18/07/24

Unit 1 : Matrix Algebra - I

Matrix :-

→ A matrix is a collection of elements are arranged by rows (—) & columns (|)

→ The matrices are denoted by capital Letters only.

→ The matrix are represent by [ ] (Square) (or) ( ) (open Bracket) (or) || || (norms)

→ The elements are small letters (or) numbers.

Ex:- $A = \begin{bmatrix} a & b & c \\ e & f & g \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$

Order of the Matrix :-

The no. of rows by (x) the no. of columns is called the order of the matrix.

Ex:- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$ $B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}_{2\times 4}$

Types of Matrices :-

1. Row matrix :-

If a matrix have single row than it is called row matrix.

Ex:- $X = \begin{bmatrix} 1 & 5 & 6 \end{bmatrix}_{1\times 3}$

2. Column matrix :-

If a matrix have single column than it is called column matrix.

Ex:- $A = \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}_{3\times 1}$

3. Square Matrix :-

A matrix A is said to be square matrix, the no. of rows is equal to the no. of columns.

Ex :- $A = \begin{bmatrix} 3 & 6 \\ 7 & 9 \end{bmatrix}_{2\times 2}$ $B = \begin{bmatrix} a & b & c \\ 1 & 5 & 6 \\ d & e & f \end{bmatrix}_{3\times 3}$

4. Rectangular Matrix :-

A matrix A is said to be rectangular matrix, the no. of rows is not equal to the no. of columns.

Ex :- $A = \begin{bmatrix} a & b & c \\ e & f & g \end{bmatrix}_{2\times 3}$

5. Null matrix (or) Zero matrix :-

In a matrix all the elements are zeros than it is called a null matrix. It is denoted by "0".

Ex :- $O_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

6. Unit matrix (or) Identity matrix :-

In a square matrix has all elements are zeros and each diagonal element is unity (1) ~~are zeros~~ ~~and~~ is called a unit matrix. It is denoted by "I".

Ex :-

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. Diagonal matrix :-

A square matrix in which all the elements except those in the principle diagonal are zero is called a Diagonal matrix.

Ex :-

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

8. Scalar Matrix :-

A square matrix in which the diagonal elements are all ~~zero~~ equal, and other elements are zeros, is called a scalar matrix.

Ex :-

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Note :- The sum of the diagonal elements of a square matrix ~~coo~~ A is called the "trace of A", and it is denoted by Tr(A)

Ex :-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix} \qquad Tr(A) = 1+2+1 = 4.$$

9. Triangular Matrices -

If all the elements below (or above) the principle diagonal of a square matrix are zeroes. It is called upper (or lower). triangular matrix.

Ex:-

1. Upper triangular matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

2. Lower triangular matrix.

$$B = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 5 \\ 0 & 0 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} -5 & 0 & 0 \\ 7 & 3 & 0 \\ 6 & 5 & 4 \end{bmatrix}$$

Equality of Matrices :-

Two matrices capital A & B are said to be equal if.

1. They are of the same order.
2. Each element of ~~Eq~~ A is equal to the corresponding elements of B

→ Symbolically, if two matrices are equal that is $A = B$

Ex:-

1. If A & B are ~~Eq~~ equal matrices then find a, b, c, d, e, f

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix}_{2\times3}, \quad B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times3}$$

Here $A = B$, the corresponding elements are equal

$a = 1, \; b = 2, \; c = 5$

$d = 3, \; e = 4, \; f = 6.$

2. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A = B$.

Addition & Subraction of matrices :-

→ If A & B are two matrices of the same order then their sum $A+B$ is define to be the matrix of the same order obtained by adding corresponding elements of ~~capital~~ A & B.

Similarly $A-B$ is defined as a matrix whose elements are obtained by subracting the elements of B from the corresponding elements of A.

Ex:-

1. If $A = \begin{bmatrix} 1 & 3 & 7 \\ 5 & 6 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 6 & 9 \\ 7 & 8 & 10 \end{bmatrix}$, then find $A+B$ and $A-B$.

A)

$$A+B = \begin{bmatrix} 1 & 3 & 7 \\ 5 & 6 & 8 \end{bmatrix} + \begin{bmatrix} 5 & 6 & 9 \\ 7 & 8 & 10 \end{bmatrix} = \begin{bmatrix} 6 & 9 & 16 \\ 12 & 14 & 18 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1 & 3 & 7 \\ 5 & 6 & 8 \end{bmatrix} - \begin{bmatrix} 5 & 6 & 9 \\ 7 & 8 & 10 \end{bmatrix} = \begin{bmatrix} -4 & -3 & -2 \\ -2 & -2 & -2 \end{bmatrix}$$

Problems :-

1. $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & 3 \end{bmatrix}$ then find

i) $A+B$ ii) $A-B$.

