mine blocks can test a certain combination of variables per second, or H/s.

The difficulty of mining a block is adjusted about every 2 weeks. The difficulty is so that the combined hashpower of the entire world is enough to mine one block every 10 minutes on average.[7]

The accepted order of transactions is the order going backwards from the latest block on the longest chain. The longest chain is always the one that the majority is mining towards. This is simplified to the extreme but what it basically means is that as long as you trust 51% of the participants in the network you can also trust that the order of transactions is correct. This mechanism prevents things like double spending the same money and holds a property called emergent consensus, which means that eventually the entire network will agree on the order of transactions.

In figure 1.3 is a diagram showing the longest chain, meaning the chain with most proof of work. The block in green is the latest block on that chain. The yellow block is what is known as a stale block, a block that is part of the chain but not part of the longest chain of blocks. Transactions in a stale block are not considered valid, and eventually they can be pruned (deleted) because they do not effect the future in any way. A stale block occurs if a new block is found in different parts of the network at almost the same time. Then the network will be split, each node tries to find the next block on the chain of whatever block they received first. In the case shown below the chain on the left "won" and the green block is now the longest chain.

The red block in figure 1.3 is what is known as an orphan block, that is it has no known parent in the chain. This can happen if someone mines a block that is malformed or when building the chain for a new node and the blocks are received out of order.

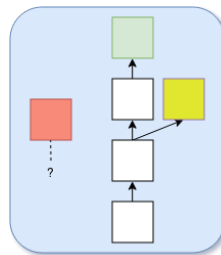


Figure 1.3: A diagram showing the longest chain, The green block is the latest block on the longest chain. The red block is a an orphan block, the yellow block is a stale block

1.2.4 Software

Just like how you can use regular money without knowing the underlying process and technology of for example the banking system, you can use bitcoin without knowing the details of how it works. There is plenty of software that handles wallets, transactions etc for you.

A good example is mycelium wallet for Android.

1.2.5 Programmable transactions

Bitcoin has another feature that makes it very versatile in what it can do and that is that every transaction is programmable. As mentioned earlier the most common transaction is sending the money to someones public key. What really goes on here is that the person who wants to spend whatever money was sent to them has to prove programmatic that they own the private key related to that public key.

This is not the only type of transaction possible, bitcoin has its own little programming language called **Script**. Any type of transaction that can be described in script is possible

as a transaction. This is basis for both lightning netowrk and atomic swap, so it is very central to the entire project.

1.3 Lightning network

One of the biggest problems facing bitcoin is scalability. At time of writing onchain transactions is capped at about ≈ 7 T/s (transactions er second).[2] This has to do with network propagation and the hard cap on block sizes, a block can only contain so many transactions before it is full. There are a couple of propsed solutions to this however, and lightning is one of them.

Lightning network is a relatively recent development in the bitcoin community. Many types of experimental transactions have been known for a long time. But in January of 2016 a white paper was released detailing a brand new type.[10] It showed that with a few changes to the bitcoin protocol a new type of transaction that opens a channel between two parties can be made. In this channel an unlimited number of transactions could be potentially made between the two parties. [10]

