

Set theory

Set :-

Well defined collection of objects together is called as a set.

These objects may be numbers, alphabets.

Eg:- $A = \{1, 4, 7, 8\}$, $B = \{a, b, c\}$

Types of sets :-

1. Null set :- An empty set contains no elements and the set is denoted by \emptyset .

2. Finite set :-

A set which contains countable numbers of elements is called a finite set.

Eg:- $B = \{1, 2, 3, 4\}$

3. Infinite set :-

A set which contains uncountable no. of elements is called a infinite set.

Eg:- $N = \{1, 2, 3, \dots\}$

4. Subset :-

If A and B are two sets such that every element of A is also the element of B. then A is said to be a subset of B. and it is denoted by $A \subset B$.

Eg:- $A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4, 5\}$

$\therefore A \subset B$

5 Superset -

V-line

If A is subset of B then, B is called superset of A
And it is denoted by $B \supset A$

Eg:- $A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4, 5\}$

$\therefore B \supset A$

proper subset:-

Any subset A is said to be proper subset of another set B. If A is subset of B, But there is atleast one element of B does not belongs to A. i.e ACB.

$A \neq B$

Eg:- $A = \{1, 2, 3\}, B = \{1, 2, 3, 4, 5\}$

ACB, $A \neq B$.

Equal set:-

Two sets A & B are said to be equal if and only if every elements of A is in B and every element in B is in A.

Eg:- $A = \{a, b, c\}, B = \{b, c, a\}$

ACB, BCA

$A = B$

Note:-

* Every set is subset of itself

* Null set is a subset of any set.

$$\{e, n, s, t\} = A$$

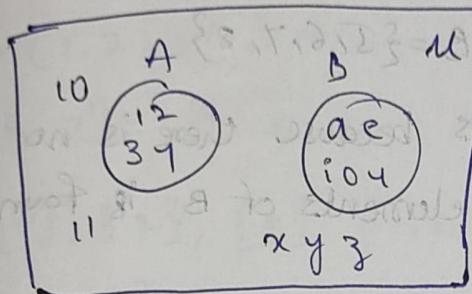
$$\{\} = B$$

$\therefore A \supset B$

Universal set :-

All the sets under investigation are likely to be considered as a subset of a particular set. This set is called universal set and it is represented as U or U_{univ} .

Eg:-



$$A \subset U, B \subset U$$

$$B \subset U$$

Union of sets :-
The union of sets A and B denoted by " $A \cup B$ " is read as A union B . is a set of all elements which belongs to set A & set B .

Eg:- ① $A = \{1, 2, 3\}, B = \{1, 2, 3, 4, 5\}$

$$A \cup B = \{1, 2, 3, 4, 5\}$$

② $A = \{1, 3, 5\}, B = \{2, 4, 6\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

Intersection of sets :-

The intersection of 2 sets $A \cap B$ is denoted by " $A \cap B$ " and it is read as A intersection B . It is the set of all elements which belongs to $A \cap B$.
Common elements

Eg:- ① $A = \{1, 2, 3\}$ ② $A = \{1, 3, 5\}$

$$B = \{1, 2, 3, 4, 5\}$$

$$B = \{2, 4, 6\}$$

$$A \cap B = \{1, 2, 3\}$$

$$A \cap B = \{\} = \emptyset$$

Disjoint sets:-

Two sets are called disjoint sets, if they do not have any common element between them it implies that the elements of one set are totally different from other set.

Eg:- $A = \{1, 2, 3, 4\}$; $B = \{5, 6, 7, 8\}$

A, B are disjoint sets because there is no element of A in set B and no elements of B is found in set A .

Equivalent sets:-

If $A \& B$, are 2 sets and if total no. of elements of A and B are same they are called equivalent sets.

Eg:- $A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7, 8\}$

$A \& B$ are equivalent sets and $A \neq B$.

Comparable & non-comparable sets:-

If $A \& B$ are two sets and if one of $A \& B$ is subset of another set $A \& B$ are comparable.

Sets:-

If none of two sets are subset of another set then they are non comparable sets.

Power set:-

If A is a set then the group of all possible subsets of A is called powerset of A . It is denoted by $P(S)$.

If a set has n elements then the power set has 2^n elements where small n denotes the no. of elements.

in the set.