A) $A+B = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -5 & -2 \\ 1 & 4 & 1 \\ 5 & -3 & 6 \end{bmatrix}$

$A-B = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 6 \\ 3 & -2 & -7 \\ 3 & 1 & 0 \end{bmatrix}$

Multiplication of a matrix by a scalar :-

Scalar matrix :-

If every element of a matrix A is multiplied by a scalar "K" (real) or (complex). The matrix obtained is K times the matrix capital A and it is denoted by "KA".

Ex :- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow 2A = 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

Problems :-

1. If $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 5 & 4 \\ -2 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $3A + 2X = 5B$ then find X.

A) Given matrices $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 5 & 4 \\ -2 & 3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

$3A = 3\begin{bmatrix} 3 & 2 & -1 \\ 1 & 5 & 4 \\ -2 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 9 & 6 & -3 \\ 3 & 15 & 12 \\ -6 & 9 & 21 \end{bmatrix}$

$5B = 5\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 5 \\ 10 & 5 & 10 \\ 5 & 10 & 5 \end{bmatrix}$

Now $3A + 2X = 5B \Rightarrow 2x = 5B - 3A$

$$2X = \begin{bmatrix} 5 & 10 & 5 \\ 10 & 5 & 10 \\ 5 & 10 & 5 \end{bmatrix} - \begin{bmatrix} 9 & 6 & -3 \\ 3 & 15 & 12 \\ -6 & 9 & 21 \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & 4 & 8 \\ 7 & -10 & -2 \\ 11 & 1 & -16 \end{bmatrix}$$

$$X = \frac{1}{2} \begin{bmatrix} -4 & 4 & 8 \\ 7 & -10 & -2 \\ 11 & 1 & -16 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 2 & 4 \\ 7/2 & -5 & -1 \\ 11/2 & 1/2 & -8 \end{bmatrix}$$

(2.) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -3 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 2 & 1 \\ 4 & 3 & 2 \end{bmatrix}$ then find $2A + 3B$.

A) Given ~~matrices~~ $2A + 3B$

$$2A + 3B = 2\begin{bmatrix} 0 & 1 & 2 \\ 1 & -3 & 4 \end{bmatrix} + 3\begin{bmatrix} 0 & 2 & 1 \\ 4 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 4 \\ 2 & -6 & 8 \end{bmatrix} + \begin{bmatrix} 0 & 6 & 3 \\ 12 & 9 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 8 & 7 \\ 14 & 3 & 14 \end{bmatrix}.$$

(3) If $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 4 \\ 1 & 6 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 2 \\ -2 & 1 & 5 \\ 0 & -2 & 4 \end{bmatrix}$ then find X.

Such that $2A + 3B - 2X = 0$.

A) $$2A = 2\begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 4 \\ 1 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 2 \\ 4 & 10 & 8 \\ 2 & 12 & 2 \end{bmatrix}$$

$$3B = 3\begin{bmatrix} 2 & 0 & 2 \\ -2 & 1 & 5 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 6 \\ -6 & 3 & 15 \\ 0 & -6 & 12 \end{bmatrix}$$

Given $2A + 3B - 2x = 0 \Rightarrow 2A + 3B = 2X$

$$\begin{bmatrix} 2 & 6 & 2 \\ 4 & 10 & 8 \\ 2 & 12 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 6 \\ -6 & 3 & 15 \\ 0 & -6 & 12 \end{bmatrix} = \cancel{2X} \begin{bmatrix} 8 & 6 & 8 \\ -2 & 13 & 23 \\ 2 & 6 & 14 \end{bmatrix}$$

$$x = \frac{1}{2}\begin{bmatrix} 8 & 6 & 8 \\ -2 & 13 & 23 \\ 2 & 6 & 14 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 4 \\ -1 & 13/2 & 23/2 \\ 1 & 3 & 7 \end{bmatrix}$$

(4) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 1 \end{bmatrix}$ then compute

i) $3A - 4B$ ii) $A+B$.

A)

$$3A - 4B = 3\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & 6 \\ 6 & 9 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 8 & -12 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 & 6 \\ -2 & 21 & 8 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 0 & 5 \end{bmatrix}$$

Multiplication of Matrices :-

2 matrices can be multiply only when the "No. of columns in the first matrix is equal to the no. of rows of second matrix". Such matrices are said to be Conformable for multiplication. otherwise the product of two matrices is undefine.