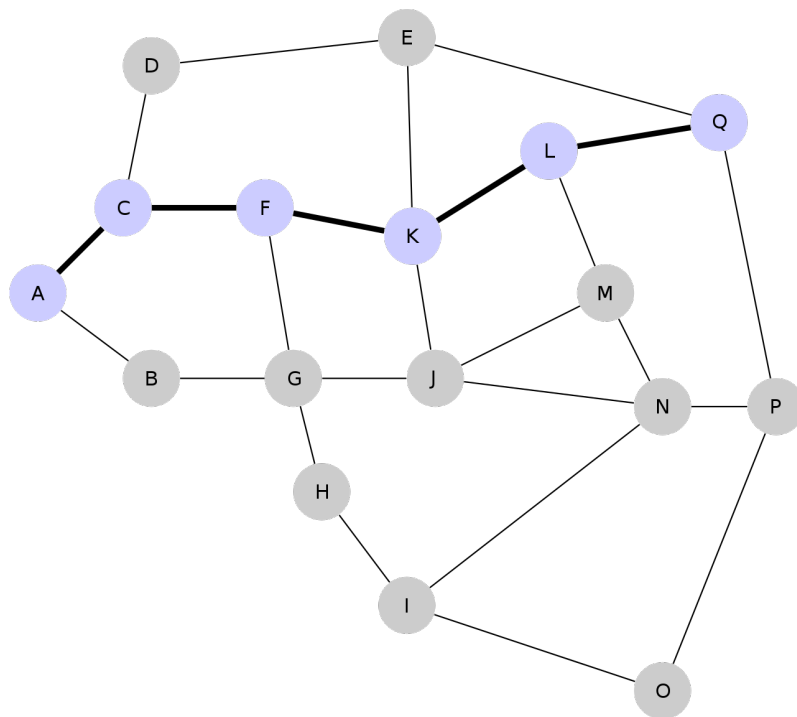


Figure 1.4: A basic overview of a lightning network, each node represent someone and each edge represents a channel between two people.

The name lightning network most likely a combination of lightning, as in lightning fast transactions. And network from the network that is formed by participants and their channels. One very useful feature of lightning network is that two parties do not need to have direct channel between them to be able to transact. Transactions can safely and atomically travel through the network via other participants, as long as there is a path between the two and there is enough funds in all the channels in between.[10]

1.4 Atomic swaps

Another recent development in bitcoin and other cryptocurrency is atomic swaps. A regular swap can for example be two parties exchanging currencies. before this could be

done in person, or via a trusted third-party handling all the transactions. Atomic swaps however is as the name implies atomic. Meaning that they either go through completely or no assets change hands at all, they are also completely trust-less meaning that you don't have to trust neither the other party or any third-party.

Just like lightning network atomic swaps use clever transaction scripts to achieve new functionality. Atomic swaps have been shown to be possible over lightning network, but it is mostly small projects and random writings on forums etc

Atomic swaps was first described by Tier Nolan in a post on bitcointalk.org on May 21, 2013.^[3]

1.4.1 The goal of the project

The goal of the project is to evaluate all the methods of atomic swaps available and compare them to a couple of different scenarios and use cases. Also I intend to become an expert in both atomic swaps and lightning network.

Chapter 2

Bitcoin and Smart contracts

2.1 Bitcoin: a peer to peer electronic cash

As described in the title of the original white paper bitcoin is a peer to peer electronic cash system, which does not rely on any centralized third-party to neither verify the validity of any transactions or handling of transaction completion. Instead it is entirely decentralized and trust-less. The mechanisms and mathematics that makes this possible is a relatively recent discovery in computer-science.

2.1.1 Network wide consensus

One of the main problems facing decentralized currencies before bitcoin was thought up is the byzantine generals problem (Byzantine fault). This refers to independent agents in a system being unable to reach a consensus on what has transpired and what actions to take next. The problem can be imagined as the network being split, where one subsection of the network believes transaction order **A** is correct while another non-overlapping subsection thinks transaction order **B** is correct. How can this be resolved, and how will a new node joining the network know which order is the right one.

Bitcoin was the first cryptocurrency to properly solve this problem once and for all, with the help of something called proof-of-work. proof of work originates from the slightly older idea of hashcash.

Hashcash was/is a means to limit email spam and DDoS-attacks by a proof of work system. For example a sender could be required to produce a hash of message and nonce with a certain number of bits set to 0 at the start of the hash sequence. This (statistically) should take several attempts to produce. But correctness could be checked in a single step. Thus the hash sent together with the message could be considered proof of work, because there is no known way to produce such a valid hash of a message without trying it randomly, thus the only realistic way to have such a hash is if you worked for it. Looking for a valid hash in this kind of proof of work systems is often referred to as **mining**.

Bitcoin used this proof of work concept for another purpose however. Rather than combating spam the proof of work mechanism is used for reaching consensus. When ever a new block is added to the chain it needs to meet a certain proof of work requirement, called difficulty. This difficulty is set so that it should take the combined hashing power of all participants on average 10 minutes to find a valid hash of the next block in the chain.

