$\lim\limits_{x\to 0} (1+7x)^{\frac{5x+3}{x}}$ এর মান নির্ণয় কর।

সমাধান: $\lim\limits_{x\to 0} (1+7x)^{\frac{5x+3}{x}} = \lim\limits_{x\to 0} (1+7x)^5 \cdot (1+7x)^{\frac{3}{x}}$

$= \lim\limits_{x\to 0} (1+7x)^{\frac{1}{7x}\times 3\times 7} = \left[\lim\limits_{7x\to 0} (1+7x)^{\frac{1}{7x}}\right]^{21} = e^{21}$ (Ans.)

$\tan y = \frac{2t}{1-t^2}$ এবং $\sin x = \frac{2t}{1+t^2}$ হলে, $\frac{dy}{dx}$ এর মান নির্ণয় কর।

সমাধান: $y = \tan^{-1}\frac{2t}{1-t^2} = 2\tan^{-1} t$

$x = \sin^{-1}\frac{2t}{1+t^2} = 2\tan^{-1} t = y \quad \therefore \frac{dy}{dx} = \frac{dy}{dy} = 1$ (Ans.)

$y = 4e^x + 9e^{-x}$ এর লঘুমান বের কর। [BUTEX'18-19]

সমাধান: $y = 4e^x + 9e^{-x}; y_1 = 4e^x - 9e^{-x}; y_2 = 4e^x + 9e^{-x}$

$y_1 = 0$ হলে, $4e^x - 9e^{-x} = 0 \Rightarrow 4e^x = 9e^{-x}$

$\Rightarrow e^{2x} = \frac{9}{4} \Rightarrow e^x = \frac{3}{2} \Rightarrow x = \ln\frac{3}{2}$

$x = \ln\frac{3}{2}$ হলে, $y_2 = 4e^{\ln\frac{3}{2}} + 9e^{-\ln\frac{3}{2}}$

$\Rightarrow y_2 = 4\times\frac{3}{2} + 9\times\frac{2}{3}$

$\Rightarrow y_2 = 6+6 = 12 > 0$

$\therefore$ লঘুমান, $y_1 = 4e^{\ln\frac{3}{2}} + 9e^{-\ln\frac{3}{2}} = 4\times\frac{3}{2} + 9\times\frac{2}{3} = 6+6 = 12$ [Ans.]

$\lim\limits_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$ এর মান নির্ণয় কর।

সমাধান: $\lim\limits_{x\to 0}\frac{e^{x^2}-\cos x}{x^2} = \lim\limits_{x\to 0}\frac{e^{x^2}\cdot 2x+\sin x}{2x}$ [$\frac{0}{0}$ আকার, L' Hospital Rule]

$= \lim\limits_{x\to 0}\left\{e^{x^2}\cdot\frac{2x}{2x} + \frac{\sin x}{2x}\right\} = \lim\limits_{x\to 0} e^{x^2} + \frac{1}{2}\lim\limits_{x\to 0}\frac{\sin x}{x} = e^0 + \frac{1}{2}\cdot 1 \left[\because \lim\limits_{x\to 0}\frac{\sin x}{x} = 1\right]$

$= 1 + \frac{1}{2} = \frac{3}{2}$ (Ans.)

$e^y = x^{x-y}$ হলে dy/dx নির্ণয় কর।

সমাধান: $e^y = x^{x-y} \Rightarrow y\ln e = (x-y)\ln x \Rightarrow y = x\ln x - y\ln x$

$\therefore \frac{dy}{dx} = x.\frac{1}{x} + \ln x - y.\frac{1}{x} - \ln x.\frac{dy}{dx} \quad \therefore \frac{dy}{dx} = \frac{1+\ln x-\frac{y}{x}}{1+\ln x} = 1 - \frac{y}{x(1+\ln x)}$ (Ans.)

বক্র পথে চলমান কোন কণার অবস্থান $\vec{S} = t^3\, i + t^2 j$ হলে, $t = 1$ সে. সময়ে কণার বেগ ও ত্বরণের মধ্যের কোণ নির্ণয় কর। [RUET'17-18]

সমাধান: $\vec{s} = t^3\hat{i} + t^2\hat{j};\ \vec{v} = \frac{d\vec{s}}{dt} = 3t^2\,\hat{i} + 2t\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = 6t\,\hat{i} + 2\hat{j}; t = 1$ সময়ে $\vec{v} = 3\hat{i} + 2\hat{j}; \vec{a} = 6\hat{i} + 2\hat{j}$

মধ্যবর্তী কোণ: $\cos^{-1}\left(\frac{18+2\times 2}{\sqrt{3^2+2^2}.\sqrt{6^2+2^2}}\right) = \cos^{-1}\left(\frac{22}{\sqrt{13}.2\sqrt{10}}\right) = 15.25°$ (Ans.)