Eg:- ① $A = \{1, 2\}$

$$P(A) = 2^2 = 4$$

So $P(A) = \{\{1\}, \{2\}, \{1, 2\}, \{\}\}$

② $A = \{1, 2, 3\}$

$$P(A) = 2^3 = 8$$

$P(A) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \{\}\}$

Complement of a set:-

If set A is a subset of universal set μ , if some elements of μ are not found in set A, then a set of those elements is called a complement of set A. It is denoted by A' or A^c .

$$A' = \mu - A$$

Eg:- $\mu = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 4, 6\}$

$$A' = \mu - A = \{1, 3, 5\}$$

Presentation of sets:-

A set is an collection of element structured with in brackets. There are two ways of presentation of sets.

① Roaster method.

② Set builder method.

① Roaster form:-

Under the roaster form all the elements of a set are written in a continuous row. Each element is separated by comma and total elements are enclosed in brackets.

Eg:- A set of even numbers below ten is.

$$A = \{2, 4, 6, 8\}$$

② Set Builder form:-

Under this method a set may be specified by stating its properties. In this form of presentation, instead of writing the elements directly. All the elements of a set should satisfy the properties and is within this common rule. This method is when the elements in the set are large & infinite.

Roaster form

$$V = \{a, e, i, o, u\}$$

$$A = \{-2, -1, 0, 1, 2\}$$

$$B = \left\{ \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \right\}$$

Set builder form

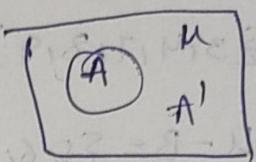
$$V = \{x : x \text{ is a vowel in English alphabets}\}$$

$$A = \{x : -2 \leq x \leq 2, x \in \mathbb{Z}\}$$

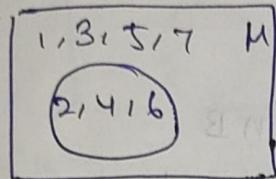
~~$$B = \{x : x = \frac{1}{n}, n \leq 5, n \in \mathbb{N}\}$$~~

Venn diagram:-

In Venn Diagram, universal set is presented in the form of a rectangle and set is shown in circle. The remaining part is complement of set.



Ex:-



$$M = \{1, 2, 3, 4, 5, 6, 7\} \quad (A') = \{2, 4, 6\}$$

$$A = \{1, 3, 5, 7\}$$

$$(A')' = A$$

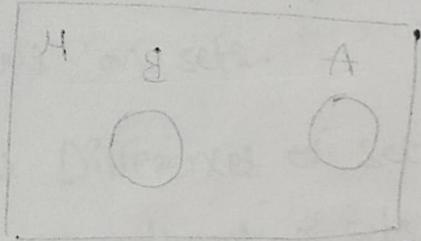
$$A' = \{2, 4, 6\}$$

Ex:- If $M = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{1, 3, 5, 6\}$ then prove that $A' \cup A = M$ and $A' \cap A = \emptyset$

a) $M = \{1, 2, 3, 4, 5, 6, 7\}$

$$A = \{1, 3, 5, 6\}$$

$$A' = M - A = \{2, 4, 7\}$$



$$A' \cup A = \{2, 4, 7\} \cup \{1, 3, 5, 6\} = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A' \cap A = \{2, 4, 7\} \cap \{1, 3, 5, 6\} = \{3\} = \emptyset$$

$$A' \cap A = \{2, 4, 7\} \cap \{1, 3, 5, 6\} = \{3\} = \emptyset$$

Ex:-

If M is universal set, A and B are subsets

$$M = \{3, 4, 5, 6, 7, 8, 9, 10, 11\}, A = \{4, 7, 9\}, B = \{3, 4, 7, 8, 9\}$$

Then prove that $A \subset B$ and $B' \subset A'$.

a) $M = \{3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$$A = \{4, 7, 9\}$$

$$A' = \mu - A = \{3, 5, 6, 8, 10, 11\}$$

$$B = \{3, 4, 7, 8, 9\}$$

$$B' = \mu - B = \{5, 6, 10, 11\}$$

$A \subset B$, since all elements of A are in B

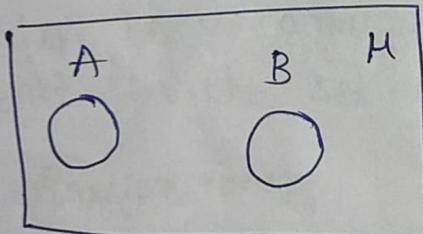
$B \subset A'$, since all the elements of B are in A'

Note:-

→ If $A \subset B$ are two subsets of μ and both sets are disjoint sets.