Note:-

1. multiplication of matrices is distributed (distibutive)

i) $A(B+C) = AB + AC$

ii) $(A+B)C = AC + BC$

Multiplication of matrices is assosiative. that is $(AB)C = A(BC)$

Problems:-

(1) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 0 & 5 \\ 1 & 2 & 0 \end{bmatrix}$ then find AB.

A) $AB = \begin{bmatrix} 3\times3+(-3)\times2+4\times1 & 3\times1+(-3)\times0+4\times2 & 3\times2+(-3)\times5+4\times0 \\ 2\times3+(-3)\times2+4\times1 & 2\times1+(-3)\times0+4\times2 & 2\times2+(-3)\times5+4\times0 \\ 0\times3+(-1)\times2+1\times2 & 0\times1+(-1)\times0+1\times2 & 0\times2+(-1)\times5+1\times0 \end{bmatrix}$

$AB = \begin{bmatrix} 9-6+4 & 3-0+8 & 6-15+0 \\ 6-6+4 & 2-0+8 & 4-15+0 \\ 0-2+1 & 0-0+2 & 0-5+0 \end{bmatrix} \Rightarrow \begin{bmatrix} 7 & 11 & -9 \\ 4 & 10 & -11 \\ -1 & 2 & -5 \end{bmatrix}$

(2) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then show that $A^2 - 4A - 5I = 0$.

A)

$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

$4A = 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$

$5I = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0.$

(3) If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, then show that

$AB = A$ and $BA = B$

A) $$AB = \begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & +2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+8 & -4-12+12 \end{bmatrix}$$

$$AB = \begin{bmatrix} +2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} = A$$

$-4+9$
$= -3$ $-14+12$
12
-9

$$BA = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2-4 & -6-8+12 & -10-10-16 \\ -2-3+4 & +3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} =$$

(4) If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ then find $A^2 - 3A + 2I$

A) $$A^2 = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+6 & 3+3 \\ 2+2 & 6+1 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 6 & 3 \end{bmatrix} \qquad 2I = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^2 - 3A + 2I = \begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 9 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7-3+2 & 6-9+0 \\ 4-6+0 & 7-3+2 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ -2 & 6 \end{bmatrix}$$

$$\therefore A^2 - 3A + 2I = \begin{bmatrix} 6 & -3 \\ -2 & 6 \end{bmatrix}$$

(5) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 4A + 7I = 0$

(6) If $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$, then find AB and BA

what is your inference about it?

5A) $$A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 4-3 & 6+6 \\ -2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$

$$4A = 4 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix}, \quad 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 1-8+7 & 12-12+0 \\ -4+4+0 & 1-8+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \underline{\underline{0}}$$

6A) $$AB = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 0+2 & 0+(-3) \\ 2+6 & 0+9 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 8 & -9 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 0+0 & 1+0 \\ 0+(-6) & 2-9 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -7 \end{bmatrix}$$

$$\therefore \underline{\underline{AB \neq BA}}.$$

(7) If $A = \begin{bmatrix} -1 & -2 & 0 \\ 3 & 4 & 5 \\ 1 & 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 4 & 3 \\ 4 & 5 & 2 \\ 2 & 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -2 & -3 & 1 \\ 2 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ then

prove that $A(B+C) = AB + AC$

$A(B+C) = AB + AC$

$$B+C = \begin{bmatrix} 4 & 4 & 3 \\ 4 & 5 & 2 \\ 2 & 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & -3 & 1 \\ 3 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 7 & 9 & 4 \\ 4 & 2 & 5 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -1 & -2 & 0 \\ 3 & 4 & 5 \\ 1 & 12 & -3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 7 & 9 & 4 \\ 4 & 2 & 5 \end{bmatrix} =$$

$$= \begin{bmatrix} -2-14+0 & -1-18+0 & -4-8+0 \\ 6+28+20 & 3+36+10 & 12+16+25 \\ 2+14-12 & 1+18-6 & 4+8-15 \end{bmatrix} = \begin{bmatrix} -16 & -19 & -12 \\ 54 & 49 & 53 \\ 4 & 13 & -3 \end{bmatrix} = \text{LHS}$$

$$AB = \begin{bmatrix} -1 & -2 & 0 \\ 3 & 4 & 5 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} 4 & 4 & 3 \\ 4 & 5 & 2 \\ 2 & 1 & 2 \end{bmatrix} =$$