There is a term called longest proof of work in bitcoin, this refers to the chain of blocks that has the most hash-power supporting it. The chain with the most work done is statistically the one with the most participants. As long as 51% or more of the participants in the network are honest the longest chain can be trusted. So any new members can accept the

chain with most work as the truth.

Splits

Contention for the longest chain can arise if a new block is found in two different parts of the network at (almost) the same time. This is not a problem and will eventually be resolved. If you imagine two blocks (**A1** and **B1**) being mined in different parts of the network with the same parent block and half the network got **A1** first and the other half of the network got **B1** first. While the entire network will accept all valid blocks they will only mine towards continuing the chain on the block they received first. So if a new block **A2** with the parent block **A1** is found first the chain formed by block **B1** will be considered invalid and the network continues the chain on the **A** side.

Such contention is called a split, or a fork, and happens naturally once every week or so. As explained they will eventually resolve themselves. The split becomes increasingly more unlikely to survive the longer it goes on. To begin with it is unlikely that a new block will form a split in the first place, and for the split to survive another block both new chains will have to receive a new block at almost the same time.

In common bitcoin lingo there is no difference between a split and a fork. but in this report a **split will referred to as one occurring unintentionally** and a **fork being intentional** just so there is no confusion

How long was the longest split?

The longest splits that occurred by chance were 4 block long and has occurred at least at 3 different occasions.

The longest split ever was caused by an update to the bitcoin core reference implementation (**0.8.0**) that rejected a block that the other implementations did not reject, the nodes accepting the new block kept building on it while those who had updated built on a different chain. The split lasted for 52 blocks before it was resolved.

Forks

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2.3 Elliptic-curve cryptography & ECDSA

As covered in the introduction, Elliptic-curve cryptography (**ECC**) and ECDSA is a fundamental building block of bitcoin. Elliptic curve cryptography relies on intractability of calculating the discrete logarithm of a elliptic curve element with respect to a publicly

known base point. Or put another way: It is easy to calculate elliptic curve multiplication with multiplicand n . But calculating n from the resulting point is considered infeasible with sufficiently large curves and multiplicands.

An elliptic curve is defined by the equation $Y^2 = x^3 + ax + b$ and six domain parameters $E(p, a, b, G, n, h)$. \mathbf{p} is the field that the curve is defined over, this is usually a very large prime number. The curve being defined over a field simply means that the points on the curve fall within $[0, p]$ rather than within the real numbers \mathbb{R} . In other words the curve is defined over the field \mathbb{F}_p . \mathbf{a} and \mathbf{b} are whatever number you put into the equation. \mathbf{G} is the generator point, that is the point on the curve that will be used in point multiplication later. \mathbf{n} is the order of G . What that means is that n is the largest number that G can be multiplied by before a point at infinity is produced. n pretty much tells you the limit on how points on the curve that can be generated from G . \mathbf{h} is the co-factor of the curve. It can be calculated as follows: $h = \frac{1}{n}|(E(\mathbb{F}_p))|$, where $|E(\mathbb{F}_p)|$ is the order/cardinality of the group of points possible on the curve over field \mathbb{F}_p . n is derived from G , G and p should be chosen in such a way that $h \leq 4$, preferably $h = 1$.

These domain parameters can be chosen manually or you can use predefined parameters. Elliptic curves that used predefined domain parameters are called named-curves. The named curve used by Bitcoin is called **Secp256k1**

2.3.1 Secp256k1

Secp256k1 is defined with the following domain parameters (hexadecimal):

$p = \text{FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFC2F}$
or alternatively:

$$p = 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1$$

$$a = 0$$

$$b = 7$$

$$G = (\text{79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9 59F2815B 16F81798,} \\ \text{483ADA77 26A3C465 5DA4FBFC 0E1108A8 FD17B448 A6855419 9C47D08F FB10D4B8})$$

$$n = \text{FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF BAAEDCE6 AF48A03B BFD25E8C D0364141}$$
$$h = 1$$

2.3.2 Math on the elliptic curve

Two mathematical operations needs to be defined to operate on the elliptic curve: addition and multiplication