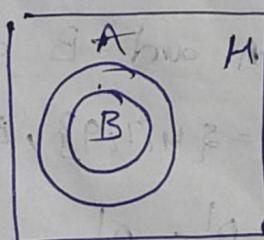
$$A \subset B$$

$$A \cap B = \emptyset$$

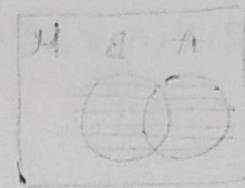
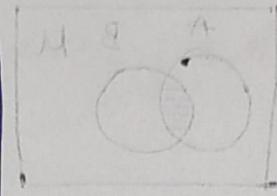
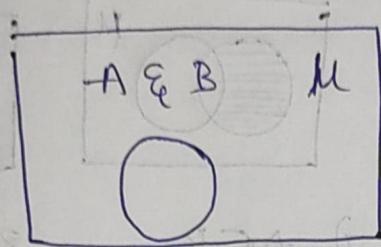


→ If $A \subset B$ are subset of μ and B is a subset of A then the circle representing B is inside the circle A .

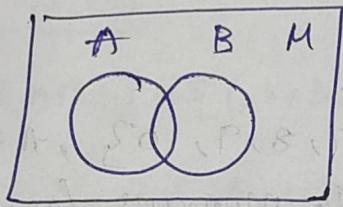
$$B \subset A$$



If $A \& B$ are subsets of μ and the two sets are equal sets then the same circle representing both $A \& B$



If $A \& B$ are not disjoint sets, they will have common elements, the circles are drawn in the rectangle in such a way that both $A \& B$ have some common area.



Operations on sets:-

There are four operations on sets.

1. Union of sets

2. Differences of sets

2. Intersection of sets

3. Complement of sets.

If $A = \{5, 6, 7\}$ and $B = \{6, 8, 10, 12\}$ then find $A \cup B$ and $A \cap B$

$$A \cup B = \{5, 6, 7\} \cup \{6, 8, 10, 12\} = \{5, 6, 7, 8, 10, 12\}$$

$$A \cap B = \{5, 6, 7\} \cap \{6, 8, 10, 12\} = \{6\}$$

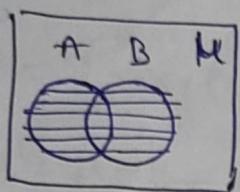
If $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3\}$ then find A^1 or A^c .

$$A^1 = \mu - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3\}$$

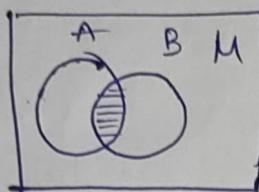
$$A^1 = \{4, 5, 6, 7, 8, 9, 10\}$$

Draw the Venn Diagrams of

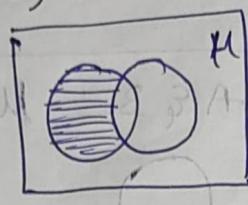
$$1) A \cup B$$



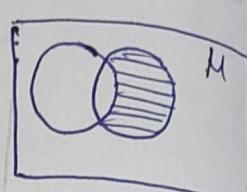
$$2) A \cap B$$



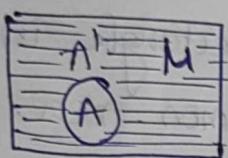
$$3) A - B$$



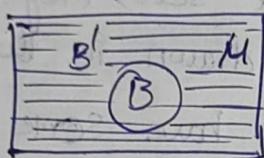
$$4) B - A$$



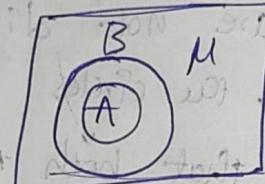
$$5) A' = M - A$$



$$6) B' = M - B$$



$$7) A \subset B$$



$$8) B \subset A$$



De. morgan's law :-

If $M = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8\}$, $B = \{1, 3, 5, 7\}$ then verify De.morgan's law.

A) w.r.t. De.morgan's law $(A \cup B)' = A' \cap B'$
 $(A \cap B)' = A' \cup B'$

Given that.