$$= \begin{bmatrix} -4-8+0 & -4-10+0 & -3-4+0 \\ 12+16+10 & 12+20+5 & 9+8+10 \\ 4+8-6 & 4+10-3 & 3+4-6 \end{bmatrix} = \begin{bmatrix} -12 & -14 & -7 \\ 38 & 37 & 27 \\ 6 & 11 & 1 \end{bmatrix}$$

$$BC = \begin{bmatrix} 4 & 4 & 3 \\ 4 & 5 & 2 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} -2 & -3 & 1 \\ 3 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -8+12+6 & -12+16+3 & 4+8+9 \\ -8+15+4 & -12+20+2 & 4+10+6 \\ -4+3+4 & -6+4+2 & 2+2+6 \end{bmatrix} = \begin{bmatrix} 10 & 7 & 21 \\ 11 & 10 & 20 \\ 3 & 0 & 10 \end{bmatrix}$$

$$AB+BC = \begin{bmatrix} -12 & -14 & -7 \\ 38 & 37 & 27 \\ 6 & 11 & 1 \end{bmatrix} \begin{bmatrix} 10 & 7 & 21 \\ 11 & 10 & 20 \\ 3 & 0 & 10 \end{bmatrix} =$$

$$= \begin{bmatrix} -12+10 & -14+7 & -7+21 \\ 38+11 & 37+10 & 27+20 \\ 6+3 & 11+0 & 1+10 \end{bmatrix} = \begin{bmatrix} -2 & -7 & 14 \\ 49 & 47 & 47 \\ 9 & -11 & 11 \end{bmatrix} = RHS.$$

$\therefore A(B+C) \neq AB + BC$ LHS $\neq$ RHS

38
11
49

11
15
26
16
15
21

12
10
22
6
16

9
12

$$AC = \begin{bmatrix} -1 & -2 & 0 \\ 3 & 4 & 5 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} -2 & -3 & 1 \\ 3 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} +2-6+0 & 3-8+0 & -1-4+0 \\ -6+12+10 & -9+16+5 & 3+8+15 \\ -2+6-6 & -3+8-3 & 1+4-9 \end{bmatrix} = \begin{bmatrix} -4 & -5 & -5 \\ 16 & 12 & 26 \\ -2 & +2 & -4 \end{bmatrix}$$

$$AB+AC = \begin{bmatrix} -12 & -14 & -7 \\ 38 & 37 & 27 \\ 6 & 11 & 1 \end{bmatrix} + \begin{bmatrix} -4 & -5 & -5 \\ 16 & 12 & 26 \\ -2 & +2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -12-4 & -14-5 & -7-5 \\ 38+16 & 37+12 & 27+26 \\ 6-2 & 11+2 & 1-4 \end{bmatrix} = \begin{bmatrix} -16 & -19 & -12 \\ 54 & 49 & 53 \\ 4 & 13 & -3 \end{bmatrix}$$

$\therefore$ LHS = RHS.

(8) $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ then prove that

i) $(AB)C = A(BC)$ ii) $A(B+C) = AB \pm AC$

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A(i) $AB = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2+4 & 1+6 \\ -4+6 & -2+9 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix}$

$(AB)C = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -18+14 & 6+0 \\ -6+14 & 2+0 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ 8 & 2 \end{bmatrix} = LHS$

$BC = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -6+2 & 2+0 \\ -6+6 & 2+0 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 0 & 2 \end{bmatrix}$

$A(BC) = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -4+0 & 2+4 \\ 8+0 & -4+6 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ 8 & 2 \end{bmatrix} = RHS$

$\therefore (AB)C = A(BC)$ $\therefore LHS = RHS$

(ii) $B+C = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2-3 & 1+1 \\ 2+2 & 3+0 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$

$A(B+C) = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} -1+8 & 2+6 \\ 2+12 & -4+9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix}$

$= LHS$

$AB = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2+4 & 1+6 \\ -4+6 & -2+9 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix}$

$AC = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -3+4 & 1+0 \\ 6+6 & -2+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix}$

$AB + AC = \begin{bmatrix} 6 & 7 \\ 2 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 12 & -2 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 14 & 5 \end{bmatrix} = RHS$ $\therefore LHS = RHS$

(9) If $A=\begin{bmatrix}1&2&0\\-3&7&8\end{bmatrix}$, $B=\begin{bmatrix}-7&4\\1&3\\5&6\end{bmatrix}$ then find AB and BA.

