1. Instrumentation Amplifier

   A three Op-Amps instrumentation Amplifer is shown in Figure 1. Three Op-Amps instrumentation amplifiers are popular because they offer high input resistance, adjustable differential gain, and high common mode rejection ratio (CMRR).

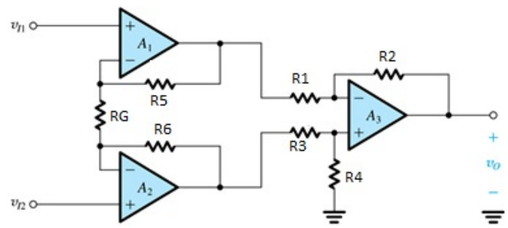

Figure 1: An Instrumentation Amplifier

To decrease the number of unknowns, R1, R2, R3, R4, R5, and R6 are usually made equal R. The differential gain of the amplifier ($A_d = \frac{v_o}{v_{I2}-v_{I1}}$) can be determined by Eq.1.

$$A_d = 1 + \frac{2R}{R_G} \tag{1}$$

The common mode gain of the instrumentation amplifier can be determined by driving both v_{I2} and v_{I1} with the same source (e.g. v_{icm}) and and calculating v_o/v_{icm}.

$$A_c = \frac{v_o}{v_{icm}} = \frac{R_4}{R_3 + R_4}\frac{R_1 + R_2}{R_1} - \frac{R_2}{R_1} \tag{2}$$

A_c is zero in theory since $R_i = R$ for $i = 1, 2...6$. But in practice, A_c is not zero because the resistors can not be perfectly matched.

The common rejection ratio (CMRR) in decibles can be calculated by

$$CMRR = 20\log_{10}\frac{A_d}{|A_c|} \tag{3}$$

The probability that a resistor is within 1 percent of its nominal value is

$$P[0.99R_n < R_n \leq 1.01R_n] = \Theta(\frac{0.01R_n}{\sigma_{R_n}}) - \Theta(\frac{-0.01R_n}{\sigma_{R_n}}) = 2\Theta(\frac{0.01R_n}{\sigma_{R_n}}) - 1 \tag{4}$$

R_n is the nominal value of a resistor. $\frac{0.01R_n}{\sigma_{R_n}}$ is the standard normal random variable (i.e. z).

Submission checklist:

(a) (Extra credit, 1 point) Derive Eq. 1.

(b) (Extra credit, 1 point) Derive Eq. 2.

(c) (1 points) Let $R = 10$ KΩ. What should be the value of R_G in order to obtain 60 dB of differential gain?

(d) (2 points) Model R_i, $i = 1, 2...6$ as a Gaussian random variable with a mean of μ_{R_i} and a standard deviation of σ_{R_i}. How is μ_{R_i} related to σ_{R_i} if the probability that R_i is within one percent of μ_{R_i} is 0.99?

(e) (3 points)Generate a histogram of CMRR of one million instrumentation amplifiers in matlab. Please submit the plot along with the code.

2. Common Rejection Ratio of a Differential Pair

   The small signal equivalent circuit of a differential pair is shown in Figure 2. R_{SS} represents the resistance of the current source. If all of the transistors and resistors are perfectly matched, then the common rejection ratio of a differential pair is ∞. Otherwise, the common rejection ratio is finite. We will explore the mismatch due to the transconductance of the transistor in this problem. Let g_{m1} and g_{m2} be the transconductance of Q_1 and Q_2 respectively.

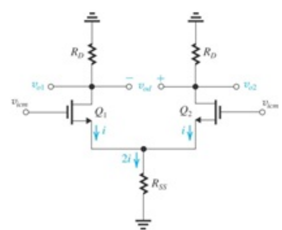

Figure 2: A Differential Amplifier

The output voltage of v_{o1} is

$$v_{o1} = -\frac{g_{m1} R_D}{1 + (g_{m1} + g_{m2}) R_{SS}} v_{icm} \tag{5}$$

The output voltage of v_{o2} is

$$v_{o2} = -\frac{g_{m2} R_D}{1 + (g_{m1} + g_{m2}) R_{SS}} v_{icm} \tag{6}$$

The differential output voltage v_{od} is then

$$v_{od} = \frac{(g_{m1} - g_{m2}) R_D}{1 + (g_{m1} + g_{m2}) R_{SS}} v_{icm} \tag{7}$$

The common-mode gain of the differential amplifier is

$$A_c = \frac{(g_{m1} - g_{m2})R_D}{1 + (g_{m1} + g_{m2})R_{SS}} \tag{8}$$

It can be shown that the differential gain of the diff pair is

$$A_d = g_m R_D \tag{9}$$

The common mode rejection ratio (CMRR) is

$$CMRR = \frac{A_d}{A_c} \tag{10}$$

CMRR can be expressed in dB by $20\log_{10}(CMRR)$.

Assume that R_D=5 KΩ and R_{SS}=25 KΩ. Assume that g_{m1} and g_{m2} can be modeled as Gaussian random variable. Let g_m (i.e. μ_{g_m} be 1 mA/V with a standard deviation of 0.1 mA/V. Generate a hisotram of one million differential pairs. What is the average CMRR in dB? (5 points)