$M = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8\}$,
 $B = \{1, 3, 5, 7\}$

$$A' = M - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 4, 6, 8\}$$

$$= \{1, 3, 5, 7, 9, 10\}$$

$$B' = M - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7\}$$

$$= \{2, 4, 6, 8, 9, 10\}$$

$$A \cup B = \{2, 4, 6, 8\} \cup \{1, 3, 5, 7\} = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A \cap B = \{2, 4, 5, 8\} \cap \{1, 3, 5, 7\} = \{5\}$$

$$\text{D}(A \cup B)' = A' \cap B' = \{2, 4, 8\} \cap \{1, 3, 6, 7\} = \{1, 3\}$$

$$(A \cup B)' = U - (A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 4, 5, 6, 7, 8\} = \{9, 10\}$$

$$= \{9, 10\}$$

$$\{P, T, 2, 8\} = \{P, T, 2, 3\} \cup \{2, 8\} = S \cup A$$

$$\{2, 8\} = \{P, T, 2, 3\} \cap \{2, 8\} = S \cap A$$

$$A' \cap B' = \{1, 3, 5, 7, 9, 10\} \cap \{2, 4, 6, 8, 9, 10\} = \{9, 10\}$$

$$\therefore (A \cup B)' = A' \cap B'$$

$$\text{ii) } \underline{(A \cap B)'} = A' \cup B'$$

$$(A \cap B)' = U - (A \cap B) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{3\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$S \cup A = (S \cap A) \cup A$$

$$A' \cup B' = \{1, 3, 5, 7, 9, 10\} \cup \{2, 4, 6, 8, 9, 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\therefore (A \cap B)' = A' \cup B'$$

~~De morgan's laws are proved~~

$$\begin{aligned} (A \cup B)' &= A' \cap B' \\ (A \cap B)' &= A' \cup B' \end{aligned}$$

$$\textcircled{2} \text{ If } U = \{1, 2, 3, 4, 5, 6, 7, 9, 11\}, A = \{3, 5\}, B = \{5, 7, 9\}$$

Then prove that De morgan's theorem.

$$\textcircled{3} \text{ If } U = \{1, 2, 3, 4, 5, 6, 7, 8\}, A = \{3, 4, 5\}, B = \{4, 5, 6\}$$

Then prove that De morgan's theorem.

A) W.R.T Demorgan's law $(A \cup B)' = A' \cap B'$
 $(A \cap B)' = A' \cup B'$

Given that .

$$\mu = \{1, 3, 5, 7, 9, 11\}, A = \{3, 5\}, B = \{5, 7, 9\}$$

$$A' = \mu - A = \{1, 3, 5, 7, 9, 11\} - \{3, 5\} = \{1, 7, 9, 11\}$$

$$B' = \mu - B = \{1, 3, 5, 7, 9, 11\} - \{5, 7, 9\} = \{1, 3, 11\}$$

$$A \cup B = \{3, 5\} \cup \{5, 7, 9\} = \{3, 5, 7, 9\}$$

$$A \cap B = \{3, 5\} \cap \{5, 7, 9\} = \{5\}$$

$$\textcircled{1} (A \cup B)' = A' \cap B'$$

$$(A \cup B)' = \mu - (A \cup B) = \{1, 3, 5, 7, 9, 11\} - \{3, 5, 7, 9\} = \{1, 3, 11\}$$

$$A' \cap B' = \{1, 7, 9, 11\} \cap \{1, 3, 11\} = \{1, 11\}$$

$$\therefore (A \cup B)' = A' \cap B'$$

$$\textcircled{2} (A \cap B)' = A' \cup B'$$

$$(A \cap B)' = \mu - (A \cap B) = \{1, 3, 5, 7, 9, 11\} - \{5\} = \{1, 3, 7, 9, 11\}$$

$$A' \cup B' = \{1, 7, 9, 11\} \cup \{1, 3, 11\} = \{1, 3, 7, 9, 11\}$$

$$\therefore (A \cap B)' = A' \cup B'$$

∴ DeMorgan's law proved.