(A) $AB=\begin{bmatrix}1&2&0\\-3&7&8\end{bmatrix}\begin{bmatrix}-7&4\\1&3\\5&6\end{bmatrix}=\begin{bmatrix}-7+2+0 & 4+6+0\\21+7+40 & -12+21+48\end{bmatrix}=\begin{bmatrix}-5&10\\68&57\end{bmatrix}$

$BA=\begin{bmatrix}-7&4\\1&3\\5&6\end{bmatrix}\begin{bmatrix}1&2&0\\-3&7&8\end{bmatrix}=\begin{bmatrix}\cancel{-7+12}\ -7-12 & -14+28 & -0+8\\1-9 & 2+21 & 0+24\\5-18 & 10+42 & 0+48\end{bmatrix}=\begin{bmatrix}-19&14&8\\-8&23&24\\-13&52&48\end{bmatrix}$

(10) If $A=\begin{bmatrix}1&0&5\\7&-3&2\end{bmatrix}$, $B=\begin{bmatrix}2&3\\4&-8\end{bmatrix}$, then find AB. and BA

A) AB :-

Given matrices $A=\begin{bmatrix}1&0&5\\7&-3&2\end{bmatrix}_{2\times3}$ $B=\begin{bmatrix}2&3\\4&-8\end{bmatrix}_{2\times2}$

Here AB is not possible, since. The no. of columns in A matrix is not equal to the no. of rows in B matrix.

BA :- $\begin{bmatrix}2&3\\4&-8\end{bmatrix}\begin{bmatrix}1&0&5\\7&-3&2\end{bmatrix}=\begin{bmatrix}2+21 & 0-9 & 10+6\\4-56 & 0+24 & 20-16\end{bmatrix}$

$=\begin{bmatrix}23&-9&16\\-52&24&4\end{bmatrix}$

Transpose of a matrix :-

The matrix obtained from any given matrix "A by interchanging its rows and columns" is called the Transpose of A ~~matrix~~. and it is denoted by A^T (or) A'

Ex:-

$A=\begin{bmatrix}1&3&7\\-5&6&0\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}1&-5\\3&6\\7&0\end{bmatrix}$

Properties of Transpose:-

~~1. $(A^T)^T = A$~~

2\. If A^T and B^T are the transposes of the matrices A and B respectively, then

1. $(A^T)^T = A$

2. $(A \pm B)^T = A^T \pm B^T$, A, B are same size,

3. $(AB)^T = B^T A^T$, A, B are multicable.

4. $(KA)^T = KA^T$, K is constant.

Problems:-

① If $A = \begin{bmatrix} 2 & 5 & -3 \\ 7 & 6 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 7 \\ 3 & -5 & 4 \end{bmatrix}$, then verify that

~~(AB)~~ $(A+B)^T = A^T + B^T$.

A) LHS:-

$$A+B = \begin{bmatrix} 2 & 5 & -3 \\ 7 & 6 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 7 \\ 3 & -5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 7 & 4 \\ 10 & 1 & 6 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 1 & 10 \\ 7 & 1 \\ 4 & 6 \end{bmatrix}$$

RHS:-

$$A^T = \begin{bmatrix} 2 & 7 \\ 5 & 6 \\ -3 & 2 \end{bmatrix}, \quad B^T = \begin{bmatrix} -1 & 3 \\ 2 & -5 \\ 7 & 4 \end{bmatrix}.$$

$$A^T + B^T = \begin{bmatrix} 2 & 7 \\ 5 & 6 \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 2 & -5 \\ 7 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 10 \\ 7 & 1 \\ 4 & 6 \end{bmatrix}$$

$\therefore$ LHS = RHS

$(A+B)^T = A^T + B^T$

(2) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ Then verify that

$(AB)^T = B^T A^T$

A) LHS :- AB :-

$$= \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4-0 & 0+2-1 & 0+0-3 \\ 3+0+0 & 0+0+2 & 0+0+6 \\ 4+10+0 & 0+5+0 & 0+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 5 & 3 & 14 \\ 1 & 2 & 5 \\ -3 & 6 & 0 \end{bmatrix}$$

RHS :-

$$B^T = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 1+4-0 & 3+2+0 & 4+10+0 \\ 0+2-1 & 0+0+2 & 0+5+0 \\ 0+0-3 & 0+0+6 & 0+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 & 14 \\ 1 & 2 & 5 \\ -3 & 6 & 0 \end{bmatrix}$$

$\therefore$ LHS = RHS

$\underline{(AB)^T = B^T A^T}$

(3) If $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$ then verify $(A+B)^T = A^T + B^T$