Point addition

Let's say you have to distinct points P and Q that both fall on curve $E(p, a, b, G, n, h)$ ($Y^2 = x^3 + ax + b$).

$$P + Q = R \Rightarrow (X_P, Y_P) + (X_Q, Y_Q) = (X_R, Y_R)$$

$$X_R = \lambda^2 - X_P - X_Q$$

$$Y_R = \lambda(x_P - X_R) - Y_P$$

where λ :

$$\lambda = \frac{Y_Q - Y_P}{X_Q - X_P} \mod p$$

Point multiplication

If P and Q are coincident, meaning that they have the same coordinates the equation is slightly different.

$$P + Q = R \Rightarrow P + P = R \Rightarrow 2P = R$$

This could be seen as P being multiplied with scalar 2. Most of the equation is the same as with addition, the difference is that:

$$\lambda = \frac{(3X_P^2 + a)}{(2Y_P)} \mod p$$

Faster multiplication with large scalars

Take $xP = R$ that could be calculated by summing P x times:

$$\sum_{n=1}^x P = R$$

This might work fine for smaller numbers but for a very large number, like $x = 2^{100}$ it will take infeasible amount of time to calculate. Luckily there is a convenient short cut that you can take called double and add.

First remember that: $P + P = 2P \Rightarrow 2P + P = 3P \Rightarrow 4P = 2(2P) \Rightarrow 8P = 2(2(2P))$

Lets say $x = 200$ in binary terms this could be written as $x = 128 + 64 + 8$ or $x = 2^7 + 2^6 + 2^3$ thus $200P = R$ could be written as

$$2^7P + 2^6P + 2^3P = R$$

which could be shorten to:

$$2(2(2(2(2(2(2P)))))) + 2(2(2(2(2P)))) + 2(2(2P)))$$

which looks cumbersome but now instead of 200 calculations you only have to do 18.

2.3.3 Private and public key

Just as RSA cryptography, ECC relies on public-private key encryption and signatures. The public key can be shared freely to everyone, while the private key should, as the name implies, be kept private. Each unique private key has a corresponding public key, through mathematics it can be proven that someone holds the private key paired with a certain public key, without actually revealing the private key.

In ECC **a private key is a really large number**. Imagine you have curve $E(p, a, b, G, n, h)$ and you want to generate a brand new private key k. k could be any number between 0 and n. Any $k > n$ will produce the exact same public key so that will not work. **A public key in ECC is represented by a point in 2D space**, more specifically a point that falls on the curve. To generate a public key P from a private key k you perform $kG = P$ as described in the section above.

Compressed key

The public key is quite large, with two 256-bit numbers representing coordinates. But there is a clever trick we can use to compress the size of the key. Take the `Secp256k1` curve for example ($Y^2 = x^3 + ax + b$). It is mirrored around the x-axis, meaning that for each x value there are two possible y values. Thus a public key can be represented by only its x value plus a prefix telling you which resulting y-value to choose.

Note that because y and x is over \mathbb{F}_p there is no negative value, instead the y value is referred to as even or odd.

2.3.4 ECDSA

The main usage of ECC in cryptocurrency

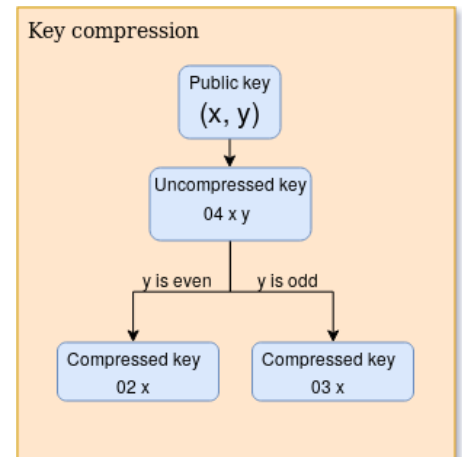


Figure 2.1: How to compress the public key in ecc

2.4 Transactions

2.4.1 Pay to public key hash (P2PKH)

2.4.2 Pay to script hash (P2SH)

2.5 Script

2.6 Lightning network

2.7 On-chain Atomic swaps

2.8 Off-chain atomic swaps

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Implementation

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Future research

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