(3A)

Given that

$$\mu = \{1, 2, 3, 4, 5, 6, 7, 8\}, A = \{3, 4, 5\}, B = \{4, 5, 6\}$$

W.K.T. DeMorgan's law $(A \cup B)' = A' \cap B'$

$$(A \cap B)' = A' \cup B'$$

$$A' = \mu - A = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{3, 4, 5\}$$

$$\cong \{1, 2, 6, 7, 8\}$$

$$B' = \mu - B = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{4, 5, 6\} = \{1, 2, 3, 7, 8\}$$

$$A \cup B = \{3, 4, 5\} \cup \{4, 5, 6\} = \{3, 4, 5, 6\}$$

$$A \cap B = \{3, 4, 5\} \cap \{4, 5, 6\} = \{4, 5\}$$

① $(A \cup B)'$ = $A' \cap B'$

$$(A \cup B)' = \mu - (A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{3, 4, 5, 6\} = \{1, 2, 7, 8\}$$

$$\begin{aligned} A' \cap B' &= \cancel{\{3, 4, 5, 6\}} \quad \{1, 2, 7, 8\} \cap \{1, 2, 3, 7, 8\} \\ &= \{1, 2, 7, 8\} \end{aligned}$$

$$\therefore (A \cup B)' = A' \cap B'$$

② $(A \cap B)'$ = $A' \cup B'$

$$(A \cap B)' = \mu - (A \cap B) = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{4, 5\} = \{1, 2, 3, 6, 7, 8\}$$

$$A' \cup B' = \cancel{\{3, 4, 5, 6\}} \cup \{4, 5\}$$

$$A' \cup B' = \{1, 2, 3, 6, 7, 8\} \cup \{1, 2, 3, 7, 8\} = \{1, 2, 3, 6, 7, 8\}$$

$$\therefore (A \cap B)' = A' \cup B'$$

∴ Demorgan's law proved

Ex If $\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, A = \{2, 4, 6, 8, 10\}$

$$B = \{1, 3, 5, 7, 9\}$$

∴ w.r.t Demorgan's law

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

$$A' = \mu - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 4, 6, 8, 10\} = \{1, 3, 5, 7, 9\}$$

$$= \{1, 3, 5, 7, 9\}$$

$$B' = \mu - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\} = \{2, 4, 6, 8, 10\}$$

$$A \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 3, 5, 7, 9\} = \{1, 2, 3, 4, 5, 6, 7, 8, \\ 9, 10\}$$

$$A \cap B = \{2, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\} = \emptyset$$

$$(A \cup B)' = A' \cap B'$$

$$(A \cup B)' = U - (A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 4, 5, 6, \\ 7, 8, 9, 10\}$$

$$= \emptyset$$

$$A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8, 10\} = \emptyset$$

$$\therefore (A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

$$(A \cap B)' = U - A \cap B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{3\\ \\ \} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A' \cup B' = \cancel{\{2, 4, 6, 8, 10\}} \cup \cancel{\{1, 3, 5, 7, 9\}} \cup \{2, 4, 6, 8, 10\} \\ = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\therefore (A \cap B)' = A' \cup B'$$

Hence the DeMorgan's laws are proved

Distributive Laws :-

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

① If $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 5, 6, 7\}$, $C = \{2, 3\}$ then prove that distributive laws.

(A) w.r.t the distributive laws.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup B = \{1, 2, 3, 4, 5\} \cup \{4, 5, 6, 7\} = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A \cup C = \{1, 2, 3, 4, 5\} \cup \{2, 3\} = \{1, 2, 3, 4, 5\}$$

$$B \cap C = \{4, 5, 6, 7\} \cap \{2, 3\} = \{2, 3, 4, 5, 6, 7\}$$

$$A \cap B = \{1, 2, 3, 4, 5\} \cap \{4, 5, 6, 7\} = \{4, 5\}$$

$$A \cap C = \{1, 2, 3, 4, 5\} \cap \{2, 3\} = \{2, 3\}$$

$$B \cap C = \{4, 5, 6, 7\} \cap \{2, 3\} = \{2, 3\}$$

∴ $A \cup (B \cap C) = \{1, 2, 3, 4, 5\} \cup \{2, 3\} = \{1, 2, 3, 4, 5, 6, 7\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7\} \cap \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5\}$

$$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

② $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$A \cap (B \cup C) = \{1, 2, 3, 4, 5\} \cap \{2, 3, 4, 5, 6, 7\} = \{2, 3, 4, 5\}$$

$$(A \cap B) \cup (A \cap C) = \{4, 5\} \cup \{2, 3\} = \{2, 3, 4, 5\}$$

$$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Hence the distributive laws are proved.