A) LHS :-

$$A+B = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 10 \\ 6 & 10 & 14 \\ 10 & 14 & 9 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 2 & 6 & 10 \\ 6 & 10 & 14 \\ 10 & 14 & 9 \end{bmatrix}$$

RHS :-

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 0 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 2 & 6 & 10 \\ 6 & 10 & 14 \\ 10 & 14 & 9 \end{bmatrix}$$

$\therefore$ LHS = RHS

$\underline{(A+B)^T = A^T + B^T}$

(5) If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 & 0 \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ then verify that

i) $(A+B)^T = A^T + B^T$ ii) $(AB)^T = B^T A^T$

LHS:-

$$A+B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 1 & 0 \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 4 & -2 & 4 \\ 5 & 2 & 7 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 5 & 4 & 5 \\ 0 & -2 & 2 \\ 0 & 4 & 7 \end{bmatrix}$$

RHS:- $A^T = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 1 & 1 \\ 0 & 3 & 8 \end{bmatrix}$, $B^T = \begin{bmatrix} 4 & 2 & 1 \\ 1 & -3 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

$$A^T + B^T = \begin{bmatrix} 5 & 4 & 5 \\ 0 & -2 & 2 \\ 0 & 4 & 7 \end{bmatrix}$$

$\therefore$ LHS = RHS

$(A+B)^T = A^T + B^T$

ii)
$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 8 \end{bmatrix} \begin{bmatrix} 4 & 1 & 0 \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 4-2+0 & 1+3+0 & 0-1-0 \\ 8+2+3 & 2-3+3 & 0+1-3 \\ 16+2+8 & 4-3+8 & 0+1-8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & -1 \\ 13 & 2 & -2 \\ 24 & 9 & -7 \end{bmatrix}, \quad (AB)^T = \begin{bmatrix} 2 & 13 & 24 \\ 4 & 2 & 9 \\ -1 & -2 & -7 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 4 & 2 & 1 \\ 1 & -3 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ -1 & 1 & 1 \\ 0 & 3 & 8 \end{bmatrix} = \begin{bmatrix} 4-2+0 & 8+2+3 & 16+2+8 \\ 1+3+0 & 2-3+3 & \cancel{0+1-3}\ 4-3+8 \\ 0-1-0 & 0+1-3 & 0+1-8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 13 & 24 \\ 4 & 2 & 9 \\ -1 & -2 & -7 \end{bmatrix}$$

$\therefore$ LHS = RHS

$(AB)^T = B^T A^T$

Symmetric matrix :-

A Square matrix A is said to be symmetric if "$A^T = A$"

Ex :- 1. $A = \begin{bmatrix} a & e & f \\ e & b & d \\ f & d & c \end{bmatrix}$, $A^T = \begin{bmatrix} a & e & f \\ e & b & d \\ f & d & c \end{bmatrix}$ $\therefore A^T = A$

A is symmetric

2. $B = \begin{bmatrix} 4 & 3 & 9 \\ 3 & 5 & 1 \\ 9 & 1 & 6 \end{bmatrix}$, $B^T = \begin{bmatrix} 4 & 3 & 9 \\ 3 & 5 & 1 \\ 9 & 1 & 6 \end{bmatrix}$ $\therefore B^T = B \Rightarrow B$

B is symmetric.

Skew Symmetric matrix :-

A square matrix A is said to be skew symmetric matrix if "$A^T = -A$"

Ex:-

1. $A = \begin{bmatrix} 0 & -3 & 4 \\ 3 & 0 & 6 \\ -4 & -6 & 0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 0 & 3 & -4 \\ -3 & 0 & -6 \\ 4 & 6 & 0 \end{bmatrix} \Rightarrow -A = \begin{bmatrix} 0 & +3 & -4 \\ -3 & 0 & -6 \\ +4 & +6 & 0 \end{bmatrix} = A$

$\therefore A^T = -A \Rightarrow A$ is skew symmetric matrix.

2.

$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -9 \\ -2 & 9 & 0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 9 \\ 2 & -9 & 0 \end{bmatrix} \Rightarrow -A = \begin{bmatrix} 0 & 1 & 2 \\ +1 & 0 & +9 \\ +2 & -9 & 0 \end{bmatrix} = A$

$A^T = -A$

$-A = \begin{bmatrix} 0 & -1 & -2 \\ +1 & 0 & 9 \\ 2 & -9 & 0 \end{bmatrix}$

① If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is a skew symmatric matrix then find the value of x.

A) Given that A is skew symmatric matrix that is

$A^T = -A$ $\therefore$ $x = 2$

Orthogonal Matrix :-

A is a square matrix, it is said to be a orthogon if "$AA^T = A^TA = I$"

Ex:-

① $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ then find A is orthogonal.