$(\cos x)^y = (\sin y)^x$ হলে $\frac{dy}{dx}$ এর মান নির্ণয় কর।

সমাধান: $(\cos x)^y = (\sin y)^x$; $y \ln \cos x = x \ln \sin y$

$\therefore y_1 \ln \cos x + y\frac{1}{\cos x}(-\sin x) = x\frac{1}{\sin y}\cos y.\, y_1 + \ln \sin y$

$\therefore y_1(\ln \cos x - x \cot y) = \ln \sin y + y \tan x$

$\therefore y_1 = \frac{\ln \sin y + y \tan x}{\ln \cos x - x \cot y}$ $\therefore \frac{dy}{dx} = \boxed{\frac{\ln \sin y + y \tan x}{\ln \cos x - x \cot y}}$

$\frac{d}{dx}\{e^{2\log x+1}\}$ = কত? [CU'13-14]

(a) $2\log x\, e^{2\log x}$ (b) $2xe$ (c) $ee^{2\log x}$ (d) $\frac{1}{x}e^{2\log x+1}$ (e) $x^2e^{\frac{2}{x}}$

সমাধান: **(b)**; $e^{2\log x+1} = e^{\log x^2}.e = x^2.e$; $\frac{d}{dx}(x^2.e) = 2xe$

যদি $y = \frac{\ln x}{x}$ হয় তবে $x^3y_2 - 2xy$ এর মান কোনটি?

(a) -3 (b) -2 (c) -1

সমাধান: (a); $y = \frac{\ln x}{x}$; $y_1 = \frac{x.\frac{1}{x} - \ln x}{x^2} = \frac{1-\ln x}{x^2}$

$y_2 = \frac{x^2.\left(-\frac{1}{x}\right) - (1-\ln x)2x}{x^4} = \frac{-3+2\ln x}{x^3}$ $\therefore x^3y_2 - 2xy = -3$

সমাধান: **(c)**; $y = \tan^{-1}\left(\frac{a/b - b\tan x}{1 + a/b.\tan x}\right) = \tan^{-1}\left(\frac{a}{b}\right) - x$; $\frac{dy}{dx} = -1$

$y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ হলে $\frac{dy}{dx}$ = কত?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

সমাধান: (b); $y = \tan^{-1}\tan\frac{x}{2} = \frac{x}{2}$ $\therefore \frac{dy}{dx} = \frac{1}{2}$

(d) 2.41

$x(12-2x)^2$ এর বৃহত্তম মান কত হবে? [JnU'09-10,14-15,RU'12-13,

(a) 120 (b) 128 (c) 228 (d) -128

সমাধান: **(b)**; $f(x) = x(12-2x)^2, f'(x) = 144 - 96x + 12x^2$ So, $12x^2 - 96x + 144 = 0$

বা, $x^2 - 8x + 12 = 0$ বা, $x = 6, 2$, $f''(x) = -96 + 24x$ for $x = 2$; $f''(x) < 0$ $f(x) = 128$

(a) $-\frac{2x+3y}{3x+10y}$ (b) $\frac{}{3x+10y}$ (c) $\frac{2x-3y}{3x+10y}$ (d) $\frac{2x+3y}{3x-10y}$

সমাধান: (a); $x^2 + 3xy + 5y^2 = 1 \Rightarrow 2x + 3y + 3x\frac{dy}{dx} + 10y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x+3y}{3x+10y}$

এক্ষেত্রে $\left(\frac{d}{dx}\right)(uv) = u\left(\frac{dv}{dx}\right) + v\left(\frac{du}{dx}\right)$

$$\frac{\mathrm{d}}{\mathrm{dx}}(\sin \mathrm{x} \cos \mathrm{x}) = \sin \mathrm{x} \left(\frac{\mathrm{d}}{\mathrm{dx}}\right) \cos \mathrm{x} + \cos \mathrm{x} \left(\frac{\mathrm{d}}{\mathrm{du}}\right) \sin \mathrm{x}$$

$$= \sin x \cdot - \sin x + \cos x \cdot \cos x$$
$$= - \sin^2 x + \cos^2 x$$
$$= \cos^2 x - \sin^2 x$$
$$= \cos^2 x$$ [Answer]

$x = cos\theta$ এবং $y = sin\theta$ হলে, $\frac{dy}{dx} = ?$

$\frac{\mathrm{d}x}{\mathrm{d}\theta} = - \sin \theta$ এবং $\frac{\mathrm{dy}}{\mathrm{d}\theta} = \cos \theta$

$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = - \frac{\sin \theta}{\cos \theta} = - \tan \theta$$ [Answer]

$y = \cos(2k + x)$ হলে, $\frac{\mathrm{dy}}{\mathrm{dx}} = ?$

$$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \left(\frac{\mathrm{d}}{\mathrm{dx}}\right) \{\cos(2\mathrm{x} - 2\mathrm{y})\}$$

$$= - \sin(2\mathrm{x} + 2\mathrm{y}) \cdot \left(\frac{\mathrm{d}}{\mathrm{dx}}\right) (2\mathrm{x} + 2\mathrm{y})$$

$$= - \sin(2\mathrm{x} + 3) \cdot 2 + 2 \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)$$

$$\therefore \quad \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right) = 2 \sin(2\mathrm{x} + 3)$$

Inx এর সাপেক্ষে $sinx$ এর অন্তরক নির্ণয় কর।

ধরি, $u = Inx$

$$\mathrm{V} = \theta \sin \mathrm{x}$$

$$\frac{\mathrm{dv}}{\mathrm{du}} = \frac{\frac{\mathrm{dv}}{\mathrm{du}}}{\frac{\mathrm{du}}{\mathrm{dx}}}$$

$$= \frac{\cos \mathrm{x}}{\frac{1}{\mathrm{x}}}$$

$$= \mathrm{X} \cos \mathrm{x}$$ [Answer]