② If $A = \{1, 3, 4, 8, 9, 12\}$, $B = \{1, 4, 9\}$, $C = \{2, 4, 8, 10\}$

then prove that distributive laws.

(A) w.r.t the distributive laws.

$$\textcircled{1} \quad A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$(B \cap C) = \{1, 4, 9\} \cap \{2, 4, 8, 10\} = \{4\}$$

$$(A \cup B) = \{1, 3, 4, 8, 9, 12\} \cup \{1, 4, 9\} = \{1, 3, 4, 8, 9, 12\}$$

$$(A \cap C) = \{1, 3, 4, 8, 9, 12\} \cap \{2, 4, 8, 10\} = \{1, 2, 3, 4, 8, 9, 10, 12\}$$

$$\textcircled{1} \quad A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$$

$$A \cup (B \cap C) = \{1, 3, 4, 8, 9, 12\} \cup \{4\} = \{1, 3, 4, 8, 9, 12\}$$

$$(A \cup B) \cap (A \cap C) = \{1, 3, 4, 8, 9, 12\} \cap \{1, 2, 3, 4, 8, 9, 10, 12\} \\ = \{1, 3, 4, 8, 9, 12\}$$

$$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$$

$$\textcircled{2} \quad (B \cup C) = \{1, 4, 9\} \cup \{2, 4, 8, 10\} = \{1, 2, 4, 8, 9, 10\}$$

$$(A \cap B) = \{1, 3, 4, 8, 9, 12\} \cap \{1, 4, 9\} = \{1, 4, 9\}$$

$$(A \cap C) = \{1, 3, 4, 8, 9, 12\} \cap \{2, 4, 8, 10\} = \{4, 8\}$$

$$\textcircled{2} \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cap (B \cup C) = \{1, 3, 4, 8, 9, 12\} \cap \{1, 2, 4, 8, 9, 10\} = \{1, 4, 8, 9\}$$

$$(A \cap B) \cup (A \cap C) = \{1, 4, 9\} \cup \{4, 8\} = \{1, 4, 8, 9\}$$

$$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Hence the Distributive laws are proved

If $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 3, 6, 7\}$, $C = \{1, 2, 4, 6, 8\}$ then verify Distributive laws.

i) Work for distributive laws $AU(B \cap C) = (A \cup B) \cap (A \cup C)$
 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$A \cup B = \{1, 2, 3, 4, 5\} \cup \{1, 2, 3, 6, 7\} = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A \cup C = \{1, 2, 3, 4, 5\} \cup \{1, 2, 4, 6, 8\} = \{1, 2, 3, 4, 5, 6, 8\}$$

$$B \cup C = \{1, 2, 3, 6, 7\} \cup \{1, 2, 4, 6, 8\} = \{1, 2, 3, 4, 6, 7, 8\}$$

$$A \cap B = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 6, 7\} = \{1, 2, 3\}$$

$$A \cap C = \{1, 2, 3, 4, 5\} \cap \{1, 2, 4, 6, 8\} = \{1, 2, 4\}$$

$$B \cap C = \{1, 2, 3, 6, 7\} \cap \{1, 2, 4, 6, 8\} = \{1, 2, 6\}$$

i) $AU(B \cap C) = (A \cup B) \cap (A \cup C)$

$$AU(B \cap C) = \{1, 2, 3, 4, 5\} \cup \{1, 2, 6\} = \{1, 2, 3, 4, 5, 6\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7\} \cap \{1, 2, 3, 4, 5, 6, 8\} = \{1, 2, 3, 4, 5, 6\}$$

ii) $AU(B \cap C) \neq (A \cup B) \cap (A \cup C)$

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$A \cap (B \cup C) = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 6, 7, 8\} = \{1, 2, 3, 4\}$$

$$(A \cap B) \cup (A \cap C) = \{1, 2, 3\} \cup \{1, 2, 4\} = \{1, 2, 3, 4\}$$

$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence Distributive laws are proved.