A) $A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\therefore AA^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} =$

$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta(\sin\theta) + \sin\theta \cos\theta \\ (-\sin\theta)(\cos\theta) + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & -\cancel{\cos\theta\sin\theta} + \cancel{\sin\theta\cos\theta} \\ -\cancel{\sin\theta\cos\theta} + \cancel{\cos\theta\sin\theta} & \sin^2\theta + \cos^2\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

$$A^TA = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta \cdot \cos\theta + (-\sin\theta)(\sin\theta) & \cos\theta \sin\theta + (-\sin\theta)\cos\theta \\ \sin\theta \cos\theta + \cos\theta(-\sin\theta) & \sin\theta \sin\theta + \cos\theta \cdot \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cancel{\cos\theta \sin\theta} - \cancel{\sin\theta \cos\theta} \\ \cancel{\sin\theta \cos\theta} - \cancel{\cos\theta \sin\theta} & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$\therefore AA^T = A^TA = I_2$ $\therefore A$ is orthogonal matrix.

(2) verify $A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & +2 \end{bmatrix}$ is a orthogonal matrix

A)

$$A = \begin{bmatrix} -6 & 3 & 6 \\ 6 & 6 & 3 \\ 3 & -6 & 6 \end{bmatrix}, \quad A^T = \begin{bmatrix} -6 & 6 & 3 \\ 3 & 6 & -6 \\ 6 & 3 & 6 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} -6 & 3 & 6 \\ 6 & 6 & 3 \\ 3 & -6 & 6 \end{bmatrix} \begin{bmatrix} -6 & 6 & 3 \\ 3 & 6 & -6 \\ 6 & 3 & 6 \end{bmatrix} = \begin{bmatrix} +36+9+36 & -36+18+18 & -18-18+36 \\ -36+18+18 & 36+36+9 & 18-36+18 \\ -18-18+36 & 18-36+18 & 9+36+36 \end{bmatrix}$$

$$AA^T = \frac{1}{3}\begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix} = I_3$$

$\cancel{\frac{1}{3}} \times \cancel{81}$

$$A^TA = \begin{bmatrix} -6 & 6 & 3 \\ 3 & 6 & -6 \\ 6 & 3 & 6 \end{bmatrix} \begin{bmatrix} -6 & 3 & 6 \\ 6 & 6 & 3 \\ 3 & -6 & 6 \end{bmatrix} = \begin{matrix} +36+36+9 & -18+36-18 & -36+18+18 \\ -18 & & \end{matrix}$$

$A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$; $A^T = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$

$AA^T = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix} \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 4+1+4 & -4+2+2 & -2-2+4 \\ -4+2+2 & 4+4+1 & 2-4+2 \\ -2-2+4 & 2-4+2 & 1+4+4 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$

$A^TA = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix} \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 4+4+1 & -2+4-2 & -4+2+2 \\ -2+4-2 & 1+4+4 & 2+2-4 \\ -4+2+2 & 2+2-4 & 4+1+4 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$ $\therefore AA^T = A^TA = I$

$\therefore$ A is orthogonal matrix.

(3) $A = \frac{1}{7}\begin{bmatrix} 3 & 2 & 6 \\ -6 & 3 & 2 \\ 2 & 6 & -3 \end{bmatrix}$ verify that A is orthogonal.

$A^T = \frac{1}{7}\begin{bmatrix} 3 & -6 & 2 \\ 2 & 3 & 6 \\ 6 & 2 & -3 \end{bmatrix}$ $A = \frac{1}{7}\begin{bmatrix} 3 & 2 & 6 \\ -6 & 3 & 2 \\ 2 & 6 & -3 \end{bmatrix}$

$$A^TA = \frac{1}{7}\begin{bmatrix} 3 & -6 & 2 \\ 2 & 3 & 6 \\ 6 & 2 & -3 \end{bmatrix} \frac{1}{7}\begin{bmatrix} 3 & 2 & 6 \\ -6 & 3 & 2 \\ 2 & 6 & -3 \end{bmatrix} = \frac{1}{49}\begin{bmatrix} 9+36+4 & 6-18+12 & 18-12-6 \\ 6-18-12 & 4+9+36 & 12+6-18 \\ 18-12-6 & 12+6-18 & 36+4+9 \end{bmatrix}$$

$$= \frac{1}{49}\begin{bmatrix} 49 & 0 & 0 \\ 0 & 49 & 0 \\ 0 & 0 & 49 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