If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{1, 2, 3, 4, 5\}$, $C = \{1, 9\}$ then

prove the Distributive laws i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ ii) $AU(B \cap C) = (A \cup B) \cap (A \cup C)$

$$A \cap (B \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5, 6\}$$

$$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5, 6\}$$

$$A \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{1, 9\} = \{1, 2, 3, 4, 5, 6, 9\}$$

$$A \cap (B \cup C) = \{1, 2, 3, 4, 5, 6\} \cup \{1, 9\} = \{1, 2, 3, 4, 5, 6, 9\}$$

$$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5\}$$

$$A \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{1, 9\} = \{1\}$$

$$B \cap C = \{1, 2, 3, 4, 5\} \cap \{1, 9\} = \{1\}$$

i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$A \cup (B \cap C) = \{1, 2, 3, 4, 5\} \cup \{1\} = \{1, 2, 3, 4, 5, 6\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 5, 6, 9\} =$$

$$\{1, 2, 3, 4, 5, 6\}$$

$$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$A \cap (B \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 5, 9\} = \{1, 2, 3, 4, 5\}$$

$$(A \cap B) \cup (A \cap C) = \{1, 2, 3, 4, 5\} \cup \{1\} = \{1, 2, 3, 4, 5\}$$

$\therefore A \cap$

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$A \cap (B \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4, 5, 9\} = \{1, 2, 3, 4, 5\}$$

$$(A \cap B) \cup (A \cap C) = \{1, 2, 3, 4, 5\} \cup \{1\} = \{1, 2, 3, 4, 5\}$$

\therefore Hence the Distributive law is proved

If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{3, 4, 5\}$ the proved
Distributive laws.

A) $A \cup B = \{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\}$

$$A \cup C = \{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$$

$$B \cup C = \{2, 3, 4\} \cup \{3, 4, 5\} = \{2, 3, 4, 5\}$$

$$A \cap B = \{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$$

$$A \cap C = \{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$$

$$B \cap C = \{2, 3, 4\} \cap \{3, 4, 5\} = \{3, 4\}$$

i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$A \cup (B \cap C) = \{1, 2, 3\} \cup \{3, 4\} = \{1, 2, 3, 4\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4\} \cap \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4\}$$

$$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \quad \text{i) } A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$A \cap (B \cup C) = \{1, 2, 3\} \cap \{2, 3, 4, 5\} = \{2, 3\}$$

$$(A \cap B) \cup (A \cap C) = \{2, 3\} \cup \{3\} = \{2, 3\}$$

$$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

\therefore Hence distributive laws are proved.

Properties of union of sets :-

1. $A \cup A = A$

Eg:-

$$A = \{1, 2, 3, 4, 5\}$$

$$A \cup A = \{1, 2, 3, 4, 5\} \cup \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5\}$$

$$\therefore A \cup A = A$$

2. $A \cup B = B \cup A$

Eg:- $A = \{1, 2, 3\}, B = \{4, 5, 6\}$

$$A \cup B = \{1, 2, 3\} \cup \{4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$$

$$B \cup A = \{4, 5, 6\} \cup \{1, 2, 3\} = \{1, 2, 3, 4, 5, 6\}$$

i. $A \cup B = B \cup A$

3. $A \cup \emptyset = A$

Eg:- $A = \{1, 2, -1, 0\} \cup \{\} = \{1, 2, -1, 0\}$

i) $A \cup A = A$

$$A \cup B = B \cup A$$

$$A \cup \emptyset = A$$

$$A \cup M = M$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

4. $A \cup M = M$

Eg:- $M = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$

$$A \cup M = \{1, 2\} \cup \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5\}$$

∴ $A \cup M = M$

5. $A \cup (B \cup C) = (A \cup B) \cup C$ associative

Eg:- $A = \{2, 4, 6\}$, $B = \{3, 5, 7\}$, $C = \{7, 8, 9\}$

$$B \cup C = \{4, 5, 6\} \cup \{7, 8, 9\} = \{4, 5, 6, 7, 8, 9\}$$

$$A \cup (B \cup C) = \{1, 2, 3\} \cup \{4, 5, 6, 7, 8, 9\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A \cup B = \{1, 2, 3\} \cup \{4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$$

$$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{7, 8, 9\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

Properties of intersection :-

i. $A \cap A = A$

Eg:- $A = \{1, 2, 3, 4, 5\}$

$$A \cap A = \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5\} = A$$

(2) $A \cap B = B \cap A$

Eg:- $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$

$$A \cap B = \{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$$

$$B \cap A = \{3, 4, 5\} \cap \{1, 2, 3\} = \{3\}$$

(3) $A \cap \emptyset = \emptyset$

Eg:- $A = \{1, 2, 3, 4, 5\}$

$$A \cap \emptyset = \{1, 2, 3, 4, 5\} \cap \{\} = \{\}$$

$$\therefore A \cap \emptyset = \emptyset$$

i) $A \cap A = A$

ii) $A \cap B = B \cap A$

iii) $A \cap \emptyset = \emptyset$

iv) $A \cap M = A$

v) $A \cap (B \cap C) = (A \cap B) \cap C$

$$\textcircled{4} \quad A \cap \emptyset = \emptyset$$

e.g. $\mu = \{1, 2, 3, 4, 5\}$, $A = \{1, 2\}$

$$A \cap M = \{1, 2\} \cap \{1, 2, 3, 4, 5\} = \{1, 2\} = A$$

$$\textcircled{5} \quad A \cap (B \cap C) = (A \cap B) \cap C$$

$A = \{1, 2, 3, 4\}$, $B = \{4, 5, 6\}$, $C = \{4, 5, 6, 7\}$

$$B \cap C = \{4, 5, 6\} \cap \{4, 5, 6, 7\} = \{4, 5, 6\}$$

$$A \cap (B \cap C) = \{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\}$$

$$A \cap B = \{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\} = A - 8$$

$$(A \cap B) \cap C = \{4\} \cap \{4, 5, 6, 7\} = \{4\}$$

Differences of two sets :-

If A & B are two sets, the set of elements of ~~set A~~ which are not contained in B . is difference of set A & B . it is denoted by $A - B$.

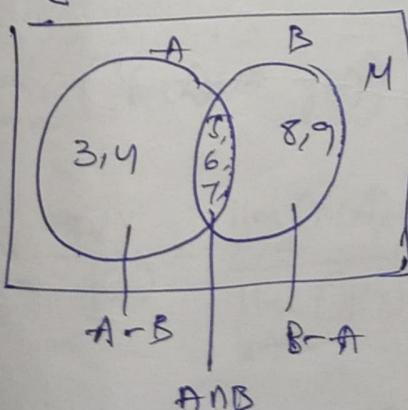
e.g. If $A = \{3, 4, 5, 6, 7\}$, $B = \{5, 6, 7, 8, 9\}$ then find $A - B$.

and $B - A$

e.g. $A = \{3, 4, 5, 6, 7\}$, $B = \{5, 6, 7, 8, 9\}$

$$A - B = \{3, 4, 5, 6, 7\} - \{5, 6, 7, 8, 9\} = \{3, 4\}$$

$$B - A = \{5, 6, 7, 8, 9\} - \{3, 4, 5, 6, 7\} = \{8, 9\}$$



Properties of Differences of sets:-

① $A - B = A \cap B'$

$$A = \{3, 4, 5, 6\}, B = \{5, 6, 7\}$$

$$A - B = \{3, 4, 5, 6\} - \{5, 6, 7\} = \{3, 4\}$$

$$\text{And } A - B = \{3, 4, 5, 6\} \cap \{3, 4\} = \{3, 4\}$$

$$M = \{3, 4, 5, 6, 7\}$$

$$B' = M - B = \{3, 4, 5, 6, 7\} - \{5, 6, 7\}$$

$$= \{3, 4\}$$

② If $A = \{1, 2, 3, 4, 5\}, B = \{3, 4, 5, 6\}$ then find $(A - B) \cup (B - A)$

$$A - B = \{1, 2, 3, 4, 5\} - \{3, 4, 5, 6\} = \{1, 2\}$$

$$B - A = \{3, 4, 5, 6\} - \{1, 2, 3, 4, 5\} = \{6\}$$

$$(A - B) \cup (B - A) = \{1, 2\} \cup \{6\} = \{1, 2, 6\}$$

C. Cloudy a day

To proceed to the next, click out in a β or γ $\in \mathbb{R}$
 angle θ . θ is the boundary for α below it.
 $\beta - \alpha$ is the boundary of β . $\beta - \alpha$ is the
 $\beta - \alpha$ without $\beta \cap \alpha$. $\beta - \alpha = \beta \setminus \alpha$

$\beta - \alpha$

$$\beta - \alpha = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3, 4\} = \{5, 6, 7\}$$

$$\beta - \alpha = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3, 4\} = \{5, 6, 7\}$$

$$\beta - \alpha = \{1, 2, 3, 4, 5, 6, 7\} - \{1, 2, 3, 4\} = \{5, 6, 7\}$$