$$AA^T = \frac{1}{7}\begin{bmatrix} 3 & 2 & 6 \\ -6 & 3 & 2 \\ 2 & 6 & -3 \end{bmatrix} \frac{1}{7}\begin{bmatrix} 3 & -6 & 2 \\ 2 & 3 & 6 \\ 6 & 2 & -3 \end{bmatrix} =$$

$$= \frac{1}{49}\begin{bmatrix} 9+4+36 & -18+6+12 & 6+12+18 \\ -18+6+12 & 36+9+4 & -12+18-6 \\ 6+12-18 & -12+18-6 & 4+36+9 \end{bmatrix} = \begin{bmatrix} 49 & 0 & 0 \\ 0 & 49 & 0 \\ 0 & 0 & 49 \end{bmatrix}$$

$$= \frac{1}{49}\begin{bmatrix} 49 & 0 & 0 \\ 0 & 49 & 0 \\ 0 & 0 & 49 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

$\therefore AA^T = A^TA = I$ is orthogonal matrix.

④ $A = \begin{bmatrix} -1 & -2 & 3 \\ 1 & 2 & 4 \\ 2 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 & 5 \\ 0 & -2 & 2 \\ 1 & 2 & -3 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$, then find $A+B+C$

A) $A+B+C =$

$$\begin{bmatrix} -1+1-2 & -2-2+1 & 3+5+2 \\ 1+0+1 & 2-2+1 & 4+2+2 \\ 2+1+2 & -1+2+0 & 3-3+1 \end{bmatrix} = \begin{bmatrix} -2 & -3 & 10 \\ 2 & 1 & 8 \\ 5 & 1 & 1 \end{bmatrix}$$

⑤ If $A = \begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$, are equal matrices then find the values of x, z, a.

A) $x-1 = 1 \Rightarrow x = 1+1 \quad x = 2$

$z = 5, \quad a = 5, \quad y = 2$

$5-y = 3$
$5-3 = y$
$2 = y$

Determinants of a square matrix :-

In a square matrix A, the sum of the products of the elements of any row or column with their corresponding co-factors is called Determinant of a Matrix A and it is denoted by "det A" or "|A|".

→ If det A ≠ 0 ($|A| \neq 0$) then the matrix A is called "non-singular matrix"

→ If $|A| = 0$ then the matrix A is called "Singular matrix"

① find the Determinant of the following matrices.

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & -2 & -2 \\ 1 & -3 & -3 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|A| = ad - bc$$

A)
$$|A| = \begin{vmatrix} 2 & 1 & -1 \\ 3 & -2 & -2 \\ 1 & -3 & -3 \end{vmatrix}$$

$$= 2\begin{bmatrix} -2 & -2 \\ -3 & -3 \end{bmatrix} - 1\begin{bmatrix} 3 & -2 \\ 1 & -3 \end{bmatrix} + (-1)\begin{bmatrix} 3 & -2 \\ 1 & -3 \end{bmatrix}$$

$$= 2\left(6 - (+6)\right) - 1\left(-9+2\right) - 1\left(-9+2\right)$$

$$= 2(0) - 1(-7) - 1(-7)$$

$$= 0 + 7 + 7 = 14 \quad \boxed{14 \neq 0}$$

∴ $|A| \neq 0$ A is a non singular matrix.

(2) $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}$

A) $|A| = 1\begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} + 1\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$= 1(4-2) - 2(6-2) + 1(3-2)$

$= 1(2) - 2(4) + 1(1) = 2 - 8 + 1 = -5 \neq 0$

$|A| \neq 0$ A is non singular matrix.

(3) $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 3 & 5 & 2 \end{bmatrix}$

$|A| = 1\begin{bmatrix} 6 & 10 \\ 5 & 2 \end{bmatrix} - 3\begin{bmatrix} 2 & 10 \\ 3 & 2 \end{bmatrix} + 5\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$

$= 1(12-50) - 3(4-30) + 5(10-18)$

$= 1(-38) - 3(-26) + 5(-8)$

$= -38 + 78 - 40 = 0$

$|A| = 0$ $\therefore$ A is singular matrix.

$\begin{array}{r} 50 \\ 12 \\ \hline 38 \end{array}$ $\begin{array}{r} 30 \\ 4 \\ \hline 26 \end{array}$ $\begin{array}{r} 18 \\ 10 \\ \hline 8 \end{array}$ $\begin{array}{r} 26 \times 3 \\ \hline 78 \end{array}$ $\begin{array}{r} 40 \\ 38 \\ \hline 78 \end{array}$